Ontology and physics

Jean Louis Van Belle, *Drs, MAEc, BAEc, BPhil*

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Abstract

This papers concludes our excursions into the epistemology/ontology of physics. We provide a basic overview of the basic concepts as used in the science of physics, with practical models based on orbital energy equations. We hope to make a difference by offering an alternative particle classification based on measurable form factors.

Contents

Prolegomena

Why is it that we want to understand quarks and wave equations, or delve into complicated math (perturbation theory[1], for example)? We believe it is driven by the same human curiosity that drives philosophy. Physics stands apart from other sciences because it examines the smallest of smallest—the *essence* of things, so to speak.

Unlike other sciences (the human sciences in particular, perhaps), physicists also seek to *reduce* the number of concepts, rather than multiply them—even if, sadly, enough, they do not always a good job at that. The goal is to arrive at a *minimal* description or *representation* reality. Physics and math may, therefore, be considered to be the King and Queen of Science, respectively.

The Queen is an eternal beauty, of course, because Her Language may mean anything. Physics, in contrast, talks specifics: *physical* dimensions (force, distance, energy, etcetera), as opposed to *mathematical* dimensions—which are mere quantities (scalars and vectors).

Science differs from religion in that it seeks to *experimentally* verify its propositions. It *measures* rather than *believes*. These measurements are cross-checked by a global community and, thereby, establish a non-subjective reality. The question of whether reality exists outside of us, is irrelevant—a *category mistake* (Ryle, 1949). All is in the fundamental equations. We are part of reality.

An equation relates a measurement to Nature's constants. Measurements – such as the energy/mass of particles, or their velocities – are relative but that does not mean they do not *represent* anything real. On the contrary.

Nature's constants do *not* depend on the frame of reference of the observer and we may, therefore, label them as being *absolute*. The difference between relative and absolute concepts corresponds to the difference between *variables* and *parameters* in equations. The speed of light (c) and Planck's quantum of action (h) are parameters in the $E/m = c^2$ and $E = h \cdot f$, respectively. In contrast, energy (E), mass (m), frequency (f) are *measured* quantities.

Feynman (II-25-6) is right that the Great Law of Nature may be summarized as U = 0 but that "this simple notation just hides the complexity in the definitions of symbols is just *a trick*." It is like talking of "the night in which all cows are equally black" (Hegel, *Phänomenologie des Geistes*, *Vorrede*, 1807). Hence, the U = 0 equation needs to be separated out. We would separate it out as:

[1] Analyzing phenomena in terms of first-, second-,… n^{th}-order effects is useful as a rough approximation of reality (especially when analyzing experimental data) but, as Dirac famously said, "neglecting infinities […] is not sensible. Sensible mathematics involves neglecting a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it!" (Dirac, 1975) Perturbative theory often relies on a series expansion, such as the series expansion of relativistic energy/mass::

$$\mathrm{m}c^2 = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

We do not immediately see the relevance (need) of this formula when solving practical problems.

$$\frac{\begin{array}{c}\mathrm{E} = \mathrm{m}c^2 \\ \mathrm{E} = hf\end{array}}{\mathrm{m}c^2 = hf \Leftrightarrow \frac{\mathrm{m}}{f} = \frac{h}{c^2}}$$

Energy is measured as a force over a distance: we do *work* with or against the force.[2]

$$\mathrm{W} = \mathrm{E} = \int_a^b \mathbf{F} \cdot d\boldsymbol{s}$$

Forces are forces between charges. If there is an *essence* in Nature, it corresponds to the concept of charge. We think there is only one *type* of charge: the electric charge q. Charge is absolute: an electron in motion or at rest has the same charge. That is why Einstein did not think much of the concept of mass: the mass of a particle measures its *inertia* to a change in its state of motion, and gravitation is likely to reflect the geometry of the Universe: a *closed* Universe, which very closely *resembles* Cartesian spacetime but *not quite*.

We imagine things in 3D space and one-directional time (Lorentz, 1927, and Kant, 1781). The imaginary unit operator (i) represents a rotation in space. A rotation takes time and involves distance: we rotate a charge from point a to point b. A *radian*, therefore, measures an angle (θ) as well as a distance and a time. We usually think of angular velocity as a derivative of the phase with respect to time, though:

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

The Lorentz force on a charge is equal to:

$$\mathbf{F} = \mathrm{q}\mathbf{E} + \mathrm{q}(\boldsymbol{v}\times\mathbf{B})$$

If we know the (electric field) **E**, we know the (magnetic field) **B**: **B** is perpendicular to **E**, and its magnitude is $1/c$ times the magnitude of **E**. We may, therefore, write:

$$\mathbf{B} = -i\cdot\mathbf{E}/c$$

To make the dimensions come out alright[3], we need to associate the s/m dimension with the imaginary unit i. This reflects Minkowski's metric signature and counter-clockwise evolution of the argument of

[2] *Potential* energy is defined with respect to a reference point. The reference point may be taken at an infinite distance (∞) of the charge at the center of the potential field, or at the charge itself (r = 0). Sign conventions depend on the choice of the reference point.

[3] **E** is measured in newton per coulomb (N/C). **B** is measured in newton per coulomb divided by m/s, so that's (N/C)·(s/m). Note the minus sign in the $\mathbf{B} = -i\mathbf{E}/c$ expression is there because we need to combine several conventions here. Of course, there is the classical physical right-hand rule for E and B, but we also need to combine the right-hand rule for the coordinate system with the convention that multiplication with the imaginary unit amounts to a counterclockwise rotation by 90 degrees. Hence, the minus sign is necessary for the consistency of the description. It ensures that we can associate the $a\cdot e^{i\theta}$ and $a\cdot e^{-i\theta}$ functions with left and right-handed polarization, respectively.

complex numbers, which represent the (elementary) wavefunction $\psi = ae^{i\theta}$.[4] The *nature* of the nuclear force is different, but its *structure* should incorporate relativity as well.[5]

The illustration below provides the simplest of simple visualizations of what an elementary particle might *be*—an oscillating pointlike charge:

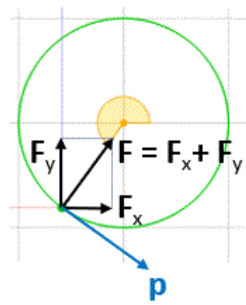


Figure 1: The ring current model[6]

Erwin Schrödinger referred to it as a *Zitterbewegung* [7], and Dirac highlighted its significance at the occasion of his Nobel Prize lecture:

> "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high, and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

The actual motion of the pointlike charge might be chaotic but this cannot be verified: we measure averages (*cycles*) only. The *regularity* (periodicity) of motion makes it *deterministic*. High velocities introduce probability: quantum physics adheres to probabilistic determinism. H.A. Lorentz told us there is no need to elevate *indeterminism* to a philosophical principle:

> "Je pense que cette notion de probabilité [in the new theories] serait à mettre à la fin, et comme conclusion, des considérations théoriques, et *non pas comme axiome a priori*, quoique je veuille bien admettre que cette indétermination correspond aux possibilités expérimentales. Je pourrais toujours garder ma foi déterministe pour les phénomènes fondamentaux, dont je n'ai pas parlé. Est-ce qu'un esprit plus profond ne pourrait pas se rendre compte des mouvements de ces électrons ? Ne pourrait-on pas

[4] 720-degree symmetries and the boson/fermion dichotomy are based on a misunderstanding of the imaginary unit representing a 90-degree rotation in this *or* that direction.

[5] For an analysis of the relativity of magnetic and electric fields, see Feynman, II-13-6.

[6] The British chemist and physicist Alfred Lauck Parson (1915) proposed the *ring current* or *magneton* model of an electron, which combines the idea of a charge and its motion to *represent* the reality of an electron. The combined idea effectively accounts for both the particle- as well as the wave-like character of matter-particles. It also explains the magnetic moment of the electron.

[7] *Zitter* (German used to be a more prominent language in science) refers to a rapid trembling or shaking motion.

garder le déterminisme en en faisant l'objet d'une croyance? Faut-il nécessairement ériger l' indéterminisme en principe?" (H.A. Lorentz, Solvay Conference, 1927)

Velocities can be linear or tangential (orbital), giving rise to the concepts of linear versus angular momentum. Angular momentum and Planck's quantum of action have the same physical dimension. It is that of a *Wirkung*: *force* (N) times *distance* (m) times *time* (s). Orbitals imply a centripetal force, and the distance and time variables becomes the length of the loop and the cycle time, respectively. When motion is linear, the length of the loop is a (linear) wavelength, which is 2π times the radius: we distinguish h and its reduced version $\hbar = h/2\pi$.

The ring current model of elementary particles

The ring current model is a mass-without-mass model of elementary particles. It analyzes them as harmonic oscillations whose total energy – at any moment (KE + PE) or over the *cycle* – is given by $E = m \cdot a^2 \cdot \omega^2$. One can then calculate the radius or amplitude of the oscillation directly from the mass-energy equivalence and Planck-Einstein relations, as well as the tangential velocity formula—interpreting c as a tangential or orbital (escape[8]) velocity.

$$\left.\begin{array}{r}\left.\begin{array}{r}\mathrm{E} = \mathrm{m}c^2 \\ \mathrm{E} = \hbar\omega\end{array}\right\} \Rightarrow \mathrm{m}c^2 = \hbar\omega \\ c = a\omega \Leftrightarrow a = \dfrac{c}{\omega} \Leftrightarrow \omega = \dfrac{c}{a}\end{array}\right\} \Rightarrow \mathrm{m}a^2\omega^2 = \hbar\omega \Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2 = \hbar\frac{c}{a} \Leftrightarrow a = \frac{\hbar}{\mathrm{m}c}$$

Such models assume a centripetal force whose *nature*, in the absence of a charge at the center, can only be explained with a reference to the quantized energy levels we associate with atomic or molecular electron orbitals[9], and the *physical* dimension of the oscillation in space and time may effectively be understood as a *quantization* of spacetime.

Tangential velocities imply orbitals: circular and elliptical orbitals are closed. Particles are pointlike charges in closed orbitals. We do not think non-closed orbitals correspond to some reality: *linear* oscillations are field particles, but we do not think of lines as non-closed orbitals: the curvature of real space (i.e. the Universe we happen to live in) suggest we should—but we are not sure such thinking is productive (efforts to model gravity as a *residual* force have failed so far).

Space and time are innate or *a priori* categories (Kant, 1781). Elementary particles can be modeled as pointlike charges oscillating in space and in time. The concept of charge could be dispensed with if there were not lightlike particles: photons and neutrinos, which carry energy but no charge.

The pointlike charge which is oscillating is pointlike but may have a finite (non-zero) physical dimension, which explains the anomalous magnetic moment of the free (*Compton*) electron. However, it only appears to have a non-zero dimension when the electromagnetic force is involved (the proton has no

[8] The concepts of orbital, tangential and escape velocity are not always used as synonyms. For a basic but complete introduction, see the MIT OCW reference course on orbital motion.

[9] See, for example, Feynman's analysis of quantized energy levels or his explanation of the size of an atom. As for the question why such elementary *currents* do not radiate their energy out, the answer is the same: persistent currents in a superconductor do not radiate their energy out either. The general idea is that of a *perpetuum mobile* (no external driving force or frictional/damping terms). For an easy mathematical introduction, see Feynman, Chapter 21 (the harmonic oscillator) and Chapter 23 (resonance).

anomalous magnetic moment and is about 3.35 times smaller than the calculated radius of the pointlike charge inside of an electron). What explains ratios like this? There is no answer to this: we just find these particles are there: their *rest* mass/energy behave like Nature's constants: they are *simply there*.

We have two forces acting on the same (electric) charges: electromagnetic and nuclear. One of the most remarkable things is that the E/m = c^2 holds for both electromagnetic and nuclear oscillations, or combinations thereof (superposition theorem). Combined with the oscillator model ($E = m \cdot a^2 \cdot \omega^2 = m \cdot c^2 \Leftrightarrow c = a \cdot \omega$), this makes one think of c^2 as an *elasticity* or *plasticity* of space.

Why two *oscillatory modes* only? In 3D space, we can only *imagine* oscillations in one, two and three dimensions (line, plane, and sphere).

Photons and neutrinos are *linear* oscillations and, because they carry no charge, travel at the speed of light. Electrons and muon-electrons (and their antimatter counterparts) are 2D oscillations packing electromagnetic and nuclear energy, respectively. The proton (and antiproton) pack a 3D nuclear oscillation. Neutrons combine positive and negative charge and are, therefore, neutral. Neutrons may or may not combine the electromagnetic and nuclear force: their size (more or less the same as that of the proton) suggests the oscillation is nuclear.

	2D oscillation	**3D oscillation**
electromagnetic force	$e^{\pm}$ (electron/positron)	orbital electron (e.g.: ^{1}H)
nuclear force	$\mu^{\pm}$ (muon-electron/antimuon)	$p^{\pm}$ (proton/antiproton); n^{0} (neutron)
Composite (stable or transient)	?	D^{+} (deuteron)? pions ($\pi^{\pm}$/ π^{0})?
corresponding field particle	γ (photon)	ν (neutrino)

The theory is complete: each theoretical/mathematical/logical possibility corresponds to a physical reality, with spin distinguishing matter from antimatter for particles with the same *form factor*.

Time and relativity

Panta rhei (Heraclitus, fl. 500 BC). Motion relates the ideas of space (position) and time. Spacetime trajectories need to be described by well-defined function: for every value of *t*, we should have one, and only one, value of *x*. The reverse is not true, of course: a particle can travel back to where it was. That is what it is doing in the graph on the right. The *force* that makes it do what it does is some wild oscillation but it is *possible*: not only theoretically but also practically.

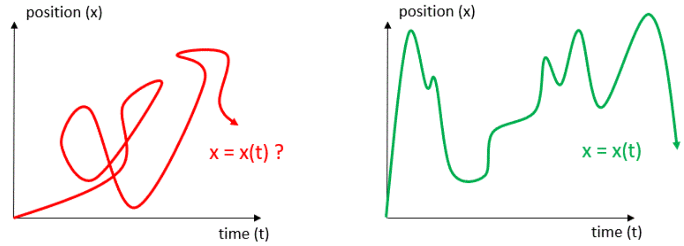


Figure 2: A well- and a not-well behaved trajectory in spacetime

Time has one direction only because we describe motion (trajectories) by well-behaved functions. In short, the idea of *motion* is what gives space and time their *meaning*. The alternative idea is spaghetti (first graph).

The idea of an *infinite* velocity makes no sense: our particle would be everywhere and we would, therefore, not be able to *localize* it. Likewise, the idea of an infinitesimally small *distance* is a *mathematical* idea only: it underlies differential calculus (the logic of integrals and derivatives) but Achilles does overtake the tortoise: motion is real, and the arrow reaches its goal (Zeno of Elea).

Light-particles (photons and neutrinos, perhaps[10]) have zero rest mass and, therefore, travel at the speed of light (c): the slightest acceleration accelerates them to lightspeed. Light-particles, therefore, acquire relativistic mass or *momentum* ($\mathbf{F} = d\mathbf{p}/dt$).

The $p = mc = \gamma m_0 c$ function behaves in a rather weird way (Figure 3): the Lorentz factor (γ) goes to infinity as the velocity goes to c, and m_0 is equal to zero. Hence, we are multiplying zero by infinity.

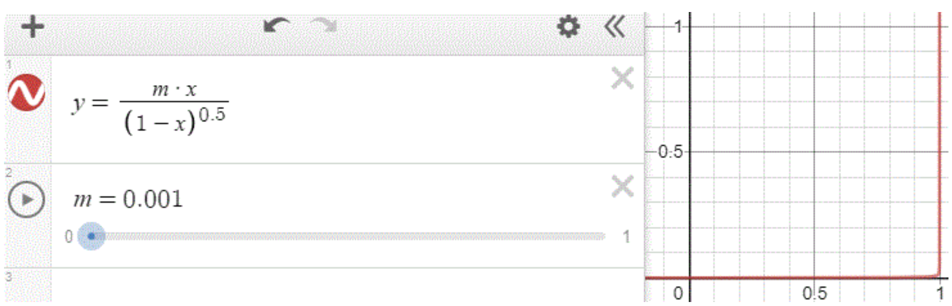


Figure 3: $p = m_v v = \gamma m_0 v$ for $m \to 0$

The function reminds one of the Dirac function $\delta(\boldsymbol{x})$: the sum of probabilities must always add up to one. If we *measure* the position of a particle at $\boldsymbol{x} = \boldsymbol{x}$ at time $t = t$, then the probability function collapses at $P(\boldsymbol{x}, t) = 1$.

[10] We think of neutrinos as 3D oscillations and they may, therefore, have some non-zero rest mass or, to be precise, some inertia to a change in their state of motion along *all* possible directions of motion. In contrast, the two-dimensional oscillation of the electromagnetic field vector (photon) is perpendicular to the direction of motion and we therefore have no inertia in the direction of propagation.

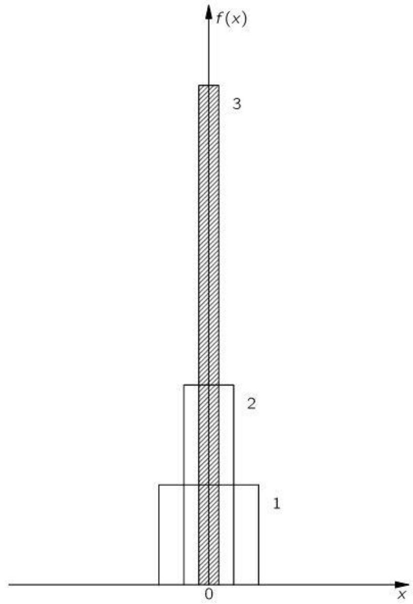


Figure 4: The Dirac function $\delta(\boldsymbol{x})$ as the limit of a probability distribution (Feynman, III-16-4)

We may imagine a wavefunction which comes with *constant* probabilities: $|\psi|^2 = |a\cdot e^{i\theta}|^2 = a^2$. The wavefunction ψ is zero outside of the space interval (x_1, x_2). We have an oscillation in a spatial *box* (Figure 1), which packs a finite amount of energy. All probabilities have to add up to one, and so we must *normalize* the distribution.

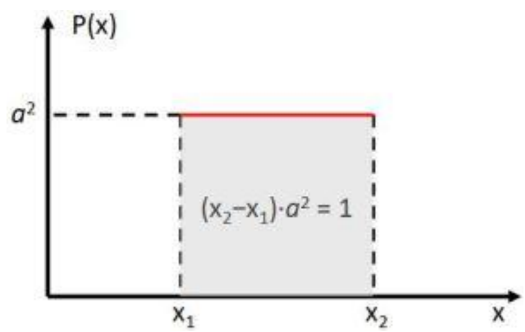


Figure 5: Elementary particle-in-a-box model

The energy (and equivalent mass) of a harmonic oscillation is given by $E = m\cdot a^2\cdot\omega^2 = m\cdot\lambda^2\cdot f^2$. We can, therefore, write:

$$a^2 = \frac{E}{m\omega^2} = \frac{c^2\hbar^2}{E^2}$$

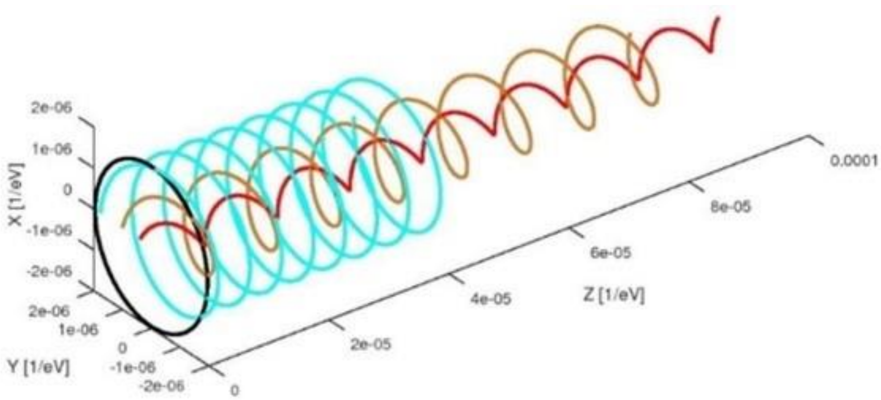
This gives us a *physical* normalization condition based on the *total* energy of the particle and the physical constants *c* and *ħ*. The wavefunction itself represents energy *densities*—energy per unit volume (V) unit, or force per area unit (A):

$$\rho_E = E/V, \text{ and } [\rho_E] = [E/V] = N\cdot m/m^3 = N/m^2 = [F/A]$$

$$\boldsymbol{r} = \boldsymbol{a} \cdot e^{i\theta} = \psi(\boldsymbol{x}) \sim \rho_{\mathrm{E}} = \frac{\mathrm{E}}{\mathrm{V}} = \frac{\mathrm{F}}{\mathrm{A}}$$

The volume V and the energy E are the volume and energy of the particle, respectively—and the area A and force F are the orbital area and the centripetal force, respectively. The physical dimension of the *components* of the wavefunction is, therefore, equal to [ρ] = N/m²: force per unit area. All other things being equal (same mass/energy), stronger forces make for smaller particles.[11]

The illustration below (**Figure 6**) imagines how the *Zitterbewegung* radius of an elementary particle decreases as one adds a lateral (linear) velocity component to the motion of the pointlike charge: it decreases as it gains linear momentum. Why is that so? Because the speed of light is the speed of light: the pointlike charge cannot travel any faster if we are adding a linear component to its motion. Hence, some of its lightlike velocity is now linear instead of circular and it can, therefore, no longer do the original orbit in the same cycle time.



Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

Figure 6: The Compton radius must decrease with increasing velocity[12]

Needless to say, the *plane* of oscillation of the pointlike charge is not necessarily perpendicular to the direction of motion. In fact, it is most likely *not* perpendicular to the line of motion, which explains why we may write the *de Broglie* relation as a *vector* equation: $\lambda_L = \boldsymbol{h}/\mathbf{p}$. Such vector notation implies $\boldsymbol{h}$ and $\mathbf{p}$ can have *different* directions: $\boldsymbol{h}$ may not even have any fixed direction! It might *wobble* around in some regular or irregular motion itself!

[11] The time dependency is in the phase (angle) of the wavefunction $\theta = \omega\cdot t = \mathrm{E}\cdot t/\hbar$. We may say that Planck's quantum of action *scales* the energy as per the Planck-Einstein relation $\mathrm{E} = \hbar\cdot\omega = h\cdot f = h/\mathrm{T}$, with T the cycle time. We may say Planck's quantum of action *expresses* itself as some energy over some time ($h = \mathrm{E}\cdot\mathrm{T}$) or as some momentum over a distance ($h = \mathrm{p}\cdot\lambda$). If the pointlike charge spends more *time* in a volume element (or passes through more often), the energy density in this volume element will, accordingly, be larger.

[12] We borrow this illustrations from G. Vassallo and A. Di Tommaso (2019).

Figure 6 also shows that the *Compton* wavelength (the circumference of the circular motion becomes a *linear* wavelength as the classical velocity of the electron goes to *c*. It is now easy to derive the following formula for the *de Broglie* wavelength[13]:

$$\lambda_L = \frac{h}{p} = \frac{h}{mv} = \frac{hc^2}{Ev} = \frac{hc}{E\beta} = \frac{1}{\beta}\cdot\frac{h}{mc} = \frac{1}{\gamma\beta}\cdot\frac{h}{m_0 c}$$

The graph below shows how the $1/\gamma\beta$ factor behaves: it is the **green curve**, which comes down from infinity (∞) to zero (0) as *v* goes from 0 to *c* (or, what amounts to the same, if β goes from 0 to 1). Illogical? We do not think so: the classical momentum **p** in the λ_L = **h**/**p** is equal to zero when *v* = 0, so we have a division by zero. We may also note that the *de Broglie* wavelength approaches the *Compton* wavelength of the electron only if *v* approaches *c*.

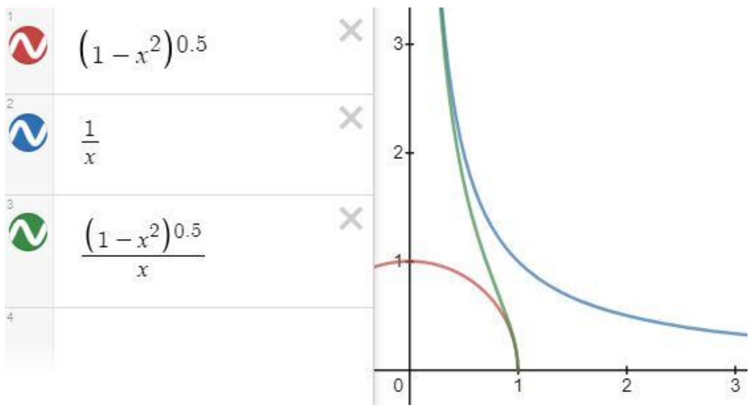


Figure 7: The 1/γ, 1/β and 1/γβ graphs[14]

The combination of circular and linear motion explains the argument of the wavefunction, which we will now turn to.

The wavefunction and its (relativistically invariant) argument

We will talk a lot about wavefunctions and probability amplitudes in the next section, so we will be brief here. When looking at **Figure 6**, it is obvious that we can use the elementary wavefunction (Euler's formula) to represents the motion of the pointlike charge by interpreting $\boldsymbol{r} = a\cdot e^{i\theta} = a\cdot e^{i\cdot(E\cdot t - \mathbf{k}\cdot\boldsymbol{x})/\hbar}$ as its position vector. The coefficient *a* is then, equally obviously, nothing but the Compton radius $a = \hbar/mc$.[15]

The relativistic invariance of the argument of the wavefunction is then easily demonstrated by noting that the position of the pointlike particle in its own reference frame will be equal to $x'(t') = 0$ for all t'.

We can then relate the position and time variables in the reference frame of the particle and in our

[13] You should do some calculations here. They are fairly easy. If you do not find what you are looking for, you can always have a look at Chapter VI of our manuscript.

[14] We used the free desmos.com graphing tool for these and other graphs.

[15] When discussing the concept of probability amplitudes, we will talk about the need to *normalize* them because the sum of all probabilities – as per our conventions – has to add up to 1. However, the reader may already appreciate we will want to talk about normalization based on physical realities—as opposed to unexplained mathematical conventions or quantum-mechanical *rules*.

frame of reference by using Lorentz's equations[16]:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

When denoting the energy and the momentum of the electron in our reference frame as E_v and p = $\gamma m_0 v$, the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows[17]:

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \frac{E_0}{\hbar}t'$$

E_0 is, obviously, the rest energy and, because p' = 0 in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself.

Besides proving that the argument of the wavefunction is relativistically invariant, this calculation also demonstrates the relativistic invariance of the Planck-Einstein relation when modelling elementary particles.[18] This is why we feel that the argument of the wavefunction (and the wavefunction itself) is more *real* – in a *physical* sense – than the various wave equations (Schrödinger, Dirac, or Klein-Gordon) for which it is some solution.

In any case, a wave equation usually models the properties of the medium in which a wave propagates. We do not think the medium in which the matter-wave propagates is any different from the medium in which electromagnetic waves propagate. That medium is generally referred to as the *vacuum* and, whether or not you think of it as true nothingness or some medium, we think Maxwell's equations – which establishes the speed of light as an *absolute* constant – model the properties of it sufficiently well! We, therefore, think superluminal phase velocities are *not* possible, which is why we think de Broglie's conceptualization of a matter particle as a wave*packet* – rather than one *single* wave – is erroneous.[19]

Rutherford, Bohr, Dirac, Schrödinger, and electron orbitals

A particle will always be *somewhere* but, when in motion, its position in space and time should be thought of as a *mathematical* points only. The solution to the quantum-mechanical wave *equation* are

[16] We can use these simplified Lorentz equations if we choose our reference frame such that the (classical) linear motion of the electron corresponds to our *x*-axis.
[17] One can use either the general E = mc^2 or – if we would want to make it look somewhat fancier – the pc = Ev/c relation. The reader can verify they amount to the same.
[18] The relativistic invariance of the Planck-Einstein relation emerges from other problems, of course. However, we see the *added value* of the model here in providing a *geometric* interpretation: the Planck-Einstein relation effectively models the *integrity* of a particle here.
[19] See our paper on matter-waves, amplitudes, and signals.

equations of motion (Dirac, 1930). The electron in an atomic or molecular orbital moves at an (average) velocity which is a fraction of lightspeed only. This fraction is given by the fine-structure constant and the principal quantum number *n*:

$$v_n = \frac{1}{n}\alpha c$$

The velocities go down, all the way to zero for $n \to \infty$, and the corresponding cycle times increases as the *cube* of *n*. Using totally non-scientific language, we might say the numbers suggest the electron starts to lose interest in the nucleus so as to get ready to just wander about as a free electron.

Table 1: Functional behavior of radius, velocity, and frequency of the Bohr-Rutherford orbitals

n	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
$r_n \propto n^2$	1	4	9	16	25	36	49	64	81
$v_n \propto 1/n$	1	0.500	0.333	0.250	0.200	0.167	0.143	0.125	0.111
$\omega_n \propto 1/n^3$	1	0.125	0.037	0.016	0.008	0.005	0.003	0.002	0.001
$T_n \propto n^3$	1	8	27	64	125	216	343	512	729

The important thing is the energy formula, of course, because it should explain the Rydberg formula, and it does:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{{n_2}^2}\mathrm{E}_R + \frac{1}{{n_1}^2}\mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \frac{\alpha^2 \mathrm{m}c^2}{2}$$

The calculations are based on the assumption that, besides energy, electron orbitals also *pack* a discrete amount of physical action—a multiple of Planck's quantum of action, to be precise:

$$\mathrm{S}_n = nh \text{ for } n = 1, 2, \ldots$$

The orbital energies do *not* include the rest mass/energy of the *Zitterbewegung* (zbw) electron itself (0.511 MeV). In fact, they are *tiny* as compared to the electron's rest mass: 13.6 eV for *n* = 1 orbital of the hydrogen atom ^{1}H. This is the Rydberg energy (ER) in the formula above. It is the combined kinetic and potential energy of the electron in the (first) Bohr orbital. Using the *definition* of the fine-structure constant (as per the 2019 revision of SI units) and the rest energy ($E_0 = m_0c^2$) of the electron, we can write it as:

$$\mathrm{E}_R = \frac{\alpha^2 \mathrm{m}_0 c^2}{2} = \frac{1}{2}\left(\frac{{\mathrm{q_e}}^2}{2\varepsilon_0 \mathrm{h}c}\right)^2 \mathrm{m}_0 c^2 = \frac{{\mathrm{q_e}}^4 \mathrm{m}_0}{8{\varepsilon_0}^2 \mathrm{h}^2} \approx 13.6 \text{ eV}$$

Schrödinger's model of the hydrogen atom does not fundamentally differ from the Bohr-Rutherford model[20] but includes non-elliptical/non-symmetrical orbitals, which obey the *vis-viva* (literally: '*living force*') equation. For the gravitational force, this equation is written as:

[20] Around 1911, Rutherford had concluded that the nucleus had to be very small. Hence, Thomson's model – which assumed that electrons were held in place because they were, somehow, embedded in a uniform sphere of positive charge – was summarily dismissed. Bohr immediately used the Rutherford hypothesis to explain the emission spectrum of hydrogen atoms, which further confirmed Rutherford's conjecture, and Niels and Rutherford

$$v^2 = \mathrm{GM}(\frac{2}{r} - \frac{1}{a})$$

The parameter *a* is the length of the semi-major axis: *a* > 0 for ellipses but infinite (∞) or negative (a < 0) for non-closed loops (parabolas and hyperbolas, respectively). The Universe is closed and all lightlike particles (photons and neutrinos) must, therefore, *return*. Einstein's view that the nature of the gravitation may not reside in a *force* but in the mere geometry of the Universe (*our* Universe, which *we* live in), therefore, makes sense. In any case, efforts to model the gravitational force as a *residual* force have failed—so far, at least.

The two fundamental forces (Coulomb and nuclear/strong)

The idea of a particle assumes its *integrity* in space and in time. Non-stable particles may be labeled as transients (e.g. charged pions[21]) or, when very short-lived, mere resonances (e.g. neutral pion or tau-particle[22]). Hence, the Planck-Einstein relation does not apply: we can*not* model them as *equilibrium states*. We think the conceptualization of both the muon- as well as the tau-electron in terms of particle *generations* is unproductive.

The muon's lifetime – about 2.2 *micro*seconds (10^{-6} s) – is, however, quite substantial and we may, therefore, consider it to be a *semi-stable* particle. This explains why we get a sensible result when using the Planck-Einstein relation to calculate its frequency and/or radius. Inserting the 105.66 MeV (about 207 times the electron energy) for its rest mass into the formula for the *zbw* radius[23], we get:

$$a = c/\omega = c\frac{\hbar}{\mathrm{E}} = \frac{\hbar c}{\mathrm{m}c^2} = \frac{\hbar}{\mathrm{m}c} \approx 1.87\ \mathrm{fm}$$

The mean lifetime of a neutron in the open (outside of the nucleus) is almost 15 minutes, and the Planck-Einstein relation should, therefore, apply (almost) perfectly, and it does:

$$\frac{\mathrm{E}}{\mathrm{m_n}} = c^2 = a^2\omega^2 = a^2\left(\frac{\mathrm{m_n}c^2}{2\hbar}\right)^2 \Leftrightarrow a = \frac{4\hbar}{\mathrm{m_n}c} \approx 0.84\ \mathrm{fm}$$

jointly presented the model in 1913. As Rydberg had published his formula in 1888, we have a gap of about 25 years between experiment and theory here. It should be noted that Schrödinger's model accounts for subshells but still models orbital electrons as spin-zero electrons (zero *spin* angular momentum). It, therefore, models electron *pairs*, which explains the ½ factor Schrödinger's wave equation, which – we think – is relativistically correct.

[21] The mean lifetime of charged pions is about 26 *nano*seconds (10^{-9} s), which is about 1/85 times the lifetime of the muon-electron. We have no idea why charged pions are lumped together with neutral pions, whose lifetime is of the order of 8.4×10^{-17} s only. An accident of history? If anything, it shows the inconsistency of an analysis in terms of quarks.

[22] The (mean) lifetime of the tau-electron is 2.9×10^{-13} s only.

[23] See the derivation earlier in the text:

$$\left.\begin{matrix} \left.\begin{matrix} \mathrm{E} = \mathrm{m}c^2 \\ \mathrm{E} = \hbar\omega \end{matrix}\right\} \Rightarrow \mathrm{m}c^2 = \hbar\omega \\ c = a\omega \Leftrightarrow a = \dfrac{c}{\omega} \Leftrightarrow \omega = \dfrac{c}{a} \end{matrix}\right\} \Rightarrow \mathrm{m}a^2\omega^2 = \hbar\omega \Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2 = \hbar\frac{c}{a} \Leftrightarrow a = \frac{\hbar}{\mathrm{m}c}$$

The 1/4 factor is the 1/4 factor between the surface area of a sphere (A = 4πr^2) and the surface area of a circle (A = πr^2).[24] We effectively think of an oscillation in three rather than just two dimensions only here: the oscillation is, therefore, driven by two (perpendicular) forces rather than just one, and the frequency of *each* of the oscillators would be equal to $\omega = E/2\hbar = mc^2/2\hbar$: each of the two perpendicular oscillations would, therefore, pack one *half*-unit of only.[25] According to the equipartition theorem, each of the two oscillations should each pack *half* of the total energy of the proton. This spherical view of neutrons (and protons) – as opposed to the planar picture of an electron – fits nicely with packing models for nucleons.

However, the calculation of the radius above is quick-and-dirty only. It applies perfectly well for the (stable) proton, but we cannot immediately reconcile it with the idea of a neutron consisting of consisting of a 'proton' and an 'electron', which are the final decay products of a (free) neutron. We should immediately qualify the 'proton' and 'electron' idea here: the reader should effectively think in terms of *pointlike charges* here—rather than in terms of a massive proton and a much less massive electron![26] Both the 'proton' and the 'electron' carry the elementary (electric) charge but we think *both must be simultaneously bound in a nuclear as well as in an electromagnetic oscillation*. In order to interpret *v* as an orbital or tangential velocity, we must, of course, choose a reference frame. Let us first jot down the orbital energy equation for the nuclear field, however[27]:

$$\frac{\mathrm{E}_N}{\mathrm{m}_N} = \frac{v^2}{2} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}_N r^2}$$

[24] Cf. the 4π factor in the electric constant, which incorporates Gauss' Law (expressed in integral versus differential form).

[25] This explanation is similar to our explanation of one-photon Mach-Zehnder interference, in which we assume a photon is the superposition of two orthogonal *linearly polarized* oscillations (see p. 32 of our paper on basic quantum physics, which summarizes an earlier paper on the same topic).

[26] We do not have a hydrogen-like model here!

[27] A dimensional check of the equation yields:

$$\left[\frac{v^2}{2} + \frac{a\mathrm{k_e q_e^2}}{\mathrm{m}r^2}\right] = \frac{\mathrm{m}^2}{\mathrm{s}^2} + \frac{\frac{\mathrm{Nm}^3}{\mathrm{C}^2}\mathrm{C}^2}{\mathrm{kg}\cdot\mathrm{m}^2} = \frac{\mathrm{m}^2}{\mathrm{s}^2} + \frac{\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\mathrm{C}^2}{\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

We recommend the reader to regularly check our formulas: we do make mistakes sometimes!

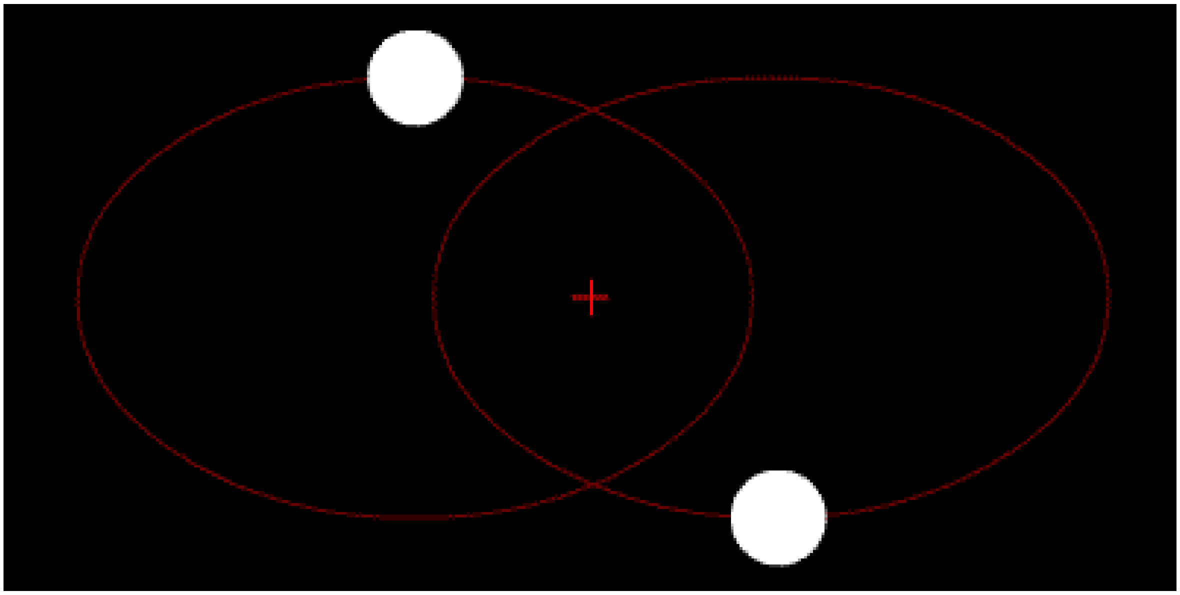


Figure 8: Two opposite charges in elliptical orbitals around the center of mass[28]

The mass factor m_N is the equivalent mass of the energy in the oscillation[29], which is the sum of the kinetic energy and the potential energy between the two charges. The velocity *v* is the velocity of the two charges (q_e^+ and q_e^-) as measured in the center-of-mass (*barycenter*) reference frame and may be written as a vector $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{r}) = \boldsymbol{v}(x, y, z) = \boldsymbol{v}(r, \theta, \varphi)$, using either Cartesian or spherical coordinates.

We have a plus sign for the potential energy term (PE = $a k_e q_e^2/mr^2$) because we assume the two charges are being kept *separate* by the nuclear force.[30] The electromagnetic force which keeps them *together* is the Coulomb force:

$$\frac{E_C}{m_C} = \frac{v^2}{2} + \frac{k_e q_e^2}{m_C r}$$

The total energy in the oscillation is given by the sum of nuclear and Coulomb energies and we may, therefore, write:

$$\frac{E}{m} = c^2 = \frac{E_C}{m_C} + \frac{E_N}{m_N} = \frac{v^2}{2} + \frac{k_e q_e^2}{m_C r} + \frac{v^2}{2} + \frac{a k_e q_e^2}{m_N r^2} \Leftrightarrow$$

$$c^2 - v^2 = \frac{k_e q_e^2}{m_C r} + \frac{a k_e q_e^2}{m_N r^2} = k_e q_e^2 \frac{m_N r + m_C a}{m_N m_C r^2} \Leftrightarrow$$

---

[28] Illustration taken from Wikipedia. For the orbital equations, see the MIT OCW reference course on orbital motion.

[29] We will use the subscripts x_N and x_C to distinguish nuclear from electromagnetic mass/energy/force. There is only one velocity, however—which should be the velocity of one charge *vis-á-vis* the other. We hope we made no logical mistakes here!

[30] We have a minus sign in the same formula in our paper on the nuclear force because the context considered two *like* charges (e.g. two protons). As for the plus (+) sign for the potential energy in the electromagnetic orbital energy, we take the reference point for zero potential energy to be the center-of-mass and we, therefore, have positive potential energy here as well.

$$c^2 = v^2 + \mathrm{k_e q_e^2}\frac{\mathrm{m}_N r + \mathrm{m}_C a}{\mathrm{m}_N \mathrm{m}_C r^2} = v^2 + \alpha\hbar c\frac{\mathrm{m}_N r + \mathrm{m}_C a}{\mathrm{m}_N \mathrm{m}_C r^2}$$

The latter substitution uses the definition of the fine-structure constant once more.[31] Dividing both sides of the equation by c^2, and substituting m_N and m_C for m/2 using the energy equipartition theorem, yields:

$$1 - \beta^2 = \frac{\alpha\hbar(r + a)}{\mathrm{m}cr^2} = \frac{\alpha\hbar}{\mathrm{m}c}\frac{r + a}{r^2}$$

It is a beautiful formula[32], and we could/should probably play with it some more by, for example, evaluating potential and kinetic energy at the *periapsis*, where the distance between the charge and the center of the radial field is closest. However, the limit values $v_\pi = c$ (for $r_\pi \to 0$) and $r_\pi = 0$ (for $v_\pi \to c$) are never reached and should, therefore, not be used.

One might hope to find a way to relate the orbital energy equations to the formula for the *zbw* radius to get a *specific* value not only for the neutron radius *a* – which should, *hopefully*, be very near to 0.84 fm (the proton/neutron diameter[33]) – but also for the *range parameter* of the nuclear force.[34] However, as we will show below, things are probably not that easy.

The nuclear range parameter and the fine-structure constant

At the very least, we have an *order of magnitude* for this range parameter now. This order of magnitude may be calculated by equating *r* to *a* in the formula above[35]:

$$1 - \beta_{r=a}^2 = \frac{\alpha\hbar}{\mathrm{m}c}\frac{2a}{a^2} = \frac{2\alpha\hbar}{a\mathrm{m}c} \Leftrightarrow 0 < \frac{2\alpha\hbar}{a\mathrm{m}c} < 1$$

$$\Rightarrow \frac{2\alpha\hbar}{a\mathrm{m}c} < 1 \Leftrightarrow \frac{2\alpha\hbar}{\mathrm{m}c} < a$$

[31] One easily obtains the $\mathrm{k_e q_e}^2 = \alpha\hbar c$ identity from the $\alpha = \frac{\mathrm{k_e q_e^2}}{\hbar c}$ formula. We think the 2019 revision of SI units *consecrates* all we know about physics.

[32] The *a* in the formula(s) above is the *range parameter* of the nuclear force, which is not to be confused with the *Zitterbewegung* (zbw) radius!

[33] The neutron radius should, in fact, be slightly *larger* than the proton radius because of the energy difference between a proton and a neutron, which is of the order of about 1.3 MeV (about 2.5 times the energy of a free electron). We note there is no CODATA value for the neutron radius. This may or may not be related to the difficulty of measuring the radius of a decaying neutral particle or, more likely, because the neutron mass/energy is not considered to be *fundamental*. However, one must get the range parameter *a* out of the formulas, *somehow*, and we, therefore, think experimental measurements of the (free) neutron radius are crucially important. As for quarks, we are happy to see NIST does not dabble too much into the quark *hypothesis*. At best, they are purely mathematical quantities (combining various *physical* dimensions) to help analyze and structure decay reactions of unstable particles, but that is being taken care of by the Particle Data Group.

[34] The reader should note that our neutron model implies a neutral (±) dipole, which relates to our previous efforts to develop an electromagnetic model of the deuteron nucleus. See our paper on the electromagnetic deuteron model.

[35] The range parameter is usually defined as the distance at which the nuclear and Coulomb potential (or the forces) equal each other. See: Ian J.R. Aitchison and Anthony J.G. Hey, *Gauge Theories in Particle Physics* (2013), section 1.3.2 (the Yukawa theory of force as virtual quantum exchange).

$$\Leftrightarrow 5.536 \times 10^{-15} < a$$

The $\alpha\hbar/\mathrm{m}c$ *constant* is, obviously, equal to the classical electron radius $r_e \approx 2.818$ fm (10^{-15} m)—which is of the order of the deuteron radius (about 2.128 fm) and which is the usual *assumed* value for the range parameter of the nuclear force.[36]

We think it is a significant result that the *lower limit* for the range parameter for the nuclear force must be at least twice at large. An *upper limit* for this range parameter must be based on the experimentally measured value for the radius of atomic nuclei. The scale for these measurements is the *picometer* (10^{-12} m). The nucleus of the very stable iron (^{26}Fe), for example, is about 50 pm.[37] The radius of the large (unstable) uranium (^{92}U) is about 175 pm.

The fine-structure constant may be involved again: 5.536 fm times $1/\alpha$ yields a value of about 77 pm. We think this is a sensible value for the (range of the upper) limit for the (nuclear) range parameter, which will, of course, depend on the shape (eccentricity) of the actual orbitals.

Of course, the stability of the nucleus of an atom is determined by other factors, most notably the magnetic coupling between the nucleons and the electrons in the atomic (sub)shells. This should, somehow, explain the 'magic numbers' explaining the (empirical) stability of nuclei, but the exact science behind this seems to be beyond us.[38]

The meaning of the fine-structure constant becomes somewhat clearer now:

- The fine-structure constants relates the classical electron radius, Compton radius and the Bohr radius of an electron: $r_e = \alpha\hbar/\mathrm{m}c = \alpha r_C = \alpha^2 r_B$
- The Bohr radius is the distance where the combined electromagnetic and nuclear potential ($1/r - a/r$) approaches the electromagnetic potential ($1/r$). Hence, we might say that the Compton radius separates nuclear from electromagnetic scale.

We may remind the reader here of the (average) radius of electron orbitals:

$$r_n = n^2 r_B = \frac{n^2 r_C}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$$

Nuclear orbitals – or combined nuclear-electromagnetic, we should say – orbitals are of the order of $\alpha\hbar/\mathrm{m}c$. We think this is *not* a coincidence but, as with magic numbers, it will take a while for the more numerology-oriented physicists to figure out a more *exact* explanation. 😊

We should, of course, raise the obvious question here: this model – combining electromagnetic and nuclear force – yields a distance scale which is *not* compatible with the neutron radius (0.84 fm). That is why we think the neutron must be a genuine *nuclear* oscillation. We are a bit at a loss, however, as to

[36] See footnote 35.
[37] This is Feynman's calculated radius of a hydrogen atom, but the *measured* radius of the hydrogen *nucleus* is about half of it. To be precise, the empirical value is about 25 pm according to the Wikipedia data article on atomic radii. We leave it to the reader to think about the 1/2 factor and the fine-structure constant as a scaling parameter.
[38] See the Wikipedia article on magic numbers (nuclei).

how to model that *exactly*? Perhaps we should return to modelling the neutron as a massive proton being enveloped by the pointlike charge.[39]

Conclusions

When reading this, my kids might call me and ask whether I have gone mad. Their doubts and worry are not *random*: the laws of the Universe are deterministic (our macro-time scale introduces probabilistic determinism only). Free will is real, however: we analyze and, based on our analysis, we determine the best course to take when taking care of business. Each course of action is associated with an anticipated cost and return. We do not always choose the best course of action because of past experience, habit, laziness or – in my case – an inexplicable desire to experiment and explore new territory. Is that *free will*?

We are not sure. Ontology is the logic of being. The separation between consciousness and its object is no more real than consciousness' inadequate knowledge of that object. The knowledge is inadequate only because of that separation.[40] Hegel completed the work of philosophy. Physics took over as the science of that what *is*. It should seek to further reduce rather than multiply concepts.

Brussels, 17 February 2021

[39] For suggestions in this regard, see our paper on the mass-without-model for protons and neutrons.
[40] Quoted from the Wikipedia article on Hegel's *Phänomenologie des Geistes* (1807).

Annex I: Dirac's energy and Schrödinger's wave equation

Dirac starts by writing the classical (relativistic) energy equation for a particle (an electron) as:

$$\mathrm{E} = \mathrm{m}c^2 = \frac{\mathrm{W}}{c^2} - p_r^2$$

This equation raises obvious questions and appears to be based on a misunderstanding of the fundamental nature of an elementary particle—which, in the context of Dirac's lecture[41], is a free or bound electron. According to the *Zitterbewegung* hypothesis (which Dirac mentions prominently) and applying the energy equipartition theorem, half of the energy of the electron will be kinetic, while the other half is the energy of the field which keeps the pointlike (*zbw*) charge localized. The pointlike charge is photon-like[42] and, therefore, has zero rest mass: it acquires a relativistic or *effective* mass $\mathrm{m}_\gamma = \mathrm{m}_e/2$. Its kinetic energy is, therefore, equal to[43]:

$$\mathrm{KE} = \mathrm{W} = \frac{\mathrm{m}_\gamma v^2}{2} = \frac{\mathrm{m}_\mathrm{e} v^2}{4}$$

Dirac refers to the p_r in the equation as *momentum*, but this must represent potential energy *in the reference frame of the particle itself*. If the oscillation's nature is electromagnetic, then this potential energy is given by[44]:

$$\mathrm{PE} = \frac{\mathrm{k_e q_e^2}}{r}$$

It is useful to write the orbital energy equation as energy *per unit mass*:

$$\frac{\mathrm{E}}{\mathrm{m}_\gamma} = c^2 = \frac{v^2}{2} + \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_\gamma r} \Leftrightarrow 1 - \frac{v^2}{2c^2} = \frac{\mathrm{k_e q_e^2}}{\mathrm{m}_\gamma c^2 r}$$

We may also write this in terms of the relative velocity $\beta = v/c$ and the fine-structure constant α[45]:

$$1 - \frac{\beta^2}{2} = \frac{2\alpha\hbar}{\mathrm{m_e} cr}$$

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[41] https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

[42] We avoid this term, however, because photons do *not* carry charge: this distinguishes light-particles (photons and neutrinos) from matter-particles.

[43] This equation is relativistically correct because (i) the velocity *v* is an orbital/tangential velocity and (ii) we use the relativistic mass concept. The velocity *v* is equal to the speed of light (*c*) but, in a more general treatment (e.g. elliptical orbitals), *v* should be distinguished from *c*.

[44] $U(r) = V(r)\cdot q_e = (k_e\cdot q_e/r)\cdot q_e = k_e\cdot q_e^2/r$ with $k_e \approx 9\times10^9$ N·m²/C². Potential *energy* (U) is, therefore, expressed in *joule* (1 J = 1 N·m), while potential (V) is expressed in *joule/Coulomb* (J/C).

[45] Since the 2019 revision of the SI units, the electric, magnetic, and fine-structure constants have been co-defined as $\varepsilon_0 = 1/\mu_0 c^2 = q_e^2/2\alpha hc$. The CODATA/NIST value for the standard error on the value ε_0, μ_0, and α is currently set at 1.5×10^{10} F/m, 1.5×10^{10} H/m, and 1.5×10^{10} (no *physical* dimension here), respectively. We use the $\mathrm{m_e} = \mathrm{m}_\gamma/2$ once more. To quickly check the accuracy and, more importantly, their *meaning*, we recommend the reader to do a dimensional check. We have a purely numerical equation here (all physical dimensions cancel):

$$\left[1 - \frac{\beta^2}{2}\right] = \left[\frac{2\alpha\hbar}{\mathrm{m_e} cr}\right] = \frac{\mathrm{Nms}}{\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}\frac{\mathrm{m}}{\mathrm{s}}\mathrm{m}}$$

When adding a linear component to the orbital motion of the pointlike charge, the electron oscillation will move linearly in space and we can, therefore, associate a *classical* velocity v_e and a classical momentum p_e with the *Zitterbewegung* oscillation. We discussed and illustrated this sufficiently in the body of our paper. We must now distinguish the *rest* energy of the electron (E_0) and its kinetic energy, which, referring to the classical momentum, we will denote by $E_p = E - E_0$. Writing E as $E = mc^2$ again, we can use the binomial theorem, to expand the energy into the following power series[46]:

$$\mathrm{m}c^2 = \mathrm{m}_0c^2 + \frac{1}{2}\mathrm{m}_0v^2 + \frac{3}{8}\mathrm{m}_0\frac{v^4}{c^2} + \cdots = \mathrm{m}_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right)$$
$$= \mathrm{m}_0c^2\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

This formula separates the rest energy $E_0 = m_0c^2$ from the kinetic energy E_p, which may, therefore, be written as:

$$\mathrm{E_p} = \mathrm{E_0}\left(\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

Schrödinger's wave equation models electron orbitals whose energy *excludes* the rest energy of the electron. We are not sure whether Dirac's wave equation correctly integrates this rest energy again: are Dirac's p_r (r = 1, 2, 3,…) references to the β^2, $(\beta^2)^2$,.. terms in the power series? We think of this series expansion as a *mathematical* exercise only: we are not able to relate them to anything real—we think of forces and/or potentials here!

We offer further comments on the use of wave equations to model motion in the Annex to our paper on the matter-wave.[47] We think it is rather telling that Richard Feynman does not bother to present Dirac's wave equation in his *Lectures on Physics* (1963). We think it is because he cannot make sense of it either. Feynman's wave equation for a free particle is the following[48]:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2\mathrm{m}}(i\hbar\boldsymbol{\nabla} + q\boldsymbol{A})^2\psi + q\phi\psi$$

This equation incorporates the *integrity* of Planck's quantum of action as the unit of the angular momentum of the oscillation (cf. the $i\hbar$ factor). The (scalar) potential ϕ can be electromagnetic, nuclear or a combination thereof, acting on the (electric) charge q. Assuming the scalar potential varies with time, the vector potential ***A*** is probably to be derived from the Lorenz gauge condition in electromagnetic theory:

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[46] See Feynman's *Lectures*, I-15-8, and I-15-9 (relativistic dynamics). The expansion is based on an expansion of $m = \gamma m_0$:

$$\mathrm{m} = \frac{\mathrm{m}_0}{\sqrt{1 + \frac{v^2}{c^2}}} = \mathrm{m}_0(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots)$$

This is multiplied with c^2 again to obtain the series in the text.

[47] Jean Louis Van Belle, *De Broglie's matter-wave : concepts and issues*, May 2020.

[48] See: Feynman, III-21, Schrödinger's equation in a magnetic field and his equation of continuity for probabilities. We took the liberty of writing $1/i$ as $-i$. We also multiplied the right-hand side of Feynman's equation with $(-1)\cdot(-1) = +1$, and substituted the dot product of the $-i\hbar\nabla - q\boldsymbol{A}$ operators for the square of the same operator.

$$\nabla \cdot \boldsymbol{A} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$$

For a time-independent scalar potential, which is what we have been modeling so far, the Lorentz gauge is zero ($\nabla \cdot \boldsymbol{A} = 0$) because the time derivative is zero: $\partial\phi/\partial t = 0 \Leftrightarrow \nabla \cdot \boldsymbol{A} = 0$.[49] The magnetic field, therefore, vanishes. The time-dependent magnetic field – or its nuclear equivalent – should absorb half the energy in accordance with relativity theory[50] and it should then be easy to develop the equivalent of Maxwell's equations for the nuclear force field using the theorems of Gauss and Stokes.

[49] The Lorenz gauge does not refer to the Dutch physicist H.A. Lorentz but to the Danish physicist Ludvig Valentin Lorenz. It is often suggested one can choose other gauges. We do not think so. We think the gauge is given by relativity theory, and that is the same for time-dependent and time-independent fields. It does vanish, however, time-independent fields (cf. electromagnetostatics). See our remarks on the vector potential and the Lorentz gauge in our paper on the electromagnetic deuteron model.

[50] When using natural units ($c = 1$), the relativity of electric and magnetic fields becomes more obvious.

Annex II: The quark *hypothesis*

Strange kaons

Kaons (aka K-mesons[51]) are supposed to consist of a *strange* quark (or its antimatter counterpart) and some other quark (the *up* or *down* quark, or its antimatter counterpart). Like pions (π-mesons), the charged kaons and the neutral kaon have very little in common, except a somewhat similar mass: a bit less than 500 MeV/c^2, so that is about half of the proton/neutron mass. However, charged kaons have a (mean) lifetime of 12.4 *nano*seconds (10^{-9} s) – quite comparable to the mean lifetime of charged pions (about 26×10^{-9} s)[52] – while the mean lifetime of a neutral kaon is… Well… We have *two* neutral kaon-particles—one with a shorter and one with a longer lifetime: K_L^0 (about 52×10^{-9} s) and K_S^0 (89×10^{-12} s).[53]

Like pions, kaons were first seen in decays of cosmic rays in bubble chambers or on photographic plates. In fact, pions and kaons are closely related, as shown in Feynman's drawings (III-11-14) of the decay reaction of a π^- and a π^0.

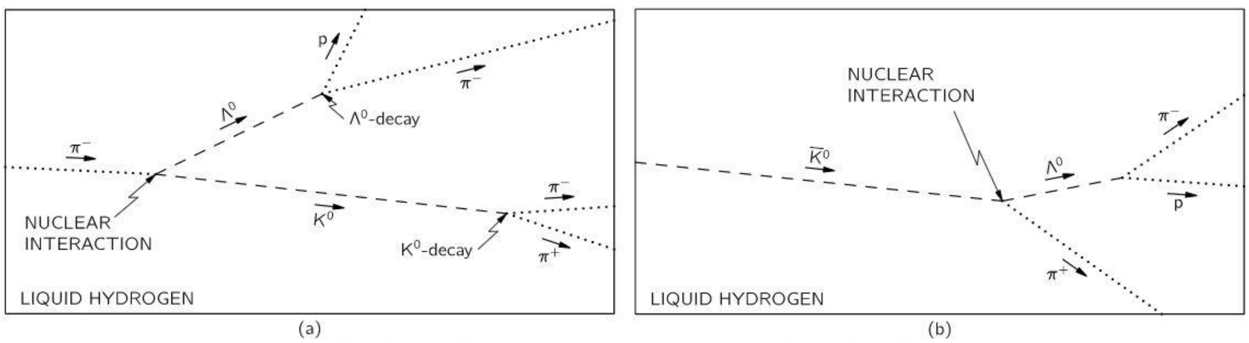


Fig. 11–5. High-energy events as seen in a hydrogen bubble chamber. (a) A π^- meson interacts with a hydrogen nucleus (proton) producing a Λ^0 particle and a K^0 meson. Both particles decay in the chamber. (b) A $\overline{K}^0$ meson interacts with a proton producing a π^+ meson and a Λ^0 particle which then decays. (The neutral particles leave no tracks. Their inferred trajectories are indicated here by light dashed lines.)

Many different reactions are possible. The Particle Data Group lists all of them. Feynman focuses very much on the hypothetical reactions that do *not* happen, such as this one:

$$K^0 + p \rightarrow \Lambda^0 + \pi^+$$

We have an intermediate (neutral) *lambda* baryon[54] (Λ^0) here: it is *very* massive – about 1115.6 MeV/c^2 – but also short-lived: as shown in figure (b) above, it decays into a π^- and a proton (p). The charged pion decays into a muon (or antimuon) and, therefore, ultimately into an electron (or positron), so we should not be concerned with it, either. The question here is: why do we observe $\underline{K}^0 + p \rightarrow \Lambda^0 + \pi^+$ reactions[55]

[51] Mesons are defined as subatomic particles composed of an equal number of quarks and antiquarks, *usually one of each*, bound together by strong interactions (read: the strong force).

[52] The mean lifetime of a neutral pion (0) is 8.4×10^{-17} s. If the charged pion can be thought of as a transient, then the neutral pion is just an extremely short-lived *resonance*.

[53] We would rather think of a K_S^0 particle as a very short-lived *resonance*, as opposed to a somewhat more robust *transient* particle, but let us go along with the argument.

[54] A baryon is supposed to consist of an *odd* number of quarks, usually three.

[55] We use an *underbar* ($\underline{K}$) instead of an overbar to denote the antimatter counterparts of a particle out of laziness (we do not want to use the equation editor all of the time).

but *not* the $K^0 + p \rightarrow \Lambda^0 + \pi^+$ reaction? We think antimatter differs from matter only because of opposite spin – or, to be precise, because of its opposite *spacetime signature*[56] – but, surely, the $\Lambda^0 + \pi^+$ come with two possible directions of spin as well, don't they?

Let us look at the [PDG listings of kaon reaction](). […] *Surprise, surprise!* These do *not* list the reactions involving $\underline{K}^0$ particles! Why not? We are not sure. It is *very* confusing: Feynman's account does not match the PDG picture. Neutral particles are supposed to be their own antiparticles, no? Yes. We think so, at least.

So what can we say? Nothing much. Let us focus on the instability part.

Transient oscillations: what is real?

Feynman argues one needs the concept of strangeness to explain why this or that reaction does *not* take place, but the argument does not convince us—especially because strangeness is *not always* conserved. When that happens, the decays are supposed to be *weak* decays (as opposed to strong, nuclear decays), which, according to Feynman (and the inventors of these strong and weak interactions) also need not respect this new strangeness conservation law. The Particle Data Group effectively invokes CP or T violation regularly: the ubiquitous symmetry-breaking which explains everything that cannot be explained.[57] *Anything goes*, it seems.

The thing that grabs *my* attention much more is the shape of the wavefunctions, which we copied from the same source (Feynman III-11-6):

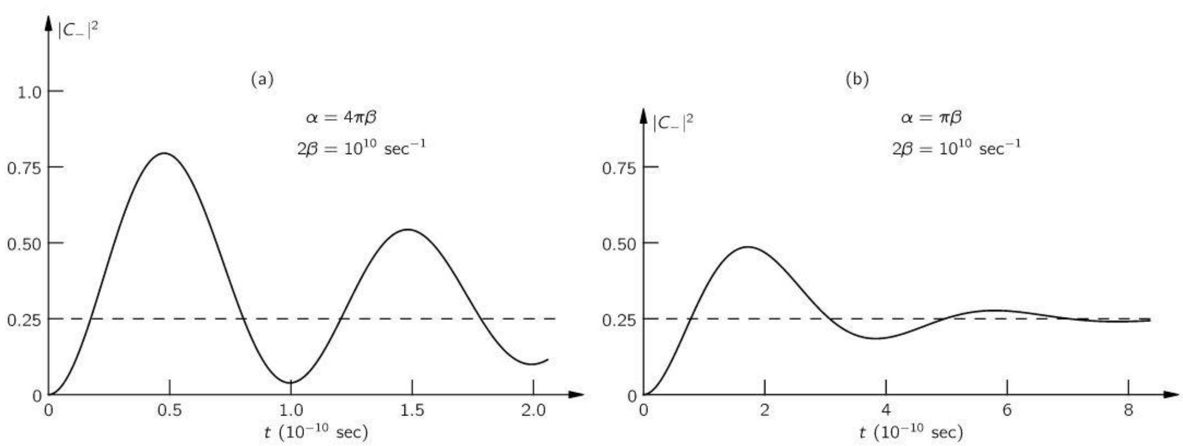


Fig. 11–6. The function of Eq. (11.56): (a) for $\alpha = 4\pi\beta$, (b) for $\alpha = \pi\beta$ (with $2\beta = 10^{10}$ sec^{-1}).

The coefficients – the C_- coefficient above – that we get out of the Hamiltonian system of equations for the K^0 and/or $\underline{K}^0$ system is not a stable wavefunction: we get a transient or – the boundary between transients and resonances is not clear-cut – a *resonance*: an *unstable energy state*, to which we can*not* apply the Planck-Einstein relation ($E = \hbar\omega$).

We admire this business of trying to reduce the *complexity* of the situation on hand through the introduction of the quark *hypothesis* but, paraphrasing H.A. Lorentz, we do not immediately see the

[56] See p. 34 to 36 of our [paper on quantum behavior]() (modeling spin and antimatter).
[57] Combined CPT-symmetry must hold, however. See the [discussion on our blog]().

need to elevate quarks (and the related *form factors*) to *ontological* status.[58] Our criticism is, therefore, not as scathing as our criticism on the 'discovery' of the Higgs field/particle[59], but the *nature* of our criticism remains the same.

It is, of course, quite OK to resort to mathematical techniques – we are dealing with some kind of factor analysis to find the *S*-matrix (scattering or *Spur*-matrix[60]) here – when we cannot explain some reaction which happens or does *not* happen on the basis of the classical conservation laws (conservation of charge, energy, physical action, and linear and angular momentum), but it is *not* OK to recognize this is just some kind of *engineering* approach to find a *numerical* approximation to a problem that, basically, amounts to a (much more complicated) three-body problem.

Was the 1969 Nobel Prize for Murray Gell-Mann justified? First, it should have been shared with others who were working on similar analyses of non-equilibrium states (Yuval Ne'eman, George Zweig and (many) others) and, second, the Nobel Prize in Physics is usually not awarded for a significant breakthrough in *numerical* or *mathematical* analysis. Gell-Mann did not discover some new physical law or *physical reality*. Englert and Higgs did not either.

The End of Science – all that is left is *engineering*, right? – is not easy to digest. :-/ In any case, we should not get too philosophical here. Let us look at those coefficients and try to find out what they might *mean*.

An analysis of non-equilibrium states

We will closely follow Feynman's treatment here but simplify and add our own remarks. It starts off with a rather typical set of Hamiltonian equations for what Feynman refers to as the $K^0\underline{K}^0$ system but we think of it as simply modelling two opposite spin states of the same neutral particle:

$$i\hbar\frac{\mathrm{d}C_+}{\mathrm{d}t} = E_0C_+ + AC_- + AC_+$$
$$i\hbar\frac{\mathrm{d}C_-}{\mathrm{d}t} = E_0C_- + AC_+ + AC_-$$

Feynman then gives you the usual *Spiel* – transformation to another set of base states and the associated *trial* solutions – but with a notable exception: the frequency ω in the $a\cdot e^{-i\omega t}$ wavefunction is *not* a real number which we get from the Planck-Einstein relation ($\omega = E_0/\hbar$ or $\omega = A/\hbar$ or some *linear* combination hereof). *No!* This time it is a complex number $\omega = \alpha + i\beta$. Of course, we know what that means: a friction term, or a *driven* oscillation—a *transient*, in short (as opposed to a pure harmonic oscillation). That is what is depicted above (Feynman's Fig. 11-6). As for the values for α and β, Feynman writes this:

> "Since nobody knows anything about the inner machinery, that is as far as Gell-Mann and Pais could go. They could not give any theoretical values for α and β. And nobody has been able to do so to this date. They were able to give a value of β obtained from the experimentally

[58] The comments of H.A. Lorentz in regard of the 'new' quantum-mechanical theories at the occasion of the last Solvay Conference (1927) he had been in charge of, were this: 'Ne pourrait-on pas garder le déterminisme en faisant l'objet d'une croyance? *Faut-il nécessairement ériger l'indéterminisme en principe?'*

[59] See our *Smoking Gun Physics* paper, July 2019.

[60] The terms are certainly *not* synonymous: *related*, at best!

observed rate of decay into two π's ($2\beta = 10^{10}$ s^{-1}), but they could say nothing about α. […] There are some rough results which indicate that the α is not zero, and that the effect really occurs—they indicate that α is between 2β and 4β. That is all there is, experimentally."

Feynman's conclusion is this:

> "The analysis we have just described is very characteristic of the way quantum mechanics is being used today in the search for an understanding of the strange particles. All the complicated theories that you may hear about are no more and no less than this kind of elementary hocus-pocus using the principles of superposition and other principles of quantum mechanics of that level."

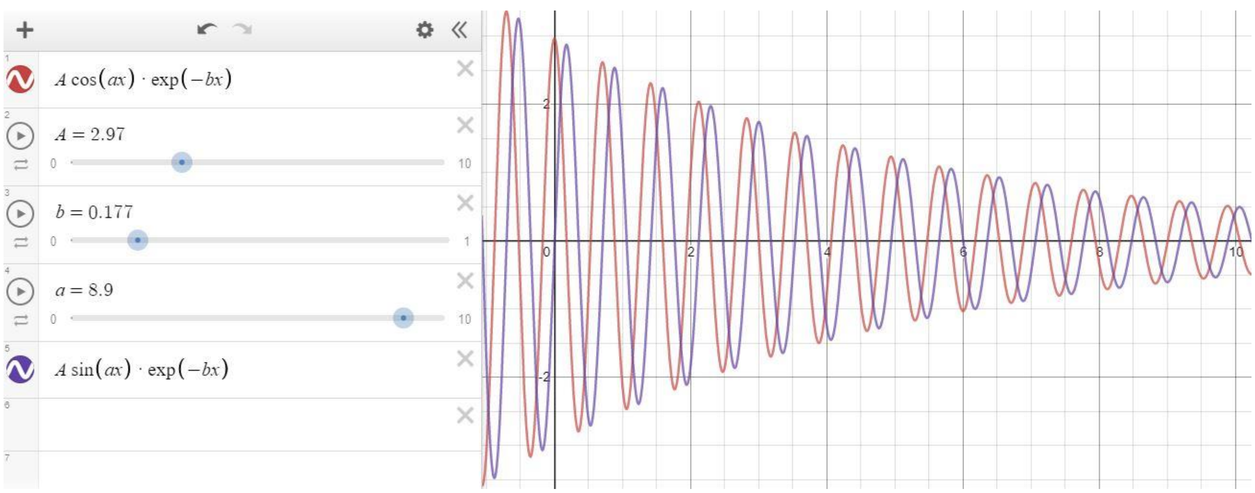
We agree—but we are even less sure now about the question of whether or not Murray Gell-Mann deserved a Nobel Prize for Physics. Perhaps the next Prize should go to the [Wolfram Physics project](). 😊

The math of transients

The math of transients is not so difficult: it suffices to multiply the wavefunction (let us refer to our unstable particle as U, so we can denote something stable as S) with a *real-valued* negative exponential:

$$\psi_U = A \cdot e^{i(\alpha + i\beta)t} = A \cdot e^{i\cdot\alpha t} \cdot e^{-\beta t}$$

The illustration below shows how this works: both the real and imaginary part of the wavefunction – think of the electric and magnetic field vector here, for example – lose amplitude and, therefore, energy.



Where does the energy go? It cannot get lost, so we must assume it goes into the field, where it contributes to progressively building up another oscillation. The combined particle-field combination will, therefore, be *something stable* (S) that conserves energy (and, therefore, mass):

$$\psi_S = A \cdot e^{i\cdot\alpha t} \cdot e^{-\beta t} \cdot e^{\beta t} = A \cdot e^{i\cdot\alpha t} = A \cdot e^{i\cdot\frac{E}{\hbar}t}$$

We may apply the usual interpretation to the α and β factors:

1. The β in the $e^{-\beta t}$ decay function gives us the mean lifetime of the unstable particle ($\tau = 1/\beta$) and, as Feynman points out, such mean lifetime will be of the order of 10^{-9} to 10^{-12} seconds.

2. The α in the $Ae^{-\alpha t}$ decay function is equal to E/ħ and will generally be a frequency (its dimension is s^{-1}) that is much *larger* than β. The frequency of an electron, for example, can be calculated as:

$$\omega_e = \frac{E_e}{\hbar} = \frac{8.187 \times 10^{-14}\ Nm}{1.054 \times 10^{-14}\ Nms} \approx 7.76 \ldots \times 10^{20}\ s^{-1}$$

As we can see, we have a difference of 10 orders of magnitude (10^{10}) between α and β here, and an electron is not very massive as compared to a proton! Of course, this explains that transient or resonant particles do not last very long, but still pack like 10^{10} cycles during their short lifetime!

We should wrap up and let us, therefore, make one final remark in regard to asymmetries in Nature. Spin may be left or right-handed, but the imaginary part – think of the magnetic field vector in an electromagnetic oscillation – will always *lag* 90 degrees behind the real part or – if you like – it will *lead* the real part by 270 degrees. This does not only define an *absolute* direction of rotation in space, but it also introduces an asymmetry, which – in our view – should also help to explain why certain reactions do *not* take place. We think this must be the cause of the CP- and T-breaking that is observed in such reactions. However, always remember we still have (combined) CPT-symmetry!

Hermiticity and reversibility

Physicists try to model these reactions and processes using the following rather general *matrix* equation, which has an *S*- or *A*-matrix at its center: it operates on some state $|\psi\rangle$ to produce some other state $|\varphi\rangle$:

$$\langle\varphi|A|\psi\rangle$$

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

$A^\dagger$ is, of course, the conjugate transpose of A – we write: $A^\dagger_{ij}=(A_{ji})^*$ – and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^\dagger = A$. Many quantum-mechanical operators are Hermitian, and we will also often *impose* that condition on the *S*-matrix.[61] Why? Because you should think of an operator or an *S*-matrix as a symmetric apparatus or a reversible process. It is as simple as that. We, therefore, think that the Hermiticity condition amounts to a simple reversibility condition[62] and, as mentioned above, we think certain processes may not be reversible because of the asymmetry in the wavefunction itself!

So do physicists really resemble econometrists modeling input-output relations?[63] We think the answer is yes, and no! The main difference is the complexity: those $\langle\varphi|$ and $|\psi\rangle$ states should probably be rewritten as multidimensional arrays, and there are a lot of constraints on that matrix *S* (or *A*)!

We will qualify this *statement* in the next section (Annex III).

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[61] See Feynman, Vol. III, Chapter 8 (*the Hamiltonian matrix*) for a rather pleasant explanation of the game.
[62] See our paper on the difference between a theory, a calculation, and an explanation. Also see our blog post on the end (?) of physics.
[63] See our (somewhat disrespectful) blog post on the end (?) of physics.

Annex III: A complete description of the Universe

We have a $\langle\varphi|$ *state* in, a $|\psi\rangle$ *state* out and we should relate both through the *S*-matrix[64]:

$$\langle\varphi|S|\psi\rangle$$

The deterministic worldview implies reversibility and we should, therefore, also be able to go back from the final state $|\psi\rangle$ (the *ket* in Dirac's *bra-ket* notation) to the initial state $\langle\varphi|$ by taking the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

The complex conjugate implies a reversal of *spin*, not of actual *time*, although both look the same from a *formal* (mathematical) point of view: time goes in one direction only and C, P and T-symmetry may be broken, but the *combined* CPT-symmetry should hold, *always*.

Both the initial as well as the final state *vectors* consist of a bunch of matter- and light-particles. Matter-particles carry charge and may be stable or not. The β-factor will be non-zero for unstable particles, which implies they have a finite decay time $\tau = 1/\beta$:

$$\psi_{\beta\neq0} = A\cdot e^{i(\alpha+i\beta)t} = A\cdot e^{i\cdot\alpha t}\cdot e^{-\beta t}$$

In contrast, the lifetime of stable particles is infinite and β is, therefore, equal to zero:

$$\psi_{\beta=0} = A\cdot e^{i(\alpha+i\beta)t} = A\cdot e^{i\cdot\alpha t}\cdot e^{-0\cdot t} = A\cdot e^{i\cdot\alpha t}$$

The α-factor may be positive or negative – representing *up* or *down* spin respectively – but β is always positive. The α-factor will also be usually much larger than β and we may, therefore, write: $|\alpha| >> \beta$.

α is the (angular) frequency of the particle and is given by the energy (for unstable particles, this energy is the *initial* energy) and the Planck-Einstein relation: $\alpha = \omega = E/\hbar$. We assume the coefficients A_k (the amplitude of the oscillation) are *normalized*: the mass/energy of all particles ($E_k = m_k c^2$) must add to the total energy of the system ($1 < k < n$), with *n* the total number of particles in the initial or final state. The $\langle\varphi|S|\psi\rangle$ *statement*, therefore, takes care of the mass/energy conservation principle.

The initial and final state may consist of a different number of particles and we take *n* to the largest number of the two. However, in general, particle interactions are rather simple – like *two* particles interacting to yield three or four other particles, or one particle decaying into a limited number of other (stable or unstable) matter-particles, with light-particles (photons) taking care of the excess energy and (linear/angular) momentum. The physicist should, therefore, consider the $\langle\varphi|S|\psi\rangle$ statement to describe a *single* event.

Events *change* the potential fields surrounding the (charged) particles and, therefore, one event is to be associated with *static* (stable) potentials. Particle-field interactions must also obey the mass-energy equivalence relation and the Planck-Einstein relation, which is why we refer to light-particles as field particles.

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[64] It is interesting, historically speaking, that John Archibald Wheeler (whom we know from the mass-without-mass models of elementary particles) and Erwin Schrödinger independently developed the idea of the *S*-matrix (the *s* stands for *scattering*, not for *Spur*) in the late 1930s/early 1940s.

The choice of a *unique* reference frame also takes care of the conservation of *linear* momentum because it splits the $E_0 \cdot t'/\hbar$ factor over its $E_v \cdot t/\hbar$ and *p·x components* (see the section on the relativistic invariance of the wavefunction in this paper).

What about the conservation of the total amount of *angular* momentum (spin)?[65] This *data* (or *information*) is implicit in the particle wavefunctions as well[66] and so we are left with the charge conservation law only:

- Neutral matter-particles (e.g. neutrons) consist of an equal number of opposite charges (the neutron, for example, consists of a positive and negative charge).
- Light-particles carry no charge but they do carry energy and (linear and angular) momentum. Photons carry electromagnetic energy/momentum, and neutrinos carry nuclear (strong) energy/momentum.

This, then, is the only *constraint* on the ⟨φ|*S*|ψ⟩ particle reaction: the total charge of the matter-particles going in must equal the total charge of the matter-particles going out.[67] There is no need for baryon, lepton, or strangeness conservation laws.

In short, we may complement Feynman's *Unworldliness* equation with Dirac's *statement* or definition of *events* as described above:

$$\mathrm{U} = 0 \Leftrightarrow \mathrm{E} - \mathrm{E} = 0 \Leftrightarrow \mathrm{m}c^2 - \boldsymbol{hf} = 0 \Leftrightarrow \frac{\mathrm{m}}{\boldsymbol{f}} = \frac{c^2}{\boldsymbol{h}}$$

$$\langle \varphi_j | S_j | \psi_j \rangle \text{ for all } events\ j\ (j = 1, 2, \dots \infty)$$

Feynman's U = 0 equation describes the laws of the Universe, which *govern* the events, happening simultaneously and/or in succession. All events that are *possible*, are real, and have been listed (in rather excruciating detail) by the Particle Data Group.

The *statements* above are not a formula for happiness. Happiness is a *state of mind* which emerges from regularly taking the best course of action when taking care of personal (human) business, which contributes to good *habits*. Taking care of others *first* (i.e. developing a sense of *duty*) is the best way to take care of oneself.

Any irreversibility of actions or processes must be rooted in the asymmetry in the wavefunction: the imagery part *lags* the real part by a right angle. Mankind should, perhaps, reverse its *course* of action. 😊

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[65] We obviously should not be measuring spin here in terms of *up* or *down* but quantify the exact *amount* of spin as well as keep track of all *directions* in space.

[66] We can apply the quantum-mechanical angular momentum to the wavefunction. More in general, the quantum-mechanical operators gives us all of the *physical* characteristics that are implicit in the wavefunction that describes the particle.

[67] For matter-antimatter *pair* creation/annihilation, see our paper on this topic.