Solvable form of the polynomial equation $x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0 = 0,(n = 2k + 1)$

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Abstract

It is known, there is no solution in radicals to general polynomial equation of degree five or higher with arbitrary coefficients [?]. In this article, we give a form of the polynomial equations with odd degree can be solved in radicals. From there, we come up some solvable equations with one or more zero coefficients,especially for the quintic and septic equations.

1.Cubic equation

$$x^3 + a_2x^2 + a_1x + a_0 = 0$$

It is known, the cubic equation is solvable.

2.Quintic equation

$$x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

We have found the coefficients below to give a solvable form.

$$a_1 = \frac{(a_3 - a_4^2)(2a_3 + a_4^2)}{9}$$

$$a_0 = \frac{(a_3 - a_4^2)(a_4^3 - a_4a_3 + 3a_2)}{9}$$

In other words , the quintic equation:

$$x^5 + px^4 + qx^3 + rx^2 + \frac{(q - p^2)(2q + p^2)}{9}x + \frac{(q - p^2)(p^3 - pq + 3r)}{9} = 0$$

is solvable.

Some special cases:

p = 0:

$$x^5 + qx^3 + rx^2 + \frac{2q^2}{9}x + \frac{qr}{3} = 0$$

q = 0:

$$x^5 + px^4 + rx^2 - \frac{p^4}{9}x - \frac{p^2(p^3 + 3r)}{9} = 0$$

r = 0:

$$x^5 + px^4 + qx^3 + \frac{(q - p^2)(2q + p^2)}{9}x - \frac{p(q - p^2)^2}{9} = 0$$

And for the case q = 0, r = 0, we obtain:

$$x^5 + px^4 - \frac{p^4}{9}x - \frac{p^5}{9} = (x+p)(x^4 - \frac{p^4}{9}) = 0$$

3.Septic equation

$$x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

We have found the coefficients below to give a solvable form.

$$a_3 = \frac{(a_5 - a_6^2)a_5}{3}$$

$$a_2 = \frac{(a_5 - a_6^2)a_4}{3}$$

$$a_1 = \frac{(a_5 - a_6^2)^2(2a_5 + a_6^2)}{27}$$

$$a_0 = \frac{(a_5 - a_6^2)^2(a_6^3 - a_6a_5 + 3a_4)}{27}$$

In other words , the septic equation:

$$x^7 + px^6 + qx^5 + rx^4 + \frac{(q-p^2)q}{3}x^3 + \frac{(q-p^2)r}{3}x^2 + \frac{(q-p^2)^2(2q+p^2)}{27}x + \frac{(q-p^2)^2(p^3-pq+3r)}{27} = 0$$

is solvable.

Some special cases:

p = 0:

$$x^7 + qx^5 + rx^4 + \frac{q^2}{3}x^3 + \frac{qr}{3}x^2 + \frac{2q^3}{27}x + \frac{q^2r}{9} = 0$$

q = 0:

$$x^7 + px^6 + rx^4 - \frac{p^2r}{3}x^2 + \frac{p^6}{27}x + \frac{p^4(p^3+3r)}{27} = 0$$

r = 0:

$$x^7 + px^6 + qx^5 + \frac{(q-p^2)q}{3}x^3 + \frac{(q-p^2)^2(2q+p^2)}{27}x - \frac{p(q-p^2)^3}{27} = 0$$

And for the case q = 0, r = 0, we obtain:

$$x^7 + px^6 + \frac{p^6}{27}x + \frac{p^7}{27} = (x+p)(x^6 + \frac{p^6}{27}) = 0$$

Generally, for the equation :

$$x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + a_{n-3}x^{n-3} + a_{n-4}x^{n-4} ... + a_1x + a_0 = 0 \text{ , n is odd}$$

There are always coefficients a_i: (i = 0;1...n-4) depend on coefficients $a_{n-1}; a_{n-2}; a_{n-3}$ to have a solvable from of the equations. Here $a_{n-1}; a_{n-2}; a_{n-3}$ are three arbitrary coefficients.

There is an algorithm for the above theorem.

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