An hypothesis for mass dependence on radial distance, with novel cosmological implications for the early universe as well as dark matter

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(Dated: April 11, 2021)

Working from first principles of special relativity and using a version of the equivalence principle, an hypothesize that an objects mass is a function of its distance to other objects is presented. Further it is posited that there is a correlation between this mass variation and gravitational time dilation in general relativity. Additionally due to the Schwarzschild metric the mass variation also distorts the spatial components of the metric which contributes to gravitational lensing. Hence this approach could be used to explain the unseen mass increase due to gravitational lensing of current dark matter exploration. This leads naturally to the hypothesis of the early rapid clumping in the early universe to produce the 'cosmic web' and dark matter halos necessary to produce galaxy formation. Certain electrically neutral MACHOS are suggested. These might include possible boson stars or primordial black holes. Radio emission spectrum would also be expected to be higher than average for various dark matter regions. We also expect that the theory is consistent with the observation that not all galaxies exhibit dark matter since not all galaxy clumping originated from the cosmic web. We propose mathematically how the theory can fit naturally with Einsteins field equation. Finally we propose a simple principle for terrestrial measurement of the theory.

I. INTRODUCTION

Einstein's analysis of the rotating disk was a key thought experiment in developing the General Theory (GR). On the rotating disk, using special relativity (SR), we find there is time dilation and mass increase relative to the centre. This time dilation is given by the standard γ factor as function of v, where v is the circumferential velocity. We note also that there is a radial acceleration and so we can apply the apply the principle of equivalence. Hence, we would expect a time dilation and mass increase, as found within an equivalent gravitational field[1].

II. A THOUGHT EXPERIMENT

Let O and P be two objects of unit mass. They are separated at a fixed distance and connected by some high tensile strength special material. They are also in free space away from all gravitational sources. Let P begin to move around O so that the tension on the material connecting them keeps P in circular orbit around O. Now in this idealised thought experiment if P begins to approach orbital relativistic speeds then P's time will slow and its mass relative to O will appear to increase.

Of course in order for O to remain in the centre of the orbit in this thought experiment, O will need to take some action. O may have for example a rocket thruster firing in the opposite radial direction to P in order to counter the growing force from P, due to it's increasing speed and increasing mass.

We note also that if both observers were placed in an enclosure so that neither could see the outside world, the result would not change, however they might now mistake their frame for a gravitational field. For example, P might conclude that he is within a gravitational field acting away from O, who is hovering above using a rocket thruster.

Now, the principle of equivalence states that acceleration is equivalent to gravity provided it is only measured locally. Hence for the purpose of the current thought experiment P cannot in their immediate vicinity distinguish the force on them from gravity or acceleration. It is therefore proposed from this rotation case that mass and time change by the same factor in gravity, where mass increases with decreasing radius.

The current view is that mass is an invariant and there is no mass increase, only a momentum increase. However, as we are in the laboratory frame, with no relative motion, we can talk in these terms. That is, issues such longitudinal or transverse mass, are not applicable in this context. So, just an observer at the centre of the disk will see a time dilation on the circumference with no relative motion (using the principle of equivalence), so similarly, we can claim a mass increase, with no relative motion. In this frame, we therefore have an increase inertial mass.

III. DERIVATION OF INCREASE MASS UNDER GRAVITY

We wish now to derive an equation for this proposed mass increase. By our initial analysis above, we begin by invoking the notion of velocity in connection to mass increase . Now in the context of gravity the most natural point to begin is the equation of the escape velocity.

Using the classical approximation in order to develop an intuitive argument, we have for the total energy

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r}. \tag{1}$$

Now the condition for the escape velocity v in the equation can be met if the energy is set to zero. Hence we have the escape velocity

$$v = \sqrt{\frac{2GM}{r}}. \tag{2}$$

Coincidentally, a full relativistic analysis using the Schwarzschild metric will produce the same escape velocity.

Now by the above approach to treat velocity as a function of gamma, we have for the standard relativistic mass equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

by simple insertion into our equation and based on our hypothesis we arrive at our proposed mass increase for a gravitational field

$$m = \frac{m_0}{\sqrt{1 - \frac{2MG}{rc^2}}}, \tag{4}$$

where M is the source mass, m_0 is the original mass at the furthest position r, and m is the increase in mass over the original mass m_0. We observe therefore that the mass m increases from m_0 as r decreases or the source mass M increases. We note that $\frac{2GM}{rc^2}$ is normally very small due the to large effect of the factor c^2. Hence it is only when M becomes very large or for a small r that the effect becomes significant. We might therefore expect to see a measurable effect for brown dwarf stars, where the mass can be upwards of 13 Jupiter masses to a limit of 80, before entering the realm of neutron stars.

Furthermore we see the factor matches the Schwarzschild metric not only for time but also length. This increases our confidence in the proposal that the mass and time are directly correlated in both special relativity and gravity.

We will now show this connection by applying the same reasoning above deriving not just the Schwarzschild coefficients for time but also for length. This will further strengthens our hypothesis that mass increase and time dilation are related.

We note that Eqn. (3) has same coefficient as the spatial Schwarzschild metric. We see therefore that for observers at various radius, their measurement of mass increase correlate to the inverse of the Schwarzschild time dilation or red shift.

We note also that since gravitational time dilation and the concurrent redshift and dimming of light increases with decreasing r, that the mass increase effect described here also correlates to this effect. From this we conclude that it is possible to use the red shift time dilation effect as a measure of mass increase. Furthermore gravitational time dilation is a function of the gravitational potential, by 1/r hence we the mass increase to follow the same relation.

Using this approach we can also derive the Schwarzschild metric. This is not however so surprising given that the Schwarzschild metric was derived assuming that it reduces to the Newtonian weak field, low velocity non relativistic limit. We note also that Schwarzschild can also describe the extreme limits of relativistic gravity. We might therefore expect our simple mass formula to also hold for extreme relativistic gravitational cases such as Neutron stars.

Now, we have for the metric of SR

$$ds^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2. \tag{5}$$

Substituting $dt = \frac{dt'}{\sqrt{1-\frac{v^2}{c^2}}}$ and $dx' = \frac{dx}{\sqrt{1-\frac{v^2}{c^2}}}$, we find

$$ds^2 = c^2 \left(1 - \frac{v^2}{c^2}\right) dt^2 - \frac{dx^2}{1 - \frac{v^2}{c^2}} - dy^2 - dz^2. \tag{6}$$

Then, using the escape velocity $v_e = \sqrt{\frac{2MG}{r}}$, for a non-rotating system, we find

$$ds^2 = \left(1 - \frac{2MG}{rc^2}\right) dt^2 - \frac{dx^2}{1 - \frac{2MG}{rc^2}} - dy^2 - dz^2, \tag{7}$$

which is the Schwarzschild metric. For rotating systems we can substitute in for the $-dy^2 - dz^2$ the ω, ϕ dependencies, which will also affect the mass.

As expected this derivation based on the Newtonian escape velocity is the same physical view as the Schwarzschild observer in flat space where the objects reaches zero velocity at the infinite observer. What makes the effect relativistic for gravity is the c^{-2} . This makes not only the mass increase very small, but also for time and length. We can then use this to predict a mass increase of the gravitating object, proportional to the red-shift measured. We can show this by looking at the redshift and beyond into infrared and radio wave. That is by simply measuring the extent of the shift due to time dilation

$$t_0 = t_f \sqrt{1 - \frac{2MG}{rc^2}}. \tag{8}$$

Hence we see that since t_f is the observer at infinity the time dilation is inverse to the mass. From the graph below we see that the a mass increase just above the event horizon is sufficient to account for dark matter which is about 5.5 the amount of ordinary mass and still below the event horizon. This is close to the amount of mass that is missing in current cosmological observations.

IV. APPLICATION

The equation $m = \frac{m_0}{\sqrt{1-\frac{2MG}{rc^2}}}$ as it stands takes M as the source mass and therefore is assumed to be quite large compared to m_0. It can therefore be used to calculate the mass increase of a body m_0. Lets us consider the mass of the earth and the a satellite orbiting it. Due

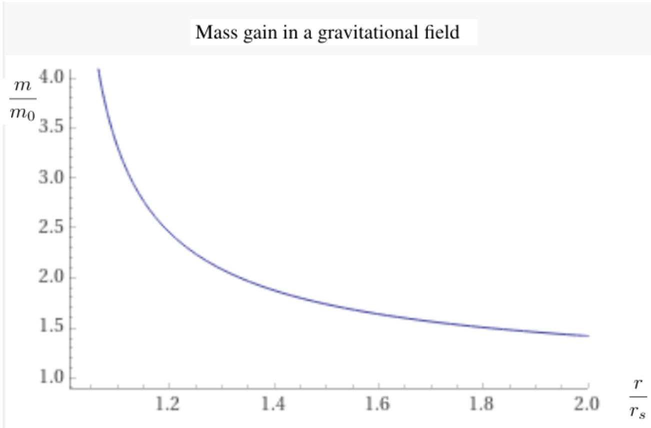

FIG. 1. Mass increase $\frac{m}{m_0}$ due to proximity to a gravitational field, at radius r/r_s, where r_s is the Schwarzschild radius. The graph is also identical but inverse to the time dilation $\frac{t}{t_0}$ expected as one approaches the event horizon.We note that the mass increase is above 4.0 while still significantly above the event horizon.

to the equality of gravitational and inertial mass a body will fall towards the earth at an acceleration independent of its mass. Hence we can use Keplers planetary law to calculate the source mass of the sun via the orbital time of the satellite. We have for Kepler

$$M = 4\pi^2 \frac{R^3}{P^2}, \tag{9}$$

where P is the period swept out and we assume a close to circular orbit. This is a standard result, giving the earth's mass to be 5.9736×10^{24} kg. For the radial orbit of a satellite...

V. A POSSIBLE MODEL FOR DARK MATTER FORMATION IN THE EARLY UNIVERSE

Given the rapidity of the self reinforcing process of mass increase as two objects come close together it is not impossible that rapid dense object formation could occur in the early universe. It would seem reasonable therefore in some early stage at a suitable temperature 'window' that the formation and rapid coalescing of baryonic and lepton/hadronic matter could have occurred. The theory therefore would predict as yet unseen MACHOs or primordial blackholes or even the proposed boson stars type objects. These may be huge collections of tiny but very dense masses as well as larger objects. The main reason for this is the gravitational origin this theory ascribes to the nature of dark matter.

This early dense matter formation, would then drive the formation of the cosmic web of dense matter seen in the very large scale structures of the universe today. This cosmic web is in turn necessary for the formation of ordinary visible and solar masses, that make up galaxies we see today. The cosmic web is needed also to allow enough time for ordinary matter to form galaxies. It is therefore proposed that if this theory is to explain dark matter that its rapidity of formation under gravitational collapse. These were formed in the hot dense early phase of the universe with the rapid collapse of more dense regions. Approximately 86 percent being formed this way before the remaining 14 percent of matter could escape via further expansion of the universe. One such candidate that fits this prediction is the formation of primordial black holes (PBH) in the early universe. With some observations confirming the link between primordial black holes and dark matter[2].

In a project named LIBRAE, a NASA group is looking for source-subtracted infrared cosmic infrared background radiation (CIB) as the signature of PBH formed in the first few seconds of the early universe. An abundance of CIB is the result of X-rays from PBH that have been stretched since the early universe.

VI. BENDING OF LIGHT AND GRAVITATIONAL LENSING

The formula for bending of light based on the angle of deflection is

$$\alpha = \frac{4MG}{dc^2}. \tag{10}$$

We note that the Einstein lensing formula needs to include both the spatial component and time component of the metric.

This formula works well for an entire galaxy or cluster of galaxies where there is contributions from many dense objects. The mass necessary to create the lensing is therefore

$$M = \frac{\alpha r c^2}{4G}. \tag{11}$$

We note that the mass increase is proportional to the gravitational redshift. So from Eq. (4) and Eq. (8) we can form the relation

$$m = m_0 \left(\frac{f_0}{f_c}\right), \tag{12}$$

where f_c is the frequency of the light at the source of emission from gravity. The photons frequency f_c near the clock τ deep in the field is $f_c = \frac{1}{\tau}$.

From standard relativity theory we expect light to be shifted toward the red and radio end for massive objects. However if we are to apply the theory to 'dark matter' then we know already the light shift is past the infrared or perhaps radio wavelength. So it might not be possible to use the gravitational time dilation to estimate the mass increase.This leads into what the theory would predict if it were to explain dark matter.

The theory would predict the presence of more than expected radio wave signals or beyond radio wavelength.

These long wavelength EMR would be a signature of dense masses. So far there are less detected very dense massive objects, such as MACHOS, but this change in the future. This does not also preclude objects such as primordial black holes formed in the very early universe. In fact as explained above this is what might also be expected. It is interesting that recently there has been discovered the so called 'Odd radio circle' (ORC). an unexplained astronomical object at radio wavelengths. Some of them contain optical galaxies in their centre. This may suggest the presence of halos of radio waves around such galaxies, giving a signature of dark matter associated with the radio wave emissions, or simply the remnants of some distant past dark matter formation or something else. This is very early work

Gravitational lensing is clearly a preferred way to estimate missing dark matter.

A. Possibility of WIMPS

Although the theory proposes dark matter to have gravitational origins, it still possible that WIMPS might be formed in the early universe via this process by some exotic and as yet unknown process.

VII. MASS DENSITIES IN GALAXIES

So far we have been making the case for the explanation of dark matter being due to its mass increase in the early universe according to our derived formula and certain conditions in the early universe that allowed for clumping. That is, as two masses approach each other, the mass increase will accelerate the clumping. We wish now to address some of the issues of how the theory might also be consistent with current theory of galactic mass density profiles. After the temperature phase change, following the initial clumping, forming the dark matter, around which the remaining matter of around 15% could form normal galaxies, in line with current observations.

It is not the purpose of this paper to give exact values to certain parts of the galaxy densities at different radii. This is because the present measurement precision is still not quite able to discern the parameters we claim. It is hoped in the next half decade this will change so that the theory will be able to tested. Here we just present an outline of the possible areas of testing in galaxy rotation we envisage will confirm or disprove the theory.

We have already proposed early clumping to create the cosmic web, in terms of explaining halo formation in galaxies, we wish to focus also on the central mass problem in galaxies and see how this may fit our proposal.

There have been many theoretical and experimental studies over the years to investigate this question. Many have shown that massive black holes live at the centre of most spiral galaxies. We would expect that there would be a natural separate process involving mass increase for galaxy core formation as well. The following figure shows the surface mass densities (SMD) of the Milky Way galaxy.

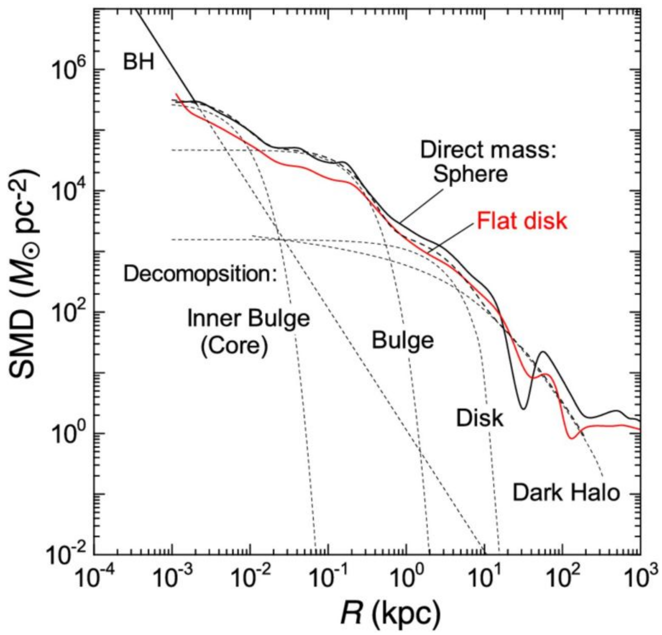

FIG. 2. Directly calculated surface mass density (SMD) of the Milky Way by spherical (black thick line) and flat-disk assumptions by log-log plot, compared with the result by deconvolution method (dashed lines)[3]. The straight line represents the black hole with mass 3.6 $\times 10^6 M\odot$. We are interested in the inner bulge and bulge as function of luminosity.

VIII. BULLET CLUSTER

The current theory is not inconsistent with the bullet cluster results. This is seen by the fact that the gas in the galaxies are found to lag compared to the more central masses. Hence the dark matter is not associated with the dust but rather the dense cores. This consistent with the prediction of the theory of more than expected dense galaxy cores.

IX. RELATION TO FIELD EQUATIONS OF GR

Since most of the effect is related to mass increase in rest mass, as well as the same order of magnitude as time or space, then we will focus our attention on the T_{00} part of the Einstein field equations before looking at the other components and the mass increase components caused by rotational motions. We now wish to see how this approach might fit with the Einstein Field equations.

For the four velocity and 4-momentum we have the following equations

$$mP_{po} \Rightarrow P_i = P.e_1 = mV_{po}\gamma_{po}, \tag{13}$$

where p is the particle as seen by the observer O

$$U_p.U_0 = -c^2\gamma_{po} \tag{14}$$

$$U_p.e_i = V_{i,po}\gamma_{po}, \tag{15}$$

where V_i is the i'th component of velocity. We have

$$E = -P_p.U_o \tag{16}$$

$$P_i = P.e_i. \tag{17}$$

Now since the above equations are independent of the coordinate system, the principle of equivalence holds, hence they hold locally in a curved spacetime as well.

For the stress energy density for the 4-momentum density we have

$$p^u = \frac{1}{c^2}T^u_v U^v, \tag{18}$$

as seen by observer O.

Since the energy density can be found from the inner product of the negative 4 momentum density and observers 4-velocity, from the 4 momentum density we have the energy density

$$T_{00} = \frac{1}{c^2}T_{uv}U^uU^v = -\frac{1}{c^2}uU_D(U_D.U_o), \tag{19}$$

where $(U_D.U_o) = -c^2\gamma$ and U_D is the energy density. We can regard it as rest mass per unit rest volume.

Since both space and time are functions of the mass change we are initially interested in the diagonal components of the stress energy tensor

$$\begin{bmatrix} \frac{e}{c^2}\gamma' & T_{01} & T_{02} & T_{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ \cdot\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}. \tag{20}$$

Hence we have

$$-c^2\gamma' = \frac{-c^2}{\sqrt{1-\frac{MG}{rc^2}}} \tag{21}$$

and so $\gamma' = \frac{1}{\sqrt{1-\frac{2MG}{rc^2}}}$. We see The $T_{11}T_{22}T_{33}$ terms are also involved as per the Schwarzschild metric.

We also want to consider the contribution of stress

$$T_{uv}e^u_i e^v_i = \frac{F^j}{A_{\perp i}} \tag{22}$$

These are perpendicular to the i-direction and are the off diagonal components. Now, since the stress energy tensor is symmetric we can write

$$\frac{F^j}{A_{\perp i}} = \frac{F^i}{A_{\perp i}} \tag{23}$$

$$T^{12}, T^{13}, T^{23} \tag{24}$$

Now pressure is not the same i direction as the momentum. We are not concerned at this stage with the energy momentum. We will concentrate on the pressure which is in the same direction as i for the diagonal components

$$T^{11}T^{22}T^{33}. \tag{25}$$

We therefore would seek solutions to the following field equations

$$R_{00} - \frac{1}{2}g_{00}R = \frac{8\pi G}{c^4}T_{00} \tag{26}$$

$$R_{11} - \frac{1}{2}g_{11}R = \frac{8\pi G}{c^4}T_{11} \tag{27}$$

$$R_{22} - \frac{1}{2}g_{22}R = \frac{8\pi G}{c^4}T_{22} \tag{28}$$

$$R_{33} - \frac{1}{2}g_{33}R = \frac{8\pi G}{c^4}T_{33}. \tag{29}$$

X. FURTHER PREDICTIONS

Due to the relationship between predicted mass increase and time dilation in dense gravitational objects we would expect a larger amount of infrared and radio emission from regions with dark matter such a galaxy halos, intergalactic regions of the cosmic web and galaxy centres.

XI. TERRESTRIAL EXPERIMENTS

We start with the simple Newtonian equation $F = \frac{MmG}{r^2}$. If we take a large source mass M and a test particle m_0 small enough, so that we can ignore its gravitational effect on M, then when both are initially separated by a distance r, and we measure the force f, then we would expect, after decreasing the radial distance of the test particle m_0, for it to have an increased mass according to our mass gain formula in Eq. (4).

Hence, classically, the final force would be the amount $F = \frac{Mm(r)G}{r^2}$. For the case, where we can no longer ignore the gravitational effect of the test particle on the corresponding mass M the effect will be a also function of M so that we would have

$$F = \frac{M(r)m(r)G}{r^2}. \tag{30}$$

We therefore expect the Newtonian formula to require a slight modification according to the theory here presented. We leave it to others to design a suitable and precise experiment based either directly or indirectly on the above principles to test the validity of the above theory.

XII. CONCLUSIONS

Using a version of the principle of equivalence for rotating bodies, the paper presents the hypothesis that the mass of a body can increase depending on the distance from other masses. It is also proposed that the well known phenomenon of time dilation in a gravitational field is accompanied by a mass increase. The effect in terrestrial gravity is expected to be very small and sifficult to detect. However, in larger gravitational fields this effect may become of significant importance. In addition to time dilation the increased density is expected to affect the space curvature components of the Einstein tensor fitting better the observed values for the bending of light

This mass increase would be significant within dense galaxy structures and thus provide an alternative explanation for dark matter. In particular if the theory is to explain some or all of dark matter it would by its very nature predict the matter is unseen or at least difficult to see. This is because as a natural consequence of mass increase in this theory we also have gravitational time dilation and this stretches the emitted light towards the extreme end of the electromagnetic spectrum. The greater the shift the greater the mass for that particular signal. We would therefore expect to see an increased region of radio waves in some regions of these sources if the theory is to be an explanation of dark matter. Also energy variations for large objects would create a time dependent metric creating additional effects.

With respect to the early universe, and its transition from the plasma to actual matter at a particular temperature and expansion, we offer the following : due to the nature of the presumably rapid positive feedback effect of decreasing radius and increasing mass, gravity could rapidly dominate over the outward radiation pressure. This could provide a mechanism for the expected matter formed in this early clumping phase.

Due also to the nature of the mechanism proposed here most of the so called missing mass would have been produced in the early universe. Hence supporting also the time frame necessary for galaxy formation.

Since most of dark matters properties in this theory seem to be gravitational alone, it would seem more natural to predict the nature of the dark matter to be MACHOS or PBH. Perhaps 'Odd radio circle' (ORC) maybe be remnants of dark matter formation.

Galaxy surface density curves are more difficult at this stage to quantify and therefore verify the theory, however within the next decade or less we would expect some confirmation. The theory is at this stage not in contradiction the the galaxy surface density analysis.

Terrestrial experiments to directly measure this mass increase due to residing in the Earth's gravitational field will be of the order of 1 part in 10^25, however it is hoped that experiments can be designed in the future to detect this.

[1] D. L. Berkahn, J. M. Chappell, and D. Abbott, European Journal of Physics (2020).

[2] A. Kashlinsky, The Astrophysical Journal **823**, L25 (2016).

[3] Y. Sofue, Publications of the Astronomical Society of Japan **69** (2016).