Notes on the Collatz conjecture.

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0- Abstract:

In this article I wanted to contribute with some of my own investigations on the Collatz conjecture. Based on the ideas of parity and analyzing the conjecture from an algebraic point of view.

1- Introduction:

- As it is a conjecture that is developed in the natural numbers, we understand throughout the article that $\forall (n, m, k, q) \in \mathbb{N}$

- When we study the conjecture from an algebraic form we have to analyze it differently depending on whether we start from an even number or an odd number. When we start from an even number we simply have to divide it by 2, the result can be even (1) or odd (2).

$$(1) \quad (2n)/2 = 2k$$

$$(2) \quad (2n)/2 = 2k+1$$

However, when we start from an odd number, in the first result, when we multiply it by 3 and add 1, we will always obtain an even number (3):

$$(3) \quad (2n+1)\cdot 3+1 = 6n+3+1 = 6n+4 \simeq 2k$$

2- Ideal conditions:

Here we will see the two forms of direct solution for both even and odd numbers, this is what I have called "ideal conditions".

- For even numbers we will simply see that starting from a number and making the necessary and appropriate iterations of divisions we reach a power of 2:

$$(4) \quad (2n)/2 = 2^k$$

- For odd numbers, we will develop a formula capable of understanding all possible iterations of odd numbers, for this we will see some examples.

$$(5) \quad (2m+1)\cdot 3+1 = 6m+3+1 = 6m+4$$

$$(6) \quad (6m+4)\cdot 3+1=18m+12+1=18m+13$$

$$(7) \quad (18m+13)\cdot 3+1=54m+29+1=54m+30$$

The formula that gives us all the combinations is:

$$(8) \quad (2\cdot 3^{(n-1)})m+(3n+1)$$

Therefore under ideal conditions this equation should be equal to a power of 2:

$$(9) \quad (2\cdot 3^{(n-1)})m+(3n+1)=2^k$$

3- Non-ideal conditions:

In this section we will see 3 possibilities capable of providing a solution to all possible combinations that do not directly achieve a power of 2:

- Even divided between 2 to make odd:

$$(10) \quad (2n)/2=(2m+1)$$

- Even divided by 2 to give torque that is not power of 2:

$$(11) \quad (2n)/2=(2m)\neq 2^k$$

- Odd to give even that is not power of 2:

$$(12) \quad (2\cdot 3^{(n-1)})m+(3n+1)=(2m)\neq 2^k$$