On the Application of Lorentz Transformation Equations to Light Speed Problems

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03 March 2021

Abstract

One of the problems among supporters and opponents of the special relativity theory is the lack of rigorous and consistent application of Lorentz transformation equations to debated light speed problems. The arguments in many cases are intuitive, lacking mathematical rigor, causing endless debates and confusions. Even relativists are usually seen to be confused regarding, for example, the time delay of a short light pulse for an observer moving at non-relativistic speeds relative to the light source. Strict and consistent application of the Lorentz transformation equations is necessary, whether to decisively refute or to defend the claims of the theory. In this paper, we present a rigorous application of Lorentz transformation equations to some light speed problems, with the aim of demonstrating the approach to be used for similar problems.

Introduction

One of the most debated topics regarding special relativity is the application of the theory to the Global Positioning System (GPS) and the Sagnac effect. Different authors have disclosed the practice in mainstream physics of making adjustments of first order effects observed in the data of a number of experiments, such as the GPS, the lunar laser ranging experiment[1] and the Venus planet radar ranging experiment[2]. One wonders how mainstream physicists accept such adjustments as consistent with special relativity theory. However, mainstream physicists have their own arguments which are usually inconsistent.

The arguments between supporters and opponents of special relativity theory are usually based on intuitive assertions about length contraction and time dilation, rather than on rigorous and consistent application of Lorentz transformations, leading to confusions in most cases. This lack of rigorous mathematical treatment and confusion is common also among opponents of special relativity theory, who in some cases unknowingly criticize the theory for what it is not, or for what it doesn't claim to be.

In this paper, we present a rigorous mathematical treatment of a light speed problem based on Lorentz transformation equations with the aim of demonstrating the approach to be followed for similar problems, whether to decisively refute or to defend the claims of the theory.

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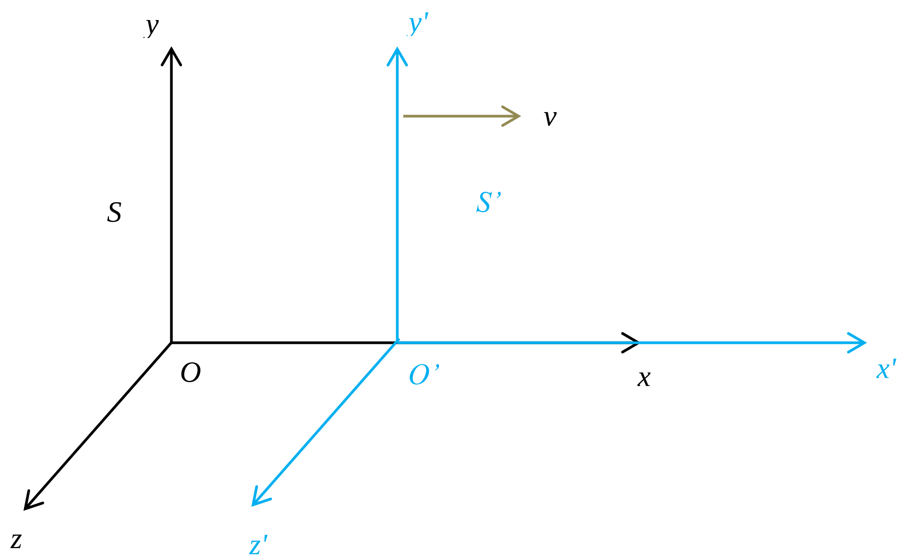
Lorentz Transformations

We briefly review Lorentz transformation equations[3].

Consider two reference frames S and S'. S' moves relative to S in the +*x* direction with velocity *v*. The origins of S and S' , which are O and O' respectively, coincide at $t = t' = 0$. An event observed in S' has coordinates (x', y', z', t'). The same event observed in S has coordinates (x, y, z, t).



The Lorentz transformation specifies that these coordinates are related in the following way:

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

$$x' = \gamma \left(x - vt \right)$$

$$y' = y$$

$$z' = z$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Writing the Lorentz transformation and its inverse in terms of coordinate differences, where for instance, one event (Event 1) has coordinates (x_1 , t_1) and (x_1' , t_1') , another event (Event 2) has coordinates (x_2 , t_2) and (x_2' , t_2') , and the differences are defined as:

$$\Delta x' = x_2' - x_1' \quad , \quad \Delta x = x_2 - x_1$$

$$\Delta t' = t_2' - t_1' \quad , \quad \Delta t = t_2 - t_1$$

we get

$$\Delta x' = \gamma \ (\ \Delta x - v\,\Delta t \ ) \quad , \quad \Delta x = \gamma \ (\Delta x' + v\,\Delta t' \ )$$

$$\Delta t' = \gamma \left(\Delta t - \frac{v\Delta x}{c^2} \right) \quad , \quad \Delta t = \gamma \left(\Delta t' + \frac{v\Delta x'}{c^2} \right)$$

Light source and observer in relative motion

We analyze the problem of light source and observer in relative motion according to Lorentz transformation equations and special relativity postulates and compare the result with experimental evidence.

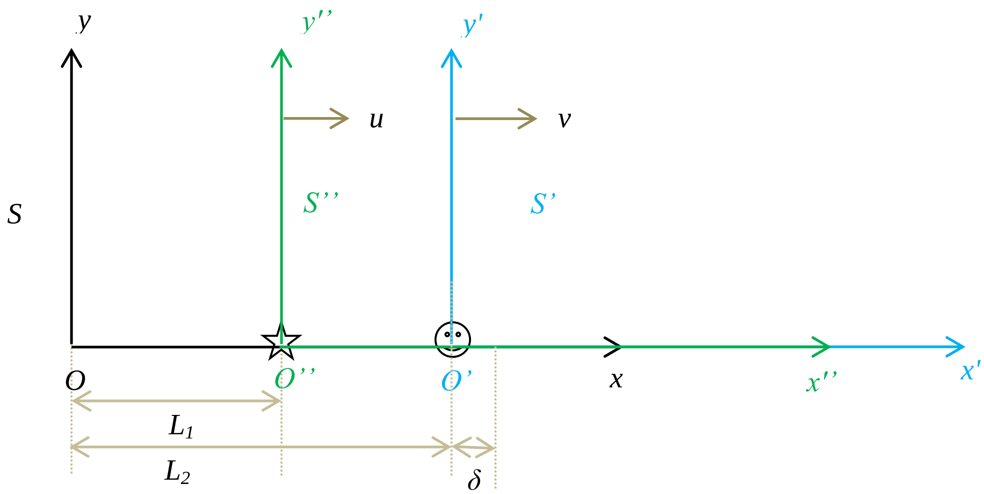
Consider a light source and an observer moving with velocities u and v, respectively, in inertial reference frame S. We consider two events: emission of a short light pulse from the source and detection of the light pulse by the observer. In general, velocities u and v will not be equal, so the light source will also be moving in the rest frame (S’) of the observer. The rest frame of the source is S’’. Assume that $v > u$. The light source is at the origin of S’’ and the observer is at the origin of S’.

At $t = t' = t'' = 0$, the origins of S, S’ and S’’ coincide and the clocks in all frames are synchronized. Let the light source emit a short light pulse when the source is at $x = L_1$ and when the observer is at $x = L_2$ in reference frame S. This means that the distance between the source and the observer is equal to $L_2 - L_1$ at the instant of light emission, in frame S.

Now, we have two events:

Event 1 is the emission of light from the source, with coordinates (x_1, y_1), (x_1', y_1'), (x_1'', y_1'') in reference frames S, S’ and S’’ respectively.

Event 2 is the detection of light by the observer, with coordinates (x_2, y_2), (x_2', y_2'), (x_2'', y_2'') in reference frames S, S’ and S’’ respectively.



Event 1 in S

Event 1, which is the emission of light, occurs in S frame at:

$$x_1 \;=\; L_1$$

$$t_1 = \frac{L_1}{u} \;=\; \frac{L_2}{v}$$

Event 1 in frame S'

The coordinates of the same event (light emission) in S' is determined from the Lorentz transformation equations :

$$x' \;=\; \gamma_v \;(\; x \;-\; vt \;)$$

$$t' \;=\; \gamma_v \;(\; t \;-\; \frac{vx}{c^2} \;)$$

By substituting the values of x_1 and t_1 obtained above:

$$x_1{}' \;=\; \gamma_v \;(x_1 \;-\; v\,t_1\;) \;=\; \gamma_v \left(\, L_1 \;-\; v\,\frac{L_1}{u} \right) \;=\; \gamma_v L_1 \,(\,1 \;-\; \frac{v}{u})$$

$$t_1{}' \;=\; \gamma_v \left(\; t_1 \;-\; \frac{v\,x_1}{c^2} \right) = \gamma_v \left(\, \frac{L_1}{u} \;-\; \frac{v * L_1}{c^2} \right) = \gamma_v\, L_1\, (\frac{1}{u} \;-\; \frac{v}{c^2})$$

where

$$\gamma_v = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

<u>Event 1 in frame S’’</u>

The coordinates of the same event (light emission) in S’’ is determined from the Lorentz transformation equations :

$$x'' = \gamma_u \, (\, x - \, ut \,)$$

$$t'' = \gamma_u \, (\, t - \frac{ux}{c^2} \,)$$

By substituting the values of x_1 and t_1 obtained above:

$$x_1'' = \gamma_u \, (x_1 - u \, t_1 \,) = \gamma_u \left(L_1 - u \, \frac{L_1}{u} \right) = 0$$

$$t_1'' = \gamma_u \left(t_1 - \frac{u \, x_1}{c^2} \right) = \gamma_u \left(\frac{L_1}{u} - \frac{u * L_1}{c^2} \right) = \gamma_u \, L_1 \, (\frac{1}{u} - \frac{u}{c^2})$$

where

$$\gamma_u = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

<u>Event 2 in frame S</u>

To determine the time delay of light to catch up with the observer in reference frame S, we proceed as follows.

During the time interval that the observer moves distance δ to the right, the light travels a distance of:

$$(L_2 - \, L_1) + \, \delta$$

From which,

$$\frac{\delta}{v} = \frac{(L_2 - \, L_1) \, + \, \delta}{c} \quad \Longrightarrow \quad \delta = (L_2 - \, L_1) \, \frac{v}{c - v}$$

and the time delay of light in frame S will be:

$$\frac{\delta}{v} = \frac{(L_2 - \, L_1) \, \frac{v}{c - v}}{v} = \frac{(L_2 - \, L_1)}{c - v}$$

Therefore,

$$x_2 \;=\; L_2 \;+\; \delta \;=\; L_2 \;+\; (\,L_2 - \; L_1)\;\frac{v}{c - v}$$

and

$$t_2 \;=\; t_1 \;+\; \frac{\delta}{v} \;=\; \frac{L_1}{u} \;+\; \frac{(\,L_2 - \; L_1)}{c - v}$$

<u>Event 2 in frame S’</u>

The same event, that is detection of light by the observer, occurs at (x_2', t_2') in frame S’.

$$x_2{}' = \gamma_v\,(\,x_2 - v\,t_2\;) = \gamma_v\left((\,L_2 \;+\;(\,L_2 - \; L_1)\;\frac{v}{c - v}) \;-\; v\;\left(\frac{L_1}{u} \;+\; \frac{(\,L_2 - \; L_1)}{c - v}\right)\right)$$

$$= \gamma_v\left(\;L_2 - \frac{vL_1}{u}\right)$$

$$\Longrightarrow\;\; x_2{}' \;=\; \gamma_v\,(\;L_2 - \; vt_1\,) = \;\gamma_v\,(\;L_2 - \;L_2\,) = 0$$

and

$$t_2{}' \;=\; \gamma_v\left(t_2 \;-\; \frac{v\,x_2}{c^2}\right) = \gamma_v\left(\;(\frac{L_1}{u} \;+\; \frac{(\,L_2 - \; L_1)}{c - v}) \;-\; \frac{v\;\,(L_2 \;+\;(\,L_2 - \; L_1)\;\frac{v}{c - v})}{c^2}\right)$$

$$\Longrightarrow\;\; t_2{}' \;\; = \gamma_v\left(\frac{(\,L_2 - \; L_1)}{c - v}\;\left(1 - \frac{v^2}{c^2}\right) + \frac{L_1}{u} - \frac{v\,L_2}{c^2}\right)$$

<u>Event 2 in frame S’’</u>

The same event, that is detection of light by the observer, occurs at (x_2'', t_2'') in frame S’’.

We determine x_2'' and t_2'' from the Lorentz transformation equations.

$$x_2{}'' \;=\; \gamma_u\;(\;x_2 - \;u\,t_2\;\;) \;=\; \gamma_u\;\left((L_2 \;+\;(\,L_2 - \; L_1)\;\frac{v}{c - v}) - \;u\,(\frac{L_1}{u} \;+\; \frac{(\,L_2 - \; L_1)}{c - v})\right)$$

$$\Longrightarrow x_2{}'' \;=\; \gamma_u\left(\frac{(\,L_2 - \; L_1)}{c - v}\;\;(\,v \;-\; u\,) \;+\; (\,L_2 - \; L_1\,)\right)$$

$$\Longrightarrow\; x_2{}'' \;=\; \gamma_u(\,L_2 - \; L_1)\;\frac{c - u}{c - v}$$

and

$$t_2'' = \gamma_u\left(t_2 - \frac{u\, x_2}{c^2} \right) = \gamma_u\left(\left(\frac{L_1}{u} + \frac{(L_2 - L_1)}{c - v}\right) - \frac{u\ (L_2 + (L_2 - L_1)\ \frac{v}{c - v})}{c^2} \right)$$

$$\Rightarrow t_2'' = \gamma_u\left(\frac{(L_2 - L_1)}{c - v} \left(1 - \frac{u\, v}{c^2}\right) + \frac{L_1}{u} - \frac{u\, L_2}{c^2} \right)$$

The time interval between Event 1 and Event 2 , that is between emission and detection of the light, in the rest frame (S') of the observer will be:

$$\Delta t' = t_2' - t_1' = \gamma_v\left(\frac{(L_2 - L_1)}{c - v} \left(1 - \frac{v^2}{c^2}\right) + \frac{L_1}{u} - \frac{v\, L_2}{c^2} \right) - \gamma_v\, L_1 \left(\frac{1}{u} - \frac{v}{c^2}\right)$$

$$\Rightarrow \Delta t' = \gamma_v\left(\frac{(L_2 - L_1)}{c - v} \left(1 - \frac{v^2}{c^2}\right) - \frac{v}{c^2}\ (L_2 - L_1) \right)$$

$$\Rightarrow \Delta t' = \gamma_v (L_2 - L_1) \left(\frac{1}{c - v} \left(1 - \frac{v^2}{c^2}\right) - \frac{v}{c^2} \right)$$

$$\Rightarrow \Delta t' = \gamma_v (L_2 - L_1) \left(\frac{c + v}{c^2} - \frac{v}{c^2} \right)$$

$$\Rightarrow \Delta t' = \frac{\gamma_v (L_2 - L_1)}{c}$$

Let us see if we will get the same value of $\Delta t'$ above by considering the light source in reference frame S'. According to the second postulate of special relativity, the speed of light in any inertial reference frame is constant *c* independent of the velocity of the source in that frame.

In frame S', the light source is at:

$$x_1' = \gamma_v\, L_1 \left(1 - \frac{v}{u}\right)$$

at the instant of light emission and moving in the $-x'$ direction. According to the second postulate of special relativity, the speed of light in any inertial reference frame is constant *c*, independent of the velocity of the source in that frame. So the time delay of light between emission and detection of light in S' is:

$$\Delta t' = \frac{\text{the distance of the point where light was emitted from the observer}}{\text{speed of light}}$$

$$\Delta t' = \frac{x_2' - x_1'}{c} = \frac{0 - \left(\gamma_v L_1 \left(1 - \frac{v}{u}\right)\right)}{c} = -\gamma_v \frac{L_1}{c} \left(1 - \frac{v}{u}\right)$$

$$\Rightarrow \Delta t' = -\gamma_v \frac{L_1}{c} \left(1 - \frac{L_2}{L_1}\right) = -\gamma_v \frac{L_1}{c} \left(\frac{L_1 - L_2}{L_1}\right) = \frac{\gamma_v (L_2 - L_1)}{c}$$

We can see that the values of *Δt'* obtained by the two approaches do agree.

The distance between Event 1 and Event 2 , that is between the point of light emission and the point of light detection, in the rest frame (S') of the observer will be:

$$\Delta x' = x_2' - x_1'$$

$$\Delta x' = 0 - (\gamma_v L_1 (1 - \frac{v}{u}))$$

$$\Rightarrow \Delta x' = -\gamma_v L_1 (1 - \frac{v}{u})$$

$$\Rightarrow \Delta x' = -\gamma_v L_1 \left(1 - \frac{L_2}{L_1}\right) = -\gamma_v L_1 \frac{(L_1 - L_2)}{L_1} = \gamma_v (L_2 - L_1)$$

Lorentz transformation of events for arbitrary locations and relative velocities of events and observers

In the analysis we made so far we chose L_1, L_2, u and v so that

$$\frac{L_1}{L_2} = \frac{u}{v}$$

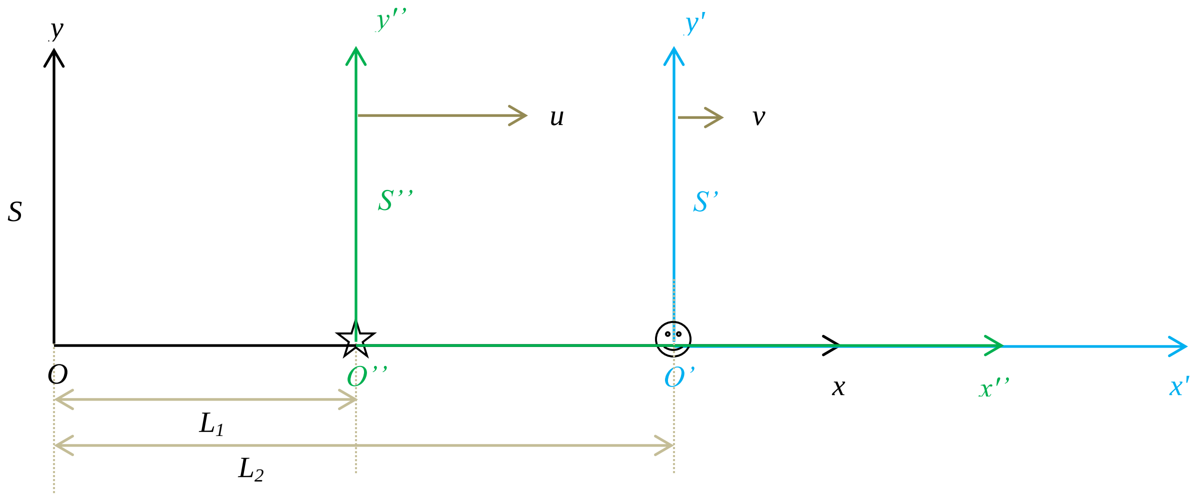
So that we could synchronize all the three clocks when the origins O, O' and O'' coincided. However, in general this is not the case.

Consider the inertial reference frames shown below. The source emits a light pulse at $x = L_1$ in frame S. At the instant of light emission, the observer is at $x = L_2$. The problem is to find the coordinates of the event in reference frames S, S' and S''.

For this, the clocks of S, S' and S'' are to be synchronized when their origins coincide. But, since

$$\frac{L_1}{L_2} \neq \frac{u}{v}$$

this is not possible. (We can see that u is much greater than v) . If we look at the velocities and relative positions of the reference frames this is not possible.



To avoid this contradiction, we start by assuming that the clocks of S and S'' are synchronized at *t = t'' = 0* when their origins O and O'' coincide. For S', we create another fictitious observer (reference frame), let us call it frame F, with the same velocity *v* as reference frame S'. It is this fictitious reference frame whose origin coincides with the origins of S and S'' at the instant of clock synchronization. The fictitious clock of F is synchronized with the clocks of S and S'' at the instant the origins of S, S'' and F coincide, at *t* = *t''* = 0. We can then determine the coordinates of any event observed in S relative to F, like any other inertial reference frame. Once the coordinates of the event are determined in F, we can easily get the coordinates of the event relative to S' because S' has the same velocity as F. The time coordinate of the event in S' will be the same as the time coordinate of the event in F. The *y* coordinate is also the same for F and S'. The *x* coordinate of the event in S' can be obtained from the distance/ relative position of F relative to S'. To determine the position of F relative to S', we use the following procedure.

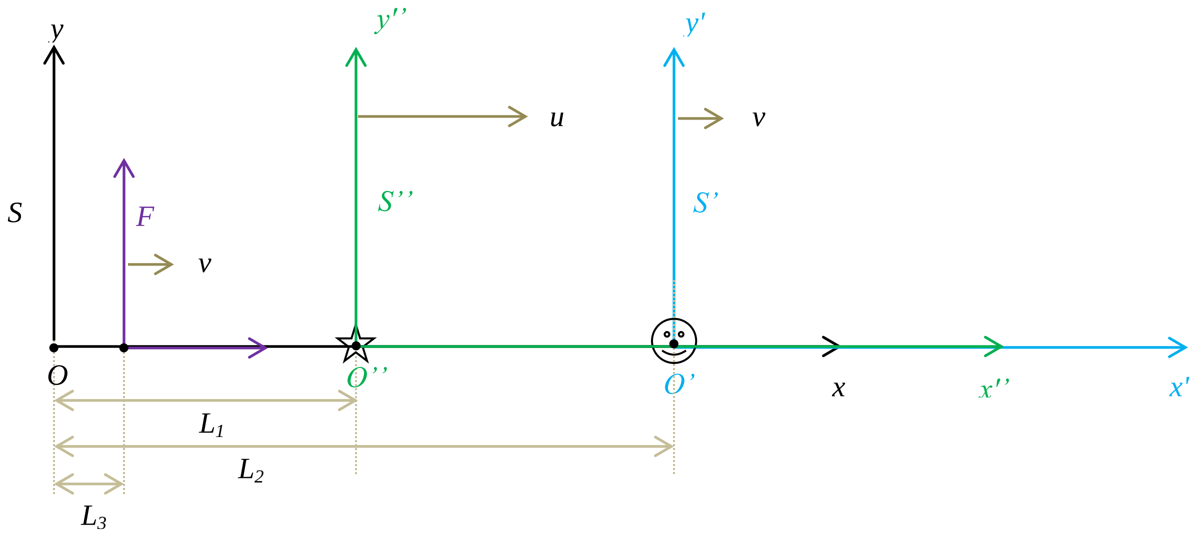
We start from the assumption that the origins of S, S'' and F coincide when *t = t' = t''= 0*. We want to know the position of F relative to S at the instant of the event (Event 1), that is light emission. In frame S, the light emission occurs at:

$$x_1 = L_1 \qquad t_1 = \frac{L_1}{u}$$

During this time interval, the origin of F will have moved a distance of :

$$L_3 = v\ t_1 = v\ \frac{L_1}{u}$$

All the four reference frames are shown below.



So the coordinates of the event (Event 1) in F are:

$$x_{F1}' = \gamma_v (x_1 - v t_1) = \gamma_v \left(L_1 - v \frac{L_1}{u} \right) = \gamma_v L_1 (1 - \frac{v}{u})$$

$$t_{F1}' = \gamma_v \left(t_1 - \frac{v x_1}{c^2} \right) = \gamma_v \left(\frac{L_1}{u} - \frac{v L_1}{c^2} \right) = \gamma_v L_1 (\frac{1}{u} - \frac{v}{c^2})$$

Now, the coordinates of the event (Event 1) in S' will be:

$$t_1' = t_{F1}' = \gamma_v L_1 (\frac{1}{u} - \frac{v}{c^2})$$

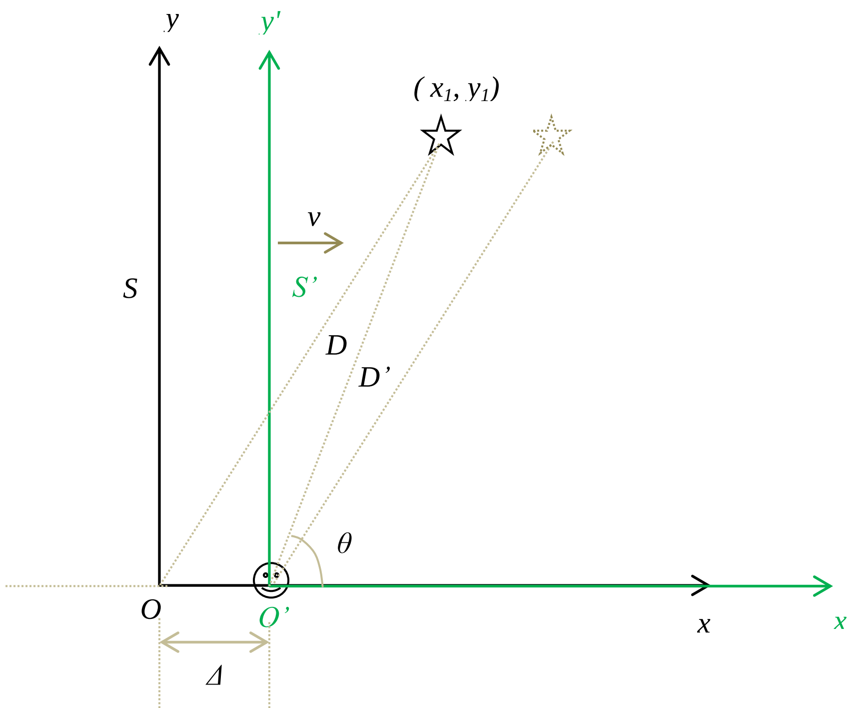
$$x_1' = -(\gamma_v (L_2 - L_3) - x_{F1}') = - (\gamma_v (L_2 - L_3) - \gamma_v L_1 \left(1 - \frac{v}{u} \right))$$

$$\Longrightarrow x_1' = - \gamma_v ((L_2 - L_3) + L_1 \left(1 - \frac{v}{u} \right))$$

Stellar aberration

As an additional exercise, let us apply Lorentz transformation equations to stellar aberration phenomenon.

Consider two inertial reference frames S and S'. At $t = t' = 0$, the origins of S and S', O and O', coincide and the clocks in both frames are synchronized. S' moves with velocity v relative to S in the + x direction. An observer is at the origin of S'.



At $t = t' = 0$ the light source emits a short light pulse. In reference frame S, the observer detects the light pulse at $x = \Delta$, where Δ can be determined as follows.

During the time interval that the observer moves a distance Δ, the light moves a distance of D'.

$$\frac{\Delta}{v} = \frac{D'}{c}$$

where

$$D' = \sqrt{(x_1 - \Delta)^2 + (y_1)^2}$$

Therefore,

$$\frac{\Delta}{v} = \frac{D'}{c} \Longrightarrow \Delta = \frac{v}{c} D'$$

$$\Longrightarrow \Delta = \frac{v}{c} \sqrt{(x_1 - \Delta)^2 + (y_1)^2}$$

$$\Longrightarrow (\frac{c}{v} \Delta)^2 = (x_1 - \Delta)^2 + (y_1)^2$$

$$\Longrightarrow \Delta^2 \left(\frac{c^2}{v^2} - 1 \right) + 2\, x_1 \Delta - ({x_1}^2 + {y_1}^2) = 0$$

$$\Longrightarrow \Delta^2 (c^2 - v^2) + 2v^2 x_1 \Delta - v^2({x_1}^2 + {y_1}^2) = 0$$

where

$$({x_1}^2 + {y_1}^2) = D^2$$

$$\Longrightarrow \Delta^2 (c^2 - v^2) + 2v^2 x_1 \Delta - v^2 D^2 = 0$$

$$\Longrightarrow \Delta = \frac{-v^2 x_1 + \sqrt{{x_1}^2 v^4 + (c^2 - v^2)\, v^2 D^2}}{c^2 - v^2}$$

Now, we have two events:

Event 1 is the emission of light from the source, with coordinates (x_1, y_1 ,t_1) and (x_1', y_1' , t_1') in reference frames S and S' respectively.

Event 2 is the detection of light by the observer, with coordinates (x_2, y_2, t_2), (x_2', y_2', t_2') in reference frames S and S' respectively.

Event 1 in S

Event 1, which is the emission of light, occurs in S frame at:

$$x = x_1 \quad , \; y = y_1$$

$$t = t_1 = 0$$

Event 1 in frame S'

The coordinates of the same event (light emission) in S' is determined from the Lorentz transformation equations :

$$x' = \gamma \; (\; x + \; vt \;)$$

$$t' \;=\; \gamma \;\left(\; t + \;\frac{vx}{c^2}\;\right)$$

By substituting the values of x_1 and t_1 obtained above:

$$x_1{}' \;=\; \gamma\,(\,x_1 \;+\; v\,t_1\;)\;=\;\gamma\,(\,x_1\;+\;v * 0\;) = \gamma\,x_1$$

$$t_1{}' \;=\; \gamma\;\left(\;t_1 + \;\frac{v\,x_1}{c^2}\right) = \gamma\;\left(\;0 + \;\frac{v\,x_1}{c^2}\right) = \gamma\;\frac{v\,x_1}{c^2}$$

$$y_1{}' \;=\; y_1$$

where

$$\gamma \;=\; \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Event 2 in frame S

The coordinates of Event 2, that is the detection of light, in frame S are as follows.

$$x \;=\; x_2 = \Delta \quad , \quad y \;=\; y_2 = 0 \quad , \qquad t = \; t_2 = \frac{\Delta}{v}$$

Event 2 in frame S'

The same event, that is detection of light by the observer, occurs at ($x_2{}'$, $y_2{}'$, $t_2{}'$) in frame S'.

$$x_2{}' = \gamma\;(\,x_2 - v\;t_2\;) = \gamma\;\left(\;\Delta - v\;\frac{\Delta}{v}\;\right) = 0$$

and

$$t_2{}' \;=\; \gamma\left(\;t_2\; - \;\frac{v\,x_2}{c^2}\;\right)\; = \gamma\left(\frac{\Delta}{v}\; - \;\frac{v\,\Delta}{c^2}\;\right)\; = \;\gamma\;\;\Delta\left(\frac{1}{v}\; - \;\frac{v}{c^2}\;\right)$$

Stellar aberration

Now, we compare the direction of light for an observer at rest in reference frame S at $x = \Delta$ and the moving observer who just detects the light at $x = \Delta$ in frame S.

For the stationary observer at $x = \Delta$ the light comes from the direction of:

$$\sin\theta = \frac{y_1}{D'} = \frac{y_1}{\sqrt{(x_1 - \Delta)^2 + (y_1)^2}}$$

The moving observer detects the light at $x = \Delta$ in frame S and for him/her the light comes from the direction of:

$$\sin\theta' = \frac{y_1'}{\sqrt{(x_1')^2 + (y_1')^2}} = \frac{y_1}{\sqrt{(\gamma\, x_1)^2 + y_1^2}}$$

The angle of aberration is the difference between θ and θ' .

The classical formula for stellar aberration is:

$$\tan\theta' = \frac{\sin\theta}{\frac{v}{c} + \cos\theta}$$

I have checked numerically (using Excel) that the relativistic prediction and the classical prediction are almost equal for $v << c$.

For example, for $y_1 = 150 \text{ x}10^6$ km , $x_1 = (v/c)\, y_1 = 15{,}000$ km, $v = 30$ km/s :

both the relativistic and classical formulas above give almost the same value for the angle of aberration (the difference between θ and θ') , the well- known 20.6 arc seconds due to Earth's velocity relative to the Sun.

Symmetry in Lorentz transformations

Let us now play a bit further with the Lorentz transformations by checking the symmetry between the different inertial frames. Suppose we have three inertial reference frames, S, S' and S''. S' and S'' are moving with velocities *u* and *v* , respectively, relative to S.

Suppose that two events occur in frame S, with coordinates (x_1, t_1) and (x_2, t_2). We will determine the coordinates of the events in S' and S'' by using two approaches. The first is the traditional approach to use Lorentz transformations between S and S', and between S and S''. The second, *indirect* approach proposed here is to use Lorentz transformation between S and one of the other two frames, say frame S', and then between S' and S'', to get the coordinates of the event in S''. We will show that the coordinates of an event in S'' obtained using the traditional and the indirect approaches agree.

Event 1 in frame S

The coordinates of Event 1 in S are :

$$x = x_1 \qquad t = t_1$$

Event 1 in frame S’

The coordinates of Event 1 in frame S’ is determined from the Lorentz transformation equations:

$$x_1{}' = \gamma_v \, (x_1 - v \, t_1)$$

$$t_1{}' = \gamma_v \left(t_1 - \frac{v \, x_1}{c^2} \right)$$

Event 1 in frame S’’

The coordinates of Event 1 in frame S’ is determined from the Lorentz transformation equations:

$$x_1{}'' = \gamma_u \, (x_1 - u \, t_1)$$

$$t_1{}'' = \gamma_u \left(t_1 - \frac{u \, x_1}{c^2} \right)$$

The above equations are colored in red for comparison with values obtained using the *indirect* approach later, which will be colored in blue.

Event 2 in frame S

The coordinates of Event 2 in S are :

$$x = x_2 \qquad t = t_2$$

Event 2 in frame S’

The coordinates of Event 2 in frame S’ is determined from the Lorentz transformation equations:

$$x_2{}' = \gamma_v \, (x_2 - v \, t_2)$$

$$t_2{}' = \gamma_v \left(t_2 - \frac{v \, x_2}{c^2} \right)$$

Event 2 in frame S’’

The coordinates of Event 2 in frame S’’ is determined from the Lorentz transformation equations:

$$x_2{}'' = \gamma_u \, (x_2 - u \, t_2)$$

$$t_2{}'' = \gamma_u \left(t_2 - \frac{u \, x_2}{c^2} \right)$$

Now we should be able to get the same coordinates of Event 1 and Event 2 in frame S’’ as above if we use Lorentz transformation *indirectly*, between frame S’ and frame S’’. Reference frame S'' is moving with velocity $w = u - v$ relative to reference frame S'. So we use the relative velocity w in the Lorentz transformations between S' and S''.

However, we will not use $w = u - v$ for the Lorentz transformations between S' and S'' !
That would be the classical, linear velocity addition law. We use the relativistic velocity addition law:

$$w = \frac{u - v}{1 - \frac{uv}{c^2}}$$

The coordinates of Event 1 in frame S', as determined above are:

$$x_1{}' = \gamma_v \left(x_1 - v\, t_1 \right)$$

$$t_1{}' = \gamma_v \left(t_1 - \frac{v\, x_1}{c^2} \right)$$

The coordinates of the same event (Event 1) in frame S'' will be:

$$x_1{}'' = \gamma_w \left(x_1{}' - w\, t_1{}' \right) = \gamma_w \left(\gamma_v \left(x_1 - v\, t_1 \right) - w\, \gamma_v \left(t_1 - \frac{v\, x_1}{c^2} \right) \right)$$

$$\Longrightarrow x_1{}'' = \gamma_w\, \gamma_v \left(\left(x_1 - v\, t_1 \right) - w \left(t_1 - \frac{v\, x_1}{c^2} \right) \right)$$

$$\Longrightarrow x_1{}'' = \gamma_w\, \gamma_v \left(x_1 \left(1 + \frac{v\, w}{c^2} \right) - t_1 \left(v + w \right) \right)$$

where

$$\gamma_w = \frac{1}{\sqrt{1 - \frac{w^2}{c^2}}}$$

and

$$t_1{}'' = \gamma_w \left(t_1{}' - \frac{w\, x_1{}'}{c^2} \right) = \gamma_w \left(\gamma_v \left(t_1 - \frac{v\, x_1}{c^2} \right) - \frac{w\, \gamma_v \left(x_1 - v\, t_1 \right)}{c^2} \right)$$

$$\Longrightarrow t_1{}'' = \gamma_w\, \gamma_v \left(\left(t_1 - \frac{v\, x_1}{c^2} \right) - \frac{w \left(x_1 - v\, t_1 \right)}{c^2} \right)$$

$$\Longrightarrow t_1{}'' = \gamma_w\, \gamma_v \left(t_1 \left(1 + \frac{w\, v}{c^2} \right) - x_1 \frac{(v + w)}{c^2} \right)$$

Comparing the coordinates of Event 1 in S'' obtained by the two approaches, we can see that they are not the same, leading to a contradiction.

$$x_1'' = \gamma_u \ (x_1 \ - \ u\,t_1 \ )$$

and

$$x_1'' \ = \ \gamma_w \, \gamma_v \left(x_1 \left(1 + \frac{v\,w}{c^2} \right) - \ t_1 \ (\ v \ + w) \ \ \right)$$

we should be able to show the equality of the two values of x_1'' by substituting:

$$w \ = \ \frac{u \ - v}{1 - \frac{uv}{c^2}}$$

in the second equation. I have checked this both analytically and numerically using Excel, and have confirmed the equality.

Also, the time coordinates obtained by the two approaches :

$$t_1'' \ = \ \gamma_u \ \left(\ t_1 \ - \ \frac{u\,x_1}{c^2} \right)$$

and

$$\Longrightarrow \ t_1'' \ = \gamma_w \, \gamma_v \left(t_1 \ \left(1 + \ \frac{w\,v}{c^2} \ \right) - \ x_1 \ \frac{(v + w)}{c^2} \right)$$

can be shown to be the same. The same applies to the coordinates of Event 2 , (x_2'', t_2'') .

Alternative theory

In the past, many authors have disclosed the logical inconsistencies of the theory of special relativity and the Lorentz transformations. The very principle of relativity has also been disproved in a number of 'ether' drift experiments, such as the Miller, the Marinov, the Silvertooth and several other experiments.

Although there are so many logical and experimental evidences against special relativity, to this date, there is no known theoretical model of the speed of light that is fully consistent with experiments. The problem is not only the lack of a correct model of the speed of light; mainstream physicists do not believe in the failure of relativity theory and the need for a new model.

This author has proposed a new theory called Apparent Source Theory (AST) in a number of papers [4][5][6]. Apparent Source Theory is consistent with (or, has the potential to consistently explain) the Michelson-Morley, the Kennedy-Thorndike, the Silvertooth, the Marinov, the Bryan G Wallace, the Sagnac and other experiments. No single known theory has achieved this so far. Conventional theories such as ether theories, emission theories and the special relativity theory have decisively failed on more than one experiments.

An extensive explanation of Apparent Source Theory is found in [4][5][6].

Proposed experiment to test special relativity theory

We propose an experiment to test the light postulate of special relativity, according to which the speed of light is constant relative to a moving observer.

There is a light source at rest at some point on the ground. A detector mounted on a car is moving with velocity v away from the light source. There are two synchronized clocks: one at S and one moving together with the detector. Since $v << c$, γ is almost equal to one, so we ignore kinematic time dilation (according to special relativity). This means that once synchronized the clocks will remain synchronized. We also ignore length contraction.

The light source emits a very short light pulse and registers the exact moment (t_0) of emission. The detector on the car continuously records its position on the ground, at every instant of time. This can be done, for example, by a continuous array of sensors along the path of the car.

The detector detects the light pulse and registers the exact time of detection (t_1) . From knowledge of the clock reading at the instant of emission (t_0) , as registered by the light source, we can know the location (x_0) of the detector at that same instant (t_0) because the detector was also continuously recording its position for every instant of time, from which we can know the distance *D at the instant of light emission*.

Thus, at the instant of light emission, the detector was at point x_0 and at the instant of light detection the detector was at point x_1 . x_0 and x_1 are measured relative to the light source. The source emitted the light pulse at time instant t_0 and the detector detected the light pulse at time instant t_1 .

The experiment is repeated with the detector at rest at the point where it was at the instant of light emission, that is at point x_0 and the time interval between emission and detection noted again. The experiment should be repeated immediately so that it is not affected by the continuously changing absolute velocity of the Earth (390 km/s).

Now, if special relativity is correct, the time intervals between emission and detection will be equal in both cases (detector moving and detector stationary). In both cases (not considering the absolute velocity of the Earth, 390 km/s):

$$\Delta t = t_1 - t_0 = \frac{x_0}{c}$$

If absolute motion theory is correct, the time intervals in the two cases (detector moving and detector stationary) will be different. In the case of moving detector:

$$\Delta t = t_1 - t_0 = \frac{x_1}{c}$$

and in the case of stationary detector:

$$\Delta t = t_1 - t_0 = \frac{x_0}{c}$$

However, distance *D* should be large enough so that it will be possible to unambiguously decide between the above cases. For example, if the light source is on the moon, *D* will be about 380,000 km.

Conclusion

There have been long standing confusions and arguments regarding the applications of special relativity theory and the Lorentz transformation equations, such as in the Global Positioning System, the Sagnac effect, stellar aberration and moving source experiments. The problem is that the arguments so far are mostly intuitive, lacking mathematical rigor, and in many cases there is no rigorous, consistent mathematical treatment of these problems strictly according to Lorentz transformation equations, whether to defend or to refute the theory. Hence, even relativists are usually seen to be confused regarding, for example, the time delay of a short light pulse for an observer moving at non-relativistic speed relative to the light source. In this paper, we have presented a strict mathematical treatment, according to Lorentz transformations, of these problems, in order to help clear long standing confusions.

Thanks to Almighty God Jesus Christ and His Mother Our Lady Saint Virgin Mary

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