Truly two-dimensional black holes under dimensional transitions of spacetime

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A sufficiently massive star in the end of its life will inevitably collapse into a black hole as more deconfined degrees of freedom make the core ever softer. One possible way to avoid the singularity in the end is by dimensional phase transition of spacetime. Indeed, the black hole interior, two-dimensional in nature, can be described well as a perfect fluid of free massless Majorana fermions and gauge bosons under a 2-d supersymmetric mirror model with new understanding of emergent gravity from dimensional evolution of spacetime. In particular, the 2-d conformal invariance of the black hole gives rise to desired consistent results for the interior microphysics and structures including its temperature, density, and entropy.

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The idea of no light escaping from a sufficiently massive astronomical object like a black hole (BH) was first proposed more than two hundred years ago by Michell and Laplace [1, 2]. Shortly after Einstein developed his theory of general relativity (GR), Schwarzschild obtained the first modern BH solution of Einstein's equations in vacuum [3]. The apparent singularity of the solution at the event horizon $r = 2GM$ does not seem to be physical and can be removed in suitable coordinates. But the singularity at $r = 0$ is genuine indicating that GR breaks down under such extreme conditions and new physics may be needed to describe the BH interior within its event horizon.

In the following, we will apply recently developed supersymmetric mirror models (SMM) and new understanding of gravity as an emergent classical phenomenon due to dimensional phase transitions of spacetime to study such extreme celestial objects. In particular, the interior of a black hole, two-dimensional in nature, can be elegantly described by the 2-d model SMM2.

The existence of a mirror sector of the Universe has been conjectured since Lee and Yang published their Nobel Prize-winning work on parity violation [4]. It is conceivable that there exist two sectors of particles sharing the same gravity but governed by two separate gauge groups under 4-d spacetime [5]. Some early works on mirror matter theory had discussed interesting perspectives mainly in cosmology [6–8]. Later attempts to introduce ad hoc feeble interactions between the two sectors might be too conservative [9–11]. Most recent works [12–22] with new understanding of mirror symmetry, supersymmetry (SUSY), and dimensional phase transitions of spacetime can consistently and quantitatively solve a variety of puzzles in fundamental physics and cosmology and may indeed lead us to new physics beyond the Standard Model we have been looking for.
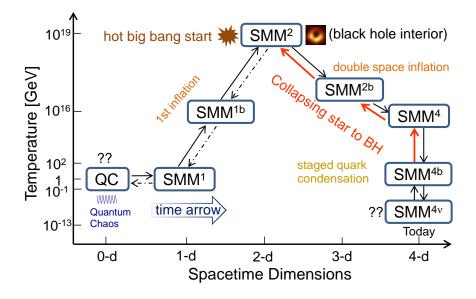
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FIG. 1. The schematic diagram (not to scale) is shown for the Supersymmetric Mirror Models at various phases of spacetime. Red arrows indicate the path of a collapsing star into a black hole. Adapted from Ref. [22].

The idea [18–22] is that spacetime was first born with one time dimension. Mirror symmetry is nothing but the orientation symmetry of the underlying geometry (same as time reversal symmetry in 1-d). The sole scalar then acquired mass in the spontaneous symmetry breaking mechanism leading to the emergence of the time arrow and another dimension of space in inflation. In the resulting 2-d spacetime, Majorana fermions and $U(1)$ gauge bosons emerged and the hot big bang started at the scale of Planck energy.

As the Universe cooled down, Majorana fermions started to condense leading to the process of double space inflation. Eventually 4-d spacetime formed at temperature of $\sim 10^{16}$ GeV and the two scalar inflatons decayed into massless quarks and leptons in two separate sectors, respectively, to reheat the Universe. At the scale of $\sim 10^2$ GeV, staged quark condensation started a series of phase transitions including electroweak and QCD phase transitions. As the Universe became cold enough, the ordinary sector has formed stars and galaxies while the mirror sector serves as the dark matter observed today. Such a picture under the proposed Supersymmetric Mirror Models is depicted in Fig. (1).

The reverse process from SMM4b to SMM2 as shown in Fig. (1) could also occur for a massive star to collapse into a black hole from 4-d to 2-d. The underlying model (SMM2) in 2-d spacetime has the on-shell Lagrangian [19, 20],

$$\mathcal{L}_{\text{SMM2}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}(\lambda_L^\dagger\bar{\sigma}^\mu\partial_\mu\lambda_L + \lambda_R^\dagger\sigma^\mu\partial_\mu\lambda_R) \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the $U(1)$ gauge tensor and the Majorana fermion λ has to be neutral and does not couple to the gauge field A_μ. Both λ and A_μ are massless and have two components or degrees of freedom ($n_f = n_b = 2$). They form the simplest $N = 1$ abelian gauge SUSY multiplet $(1, 1/2)$. As we shall use later, the effective number of relativistic degrees of freedom $g^* = n_b + 1/2n_f = 3$.

On the other hand, gravity in 2-d is described by the smooth spacetime geometry as follows [21],

$$\mathcal{R} + 2\Lambda = -8\pi G T_\mu^\mu = 0 \tag{2}$$

where $\mathcal{R}$ is the Ricci curvature scalar, Λ is the cosmological constant, and the energy-momentum tensor T_μ^μ has to be traceless. In SMM2 as shown in Eq. (1), free massless Majorana fermions and $U(1)$ gauge bosons make a perfect fluid resulting in the vanishing trace of $T_{\mu\nu}$ and therefore is consistent with the 2-d gravity / spacetime model of Eq. (2).

Eqs. (1-2) together describe a conformally invariant 2-d quantum and spacetime theory. Its effective number of relativistic degrees of freedom $g^* = 3$ is identically the central charge c of the Virasoro symmetry algebra in 2-d conformal field theory (CFT) [23]. The corresponding Weyl anomaly in 2-d CFT [24] can produce a non-vanishing trace of $2\rho_\Lambda = -c/(24\pi G)\mathcal{R}$ due to nontrivial metric, where ρ_Λ can be interpreted as vacuum energy density and related to the cosmological constant via $\Lambda = 8\pi G\rho_\Lambda$. This trace anomaly, introduced dynamically by the inflaton scalars of Majorana condensates [19], is related to a topological invariant - the Euler characteristic [25], which may be important for studying nontrivial spacetime topology during the double space inflation process SMM2b [19].

However, for a static spherically symmetric black hole, its interior should be Ricci-flat, i.e, $\mathcal{R} = 0$ and can then be studied in 2-d CFT on a torus. The Euler characteristic of a torus vanishes and makes it consistent that the black hole interior should be static and free of the trace anomaly or $\Lambda = 0$. The general solution of the 2-d metric can then be written as,

$$ds^2 = e^{2\omega(x)}(dt^2 - dx^2) \tag{3}$$

where $\omega(x)$ defines the conformal transformations of the metric.

We will present the views of the BH interior by both a distant exterior observer and an interior one. The two reference frames are connected by the global scale transformation under 2-d CFT. There are two quantities that the two observers see the same: proper energy density ρ and total entropy S_{BH}.

The view of the exterior observer is fairly simple for a Schwarzschild black hole with mass M and radius $R = 2GM$. The BH interior can be regarded as a 2-d torus with two circumferences of $2R$ (space) and $2\pi R$ (time), which can also be viewed externally as a dual of the event horizon surface, notably with the same area $A = 4\pi R^2$. Its conformally invariant 2-d energy density is,

$$\rho = \frac{M}{2R} = \frac{1}{4G} \tag{4}$$

which is the same as the CFT Casimir energy density $\rho_c = E_c/(2R) = c/(12G)$ with no negative sign under the condition of $c = g^* = 3$. This 2-d structure with a constant energy density provides a natural explanation of the linear relation between mass and radius of the black hole.

The entropy of an $n+1$ dimensional CFT on $R \times S^n$ can be given by the Cardy-Verlinde formula [26, 27],

$$S = \frac{2\pi R}{n}\sqrt{E_c(2E - E_c)} \tag{5}$$

where $E_c = 2Rc/(12G)$ is the Casimir energy in 2-d CFT (i.e., $n = 1$) and the total energy $E = 2R\rho$ is exactly the same as E_c if the central charge $c = g^* = 3$. Under the BH conditions of $n = 1$ and $c = g^* = 3$, Eq. (5) becomes the well-known Bekenstein-Hawking entropy [28, 29],

$$S_{\mathrm{BH}} = \frac{A}{4G}. \tag{6}$$

Based on Maldacena's AdS/CFT correspondence [30] and Witten's arguments [31], Verlinde proposed a more general entropy formula for the same CFT on $R \times S^n$ as [27],

$$S = \frac{cA}{12nG} \qquad (7)$$

which again gives the same entropy as Eq. (6) for $n = 1$ and $c = g^* = 3$.

The BH entropy can also have an equivalent thermodynamic explanation. Namely, the exterior observer sees no free energy from the black hole, i.e., its partition function $Z = 1$. As such, the entropy is solely determined by its total energy / mass M and temperature $1/\beta$ where $\beta = 2\pi R$ is the Euclidean time period or the time circumference of the torus. Then, the same BH entropy can be easily calculated from

$$S_{\mathrm{BH}} = \ln(Z) + \beta M = \beta M = \frac{A}{4G} \qquad (8)$$

where, notably, the interior temperature of $1/\beta$ viewed by the exterior observer is twice as high as the surface temperature that is responsible for Hawking radiation [29, 32].

From the view of the interior observer, the BH interior is composed of a perfect fluid of Majorana fermions and gauge bosons governed by SMM2. Its energy and entropy densities in 2-d spacetime can be obtained as,

$$\rho = \frac{\pi}{6} g^* T_{\mathrm{in}}^2 = \frac{\pi}{2} T_{\mathrm{in}}^2 \qquad (9)$$

$$s = \frac{\rho}{T_{\mathrm{in}}} = \frac{\pi}{2} T_{\mathrm{in}} \qquad (10)$$

where T_{in} is the interior temperature. Such an expression of 2-d entropy density in Eq. (10) has also been obtained in general 2-d CFT in the high temperature limit [26, 33].

Using the two views of the energy density in Eqs. (4,9), we can obtain the temperature of the BH interior,

$$T_{\mathrm{in}} = \frac{1}{\sqrt{2\pi G}} \simeq \frac{0.4}{\sqrt{G}} \qquad (11)$$

which is constant and just below the Planck energy of $1/\sqrt{G} \sim 10^{19}$ GeV but well above the double space inflation energy scale of 10^{16} GeV. This means that the interior temperature does meet the criteria for SMM2 in 2-d spacetime and this BH model is self-consistent. The constant energy density and temperature also indicate that such BH interior structures are indeed stable.

As viewed by the interior observer, the torus is conformally dilated as follows,

$$2R \rightarrow L_s \equiv e^{\omega} 2R, \quad 2\pi R \rightarrow L_t \equiv e^{-\omega} 2\pi R \qquad (12)$$

where L_s and L_t are the different space and time scales (or sizes of the torus) for the interior observer under the global dilation or scale transformation. From $L_t = 1/T_{\mathrm{in}}$, we can obtain the dilation factor as,

$$e^{\omega} = \sqrt{\frac{2\pi}{G}} R \simeq 4.58 \times 10^{38} \frac{M}{M_{\odot}}. \qquad (13)$$

It is now easy to calculate the entropy from the point of view of the interior observer,

$$S_{\mathrm{BH}} = e^{\omega} 2Rs = \frac{A}{4G} \qquad (14)$$

which is the same Bekenstein-Hawking entropy. To demonstrate the enormous BH size seen by the interior observer, we can calculate it for one solar mass to be $e^\omega 2R \sim 10^{42}$ m in contrast to the size of the observable Universe (10^{27} m).

In summary, black holes are presented to be 2-d in nature under the picture of dimensional transitions of spacetime. The apparent singularity at the event horizon under 4-d general relativity is most likely topological describing a transition from the exterior 4-d spacetime to the interior 2-d spacetime. Techniques especially topological ones developed in studies of renormalization group, loop quantum gravity, and other quantum gravity approaches could be applied around the event horizon and possibly elsewhere such as the space inflation process of the early Universe.

ACKNOWLEDGMENTS

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