An Idea of Fermat for the Stop and Division by Zero Calculus

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Abstract: In this note we will consider an idea of Fermat for the stop in connection with the division by zero calculus. Here, in particular, we will see some mysterious logic on the stop in connection with the concepts of differential and differential coefficient.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, isolated singular point, analytic function, Laurent expansion, $1/0 = 0/0 = z/0 = \tan(\pi/2) = \log 0 = 0$, $[(z^n)/n]_{n=0} = \log z$, $[e^{(1/z)}]_{z=0} = 1$.

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1 Introduction

For a fixed $\ell > 0$, we will consider the area S(a) by

$$S(a) = a(\ell - a)$$

and the maximum of S(a) for $0 \leq a \leq \ell$. Of course, this problem is very simple with elementary calculus.

However, Fermat (1629) considered this problem in the following way:

Assume that

$$(a+\epsilon)(\ell - a - \epsilon) = a(\ell - a). \tag{1.1}$$

Then, formally we have the identity

$$\epsilon^2 + \epsilon(2a - \ell) = 0. \tag{1.2}$$

See [2], 358-359.

From this logic and identity, could we obtain the desired result

$$a = \frac{\ell}{2} \tag{1.3}$$

?

Note that

$$S'(a) = \ell - 2a.$$

Firstly, note that the identity (1.1) is not valid except $\epsilon = 0$ and $\epsilon = \ell - 2a$, as we from the representation of $S(a)$; that is the identity is nonsense when we consider a small variation. Of course, (1.2) is valid for $\epsilon = 0$ and $\epsilon = \ell - 2a$. So, we wonder the above logic is nonsense.

The logic is not on any variation of the area $S(a)$ essentially that may be related to differential and differential coefficient.

For this question, we will be able to give our interpretation with the concept of the division by zero calculus.

2 Division by zero calculus

We will simply introduce the division by zero calculus.

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n (z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n (z-a)^n, \tag{2.1}$$

we will define

$$f(a) = C_0. \tag{2.2}$$

We define the value of the function $f(z)$ at the singular point $z = a$ by ignoring the singular parts of the Laurent expansion.

For the correspondence (2.2) for the function $f(z)$, we will call it **the division by zero calculus**. By considering derivatives in (2.1), we **can define** any order derivatives of the function f at the singular point a; that is,

$$f^{(n)}(a) = n!C_n.$$

With this assumption (definition), we can obtain many new results and new concepts. See [20] and the references in this paper.

3 Interpretation by the division by zero calculus

In the formal formula (1.2), from the identity

$$\frac{\epsilon^2}{\epsilon} + (2a - \ell) = 0, \tag{3.1}$$

we obtain the desired result (1.3) at $\epsilon = 0$

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References

[1] W. W. Däumler, H. Okumura, V. V. Puha and S. Saitoh, Horn Torus Models for the Riemann Sphere and Division by Zero, viXra:1902.0223 submitted on 2019-02-12 18:39:18.

[2] J. Fauvel and J. Gray, The History of Mathematics — A Reader — (1987/3/16).

[3] M. Kuroda, H. Michiwaki, S. Saitoh and M. Yamane, New meanings of the division by zero and interpretations on $100/0 = 0$ and on $0/0 = 0$, Int. J. Appl. Math. **27** (2014), no 2, pp. 191-198, DOI: 10.12732/ijam.v27i2.9.

[4] T. Matsuura and S. Saitoh, Matrices and division by zero $z/0 = 0$, Advances in Linear Algebra & Matrix Theory, **6**(2016), 51-58 Published Online June 2016 in SciRes. http://www.scirp.org/journal/alamt http://dx.doi.org/10.4236/alamt.2016.62007.

[5] T. Matsuura, H. Michiwaki and S. Saitoh, $\log 0 = \log \infty = 0$ and applications, Differential and Difference Equations with Applications, Springer Proceedings in Mathematics & Statistics, **230** (2018), 293-305.

[6] H. Michiwaki, S. Saitoh and M.Yamada, Reality of the division by zero $z/0 = 0$, IJAPM International J. of Applied Physics and Math. **6**(2015), 1–8. http://www.ijapm.org/show-63-504-1.html

[7] H. Michiwaki, H. Okumura and S. Saitoh, Division by Zero $z/0 = 0$ in Euclidean Spaces, International Journal of Mathematics and Computation, **2**8(2017); Issue 1, 1-16.

[8] H. Okumura, S. Saitoh and T. Matsuura, Relations of 0 and ∞, Journal of Technology and Social Science (JTSS), **1**(2017), 70-77.

[9] H. Okumura and S. Saitoh, The Descartes circles theorem and division by zero calculus, https://arxiv.org/abs/1711.04961 (2017.11.14).

[10] H. Okumura, S. Saitoh, Remarks for The Twin Circles of Archimedes in a Skewed Arbelos by Okumura and Watanabe, Forum Geom., **18** (2018) 97–100.

[11] H. Okumura, Is It Really Impossible To Divide By Zero? Biostat Biometrics Open Acc J. 2018; 7(1): 555703. DOI: 10.19080/BBOJ.2018.07.555703.

[12] H. Okumura and S. Saitoh, Harmonic Mean and Division by Zero, Dedicated to Professor Josip Pečarić on the occasion of his 70th birthday, Forum Geometricorum, **18** (2018), 155—159.

[13] H. Okumura and S. Saitoh, Remarks for The Twin Circles of Archimedes in a Skewed Arbelos by H. Okumura and M. Watanabe, Forum Geometricorum, **18**(2018), 97-100.

[14] H. Okumura and S. Saitoh, Applications of the division by zero calculus to Wasan geometry, GLOBAL JOURNAL OF ADVANCED RESEARCH ON CLASSICAL AND MODERN GEOMETRIES" (GJARCMG), **7**(2018), 2, 44–49.

[15] H. Okumura and S. Saitoh, Wasan Geometry and Division by Zero Calculus, Sangaku Journal of Mathematics (SJM), **2** (2018), 57–73.

[16] S. Pinelas and S. Saitoh, Division by zero calculus and differential equations. Differential and Difference Equations with Applications. Springer Proceedings in Mathematics & Statistics, **230** (2018), 399-418.

[17] S. Saitoh, A reproducing kernel theory with some general applications, Qian,T./Rodino,L.(eds.): Mathematical Analysis, Probability and Applications - Plenary Lectures: Isaac 2015, Macau, China, Springer Proceedings in Mathematics and Statistics, **177**(2016), 151-182.

[18] S. Saitoh, Fundamental of Mathematics; Division by Zero Calculus and a New Axiom, viXra:1908.0100 submitted on 2019-08-06 20:03:01.

[19] S. Saitoh, Essential Problems on the Origins of Mathematics; Division by Zero Calculus and New World, viXra:1912.0300 submitted on 2019-12-16 18:37:53.

[20] S. Saitoh, Introduction to the Division by Zero Calculus, Scientific Research Publishing, Inc. (2021), 202 pages.

[21] S.-E. Takahasi, M. Tsukada and Y. Kobayashi, Classification of continuous fractional binary operations on the real and complex fields, Tokyo Journal of Mathematics, **38**(2015), no. 2, 369-380.