

Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants (viXra:2102.0162v4)

Gang Chen[†], Tianman Chen, Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China

7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

[†]Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous paper “Formulas of Feigenbaum Constants and Their Physical Meanings” (viXra:2101.0187). In the previous paper, some formulas of Feigenbaum constants in fractional number format were given and the physical meanings of the factors in the formulas were exhibited, especially their relationships with nuclides, the fine-structure constant and 2π . In the previous paper, some integrated formulas of the fine-structure constant, Feigenbaum constants and 2π were also given, briefly denoted as $\alpha_1\delta^2(2\pi)\approx 1$, and their relationships with nuclides were illustrated. In this paper, some formulas for $\alpha_1\delta^2(2\pi)\approx 1$ are supplemented, some formulas for $\alpha_2(\delta\alpha)^2\approx 1$, $[\alpha_1(2\pi)]/(\alpha_2\alpha^2)\approx 1$ and $(2\pi)/\alpha^2\approx 1$ are given, some formulas of the fine-structure constant (α_1 and α_2) based on the key number 103 instead of 112, 173, 137, 83 and 29 are supplemented. In the end, by introducing correction factors γ_1 , γ_2 and γ , accurate formulas $\alpha_1(\delta/\gamma_1)^2(2\pi)=1$, $\alpha_2(\delta\alpha/\gamma_2)^2=1$ and $2\pi/(\alpha\gamma)^2=1$ are gained.

Keywords: Formulas; the fine-structure constant; Feigenbaum constants; 2π .

1. Introduction

In our previous papers^{1,2,3,4,5}, we gave or exhibited the following formulas.

$$(2\pi)_{Chen-k} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}; \quad (2\pi)_{Wallis-k} = 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{2k}{2k+1} \frac{2k+2}{2k+1}\right)$$

$$(2\pi)_{GL-k} = 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + (-1)^{k+1} \frac{1}{2k+1}\right) \quad (GL \text{ means Gregory-Leibniz})$$

$$(2\pi)_{NC-k} = 6 + \sum_{n=1}^k \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \quad (NC \text{ means Nilakantha-Chen})$$

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}$$

$$= 137.035999074626$$

$$1/\alpha_1 = 56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}} = 137.035999037435$$

$$1/\alpha_2 = 56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}} = 137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{28 - \frac{5 \cdot (4 \cdot 3 \cdot 7 \cdot 17 - 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}} = 137.035999074626$$

Note: c_{au} refers to the speed of light in vacuum in atomic units

Feigenbaum Constants: $\delta = 4.66920160910299$

$\alpha = 2.50290787509589$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$

$$= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1) + \frac{2 \cdot 23}{3 \cdot 19}}$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}$$

Note: $136=8 \cdot 17$, $138=6 \cdot 23$

$$\alpha_1 \delta^2 (2\pi) \approx 1$$

On Feb. 8, 2021, we also noticed that Hieb uploaded a paper⁶ in viXra in April of 2017, and gave an approximate formula of the fine-structure constant and Feigenbaum constant as follows, but without any explanations to its physical meanings.

$$\delta' = (1/(2\pi\alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609$$

$$\delta' - \delta = 0.000912$$

α : the fine-structure constant, $\alpha \approx 1/137.036$

2. Integrated Formulas of α_1 , δ and 2π

A Concise Deduction

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\begin{aligned} \alpha_1 &= \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} \approx \frac{36}{7 \cdot (2\pi)} \frac{1}{112} = \left(\frac{3}{14}\right)^2 \frac{1}{2\pi} \approx \frac{1}{\delta^2(2\pi)} \\ &= \frac{1}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)} \approx \frac{1}{136.982} \end{aligned}$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_1 \delta^2(2\pi) \approx 1 \text{ or } \frac{1}{\alpha_1 \delta^2(2\pi)} \approx 1$$

$$\text{Numerically: } \alpha_1 \delta^2(2\pi) = \frac{4 \cdot 6692^2 \times 6.2832}{137.036} = 0.99961 \approx 1$$

2021/2/1-3

The above approximate formula $\alpha_1 \delta^2(2\pi) \approx 1$ is assumed to be the brief form of integrated formulas of α_1 , δ and 2π . There should be some corresponding accurate forms of integrated formulas of α_1 , δ and 2π as follows.

$$\begin{aligned} \alpha_1 \delta^2(2\pi)_{Chen-25-17} &= \frac{4.66920160910299^2 \cdot \left(\frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \right)}{137.035999037435} \\ &= \frac{4.66920160910299^2 \cdot 6.28564399787948}{137.035999037435} \\ &= 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1 \end{aligned}$$

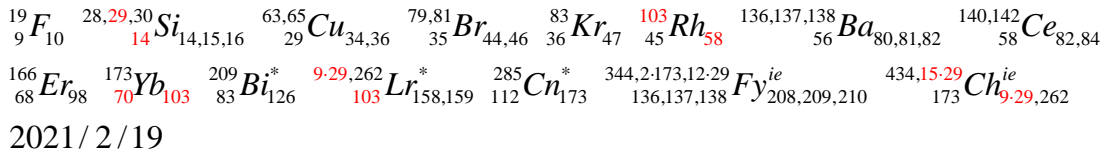
${}_{14,15}^7 N_{7,8}$ ${}_{16,17,18}^8 O_{8,9,10}$ ${}_{23}^{11} Na_{12}$ ${}_{35,37}^{17} Cl_{18,20}$ ${}_{50,51}^{23} V_{27,28}$ ${}_{64,66,67,68}^{30} Zn_{34,36,37,38}$ ${}_{80}^{34} Se_{46}$ ${}_{79,81}^{35} Br_{44,46}$
 ${}_{82,83,84,86}^{36} Kr_{46,47,48,50}$ ${}_{85,87}^{37} Rb_{48,50}$ ${}_{105,110}^{46} Pd_{59,64}$ ${}_{111,112}^{48} Cd_{63,64}$ ${}_{112,114,115-120,122,124}^{50} Sn_{62,64,65-70,72,74}$
 ${}_{121,123}^{51} Sb_{70,72}$ ${}_{126}^{52} Te_{74}$ ${}_{128}^{54} Xe_{74}$ ${}_{136,137,138}^{56} Ba_{80,81,82}$ ${}_{157}^{64} Gd_{93}$ ${}_{168,170}^{68} Er_{100,102}$ ${}_{171,173}^{70} Yb_{101,103}$
 ${}_{169}^{69} Tm_{100}$ ${}_{175,176}^{71} Lu_{104,105}$ ${}_{185,187}^{75} Re_{110,112}$ ${}_{208}^{82} Pb_{126}$ ${}_{209}^{83} Bi_{126}^*$ ${}_{209}^{84} Po_{125}^*$ ${}_{210}^{85} At_{125}^*$ ${}_{261,262}^{103} Lr_{158,159}^*$
 ${}_{257}^{100} Fm_{157}^*$ ${}_{285}^{112} Cn_{173}^*$ ${}_{4-71,286}^{113} Nh_{171,173}^{ie}$ ${}_{326}^{128} Ch_{18-11}^{ie}$ ${}_{344,2173,348}^{136,137,138} Fy_{208,209,210}^{ie}$ ${}_{6-71}^{169} Ch_{257}^{ie}$ ${}_{434,435}^{173} Ch_{261,262}^{ie}$

2021/1/31

$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Chen-25-17}} = \frac{137.035999037435}{4.66920160910299^2 \cdot \left(e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28564399787948}$$

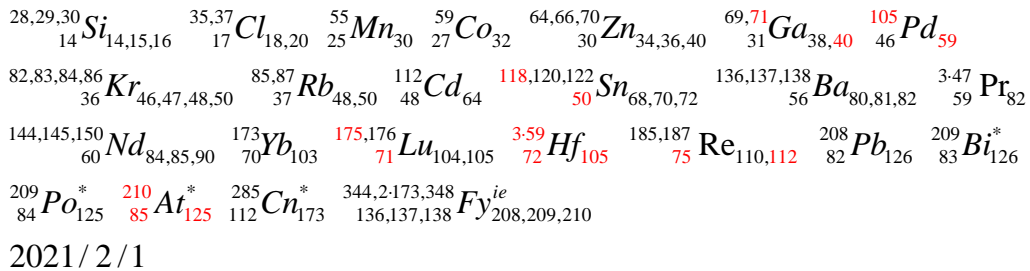
$$= 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$



$$\alpha_1 \delta^2 (2\pi)_{Wallis-9-71} = \frac{4.66920160910299^2 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1} \right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28564015562186}{137.035999037435}$$

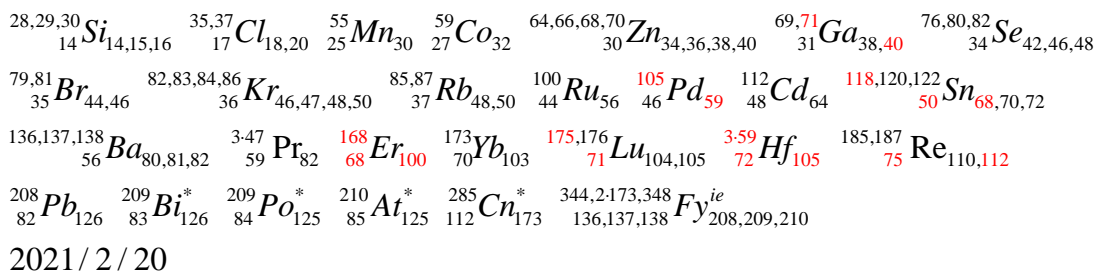
$$= 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606 \approx 1$$



$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Wallis-9-71}} = \frac{137.035999037435}{4.66920160910299^2 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1} \right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28564015562186}$$

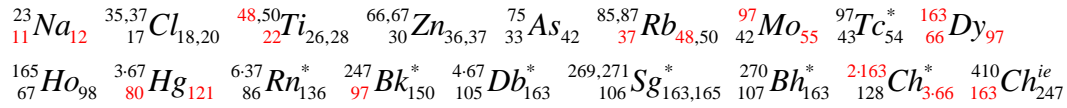
$$= 1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3 \cdot 4}{17}} = 0.999999775803991 \approx 1$$



$$\alpha_1 \delta^2 (2\pi)_{GL-22,37} = \frac{4.66920160910299^2 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1})}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28563929398602}{137.035999037435}$$

$$= 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598 \approx 1$$

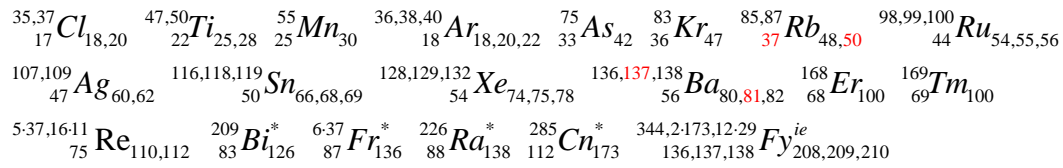


2021 / 2 / 1

$$\alpha_1 \delta^2 (2\pi)_{GL-22,37} = \frac{1}{4.66920160910299^2 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1})}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28563929398602}$$

$$= 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.999999912884025 \approx 1$$

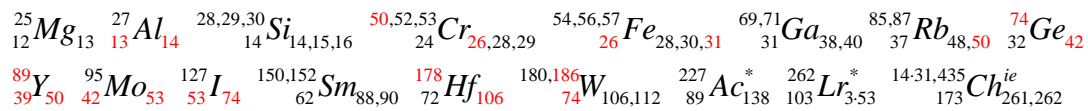


2021 / 2 / 19

$$\alpha_1 \delta^2 (2\pi)_{NC-3} = \frac{4.66920160910299^2 \cdot (6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435}$$

$$= 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352 \approx 1$$

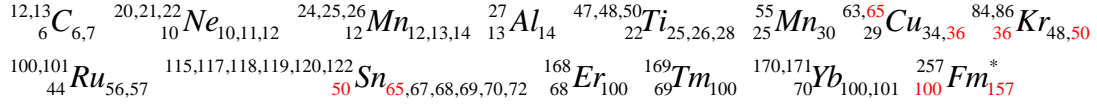


2021 / 2 / 1,3

$$\alpha_1 \delta^2 (2\pi)_{NC-3} = \frac{1}{4.66920160910299^2 \cdot (6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.29047619047619}$$

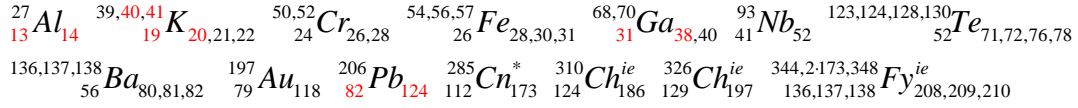
$$= 1 - \frac{1}{4 \cdot 25 \cdot 13} + \frac{1}{4 \cdot 9 \cdot 25 \cdot (2 \cdot 25 \cdot (4 \cdot 25 + 1) + 1) + \frac{2}{7}} = 0.999230989209198 \approx 1$$



2021/2/20

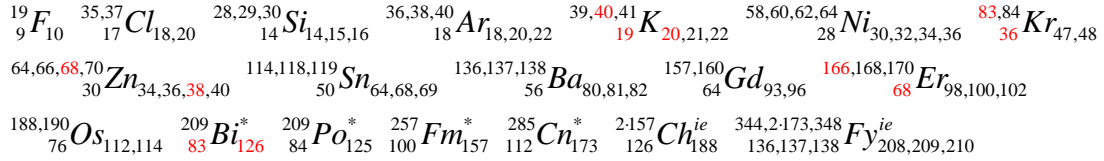
$$\alpha_1 \delta^2 (2\pi) = \frac{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}{137.035999037435}$$

$$= 1 - \frac{1}{13 \cdot 197} + \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 \cdot 31 - 1)} = 0.99960967543223 \approx 1$$



$$\frac{1}{\alpha_1 \delta^2 (2\pi)} = \frac{137.035999037435}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}$$

$$= 1 + \frac{1}{512 \cdot 5} - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83} = 1.00039047698053 \approx 1$$



2021/2/8

3. Integrated Formulas of α_2 , δ and α

The Fine-structure Constant:

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha = 2.50290787509589$$

$$\alpha_2 = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} \approx \frac{13 \cdot (2\pi)}{100} \frac{1}{112} \approx \frac{(2\pi)}{(\delta\alpha)^2 (2\pi)}$$

$$\approx \frac{1}{(\delta\alpha)^2} = \frac{1}{(4.66920160910299 \cdot 2.50290787509589)^2} \approx 1/136.575$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_2 (\delta\alpha)^2 \approx 1 \text{ or } \frac{1}{\alpha_2 (\delta\alpha)^2} \approx 1$$

$$\text{Numerically: } \alpha_2 (\delta\alpha)^2 = \frac{(4 \cdot 6692 \times 2.5029)^2}{137.036} = 0.99664 \approx 1$$

2021/2/7

The above approximate formula $\alpha_2(\delta\alpha)^2 \approx 1$ is assumed to be the brief form of integrated formulas of α_2 , δ and α . There should be some corresponding accurate forms of integrated formulas of α_2 , δ and α as follows.

$$\alpha_2(\delta\alpha)^2 = \frac{(4.66920160910299 \cdot 2.50290787509589)^2}{137.035999111818}$$

$$= 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908 \approx 1$$

^{12,13}₆C_{6,7} ^{14,15}₇N_{7,8} ¹⁹₉F₁₀ ^{24,25,26}₁₂Mg_{12,13,14} ²⁷₁₃Al₁₄ ^{28,29,30}₁₄Si_{14,15,16} ³¹₁₅P₁₆ ^{39,40,41}₁₉K_{20,21,22}
^{46,47,48,49,50}₂₂Ti_{24,25,26,27,28} ^{50,52,53}₂₄Cr_{26,28,29} ^{54,56,57,58}₂₆Fe_{28,30,31,32} ^{58,60,62,64}₂₈Ni_{30,32,34,36}
^{63,65}₂₉Cu_{34,36} ^{69,71}₃₁Ga_{38,40} ^{70,74,76}₃₂Ge_{38,42,44} ^{76,78}₃₄Se_{42,44} ^{85,87}₃₇Rb_{48,50} ⁸⁹₃₉Y₅₀ ⁹³₄₁Nb₅₂
^{94,95,96,98,100}₄₂Mo_{52,53,54,56,58} ^{112,113}₄₈Cd_{64,65} ^{113,115}₄₉In_{64,66} ⁷⁻¹⁹₅₄Xe_{76,78} ⁷⁻¹⁹₅₅Cs₇₈ ¹⁴⁹₆₂Sm₈₇
^{134,136,137,138}₅₆Ba_{78,80,81,82} ^{155,156,157,160}₆₄Gd_{91,92,93,96} ^{186,187,189,190,192}₇₆Os_{110,112,113,114,116}
^{190,192,194,195,196}₇₈Pt_{112,114,116,117,118} ²²³₈₇Fr₁₃₆ ²²⁷₈₉Ac₁₃₈ ²¹⁻¹¹₉₁Pa₁₄₀ ²³⁷₉₃Np₁₄₄ ¹⁵⁻¹⁹₁₁₂Cn₁₇₃
²²⁻¹³₁₁₃Nh₁₇₃^{ie} ^{344,2,173,12-29}_{136,137,138}Fy_{208,209,210}^{ie} ⁸⁻⁴⁷₁₄₉Ch₂₂₇^{ie} ^{14-31,15-29}₁₇₃Ch_{9-19,262}^{ie}

2021/2/7

$$\frac{1}{\alpha_2(\delta\alpha)^2} = \frac{137.035999111818}{(4.66920160910299 \cdot 2.50290787509589)^2}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (2 \cdot 9 \cdot 5 \cdot 23 - 1) - \frac{2}{5}} = 1.00336671044256 \approx 1$$

^{10,11}₅Be_{5,6} ²³₁₁Na₁₂ ^{24,25}₁₂Mg_{12,13} ^{46,47,49,50}₂₂Ti_{24,25,27,28} ^{50,51}₂₃V_{27,28} ⁵⁵₂₅Mn₃₀ ⁷⁵₃₃As₄₂
^{80,82,83,86}₃₆Kr_{44,46,47,50} ^{98,99,100,104}₄₄Ru_{54,55,56,60} ^{107,109}₄₇Ag_{60,62} ^{115,116,119,120}₅₀Sn_{65,66,69,70}
^{129,131,132}₅₄Xe_{75,77,78} ^{136,137,138}₅₆Ba_{80,81,82} ¹⁶⁹₆₉Tm₁₀₀ ^{185,187}₇₅Re_{110,112} ²²⁶₈₈Ra₁₃₈^{*}
^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie}

2021/2/9

4. Integrated Formulas of α_1 , α_2 , α and 2π

$$\alpha_1 \delta^2(2\pi) = 0.99961 \approx 1$$

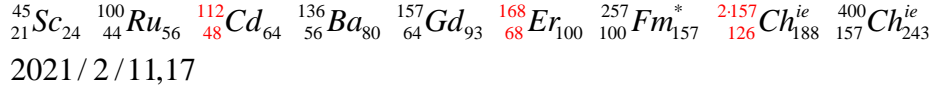
$$\alpha_2(\delta\alpha)^2 = 0.99664 \approx 1$$

$$\frac{\alpha_1 \delta^2(2\pi)}{\alpha_2(\delta\alpha)^2} = \frac{\alpha_1(2\pi)}{\alpha_2 \alpha^2} \approx \frac{2\pi}{\alpha^2} = 1.002975 \approx 1$$

2021/2/11

$$\frac{\alpha_1(2\pi)}{\alpha_2\alpha^2} = \frac{137.035999111818 \cdot (2 \cdot 3 \cdot 14159265358979)}{137.035999037435 \cdot 2.50290787509589^2}$$

$$= 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 17 \cdot (2 \cdot 157 - 1) - 1) - \frac{1}{4}} = 1.00297507176499 \approx 1$$

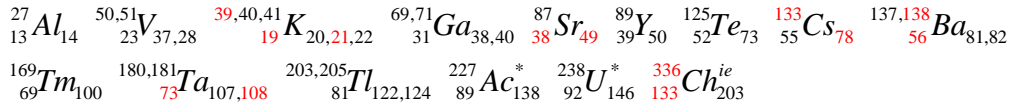


$$\frac{\alpha_2\alpha^2}{\alpha_1(2\pi)} = \frac{137.035999037435 \cdot 2.50290787509589^2}{137.035999111818 \cdot (2 \cdot 3 \cdot 14159265358979)}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3 \cdot 7^2 \cdot 23 - 1) + \frac{16}{3 \cdot 13}}$$

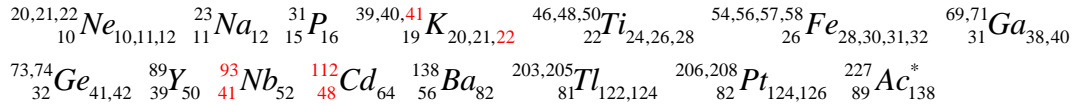
$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (8 \cdot 5 \cdot 13^2 + 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 27 \cdot 19 + 1) - \frac{23}{3 \cdot 13}} = 0.997033753032614 \approx 1$$



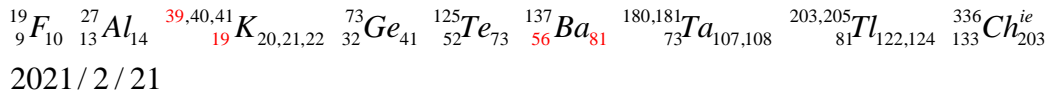
$$\frac{2\pi}{\alpha^2} = \frac{2 \cdot 3 \cdot 14159265358979}{2.50290787509589^2} = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22}}$$

$$= 1.00297507122057 \approx 1$$



$$\frac{\alpha^2}{2\pi} = \frac{2.50290787509589^2}{2 \cdot 3 \cdot 14159265358979} = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13}}$$

$$= 0.997033753573803 \approx 1$$



5. Marvelous Coincidences

There are some marvelous coincidences of factors with nuclides in the above formulas. One typical example of these coincidences is listed as follows, which indicates the methodology and the formulas in this paper should be correct.

$$\alpha_1 \delta^2 (2\pi)_{Chen-25:17} = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1$$

$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Chen-25:17}} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$

$$(2\pi)_{Chen-25:17} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}}$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{80}_{34}Se_{46}$ $^{79,81}_{35}Br_{44,46}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$
 $^{85,87}_{37}Rb_{48,50}$ $^{103}_{45}Rh_{58}$ $^{112}_{48}Cd_{64}$ $^{128}_{54}Xe_{74}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{173}_{70}Yb_{103}$ $^{209}_{83}Bi_{126}^*$
 $^{209}_{84}Po_{125}^*$ $^{210}_{85}At_{125}^*$ $^{9-29,262}_{103}Lr_{158,159}^*$ $^{285}_{112}Cn_{173}^*$ $^{344,2-173,12-29}_{136,137,138}Fy_{208,209,210}^{ie}$ $^{434,15-29}_{173}Ch_{9-29,262}^{ie}$

6. Formulas of the Fine-structure Constant based on 103

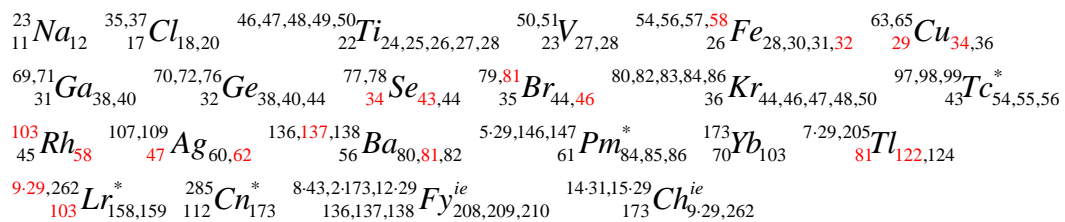
In our previous paper^{1,2,4}, many formulas of the fine-structure constant based on the key numbers 112, 173, 137, 83 and 29 were given. As shown in the above two formulas in **Section 5**, it seems 103 is another key number comparable to the above stated key numbers, so some formulas of the fine-structure constant based on the key number 103 instead of them are constructed as follows.

$$\alpha_1 = \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{32 \cdot (32 \cdot 29 + 1) - \frac{3}{2 \cdot 17}}}$$

$$= \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{81 \cdot (2 \cdot 3 \cdot 61 + 1) + \frac{31}{2 \cdot 17}}}$$

$$= \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{81 \cdot (16 \cdot 23 - 1) + \frac{31}{2 \cdot 17}}}$$

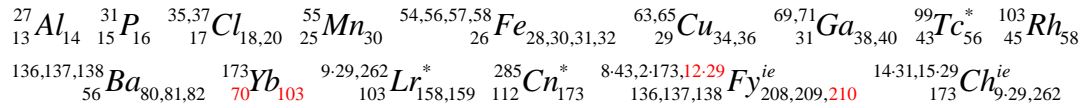
$$= 1/137.035999037435$$



2021/2/25

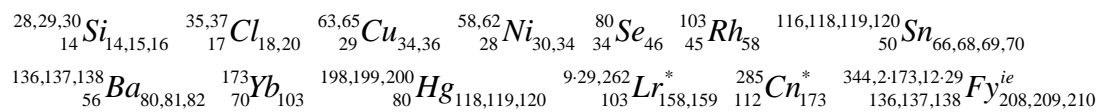
$$\alpha_1 = \frac{137}{29 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1548}{1549} \frac{2 \cdot 25 \cdot 31}{2 \cdot 2 \cdot 9 \cdot 43 + 1}\right)} \frac{1}{103 + \frac{1}{2 \cdot 3 \cdot 5 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 - 1) - \frac{3}{17}}}$$

$$= 1/137.035999037435$$



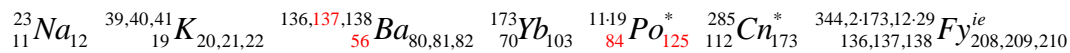
$$\alpha_1 = \frac{137}{29 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 17 \cdot 29 + 1}\right)} \frac{1}{103 + \frac{1}{7 \cdot (4 \cdot 7 \cdot 199 + 1) + \frac{4}{7}}}$$

$$= 1/137.035999037435$$



$$\alpha_1 = \frac{137}{29 \cdot \left(6 + \sum_{n=1}^7 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)} \frac{1}{103 + \frac{1}{3 \cdot 19} - \frac{1}{125 \cdot (8 \cdot 7 \cdot 11 + 1) + \frac{1}{4}}}$$

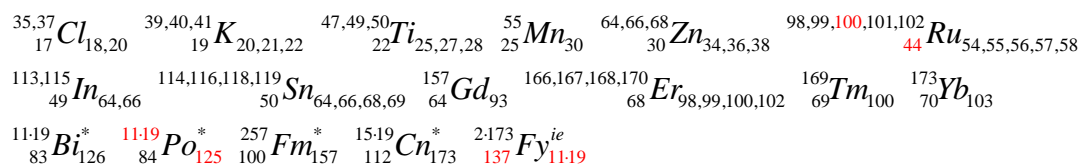
$$= 1/137.035999037435$$



2021/2/26

$$\alpha_2 = \frac{25 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2500}{3 \cdot 49 \cdot 17}\right)^{4999}}}{11 \cdot 19} \frac{1}{103 - \frac{1}{32 \cdot (512 \cdot 25 - 1) + \frac{3}{10}}}$$

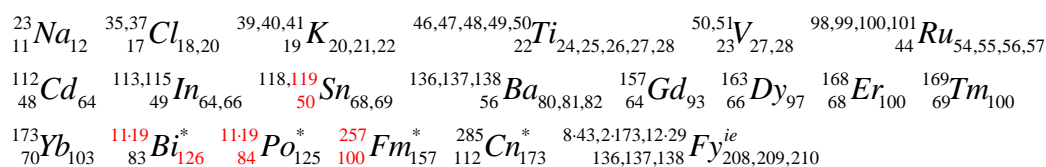
$$= 1/137.035999111818$$



2021/2/25

$$\alpha_2 = \frac{25 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{7496}{7497} \frac{2 \cdot 23 \cdot 163}{9 \cdot 49 \cdot 17}\right)}{11 \cdot 19} \frac{1}{103 - \frac{1}{7 \cdot (16 \cdot 3 \cdot 19 \cdot 257 - 1)}}$$

$$= 1/137.035999111818$$



$$\alpha_2 = \frac{25 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{5 \cdot 23 \cdot 83})}{11 \cdot 19} \frac{1}{103 - \frac{1}{2 \cdot 3 \cdot 5 \cdot 13 \cdot 23 \cdot (2 \cdot 11 \cdot 17 - 1)}}$$

$$= 1/137.035999111818$$

$$^{23}_{11}\text{Na}_{12} \quad ^{27}_{13}\text{Al}_{14} \quad ^{35,37}_{17}\text{Cl}_{18,20} \quad ^{39,40,41}_{19}\text{K}_{20,21,22} \quad ^{50,51}_{23}\text{V}_{27,28} \quad ^{55}_{25}\text{Mn}_{30} \quad ^{168}_{68}\text{Er}_{100} \quad ^{169}_{69}\text{Tm}_{100} \quad ^{119}_{83}\text{Bi}_{126}^*$$

$$\alpha_2 = \frac{25 \cdot (6 + \sum_{n=1}^{11} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{11 \cdot 19} \frac{1}{103 - \frac{1}{4 \cdot 5 \cdot 23} + \frac{1}{2 \cdot (2 \cdot 3 \cdot 5 \cdot 7 + 1) \cdot (2 \cdot 9 \cdot 17 + 1) + \frac{2}{3}}}$$

$$= 1/137.035999111818$$

$$^{23}_{11}\text{Na}_{12} \quad ^{35,37}_{17}\text{Cl}_{18,20} \quad ^{39,40,41}_{19}\text{K}_{20,21,22} \quad ^{50,51}_{23}\text{V}_{27,28} \quad ^{55}_{25}\text{Mn}_{30} \quad ^{103}_{45}\text{Rh}_{58} \quad ^{136,137,138}_{56}\text{Ba}_{80,81,82} \quad ^{173}_{70}\text{Yb}_{103}$$

$$^{119}_{83}\text{Bi}_{126}^* \quad ^{119}_{84}\text{Po}_{125}^* \quad ^{210}_{85}\text{At}_{125}^* \quad ^{238}_{92}\text{U}_{143}^* \quad ^{15-19}_{112}\text{Cn}_{173}^* \quad ^{344,2-173,348}_{136,137,138}\text{Fy}_{208,209,210}^{ie}$$

2021/2/26

7. Integrated Formulas of α_1 , δ , 2π and γ_1

By introducing a correction factor γ_1 , some integrated formulas of α_1 , δ , 2π and γ_1 in the format of $\alpha_1(\delta/\gamma_1)^2(2\pi)=1$ could be obtained as follows.

$$\alpha_1(\delta/\gamma_1)^2(2\pi) = 1$$

$$\gamma_1 = \sqrt{2\pi\alpha_1}\delta = \sqrt{\frac{6.28564399787948}{137.035999037435}} 4.66920160910299$$

$$= 1 - \frac{1}{47 \cdot 109} + \frac{1}{27 \cdot 7 \cdot (3 \cdot 8 \cdot (3 \cdot 8 \cdot (4 \cdot 137 - 1) - 1) - 1)} = 0.999804818668238$$

$$^{45}_{21}\text{V}_{24} \quad ^{59}_{27}\text{Co}_{32} \quad ^{83,84}_{36}\text{Kr}_{47,48} \quad ^{107,109}_{47}\text{Ag}_{60,62} \quad ^{3-47}_{59}\text{Pr}_{82} \quad ^{158}_{64}\text{Gd}_{94} \quad ^{183}_{74}\text{W}_{109} \quad ^{4-47}_{76}\text{Os}_{112} \quad ^{137}_{56}\text{Ba}_{81} \quad ^{209}_{83}\text{Bi}_{126}^*$$

$$^{209}_{84}\text{Po}_{125}^* \quad ^{278}_{109}\text{Mt}_{169}^* \quad ^{285}_{112}\text{Cn}_{173}^* \quad ^{2-173}_{137}\text{Fy}_{209}^{ie}$$

$$\frac{1}{\gamma_1} = \frac{1}{\sqrt{2\pi\alpha_1}\delta} = \sqrt{\frac{137.035999037435}{2 \cdot 3.14159265358979}} \frac{1}{4.66920160910299}$$

$$= 1 + \frac{1}{2 \cdot 13 \cdot 197} - \frac{16 \cdot 7 \cdot 17 \cdot (16 \cdot 3 \cdot 23 - 1)}{125 \cdot 10^{12}} = 1.00019521943495 \approx 1$$

$$^{24,25,26}_{12}\text{Mg}_{12,13,14} \quad ^{27}_{13}\text{Al}_{14} \quad ^{28,29,30}_{14}\text{Si}_{14,15,16} \quad ^{35,37}_{17}\text{Cl}_{18,20} \quad ^{46,48,50}_{22}\text{Ti}_{24,26,28} \quad ^{50,51}_{23}\text{V}_{27,28} \quad ^{54,56,58}_{26}\text{Fe}_{28,30,32}$$

$$^{58,60,62,64}_{28}\text{Ni}_{30,32,34,36} \quad ^{74,76,78,80,82}_{34}\text{Se}_{40,42,44,46,48} \quad ^{79,81}_{35}\text{Br}_{44,46} \quad ^{90,91,92,96}_{40}\text{Zr}_{50,51,52,56} \quad ^{112,116}_{48}\text{Cd}_{64,68}$$

$$^{102,104,105,106,110}_{46}\text{Pd}_{56,58,59,60,64} \quad ^{114,115,118,119,120}_{50}\text{Sn}_{64,65,68,69,70} \quad ^{135,136,137,138}_{56}\text{Ba}_{79,80,81,82} \quad ^{156,160}_{64}\text{Gd}_{92,96}$$

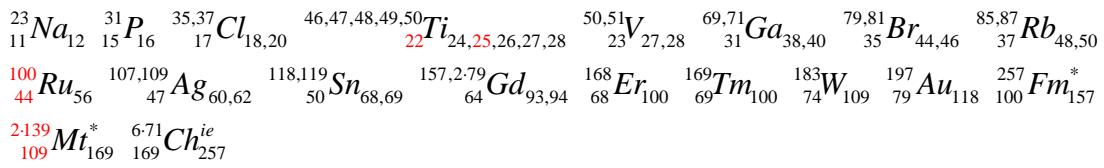
$$^{168}_{68}\text{Er}_{100} \quad ^{169}_{69}\text{Tm}_{100} \quad ^{173}_{70}\text{Yb}_{103} \quad ^{185,187}_{75}\text{Re}_{110,112} \quad ^{197}_{79}\text{Au}_{118} \quad ^{209}_{84}\text{Po}_{125}^* \quad ^{210}_{85}\text{At}_{125}^* \quad ^{222}_{86}\text{Rn}_{136}^* \quad ^{223}_{87}\text{Fr}_{136}^*$$

$$^{226}_{88}\text{Ra}_{138}^* \quad ^{227}_{89}\text{Ac}_{138}^* \quad ^{238}_{92}\text{U}_{146}^* \quad ^{285}_{112}\text{Cn}_{173}^* \quad ^{24-13}_{125}\text{Ch}_{187}^{ie} \quad ^{326}_{129}\text{Ch}_{197}^{ie} \quad ^{344,2-173,348}_{136,137,138}\text{Fy}_{208,209,210}^{ie} \quad ^{2-197}_{156}\text{Ch}_{238}^{ie}$$

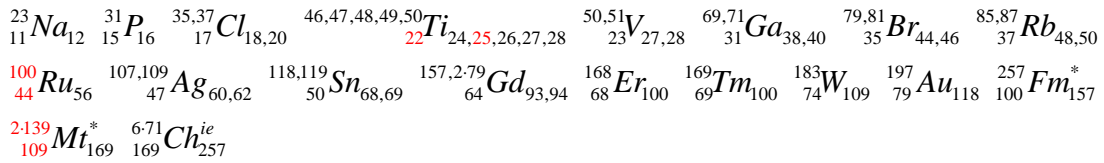
2021/2/28

$$\alpha_1(\delta / \gamma_{1-\text{Chen}-25.17})^2 (2\pi)_{\text{Chen}-25.17} = 1$$

$$\begin{aligned} \gamma_{1-\text{Chen}-25.17} &= \sqrt{(2\pi)_{\text{Chen}-25.17} \alpha_1 \delta} \\ &= \frac{\sqrt{e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= \frac{\sqrt{6.28564399787948 \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{2}{17} \text{ or } \frac{3}{25}} = 1.00000041773574 \end{aligned}$$



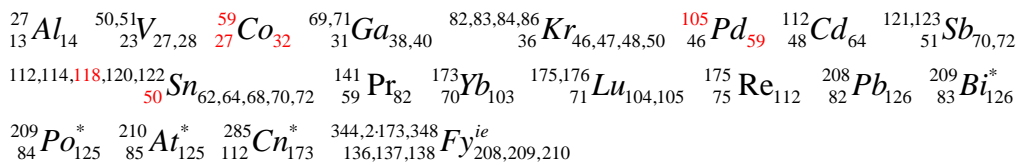
$$\begin{aligned} \gamma_{1-\text{Chen}-25.17} &= \frac{1}{\sqrt{(2\pi)_{\text{Chen}-25.17} \alpha_1 \delta}} \\ &= 1 - \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 + \frac{15}{17} \text{ or } \frac{22}{25}} = 0.999999582264432 \end{aligned}$$



2021/3/1

$$\alpha_1(\delta^2 / \gamma_{1-\text{Wallis}-9.71})(2\pi)_{\text{Wallis}-9.71}$$

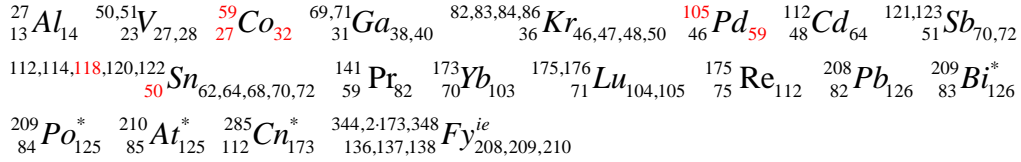
$$\begin{aligned} \gamma_{1-\text{Wallis}-9.71} &= \sqrt{(2\pi)_{\text{Wallis}-9.71} \alpha_1 \delta} \\ &= \frac{\sqrt{4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1}\right) \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= \frac{\sqrt{6.28564015562186 \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802 \end{aligned}$$



2021/3/2

$$\frac{1}{\gamma_{1-\text{Wallis}-9.71}} = \frac{1}{\sqrt{(2\pi)_{\text{Wallis}-9.71} \alpha_1 \delta}}$$

$$= 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 \cdot 59 - 1)} = 0.999999887901990$$



2021/3/2

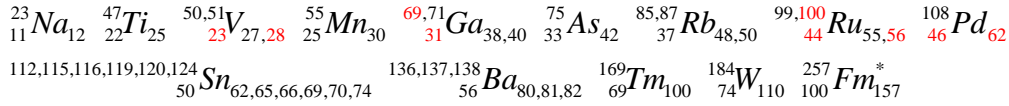
$$\alpha_1 (\delta / \gamma_{1-\text{GL}-22.37})^2 (2\pi)_{\text{GL}-22.37} = 1$$

$$\gamma_{1-\text{GL}-22.37} = \sqrt{(2\pi)_{\text{GL}-22.37} \alpha_1 \delta}$$

$$= \frac{\sqrt{8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1})} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

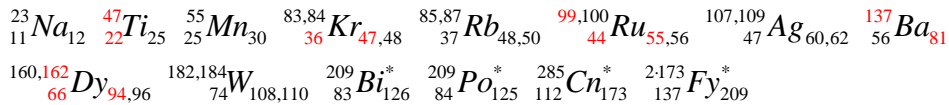
$$= \frac{\sqrt{6.28563929398602} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

$$= 1 + \frac{1}{4 \cdot 25 \cdot 7 \cdot (2 \cdot 23^2 \cdot 31 - 1) + \frac{6}{11}} = 1.00000004355799$$



$$\frac{1}{\gamma_{1-\text{GL}-22.37}} = \frac{1}{\sqrt{(2\pi)_{\text{GL}-22.37} \alpha_1 \delta}}$$

$$= 1 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{5}{11}} = 0.999999956442012$$



2021/3/2

$$\alpha_1 (\delta / \gamma_{1-\text{NC}-3})^2 (2\pi)_{\text{NC}-3} = 1$$

$$\gamma_{1-\text{NC}-3} = \sqrt{(2\pi)_{\text{NC}-3} \alpha_1 \delta}$$

$$= \frac{\sqrt{(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

$$= \frac{\sqrt{6.29047619047619} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

$$= 1 + \frac{1}{23 \cdot 113} - \frac{1}{2 \cdot 3 \cdot 257 \cdot (4 \cdot 5 \cdot 29 \cdot 31 + 1)} = 1.00038472730421$$

$^{50,51}_{23}V$ $^{63,65}_{29}Cu$ $^{69,71}_{31}Ga$ $^{104,108}_{46}Pd$ $^{113,115}_{49}In$ $^{157}_{64}Gd$ $^{169}_{69}Tm$ $^{189}_{76}Os$
 $^{226}_{88}Ra^*$ $^{257}_{100}Fm^*$ $^{284,286}_{113}Nh^{ie}$ $^{426}_{169}Ch^{ie}$

2021/3/3

$$\frac{1}{\gamma_{1-NC-3}} = \frac{1}{\sqrt{(2\pi)_{NC-3} \alpha_1 \delta}}$$

$$= 1 - \frac{1}{8 \cdot 25 \cdot 13} + \frac{1}{179 \cdot (8 \cdot 9 \cdot (2 \cdot 3 \cdot (2 \cdot 179 + 1) - 1) + 1) - \frac{3}{7}} = 0.999615420653963$$

$^{56}_{26}Fe$ $^{100}_{44}Ru$ $^{115,119}_{50}Sn$ $^{124}_{52}Te$ $^{169}_{69}Tm$ $^{179}_{72}Hf$ $^{257}_{100}Fm^*$ $^{298}_{119}Ch^{ie}$

2021/3/2

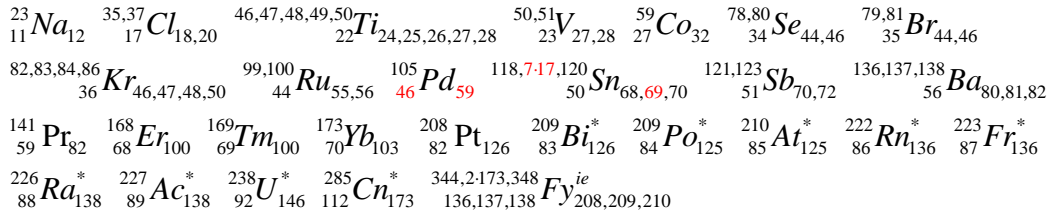
8. Integrated Formulas of α_2 , α , δ and γ_2

By introducing a correction factor γ_2 , some integrated formulas of α_2 , α , δ and γ_2 in the format of $\alpha_2(\delta\alpha\gamma_2)^2=1$ could be obtained as follows.

$$\alpha_2(\delta\alpha/\gamma_2)^2 = 1$$

$$\gamma_2 = \sqrt{\alpha_2(\alpha\delta)^2} = \frac{4.66920160910299 \cdot 2.50290787509589}{\sqrt{137.035999111818}}$$

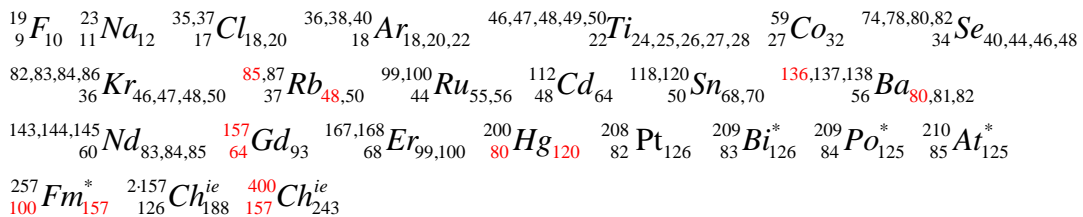
$$= 1 - \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699$$



2021/2/28

$$\frac{1}{\gamma_2} = \frac{1}{\sqrt{\alpha_2 \delta \alpha}} = \frac{\sqrt{137.035999111818}}{4.66920160910299 \cdot 2.50290787509589}$$

$$= 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892$$



2021/2/27

9. Integrated Formulas of α_1 , α_2 , α , 2π , γ_1 and γ_2

$$\frac{\alpha_1(2\pi)}{\alpha_2(\alpha\gamma_1/\gamma_2)^2} = 1$$

$$\frac{\gamma_1}{\gamma_2} = \frac{\sqrt{137.035999111818 \cdot (2 \cdot 3.14159265358979)}}{\sqrt{137.035999037435 \cdot 2.50290787509589}}$$

$$= 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1) - \frac{25}{2 \cdot 23}} = 1.00148643114372$$

^{50,51}V_{27,28} ⁵³Cr₂₉ ⁸⁹Y₅₀ ⁹⁵Mo₅₃ ²⁻⁵³Pd₆₀ ¹¹²Cd₆₄ ¹¹⁹Sn₆₉ ¹²⁷I₇₄ ^{136,137,138}Ba_{80,81,82} ^{151,153}Eu_{88,90}
¹⁶⁹Tm₁₀₀ ²⁻⁸⁹Hf₇₂ ²⁻⁵³Re₇₅ ^{185,187}Re_{110,112} ²²⁷Ac₁₃₈ ²⁸⁵Cn₁₇₃ ⁶⁻⁵³⁻³²⁰Ch₁₉₁₋₁₉₃^{ie} ^{344,2-173,348}Fy_{208,209,210}^{ie}

$$\frac{\gamma_2}{\gamma_1} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 \cdot 29 + 1) + \frac{2 \cdot 3}{23}} = 0.99851577505446$$

^{28,29,30}Si_{14,15,16} ^{50,51}V_{27,28} ⁵³Cr₂₉ ^{63,65}Cu_{34,36} ⁹⁵Mo₅₃ ²⁻⁵³Pd₆₀ ¹¹²Cd₆₄ ¹²⁷I₇₄ ^{136,137,138}Ba_{80,81,82}
²⁸⁵Cn₁₇₃ ³³⁶Ch₁₃₃^{ie} ^{6-53,11-29,320}Ch_{191,192,193}^{ie} ^{344,2-173,12-29}Fy_{208,209,210}^{ie}

2021/3/3

10. Integrated Formulas α , 2π and γ

By introducing a correction factor γ , some integrated formulas of 2π , α and γ in the format of $2\pi/(\alpha\gamma)^2=1$ could be obtained as follows.

$$\frac{2\pi}{(\alpha\gamma)^2} = 1$$

$$\gamma = \frac{\sqrt{(2 \cdot 3.14159265358979)}}{2.50290787509589^2}$$

$$= 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4 \cdot 7}} = 1.00148643087192$$

²³Na₁₂ ^{50,51}V_{27,28} ^{82,83,84}Kr_{46,47,48} ^{136,137,138}Ba_{80,81,82} ^{151,153}Eu_{88,90} ²⁰⁸Pd₁₂₆ ²⁰⁹Bi₁₂₆^{*} ²⁰⁹Po₁₂₅^{*}
²⁴⁷Cm₁₅₁^{*} ²⁸⁵Cn₁₇₃^{*} ^{344,2-173,348}Fy_{208,209,210}^{ie}

$$\frac{1}{\gamma} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1) - \frac{1}{6}} = 0.99851577532546$$

⁵⁵Mn₃₀ ⁷³Ge₄₁ ^{82,83,84,86}Kr_{46,47,48,50} ⁹³Nb₅₂ ^{140,142}Ce_{82,84} ²⁰⁸Pd₁₂₆ ²⁰⁹Bi₁₂₆^{*} ²⁰⁹Po₁₂₅^{*} ²¹⁰At₁₂₅^{*}

2021/3/3

11. Summary

The above integrated formulas of the fine-structure constant and Feigenbaum constants are summarized as follows.

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

$$\text{Feigenbaum Constants: } \delta = 4.66920160910299$$

$$\alpha = 2.50290787509589$$

$$2\pi = 2 \cdot 3.14159265358979$$

$$\alpha_1 (\delta / \gamma_1)^2 (2\pi) = 1$$

$$\gamma_1 = 1 - \frac{1}{47 \cdot 109} + \frac{1}{27 \cdot 7 \cdot (3 \cdot 8 \cdot (3 \cdot 8 \cdot (4 \cdot 137 - 1) - 1) - 1)} = 0.999804818668238$$

$$\frac{1}{\gamma_1} = 1 + \frac{1}{2 \cdot 13 \cdot 197} - \frac{16 \cdot 7 \cdot 17 \cdot (16 \cdot 3 \cdot 23 - 1)}{125 \cdot 10^{12}} = 1.00019521943495$$

$$\gamma_1^2 = 1 - \frac{1}{13 \cdot 197} + \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 \cdot 31 - 1)} = 0.99960967543223$$

$$\frac{1}{\gamma_1^2} = 1 + \frac{1}{512 \cdot 5} - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83} = 1.00039047698053$$

$$\alpha_1 (\delta / \gamma_{1-Chen-2517})^2 (2\pi)_{Chen-2517} = 1$$

$$(2\pi)_{Chen-2517} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} = 6.28564399787948$$

$$\gamma_{1-Chen-2517} = 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{3}{25}} = 1.00000041773574$$

$$\frac{1}{\gamma_{1-Chen-2517}} = 1 - \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 + \frac{22}{25}} = 0.999999582264432$$

$$\gamma_{1-Chen-2517}^2 = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717$$

$$\frac{1}{\gamma_{1-Chen-2517}^2} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$

$$\alpha_1(\delta / \gamma_{1-\text{Wallis-9.71}})^2(2\pi)_{\text{Wallis-9.71}} = 1$$

$$(2\pi)_{\text{Wallis-9.71}} = 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1}\right) = 6.28564015562186$$

$$\gamma_{1-\text{Wallis-9.71}} = 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802$$

$$\frac{1}{\gamma_{1-\text{Wallis-9.71}}} = 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 \cdot 59 - 1)} = 0.999999887901990$$

$$\gamma_{1-\text{Wallis-9.71}}^2 = 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606$$

$$\frac{1}{\gamma_{1-\text{Wallis-9.71}}^2} = 1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3 \cdot 4}{17}} = 0.99999977580399$$

$$\alpha_1 \delta^2 (2\pi / \gamma_{1-\text{GL-22.37}})_{\text{GL-22.37}} = 1$$

$$(2\pi)_{\text{GL-22.37}} = 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right) = 6.28563929398602$$

$$\gamma_{1-\text{GL-22.37}} = 1 + \frac{1}{4 \cdot 25 \cdot 7 \cdot (2 \cdot 23^2 \cdot 31 - 1) + \frac{6}{11}} = 1.00000004355799$$

$$\frac{1}{\gamma_{1-\text{GL-22.37}}} = 1 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{5}{11}} = 0.999999956442012$$

$$\gamma_{1-\text{GL-22.37}}^2 = 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598$$

$$\frac{1}{\gamma_{1-\text{GL-22.37}}^2} = 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.999999912884025$$

$$\alpha_1(\delta / \gamma_{1-\text{NC-3}})^2(2\pi)_{\text{NC-3}} = 1$$

$$(2\pi)_{\text{NC-3}} = \left(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right) = 6.29047619047619$$

$$\gamma_{1-\text{NC-3}} = 1 + \frac{1}{23 \cdot 113} - \frac{1}{2 \cdot 3 \cdot 257 \cdot (4 \cdot 5 \cdot 29 \cdot 31 + 1)} = 1.00038472730421$$

$$\frac{1}{\gamma_{1-\text{NC-3}}} = 1 - \frac{1}{8 \cdot 25 \cdot 13} + \frac{1}{179 \cdot (8 \cdot 9 \cdot (2 \cdot 3 \cdot (2 \cdot 179 + 1) - 1) + 1) - \frac{3}{7}}$$

$$= 0.999615420653963$$

$$\gamma_{1-NC-3}^2 = 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352$$

$$\frac{1}{\gamma_{1-NC-3}^2} = 1 - \frac{1}{4 \cdot 25 \cdot 13} + \frac{1}{4 \cdot 9 \cdot 25 \cdot (2 \cdot 25 \cdot (4 \cdot 25 + 1) + 1) + \frac{2}{7}} = 0.999230989209198$$

$$\alpha_2(\delta\alpha / \gamma_2)^2 = 1$$

$$\gamma_2 = 1 - \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699$$

$$\frac{1}{\gamma_2} = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892$$

$$\gamma_2^2 = 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908$$

$$\frac{1}{\gamma_2^2} = 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}} = 1.00336671044256$$

$$\frac{\alpha_1(2\pi)}{\alpha_2(\alpha\gamma_1 / \gamma_2)^2} = 1$$

$$\frac{\gamma_1}{\gamma_2} = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1) - \frac{25}{2 \cdot 23}} = 1.00148643114372$$

$$\frac{\gamma_2}{\gamma_1} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 \cdot 29 + 1) + \frac{2 \cdot 3}{23}} = 0.99851577505446$$

$$\left(\frac{\gamma_1}{\gamma_2}\right)^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 17 \cdot (2 \cdot 157 - 1) - 1) - \frac{1}{4}} = 1.00297507176499$$

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2 = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3 \cdot 7^2 \cdot 23 - 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (8 \cdot 5 \cdot 13^2 + 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 3 \cdot 73 \cdot (4 \cdot 27 \cdot 19 + 1) - \frac{23}{3 \cdot 13}} = 0.997033753032614$$

$$\frac{2\pi}{(\alpha\gamma)^2} = 1$$

$$\gamma = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4 \cdot 7}} = 1.00148643087192$$

$$\frac{1}{\gamma} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1) - \frac{1}{6}} = 0.99851577532546$$

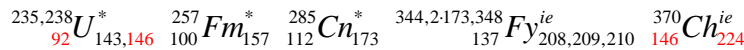
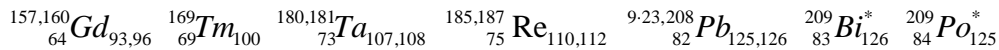
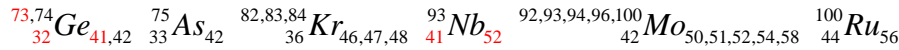
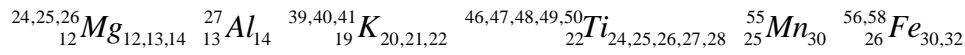
$$\gamma^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22}} = 1.0029750712205$$

$$\frac{1}{\gamma^2} = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13}} = 0.997033753573803$$

12. Some Supplement Formulas of Feigenbaum Constants

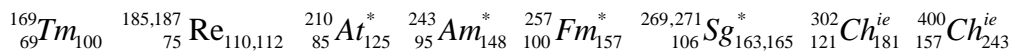
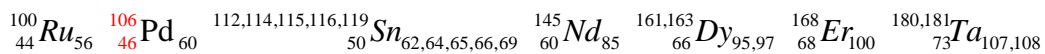
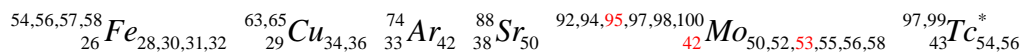
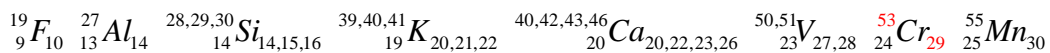
$$(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059$$

$$= \frac{100 \cdot (112 + \frac{1}{3 \cdot 5 \cdot 73} - \frac{1}{2 \cdot 5 \cdot 73 \cdot (32 \cdot 3 \cdot 23 - 1)})}{13 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \dots \frac{e^2}{(\frac{42}{41})^{83}}}$$



$$(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.57618663 \cdot 1518059$$

$$= \frac{100 \cdot (112 - \frac{1}{2 \cdot 53} + \frac{1}{25 \cdot 19 \cdot 181 + \frac{23}{29}})}{13 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \dots \frac{e^2}{(\frac{43}{42})^{85}}}$$

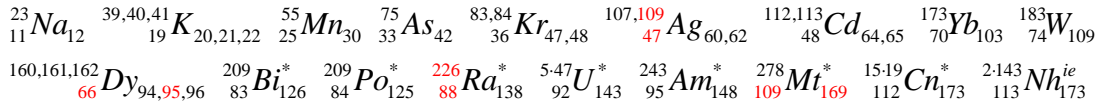


2021/3/23

$$(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059$$

$$11 \cdot 19 \cdot \left(103 + \frac{1}{2 \cdot 109} - \frac{1}{4 \cdot 3 \cdot 11 \cdot (4 \cdot 13^2 + 1) + \frac{19}{30}}\right)$$

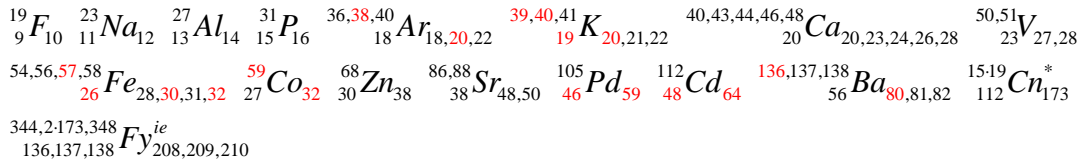
$$= \frac{25 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{48}{47}\right)^{5 \cdot 19}}}{11 \cdot 19 \cdot \left(103 + \frac{1}{2 \cdot 109} - \frac{1}{4 \cdot 3 \cdot 11 \cdot (6 \cdot 113 - 1) + \frac{19}{30}}\right)}$$



2021/3/24

$$(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.576186638059$$

$$= 137 - \frac{1}{2} + \frac{1}{13} - \frac{1}{23 \cdot 59} + \frac{1}{8 \cdot 3 \cdot 19 \cdot (16 \cdot 3 \cdot 5 \cdot 19 + 1)}$$

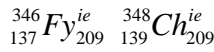
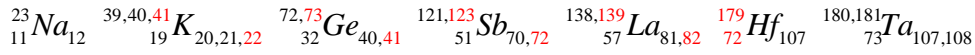


2021/3/24

$$(\delta\alpha)^2 = (4.66920160910299 \cdot 2.50290787509589)^2 = 136.5761866380594500$$

$$= 137 - \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{5 + \frac{1}{5 + \frac{1}{3 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{6}}}}}}}}}}}}}}}}}}$$

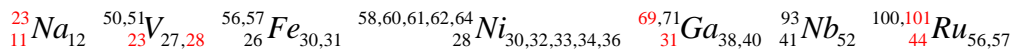
$$= 137 - \frac{11 \cdot 139 \cdot (4 \cdot 73 + 1)}{2 \cdot 3 \cdot 41 \cdot (8 \cdot 3 \cdot 179 + 1)}$$



2021/4/5

$$\delta^2 = (4.66920160910299)^2 = 21.8014436664500$$

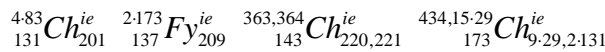
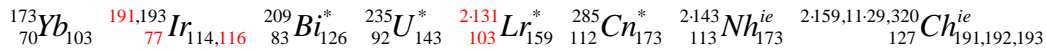
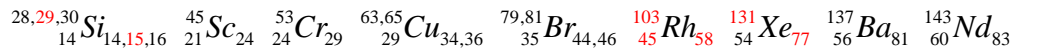
$$= \frac{7 \cdot \left(112 + \frac{1}{8} - \frac{1}{3 \cdot 101} + \frac{1}{3 \cdot 31 \cdot (4 \cdot 11 \cdot 23 + 1)} + \frac{4}{7}\right)}{36} = 21.8014436664499$$



2021/3/23

$$\delta^2 = (4.66920160910299)^2 = 21.8014436664500$$

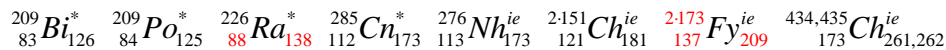
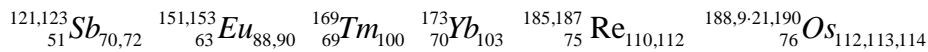
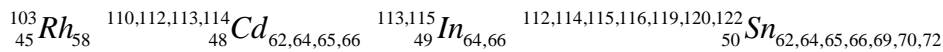
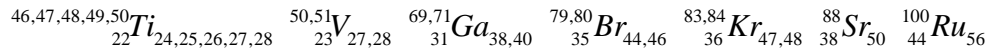
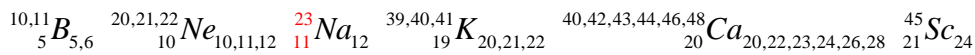
$$= \frac{29 \cdot \left(103 - \frac{1}{11 \cdot 13} + \frac{1}{2 \cdot 131 \cdot 191} + \frac{9 \cdot 5}{7 \cdot 11}\right)}{137}$$



2021/3/24

$$\delta^2 = (4.66920160910299)^2 = 21.8014436664500$$

$$= 22 - \frac{1}{5} + \frac{1}{4 \cdot 173} - \frac{1}{11^3 \cdot 23^2} - \frac{4}{19}$$

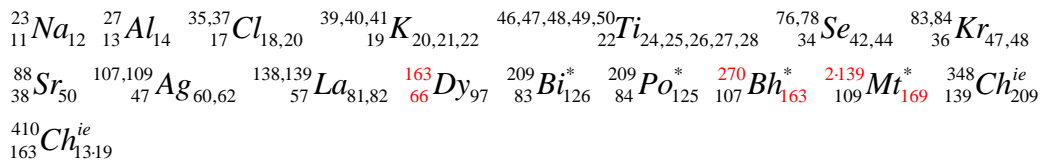


2021/4/3

$$\delta^2 = (4.66920160910299)^2 = 21.8014436664500$$

$$= 22 - \frac{1}{5 + \frac{1}{27 + \frac{1}{1 + \frac{1}{1 + \frac{1}{34 + \frac{1}{13 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{11}}}}}}}}}}}}$$

$$= 22 - \frac{1}{5 + \frac{1}{27 + \frac{1}{1 + \frac{1}{1 + \frac{1}{34 + \frac{47}{2 \cdot 17 \cdot 19}}}}}}}} = 22 - \frac{1}{5 + \frac{1}{27 + \frac{139 \cdot 163}{4 \cdot 13 \cdot (2 \cdot 3 \cdot 11 \cdot 13 + 1)}}}}$$



2021/4/4

References:

1. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2002.0203.
2. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2008.0020.
3. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2010.0252.
4. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2012.0107.
5. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2101.0187.
6. M. Hieb. viXra e-prints, viXra:1704.0365.

Acknowledgements

Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Thank Prof. Wenhao Hu, the dean of School of Pharmaceutical Sciences, Sun Yet-Sen University, for providing us an apartment in Shanghai since January of 2021 and hence facilitating the process of writing this paper.

Appendix I: Research History

Section	Page	Date	Location
Abstract	1	2021/2/20	Hanyuan
1	2	2021/2/20-21	
2	3-6	2021/1/31	Shanghai
		2021/2/1	
		2021/2/2-3	Chengdu
		2021/2/8	
3	6-7	2021/2/7	Chengdu
	7	2021/2/9	
4	7-8	2021/2/11	Chengdu – Hanyuan
		2021/2/17,21	Hanyuan
5	8-9	2021/2/25	Chengdu
6	9-11	2021/2/25-26	Chengdu
7	11-14	2021/2/28-3/3	Chengdu
8	14	2021/2/27-28	Chengdu
9	15	2021/3/3	Chengdu
10	15	2021/3/3	Chengdu
11	16-19	2021/3/7-8	Chengdu
12	19-22	2021/3/23-4/5	Chengdu
13	22-23	2021/4/4-5	Chengdu
Preparing this paper	1-24	2021/1/31-4/5	

Note: Time was recorded according to Beihing Time.