Using Narilkar Quantization of the Einstein-Hilbert First Integral to Further Bound the Cosmological Constant

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Abstract:

We take the results were we reduplicate the Book "Dark Energy" by M. Li, X-D. Li, and Y. Wang, zero-point energy calculation, as folded in with the Klauder methodology, as given in a prior publication . From there we first access the Rosen solution to a mini universe energy to ascertain an energy value of t the pre-inflationary near singularity,. Then access what would be needed as to inject information into our universe. We then close with an argument by Narilkar as to a quantum bound on the Einstein- Hilbert action integral, so as to obtain quantum Gravity. Narlikar omits the cosmological constant. We keep it in, four our overall conclusion about the cosmological constant and its relevance to Quantum gravity

Keywords: Minimum scale factor, cosmological constant, space-time bubble, bouncing cosmologies

1.Introduction- how we can link a cosmological constant to quantum effects

What we are doing is to take the results of [1] and [2] reduplicated and using the

Rosen [3] calculation to obtain a minimum energy which would be needed to move past the near singularity in order to obtain a linkage to the computed cosmological constant and quantum gravity, adopting an argument by Narlikar [4]. What Narlikar does is to bound the Einstein-Hilbert action integral for a black hole via Planck's h bar constant, to obtain $L \le 2\ell_{Planck}$ i.e. stating that quantum effects are important at lengths twice the Plank length. We will in the end duplicate this idea but with the cosmological constant included which we claim leads to $L \le \ell_{Planck}$, i.e. the cosmological constant makes the introduction of quantum gravity commensurate with Planck length. In addition we also specify arguments as to holographic limits [5],and a near quantum bounce [6]as to ascertain what conditions would be needed to have information outside a near singularity condition of space-time we think is congruent to a start to the universe for a quantum system commensurate to a black hole for the onset of information needed for inflation[5], [7],.That also involves using [8] methodologies as to the linkage between geometric optics and quantum mechanics for our system [8]

2. Methods

Our methodologies specifically refer to the formalism built up in reference [1] as given in [1] and [2] which gives both the Li, Li, and Wang solution to the cosmological constant and then from there obtain the re do of the [4] calculation with an eye toward linking our cosmological constant to the onset of quantum effects in a non singular start point for expansion of the universe

3.1 We compare two types of cosmological constant values given in [1]

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1}+2\tilde{N}+\frac{\gamma(3\gamma-1)}{8\pi G t_{Planck}}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\,d^3x}+c_1\frac{16\pi}{M_P^2}\cdot\left(\rho-\frac{\rho^2}{2|\sigma|}\right) \quad (1)$$

$$\Leftrightarrow \rho_{DE}=\frac{\Lambda}{8\pi G}\Leftrightarrow \Lambda\approx\hbar\cdot 8\pi G\cdot\left.\frac{(2\pi)^4}{\lambda_{DE}{}^4}\right|_{\lambda_{DE}\approx 10^{30}\,\ell_{Planck}} \equiv 8\pi\cdot\left.\frac{(2\pi)^4}{\lambda_{DE}{}^4}\right|_{\lambda_{DE}\approx 10^{30}}$$

To compare these two values we can state that within the bubble, that just before the bubble boundary, we have $\tilde{N}=0$, i.e. we pick c_1 so that the two are equivalent in value, with σ a surface tension of the bubble of space-time, just before cosmological expansion [1],[6]

Hence we also would be looking at

$$\Lambda\approx\gamma\cdot t^2+8\pi V_0\cdot\left.\left\{\sqrt{\frac{8\pi V_0}{\gamma(3\gamma-1)}}\cdot t\right\}^{\sqrt{\frac{\gamma}{4\pi}}-\sqrt{\frac{8\pi}{\gamma}}}\right|_{t\approx t_{Planck}=1}$$

$$\approx\frac{-\left[\frac{V_0}{3\gamma-1}+\frac{\gamma(3\gamma-1)}{8\pi}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\,d^3x}+c_1\frac{16\pi}{M_P^2}\cdot\left(\rho-\frac{\rho^2}{2|\sigma|}\right) \quad (2)$$

$$\approx\hbar\cdot 8\pi G\cdot\left.\frac{(2\pi)^4}{\lambda_{DE}{}^4}\right|_{\lambda_{DE}\approx 10^{30}\,\ell_{Planck}} = 8\pi\cdot\left.\frac{(2\pi)^4}{\lambda_{DE}{}^4}\right|_{\lambda_{DE}\approx 10^{30}}$$

In Eq. (1) we will also make the following identification $\rho\approx\rho_{DE}\approx\hbar\cdot\left.\frac{(2\pi)^4}{\lambda_{DE}{}^4}\right|_{\lambda_{DE}\approx 10^{30}}$

I.e. just before the bubble, we will factor in a very large bubble tension, σ [6]

Doing so would be to have an optimal c_1 value, which we use to obtain quantum effects in early space-time

3.2 What we obtain if we model the cosmological constant this way?

***We do not confine ourself to Isotropic backgrounds. We also do share with Rosen {4] a procedure as to* rendition a linkage between classical to quantum mechanical treatment of our problem, but our work is more in tune with using what Powell and Craseman in their book on quantum mechanics used, [8] which is similar to a limiting case of geometric optics.**

Hence this expanded treatment whereas the Rosen treatment as Corda outlined is most effective for quantum black holes is used as a short cut to analyzing a pre big bang to a big bang transformation of space-time.

3.3 Final reference to high frequency gravitational waves

From [9],Maggorie, we have

$$\begin{aligned}&\left(1+z_{initial-era}\right)\equiv\frac{a_{today}}{a_{initial-era}}\approx\left(\frac{\omega_{Earth-orbit}}{\omega_{initial-era}}\right)^{-1}\\&\Rightarrow\left(1+z_{initial-era}\right)\omega_{Earth-orbit}\approx10^{25}\,\omega_{Earth-orbit}\approx\omega_{initial-era}\end{aligned}\qquad\textbf{(3)}$$

Where $c\left(light-speed\right)\approx\omega_{initial-era}\cdot\left(\lambda_{initial-post-bubble}=\ell_{Planck}\right)$ and $\Delta E\approx\hbar\omega_{initial-era}$

and that dimensional comparison with having a temperature built up so as $\Delta E\approx\hbar\omega_{initial-era}$

where $T_{universe}\approx T_{Plank-temerature}=1.22\times10^{19}\,\textbf{GeV}$. If so then the Planck era

temperature would be extremely high leading to a change in temperature from the

Pre Planckian conditions to Planck era leading to , if d is the dimension of space-time

$$\Delta E=\frac{d(\dim)}{2}\cdot k_B\cdot T_{universe}\qquad(4)$$

In doing so, be assuming

$$\omega_{initial-era}\approx\frac{c}{\ell_{planck}}\leq1.8549\times10^{43}\,Hz\qquad(5)$$

Where we would be assuming so then we would be looking at frequencies on Earth from gravitons of mass m(graviton) less than of equal to

$$\omega_{Earth-orbit}\leq10^{-25}\,\omega_{initial-era}\qquad(6)$$

And this partly due to the transference of cosmological 'information' as given in [10]

for a phantom bounce type of construction .Further point that since we have that gravitons travel at nearly the speed of light[9], that gravitons are formed from the surface of a bubble of space-time up to the electroweak era that mass values of the order of 10^-65 grams (rest mass of relic gravitons) would increase due to extremely high velocity would lead to enormous $\Delta E \approx \hbar\omega_{initial-era}$ values per graviton, which would make the conflation of ultrahigh temperatures with gravitons traveling at nearly the speed of light as given in Eq.(5) as compared with $\Delta E \approx \hbar\omega_{initial-era}$

We can compare this with the Rosen[3] value of energy for a mini universe of(from a Schrodinger equation) with ground state mass of $m = \sqrt{\pi} M_{Planck}$ and an energy of

$$E_{\hat{n}} = \frac{-Gm^5}{2\pi^2\hbar^2\hat{n}^2} \tag{7}$$

Our preliminary supposition is that Eq. (7) could represent the initial energy of a Pre Planckian Universe and that Eq.(4) be the thermal energy dumped in due to the use of Cyclic Conformal cosmology (maybe in multiverse form) so that if there is a build up of energy greater than the magnitude of Eq.(7) due to thermal build up of temperature due to infall of matter-energy, we have a release of Gravitons in great number which would commence as a domain wall broke down about in the Planckian era with a temperature of the magnitude of Planck Energy information number of . And this ties in with release of information N(information) for which we have a total Graviton mass of

$$M_{graviton-total} = n_{graviton} \cdot m_{graviton} \tag{8}$$

Where we will be looking at a value of "information" of initially

$$\mathbb{N}(\inf) = \frac{9\pi}{\ln 2} \cdot \left(\frac{n_{graviton} \cdot m_{graviton}}{m_{planck}} \right)^2 \approx \frac{9\pi \cdot \left(n_{graviton}\right)^2}{\ln 2} \cdot 10^{-120} \tag{9}$$

Now use the following approximation of the Universe, initially having the entropy of a black hole, i.e, we are using Ng Infinite Quantum statistics, [11],[12]

$$S_{Universe} \propto S_{BH} \cong \frac{A(area)}{4 \cdot \ell_{Planck}^2} \approx \frac{9n_Q}{4} \approx n_{graviton} \tag{10}$$

In taking this step, we are making use of [3] having the following radius used, namely using in our model of a black hole, the quantum "atom" approximation

$$r(n_Q) \approx \frac{3\sqrt{n_Q} \cdot \ell_{Planck}}{2\sqrt{\pi}} \tag{11}$$

In order to have non vanishing information according to [7] we would need to specify having

$$\mathbb{N}(\inf) = \frac{9\pi}{\ln 2} \cdot \left(\frac{n_{graviton} \cdot m_{graviton}}{m_{planck}} \right)^2 \approx \frac{729\pi \cdot \left(n_Q\right)^2}{16 \cdot \ln 2} \cdot 10^{-120} \tag{12}$$

Given this, if

$$\mathbb{N}(\inf) = \frac{9\pi}{\ln 2} \cdot \left(\frac{n_{graviton} \cdot m_{graviton}}{m_{planck}} \right)^2 \approx \frac{729\pi \cdot (n_Q)^2}{16 \cdot \ln 2} \cdot 10^{-120} \geq 1$$

$$\Rightarrow E_{\hat{n}=n_Q} = \frac{-G \cdot \left(m = \sqrt{\pi n_Q} \cdot m_{planck} \right)^5}{2\pi^2 \hbar^2 n_Q{}^2} \approx \frac{-G \cdot \left(\sqrt{\pi n_Q} \cdot (m_{planck})^5 \right)}{2\hbar^2} \quad (13)$$

$$\xrightarrow[\hbar = m_{planck} = G = 1]{} E_{\hat{n}=n_Q} \simeq \frac{-\left(\sqrt{\pi n_Q} \cdot \right)}{2} \approx l\arg e - negative - value$$

We speculate that in order to have a large negative binding energy that this will mean we have then an enormous initial thermal energy from a multiverse cyclic conformal cosmology input as to undo this so as to initiate a new cycle of creation, i.e breaking the binding energy would require Planck level temerature values. And this is a topic which will be researched furthermore in great detail . And the necessary thermal heat would drive having enormously high initial frequencies

3.4 ***And now our conclusing words as to a quantization limit to pursue, if the early universe has characteristics of a pre planckian black hole***

In order to do this we adapt an argument used by [4] as using the quantization of an action which we write using [43] first in the case of no cosmological constant, namely if $\Im$ is an action integral with the form of the Einstein – Hilbert least action of which :L is a radius , and

$$\delta\Im = 0 \quad (14)$$

$$\Im \leq \hbar \quad (15)$$

Equation (15) is an imposed upon quantization limit where we use from [43]

$$\Im = \frac{c^4}{16\pi G} \int \Re \sqrt{-g} d^4 x \approx \left. \frac{2\pi^4 \rho L^4 c}{3} \right|_{t=L/c} \leq \hbar \quad (16)$$

In case of using a black hole limit and constant energy density ρ, [4] argues for L

Quantization near singularity if $L \leq 2\ell_{Planck}$ (17)

In the case of when the comological constant is NOT zero we impose

$$\Im = \frac{c^4}{16\pi G} \int (\Re - 2\Lambda) \cdot \sqrt{-g} d^4 x \approx \left. \frac{2\pi^4 \rho L^4 c}{3} \right|_{t=L/c} - \frac{2L^4}{c} \cdot \Lambda \leq \hbar \quad (18)$$

Here, ρ is an energy density and in the case of no cosmological constant we would use

$$\rho L^2 \leq \left(3c^2 / 8\pi G \right) if \quad \Lambda = 0 \quad (19)$$

And

$$\Im = \frac{c^4}{16\pi G} \int (\Re) \cdot \sqrt{-g} d^4 x \approx \left. \frac{2\pi^4 \rho L^4 c}{3} \right|_{t=L/c} \leq L^2 c^3 / 4G \quad (20)$$

We argue as does [4] that when there is no cosmological costant that Eq. (19) and Eq(15) hold we are obtaining Eq. (20) so that

$L \le 2\ell_{Planck}$ as a quantum length limit. (21)

In the case where we use Eq. (18) instead of Eq. (19) we would instead see quantization

$L \le \ell_{Planck}$ as a quantum limit (22)

This final set of arguments if the early universe had the characteristics of a Black hole will be pursued in future research. This in tandem for obtaining an optimal understanding of the quantum characteristics embodied in [5], [7] with [5] holographic

information

$$\mathbb{N}(\inf) = \frac{9\pi}{\ln 2} \cdot \left(\frac{n_{graviton} \cdot m_{graviton}}{m_{planck}} \right)^2 \approx \frac{729\pi \cdot \left(n_Q\right)^2}{16 \cdot \ln 2} \cdot 10^{-120} \ge 1 \quad (23)$$

As well as picking c_1 to give a quantum interpretation of Eq. (1) and Eq.(2). This would allow for an investigation of ideas in [7], [8], [13] , [14] and [15]

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