# Bell-test Experiments and Probabilities: EPR paradox and Bell's paradox solved

Gerard van der Ham

Abstract:

In Bell-test experiments two kinds of probabilities appear: calculated probabilities and measured probabilities. The calculated probabilities (Bell's) cannot be detected by the detectors whereas the measured probabilities (Quantum Mechanics) are being detected by the detectors indirectly. The two kinds of probabilities are very different but both can be accounted for in a local-real way. They both apply to every particle detected in the experiment. They both can be made visible as projections of one and the same vector space in different directions.

The differences between the kinds of probabilities are caused by the fact that detectors and observers perceive things like: particles, pairs of particles, vectors and vector spaces, differently. Observers perceive calculated probabilities (Bell's) and detectors detect measured probabilities (QM's) but only indirectly. In the paper both kinds of probabilities are being defined exactly. The exact definition of the probabilities makes clear why the probabilities appear as they do.

## Introduction

In this paper a simple model for Bell-test experiments is being described. The model is based on real experiments and is meant to explain what is going on in the experiments.

In Bell-test experiments two kinds of probabilities appear: calculated and measured probabilities. Both kinds of probabilities are real but very different. They both apply to every detected particle. In the model both kinds of probabilities can be made visible as projections of one and the same vector space in two different directions.

The model is classical because spin of a particle is represented by an axial vector. According to Copenhagen interpretation disciples spin is not a vector. According to them spin of a particle chooses a certain value at the moment of its measurement. In case of entangled particles, having opposite spin, this causes logical problems. If detector A measures particle 1, meaning that particle 1 chooses its spin at the moment of measurement, then when does particle 2 choose its spin? Is it at the same moment particle 1 did or is it at the moment of measurement by detector B? And what spin does it choose? Is it spin opposite to that of particle 1 or is it random spin? If it is random spin then how can the spins of both particles be correlated? If it is opposite to that of particle 1 we still have to explain Quantum Mechanic (QM) correlation. In that case spin of entangled particles might as well be represented by opposite vectors, existing from the moment the pair is produced.

Fortunately these questions and problems don't exist in a classical model. When spin of entangled particles is being considered as opposite vectors, QM correlation as well as Bell's correlation are very well explicable.

## Bell-test Experiments

Bell-test experiments are very ironically: detectors measure QM probabilities whereas one would expect them to measure Bell's probabilities and observers (theorists) perceive Bell's probabilities whereas one would expect them to perceive QM's probabilities. It all has to do with perspective and perspective is direction. The irony is, the probabilities are both real and both are where you least expect them to find.

Bell-test experiments are designed to find an answer to the EPR paradox. In Bell-test experiments sometimes entangled particles are being used. They are produced in pairs. The particles of a pair move in opposite directions and have opposite spin. In this paper spin is considered to be a vector, so opposite spin is: two vectors pointing in opposite directions. The spin of entangled particles has a random but opposite direction in space, independent of the direction of movement (line of motion) of the particles.

Each particle of a pair is being detected by a detector, for example a Stern Gerlach device. A Stern Gerlach device consists of two magnets and a detector plate. The magnets are oblong and are placed along the line of motion. They have different shapes and thus produce an inhomogeneous magnetic field where the particles are being send through. The direction of the field depends on the position of the magnets and is adjustible by rotating the magnets around the line of motion.

The particles, moving through the field, are being deflected in the direction of their spin component that corresponds to the field direction. So if the field direction is vertical, the particle can be deflected upward or downward. Leaving the field, the particle strikes the detector plate, which is placed behind the magnets, perpendicularly on the line of motion. Because of the deflection, the particle strikes the detector plate above or beneath the imaginary central perpendicular plane of the device, resulting in spin 'up' or spin 'down'.

The two particles of a pair are being detected by two Stern Gerlach devices. If the devices are adjusted in the same directions then the combination of spin results is always opposite for every pair of particles. If the devices are adjusted in opposite directions then the combination of spin results is always equal. For adjustments in between, the difference in angle of adjustment ($\varphi$) is correlated to the probabilities for certain combinations of spin results to occur. Correlation (C) is calculated from the probabilities for the combinations of equal spin results (E) and the probabilities for the combinations of opposite spin results (O): $C = E - O$.

QM's correlation is proportional to $\cos \varphi$, meaning that QM's probabilities for equal or opposite spin combinations are proportional to $\cos \varphi$ and Bell's correlation is proportional to $\varphi$, meaning that Bell's probabilities for equal or opposite spin combinations are proportional to $\varphi$.

The irony also is that Bell wanted to prove that Einstein was right in his statement that particles (quanta) should have definite properties whereas QM stated they have not. When the experiments finally could be carried out, the QM results appeared to be correct. So the general opinion was that Einstein was wrong. But with Copenhagen interpretation of QM, these QM results could not be accounted for. Assuming definite properties of particles, in agreement with Einstein, the QM results can very well be explained, as we will see. So Einstein was right after all.

## The Model

The model is a very simple and frequently used model. In fact it is the most simple possible description of a real experiment in which Stern Gerlach devices are being used to detect the particles. As already has been mentioned: in Bell-test experiments it is all about direction. It is only about direction because correlation solely depends on $\varphi$, being the angle between field directions, and on nothing else. So time and distance play no role and we can keep the set up of the experiment small and put it on a table (see fig.1)).

It is a classical model, meaning that spin of a particle is represented by a vector in a random direction. Entangled particles have spin in opposite directions, independent of the direction of their movement. Spin vectors keep a fixed direction in space. So particles have a fixed spin direction in space from the moment they come to existence.

In the model we will follow only one pair of entangled particles. This pair is produced in a source S in the middle between the detectors. From the moment the pair is produced, the particles have opposite spin vectors in a random direction. These vectors determine a sphere shaped vector space around the source as they can point in any direction.

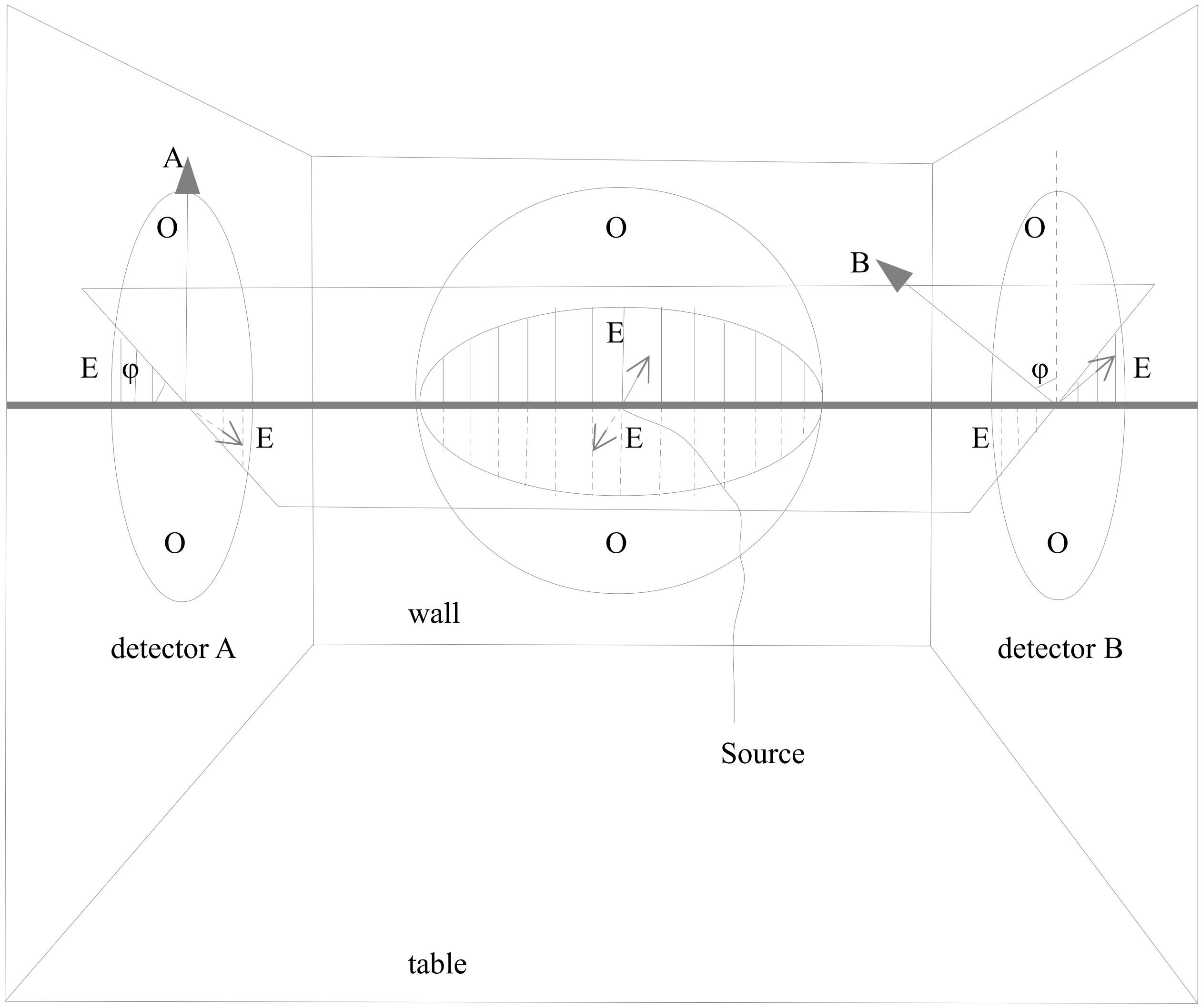



Fig. 1) Realistic model of a Bell-test experiment

The fat horizontal line is chosen to be the reference direction for the experiment. It is also the line of motion of the particles and it reprenents the perfect horizontal central perpendicular plane of detector A. The central perpendicular plane of B makes an angle φ with this horizontal plane.

Around the source in the middle is the spherical vector space, divided in the four vector spaces E and O (like the parts of an orange). The spaces E and O are being projected to the left and the right onto the detector plates and to the back onto the wall. The picture of the projection on the wall is of course the same as the picture in the middle.

The projections of the vector spaces on the detectors represent Bell's probabilities, although the detectors themselves cannot represent these probabilities. Those probabilities are calculated probabilities.
The projections of the vector spaces on the wall (from your position) represent QM's probabilities. Taking into account the rotations of the detectors, the detectors can represent these probabilities indirectly, as explained in the text.

The particles move from the source, one to the left and one to the right, along the line of motion. THIS LINE OF MOTION IS CHOSEN AS REFERENCE DIRECTION FOR THE EXPERIMENT. At the ends of the table the detectors are being placed perpendicularly on the line of motion. Suppose detector A is at the left side and adjusted in the vertical direction, meaning that its field direction is vertical. A keeps that vertical direction irrespective of B. Then B is at the right side and adjusted at an angle $\varphi$ in respect of A, meaning that its field direction makes an angle $\varphi$ in respect of the field direction of A. As particles are being deflected up- or downwards in respect of the field direction, we can attache a central perpendicular plane to the detectors. The central perpendicular plane of A is a perfect horizontal plane and the angle between the central perpendicular plane of B and this horizontal plane is of course also $\varphi$.

Stretched to infinity these planes divide the spherical vector space in four parts, two by two opposite of each other, like the parts of an orange. These partial vector spaces are called E and O. It is obvious that the opposite vectors of one pair of particles are both in space E (each in one part of E), or they are both in space O (each in one part of O). It is also not difficult to see that when the particles have their vectors (spin directions) in E then the combination of results of the detectors is a combination of equal spin. And if the particles have their vectors in O then the combination of results is a combination of opposite spin. This best visible on the detector plates, left and right.

Explanation

So far, so good. Now Bell comes along. He notices that the probability for a pair of particles to have their vectors in E, is proportional to the size of E and that is proportional to $\varphi$ and the projection of E onto the detector plates is also proportional to $\varphi$. Note that Bell perceives (in his mind (and from the position of the detectors)) the two particles of the pair together with their vectors in certain vector spaces (E or O). Then he calculates the probabilities: $E = \varphi/\pi$ and $O = (\pi - \varphi)/\pi$ and from them the correlation: $C = E - O$ so $C(\text{Bell}) = (2\varphi - \pi)/\pi \quad (0 < \varphi < \pi)$, (see diagram). The probability for equal spin combinations is also visible as the projection of the spaces E onto the detector plates, each to one side. This also goes for O.

Note also that the detectors A and B cannot perceive these probabilities for the simple reason they cannot perceive the vector spaces E and O because A doesn't 'know' anything about B and vice versa. Together the detectors can perceive both particles of a pair but they cannot perceive the vector spaces. Each detector can perceive one particle of a pair and determine its spin direction in respect of the field direction in a 50 % probability for the spin to be 'up' or 'down' (depending on the deflection of the particle). That is all both detectors do.

The measured probabilities (QM's) are being obtained by comparing the lists of results of both detectors afterwards. The combinations of equal spin results and the combinations of opposite spin results are being counted and divided by the total number of combinations. This yield the probabilities for E and O and from them the correlation is calculated as predicted by QM. These QM probabilities differ from Bell's probabilities.

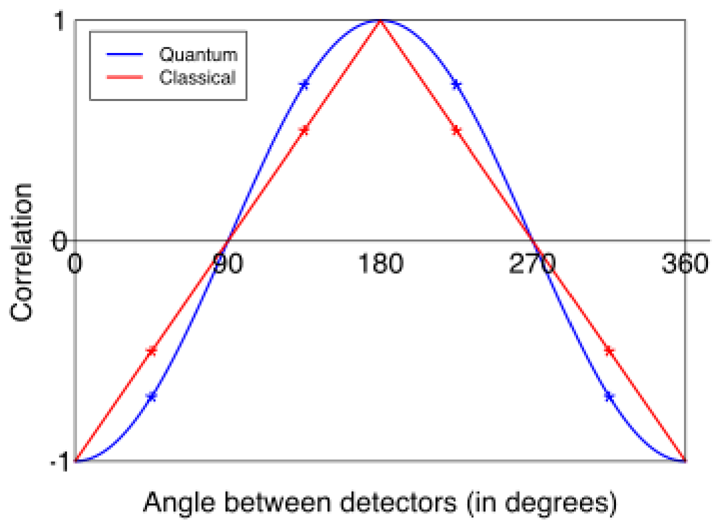
Definition: Bell's probability is the probability for the two particles of an entangled pair to have their spin directions in certain vector spaces (E or O).

As we have seen these probabilities cannot be perceived by the detectors. The probabilities measured by the detectors INDIRECTLY (by afterwards comparing the lists of results) are different: they are QM's probabilities.

Definition: QM's probability is the probability for a particular particle to belong to a pair of entangled particles that have their spin directions in certain vector spaces (E or O).

Each of the detectors measures for every particle a 50 % probability to have spin 'up' or spin

'down'. The lists of results of both detectors compared afterwards, appear to give QM's probabilities for equal and opposite spin combinations. It is easy to understand (and to see in the diagram as well as in the figure) that for small angles (0° < φ < 90°) less pairs have their spin directions in E and more pairs have their spin directions in O. So the probability for one particular pair (because a particular particle is at the same time a particle of a particular pair) to belong to one of the pairs having their spin directions in E, is relatively smaller and the probability for one particular pair to belong to one of the pairs having their spin directions in O, is relatively bigger than the probability for a pair to have its spin directions in E or O.
For angles of φ between 90° and 270° this is the other way round, and for φ between 270° and 360° it is the same as for φ between 0° and 90°. This causes the negative cosine shape in the diagram (see diagram). From the diagram and the figure we can see that at 0°, 90°, 180°, 270° and 360° QM's probabilities equal Bell's probabilities, resulting in equal correlations.




The best possible local realist imitation (red) for the quantum correlation of two spins in the singlet state (blue), insisting on perfect anti-correlation at 0°, perfect correlation at 180°. Many other possibilities exist for the classical correlation subject to these side conditions, but all are characterized by sharp peaks (and valleys) at 0°, 180°, and 360°, and none has more extreme values (±0.5) at 45°, 135°, 225°, and 315°. These values are marked by stars in the graph, and are the values measured in a standard Bell-CHSH type experiment: QM allows $\pm 1/\sqrt{2} = \pm 0.7071\ldots$, local realism predicts ±0.5 or less.

Source: Wikipedia

In this diagram, representing different correlations in Bell-test experiments, the probabilities for combinations of equal and opposite spin results can also be found. For every angle φ the probability for opposite spin combinations is the space above the curve of the correlation and the probability for equal spin combinations is the space beneath that curve.

T he horizontal axis, dividing the diagram in two equal parts, represents the correlation for each detector. Each detector yield a 50 % probability for equal and opposite spin results, visible as the equal spaces above and beneath the axis.

At all angles the probabilities for each correlation curve add up to 1.

QM's probabilities can also be made visible: as projections of the vector spaces E and O on the wall behind the table from your position. The projection P' from every point P of the sphere onto the wall is: $P' = P \sin \varphi$. The projection of vector space E is proportional to $\sin \varphi$. The projection density of E is proportional to $\sin^2 (\varphi/2)$. So the probability for a particular vector to belong to one of the vectors in E is proportional to $\sin^2 (\varphi/2)$. So the probability for a particular vector to belong to one of the vectors in O is proportional to $1 - \sin^2 (\varphi/2) = \cos^2 (\varphi/2)$. From these probabilities the QM correlation can be calculated: $C(QM) = E - O = \sin^2 (\varphi/2) - \cos^2 (\varphi/2) = - \cos \varphi$.

It is not a coïncidence that the vector spaces must be projected from your position, or that the vector spaces must be looked at from your position (which is the same), to get a picture of the QM probabilities. To describe the experiment correctly, a reference direction has to be chosen. We have chosen it to be the direction of the line of motion and now its role becomes clear. We have to describe the experiment in respect of the reference direction. That means that we have to take into account all rotations. Starting from the set up of the experiment, we have seen that the Stern Gerlach devices consist of two oblong magnets that have been placed along the reference direction. They don't need a rotation (except for B in respect of A). The detector plates, however, have been placed perpendicularly on the line of motion (the reference direction). To get there they have to start from a position along the reference direction and rotate 90°. For example they can start from your position along the reference direction. This is why the detectors represent the results as if detected from your position. The detectors must represent the QM results as if detected from your position because only then the results agree with the perspectives of the particles and thus with QM, descring the particles from one perspective: the reference direction (line of motion).

One could ask: how many particles really have their spin directions between the central perpendicular planes of the devices (E) in one run of a test? The answer is: statistically it is the number representing Bell's probabilities. The next question then is: why are these numbers then not being measured? The answer to that question is bipartite: an empirical answer and a theoretical answer.

1) (empirical): Because a detector can only perceive one particle of a pair and no vector spaces. Therefore detectors cannot determine the probability for a pair of particles to have their spin directions in certain vector spaces: they can't produce Bell numbers. Detectors can represent, indirectly, the probability for a particular particle to belong to a pair of particles having their spin directions in certain vector spaces. These are QM numbers and not Bell numbers.
2) (theoretical): Because not all rotations of the detectors in respect of the reference direction have been taken into account then. If detectors would not had to be rotated from a position along the reference direction to their position, perpendicularly, on the line of motion, then they would have definitely produced Bell numbers. But the detectors did have to undergo that rotation. Therefore the numbers produced by the combined lists of the detectors are the QM numbers. They are numbers AS IF detected from your position. Then the rotations of the detectors have been taken into account. These numbers lead to QM probabilities and to QM correlation. The QM probabilities are equally real as Bell's probabilities (and vice versa) and they are both local-real explicable.

Probabilities are no real numbers. They are meaningless ratio of numbers that need definition. All real numbers in the run of an experiment agree with each other. In fact there are no pairs of entangled particles with equal spin because they all have opposite spin. The combinations of equal and opposite spin results are solely due to the adjustments of the detectors. And the probabilities for those combinations depend on how the probabilities are being defined and on how the probabilities are being perceived.

## Perspective

Perspective is the direction of observation in respect of a reference direction. It is one of the most difficult phenomena in physics. This is because it is not about the questions: 'how do you see the universe?' or 'how do I see the universe?' but it is about the questions: 'how do you see the universe from my perspective?' or 'how do I see the universe from your perspective?'

Translated to Bell-test experiments this means: how do the detectors perceive vectors and vector spaces from the perspective of the particles? Fortunately the answer to these questions is very simple. One has to take into account all rotations of the detectors and let the universe, in this case the vectors and vector spaces, carry out the same rotations (see ref. 1)). In that way one can immediately see that the correct QM probabilities are being represented by the detectors.

## Conclusion

To be able to account for QM correlations in Bell-test experiments we need to assume definite properties of quanta, corresponding Einsteins view of physics. This is a statement I cannot prove but in the past 50 years no one succeeded in accounting for QM correlations in Bell-test experiments without definite properties of quanta. We also have to take into account perspective, as Bell-test experiments are solely about direction.

In Bell-test experiments two kinds of probabilities appear. Probabilities are meaningless ratio of numbers and they need to be defined. Doing that, one discovers that the two kinds of probabilities in Bell-test experiments are equally real and both are local-real explicable. In this way the 'EPR paradox' and the 'Bell paradox' have been solved in favour of Einstein.

## Reference:

1) https://www.youtube.com/watch?v=g1quDMTEIFE (video)

## Addendum

In a two dimensional space (a plane) perspective can be considered as phase shift: what is a sine in one direction, can be a cosine in a perpendicular direction. In directions in between perspective can even be considered as interference (= superposition): sin + cos (dubble slit experiments). That is only in two dimensions. In three dimensions even Bell-test situations can occur.