Formulas of Feigenbaum Constants and Their Physical Meanings (viXra:2101.0187v2)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper supposes that Feigenbaum constants should be rational numbers in the world of nuclides, gives their formulas in fractional number format and exhibits the physical meanings of the factors in the formulas, especially their relationships with nuclides, the fine-structure constant and 2π. This paper also supposes that there would be the third Feigenbaum constant and gives its two possible approximate values. Formulas of the fine-structure constant α_1, Feigenbaum constant δ and 2π are also given, briefly to be $\alpha_1\delta^2(2\pi)\approx 1$, and their relationships with nuclides are illustrated.

Keywords: Feigenbaum constants; the fine-structure constant; 2π; nuclides.

1. Introduction

Feigenbaum constants are characterizing constants in chaotic systems, it is assumed by scientific community that they would characterize chaos just as 2π stands for periodicity. The nuclei of nuclides except proton should be multiple-body systems and hence chaos should be the common state in the world of nuclei. So Feigenbaum constants should express some functions in nuclides.

There are two Feigenbaum constants, i. e., δ=4,6692… and α=2.5029…, and mathematically they should be irrational numbers with infinite digits.

In our previous papers[1,2,3,4], we had already given formulas of the fine-structure constant and given two values of it, i. e., α_1=1/137.035999037435 and α_2=1/137.035999111818 which are rational numbers with 15 digits. The values and formulas of the fine-structure constant show strong relationships with nuclides, and hereby some relevant examples of these relationships are listed as follows. And it shows that the values of the fine-structure constant express as integer numbers of 136, 137 and 138 in the world of nuclides.

$$^{82,83,84,86}_{36}Kr_{46,47,48,50}\quad ^{85,87}_{37}Rb_{48,50}\quad ^{86,87,88}_{38}Sr_{48,49,50}\quad ^{89}_{39}Y_{50}\quad ^{136,137,138}_{56}Ba_{80,81,82}\quad ^{208}_{82}Pb_{126}\quad ^{209}_{83}Bi^{*}_{126}$$

$$^{209}_{84}Po^{*}_{125}\quad ^{210}_{85}At^{*}_{125}\quad ^{222}_{86}Rn^{*}_{136}\quad ^{223}_{87}Fr^{*}_{136}\quad ^{226}_{88}Ra^{*}_{138}\quad ^{227}_{89}Ac^{*}_{138}\quad ^{285}_{112}Cn^{*}_{173}\quad ^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$$

In our previous paper[1,2,3,4], we also exhibited the relationships of 2π with nuclides, for example, 2π≈6.28=(4×157)/100 relates to nuclides as follows.

$$^{100}_{44}Ru_{56}\quad ^{118,119}_{50}Sn_{68,69}\quad ^{168}_{68}Er_{100}\quad ^{169}_{69}Tm_{100}\quad ^{257}_{100}Fm^{*}_{157}\quad ^{2\cdot157}_{126}Ch^{*}_{4\cdot47}\quad ^{400}_{157}Ch^{ie}_{243}$$

In this paper, we suppose Feigenbaum constants should also be rational numbers in the world of nuclides, give their formulas and exhibit their relationships with nuclides and hence with the fine-structure constant and 2π.

2. Formulas of Feigenbaum Constants in Fractional Numbers

It is supposed that Feigenbaum constants should be rational numbers with 15 digits and have the following fractional formulas and relationships with nuclides.

Note: 136=8·17, 138=6·23

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326$$

$$=\frac{1}{4}-\frac{1}{27}+\frac{1}{4\cdot 9\cdot 23}-\frac{1}{2\cdot3\cdot7\cdot23\cdot(2\cdot3\cdot(4\cdot3\cdot11-1)+1)+\frac{2\cdot23}{3\cdot19}}$$

$$^{9}_{4}Be_{5}\quad ^{19}_{9}F_{10}\quad ^{23}_{11}Na_{12}\quad ^{27}_{13}Al_{14}\quad ^{36,38,40}_{18}Ar_{18,20,22}\quad ^{39,40,41}_{19}K_{20,21,22}\quad ^{45}_{21}Sc_{24}\quad ^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$$

$$^{50,51}_{23}V_{27,28}\quad ^{54,56,57,58}_{26}Fe_{28,30,31,32}\quad ^{59}_{27}Co_{32}\quad ^{64,66}_{30}Zn_{34,36}\quad ^{69,71}_{31}Ga_{38,40}\quad ^{75}_{33}As_{42}\quad ^{80,82,83,84}_{36}Kr_{44,46,47,48}$$

$$^{84,86,88}_{38}Sr_{46,48,50}\quad ^{92,96,98}_{42}Mo_{50,54,56}\quad ^{100,101}_{44}Ru_{56,57}\quad ^{102,105,108,110}_{46}Pd_{56,59,62,64}\quad ^{119}_{50}Sn_{69}\quad ^{137,138}_{56}Ba_{81,82}$$

$$^{138,139}_{57}La_{81,82}\quad ^{169}_{69}Tm_{100}\quad ^{209}_{83}Bi^{*}_{126}\quad ^{209}_{84}Po^{*}_{125}\quad ^{226}_{88}Ra^{*}_{138}\quad ^{238}_{92}U^{*}_{146}\ {}^{257}_{100}Fm^{*}_{157}\ {}^{285}_{112}Cn^{*}_{173}\ {}^{2\cdot173,348}_{137,138}Fy^{ie}_{209,210}$$

$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135$$

$$=\frac{1}{2}-\frac{1}{9}+\frac{1}{3\cdot31}-\frac{1}{23\cdot(8\cdot3\cdot17+1)}+\frac{1}{17\cdot23\cdot(8\cdot3\cdot11^{4}-1)}$$

$$^{23}_{11}Na_{12}\quad ^{31}_{15}P_{16}\quad ^{35,37}_{17}Cl_{18,20}\quad ^{36,38,40}_{18}Ar_{18,20,22}\quad ^{46,48,50}_{22}Ti_{24,26,28}\quad ^{50,51}_{23}V_{27,28}\quad ^{56,57}_{26}Fe_{30,31}\quad ^{69,71}_{31}Ga_{38,40}$$

$$^{64,66,68}_{30}Zn_{34,36,38}\quad ^{80,82,83,84}_{36}Kr_{44,46,47,48}\quad ^{84,86,87,88}_{38}Sr_{46,48,49,50}\quad ^{89}_{39}Y_{50}\quad ^{90,91,92}_{40}Zr_{50,51,52}\quad ^{93}_{41}Nb_{52}\quad ^{100}_{44}Ru_{56}$$

$$^{112,114,116,118,119,120,124}_{50}Ra_{62,64,66,68,69,70,74}\quad ^{121,123}_{51}Sb_{70,72}\quad ^{136,137,138}_{56}Ba_{80,81,82}\quad ^{150,152}_{62}Sm_{88,90}\quad ^{168}_{68}Er_{100}$$

$$^{156,157}_{64}Gd_{92,93}\quad ^{169}_{69}Tm_{100}\quad ^{201,204}_{80}Hg_{121,124}\quad ^{208}_{82}Pt_{126}\quad ^{209}_{83}Bi^{*}_{126}\quad ^{209}_{84}Po^{*}_{125}\quad ^{210}_{85}At^{*}_{125}\quad ^{222}_{86}Rn^{*}_{136}\quad ^{223}_{87}Fr^{*}_{136}$$

$$^{226}_{88}Ra^{*}_{138}\quad ^{227}_{89}Ac^{*}_{138}\quad ^{14\cdot17}_{92}U^{*}_{146}\quad ^{237}_{93}Np^{*}_{144}\quad ^{257}_{100}Fm^{*}_{157}\quad ^{285}_{112}Cn^{*}_{173}\quad ^{2\cdot151}_{121}Ch^{ie}_{181}\quad ^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$$

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3. Formulas of Feigenbaum Constants in Continued Fractional Numbers

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326=\cfrac{3}{14+\cfrac{1}{131+\cfrac{1}{2+\cfrac{1}{54+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{18}}}}}}}}$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{96}_{42}Mo_{54}$ ${}^{131}_{54}Xe_{77}$ ${}^{2\cdot131}_{103}Lr^{*}_{158}$ ${}^{4\cdot83}_{131}Ch^{ie}_{201}$

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326=\cfrac{3}{14+\cfrac{1}{131+\cfrac{1}{2+\cfrac{1}{54+\cfrac{1}{6+\cfrac{7\cdot13}{109}}}}}}$$

${}^{27}_{13}Al_{14}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{3\cdot77}_{91}Pa^{*}_{140}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{278}_{109}Mt^{*}_{169}$

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326=\cfrac{3}{14+\cfrac{1}{131+\cfrac{1}{2+\cfrac{5\cdot149}{13\cdot29\cdot107}}}}$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{106,108}_{46}Pd_{60,62}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{149}_{62}Sm_{3\cdot29}$
${}^{270}_{107}Bh^{*}_{163}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326=\cfrac{3}{14+\cfrac{9\cdot83\cdot109}{64\cdot7\cdot(6\cdot7\cdot(6\cdot5\cdot19-1)+1)}}$$

${}^{19}_{9}F_{10}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{83}_{36}Kr_{47}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{183}_{74}W_{109}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{4\cdot83}_{131}Ch^{ie}_{201}$

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326$$

$$=\frac{64\cdot3\cdot7\cdot(2\cdot3\cdot7\cdot(2\cdot3\cdot5\cdot19-1)+1)}{(128-1)\cdot(2\cdot3\cdot7\cdot(2\cdot3\cdot7\cdot31\cdot(4\cdot227-1)-1)}$$

$$=\frac{64\cdot3\cdot7\cdot(2\cdot3\cdot7\cdot(2\cdot3\cdot5\cdot19-1)+1)}{(128-1)\cdot(2\cdot3\cdot7\cdot(2\cdot3\cdot7\cdot31\cdot(2\cdot3\cdot(8\cdot19-1)+1)-1)}=\frac{32120256}{149975951}$$

${}^{19}_{9}F_{10}$ ${}^{31}_{15}P_{16}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{69,71}_{31}Ga_{38,40}$
${}^{76}_{34}Se_{42}$ ${}^{95,96,98}_{42}Mo_{53,54,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{110,111,112,114}_{48}Cd_{62,63,64,66}$ ${}^{128}_{52}Te_{76}$ ${}^{150,152}_{62}Sm_{88,90}$
${}^{186,188,189,190,192}_{76}Os_{110,112,113,114,116}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

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$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135$$

$$=\cfrac{2}{5+\cfrac{1}{171+\cfrac{1}{1+\cfrac{1}{17+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15}}}}}}}}}}}}}$$

$$=\cfrac{2}{5+\cfrac{1}{171+\cfrac{1}{1+\cfrac{1}{17+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{16}{31}}}}}}}}}}}$$

$^{14,15}_{7}N_{7,8}$ $^{16,17,18}_{8}O_{8,9,10}$ $^{31}_{15}P_{16}$ $^{58,60,62}_{28}Ni_{30,32,34}$ $^{69,71}_{31}Ga_{38,40}$ $^{110,112,113,114}_{48}Cd_{62,64,65,68}$

$^{168,170}_{68}Er_{100,102}$ $^{170,171,172,173}_{70}Yb_{100,101,102,103}$ $^{281}_{110}Ds^{*}_{171}$ $^{285}_{112}Cn^{*}_{173}$ $^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$

$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135$$

$$=\cfrac{2}{5+\cfrac{1}{171+\cfrac{1}{1+\cfrac{1}{17+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{47}{8\cdot 9\cdot 5}}}}}}}}}$$

$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135$$

$$=\cfrac{2}{5+\cfrac{1}{171+\cfrac{1}{1+\cfrac{1}{17+\cfrac{1}{1+\cfrac{1}{4+\cfrac{11\cdot 37}{13\cdot 59}}}}}}}$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,50}_{22}Sc_{24,25,27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{85,87}_{37}Rb_{48,50}$

${}^{100,101,102,104}_{44}Ru_{56,57,58,60}$ ${}^{103}_{45}Rh_{58}$ ${}^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$

$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135$$

$$=\cfrac{2}{5+\cfrac{1}{171+\cfrac{1}{1+\cfrac{1}{17+\cfrac{25\cdot(4\cdot 5\cdot 7-1)}{2\cdot 3\cdot 7\cdot 101}}}}}=\cfrac{2}{5+\cfrac{1}{171+\cfrac{269\cdot 281}{97\cdot(6\cdot 137+1)}}}$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{94,96,97,98}_{42}Mo_{52,53,55,56}$ ${}^{137}_{56}Ba_{81}$ ${}^{170,171,172,173}_{70}Yb_{100,101,102,103}$

${}^{163}_{66}Dy_{97}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{281}_{110}Ds^{*}_{171}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135$$

$$=\cfrac{2}{5+\cfrac{97\cdot(6\cdot 137+1)}{2\cdot 5\cdot(2\cdot 113+1)\cdot(32\cdot 27\cdot 7-1)}}=\cfrac{2}{5+\cfrac{(2\cdot 49-1)\cdot(6\cdot 137+1)}{2\cdot 5\cdot(2\cdot 113+1)\cdot(32\cdot 27\cdot 7-1)}}$$

$$=\frac{4\cdot 5\cdot(2\cdot 113+1)\cdot(32\cdot 27\cdot 7-1)}{9\cdot 7\cdot 13\cdot 53\cdot(2\cdot 7\cdot 113+1)}=\frac{27453380}{68713281}$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{59}_{27}Co_{32}$ ${}^{95,96,97,98}_{42}Mo_{53,54,55,56}$ ${}^{97}_{43}Tc^{*}_{54}$

${}^{113,115}_{49}In_{64,66}$ ${}^{137}_{56}Ba_{81}$ ${}^{4\cdot 47,27\cdot 7}_{76}Os_{112,113}$ ${}^{2\cdot 113}_{88}Ra^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{284,286}_{113}Nh^{ie}_{171,173}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

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$$\frac{1}{\alpha}=\frac{4\cdot 5\cdot(2\cdot(2\cdot 3\cdot 19-1)+1)\cdot(32\cdot 27\cdot 7-1)}{9\cdot 7\cdot 13\cdot(2\cdot 27-1)\cdot(2\cdot 7\cdot(2\cdot 3\cdot 19-1)+1)}=\frac{27453380}{68713281}$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{59}_{27}Co_{32}$ ${}^{70,74,76}_{32}Ge_{38,42,44}$ ${}^{86}_{38}Sr_{48}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{112,114}_{48}Cd_{64,66}$ ${}^{130,132}_{54}Xe_{76,78}$ ${}^{138,139}_{57}La_{81,82}$

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4. The Third Feigenbaum Constant

There would be the third Feigenbaum constant γ, and we could only give some guesses at present stage as follows.

Note: 136=8·17, 138=6·23

$$\frac{1}{\alpha}=\frac{1}{2.50290787509589}=0.399535280523135=f(2,3,17,23,11,\cdots)$$

$$=\frac{1}{2}-\frac{1}{9}+\frac{1}{3\cdot 31}-\frac{1}{23\cdot(8\cdot 3\cdot 17+1)}+\frac{1}{17\cdot 23\cdot(8\cdot 3\cdot 11^4-1)}$$

$$=\cfrac{2}{5+\cfrac{1}{171+\cfrac{269\cdot 281}{97\cdot(6\cdot 137+1)}}}$$

$$\frac{1}{\delta}=\frac{1}{4.66920160910299}=0.214169377062326=f(2,3,23,11,\cdots)$$

$$=\frac{1}{4}-\frac{1}{27}+\frac{1}{4\cdot 9\cdot 23}-\cfrac{1}{2\cdot 3\cdot 7\cdot 23\cdot(2\cdot 3\cdot(4\cdot 3\cdot 11-1)+1)+\cfrac{2\cdot 23}{3\cdot 19}}$$

$$=\cfrac{3}{14+\cfrac{1}{131+\cfrac{13\cdot 29\cdot 107}{9\cdot 83\cdot 109}}}$$

$$\frac{1}{\gamma}=f(2,3,17,\cdots)\approx 1-\frac{1}{3}+\frac{1}{8\cdot 17}=\frac{25\cdot 11}{8\cdot 3\cdot 17}\approx\cfrac{2}{3-\cfrac{1}{31+\cfrac{1}{2}}}=\frac{2\cdot 9\cdot 7}{11\cdot 17}\approx\frac{1}{1.484}$$

or

$$\frac{1}{\gamma}=f(2,3,17,\cdots)\approx\frac{1}{8}-\frac{1}{81}+\frac{1}{8\cdot 3\cdot 17}=\frac{317}{2\cdot 81\cdot 17}\approx\cfrac{3}{26+\cfrac{1}{15}}=\frac{9\cdot 5}{17\cdot 23}\approx\frac{1}{8.689}$$

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$$\frac{1}{\gamma}=f(2,3,17,\cdots)\approx\frac{1}{8}-\frac{1}{101}=\frac{3\cdot 31}{8\cdot 101}\approx\cfrac{3}{26+\cfrac{1}{151}}=\frac{151}{7\cdot 11\cdot 17}\approx\frac{1}{8.6689}$$

$$\frac{1}{\gamma}=f(2,3,17,\cdots)\approx\frac{1}{8}-\frac{1}{103}=\frac{5\cdot 19}{8\cdot 103}\approx\cfrac{3}{26+\cfrac{1}{157}}=\frac{157}{6\cdot 227-1}\approx\frac{1}{8.6688}$$

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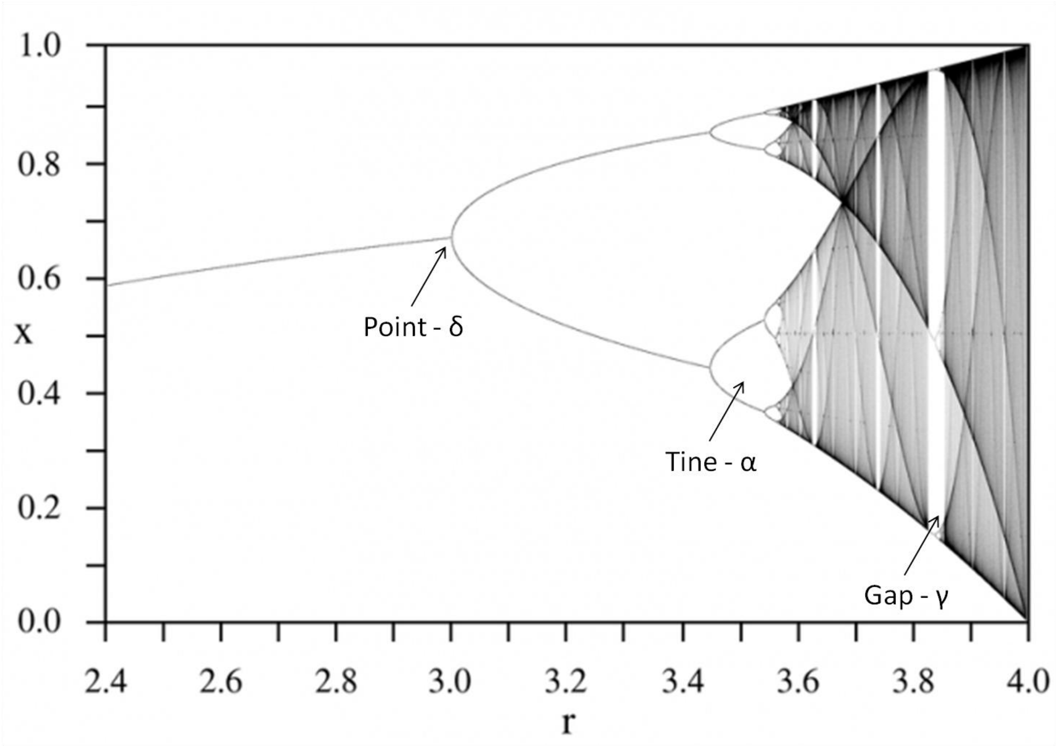
Formula of α in fractional number format contains the factors of 2, 3, 17, 23 and 11, Formula of δ in fractional number format contains the factors of 2, 3, 23 and 11, so it is supposed that there would be the third Feigenbaum constant γ which should relate to factors of 2, 3, 17 and 11.

5. Bifurcation Diagram and Feigenbaum Constants

There are three features in a typical bifurcation diagram, which are “point”, “tine” and “gap”. These three features should correspond to three Feigenbaum constants δ, α and γ (**Fig. 1**), so there should be three Feigenbaum constants.



Bifurcation Diagram and Three Feigenbaum Constants
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Fig. 1

6. Integrated Formulas of α_1, δ and 2π

$$(2\pi)_{Chen-k} = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

$$(2\pi)_{Wallis-k} = 4 \cdot (2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{2k}{2k+1} \frac{2k+2}{2k+1})$$

$$(2\pi)_{GL-k} = 8 \cdot (1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \cdots \pm \frac{1}{2k+1})$$ (GL means Gregory-Leibniz)

$$(2\pi)_{NC-k} = 6+\sum_{n=1}^{k}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}$$ (NC means Nilakantha-Chen)

$$\alpha_1=\frac{1}{137.035999037435}=\frac{1+\dfrac{1}{128\cdot 9\cdot(2\cdot 3\cdot 173+1)+\dfrac{7}{2\cdot 5}}}{4.66920160910299^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\,\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{2\cdot 3\cdot 71}{25\cdot 17})^{23\cdot 37}}}$$

${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{80}_{34}Se_{46}$ ${}^{79,81}_{35}Br_{44,46}$
${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{105,110}_{46}Pd_{59,64}$ ${}^{112}_{48}Cd_{64}$ ${}^{112,114,115-120,122,124}_{50}Sn_{62,64,65-70,72,74}$
${}^{121,123}_{51}Sb_{70,72}$ ${}^{126}_{52}Te_{74}$ ${}^{128}_{54}Xe_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{157}_{64}Gd_{93}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{171,173}_{70}Yb_{101,103}$
${}^{175,176}_{71}Lu_{104,105}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{209}_{84}Po^*_{125}$ ${}^{210}_{85}At^*_{125}$ ${}^{257}_{100}Fm^*_{157}$
${}^{285}_{112}Cn^*_{173}$ ${}^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

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$$\alpha_1=\frac{1}{137.035999037435}=\frac{1+\dfrac{1}{2\cdot 9\cdot(8\cdot 3\cdot 25\cdot 7\cdot 59-1)-\dfrac{2}{5}}}{4.66920160910299^2\cdot 4\cdot(2\,\dfrac{2}{3}\,\dfrac{4}{3}\,\dfrac{4}{5}\,\dfrac{6}{5}\cdots\dfrac{1278}{1279}\,\dfrac{1280}{2\cdot 9\cdot 71+1})}$$

${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{105}_{46}Pd_{59}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{112}_{48}Cd_{64}$ ${}^{118,120,122}_{50}Sn_{68,70,72}$ ${}^{175,176}_{71}Lu_{104,105}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{3\cdot 59}_{72}Hf_{105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{209}_{84}Po^*_{125}$ ${}^{210}_{85}At^*_{125}$
${}^{285}_{112}Cn^*_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_1=\frac{1}{137.035999037435}$$

$$=\frac{1+\dfrac{1}{163\cdot(6\cdot 11^2\cdot 97+1)}}{4.66920160910299^2\cdot 8\cdot(1+\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{1}{7}-\cdots-\dfrac{1}{2\cdot 2\cdot 11\cdot 37+1})}$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{48,50}_{22}Ti_{26,28}$ ${}^{66,67}_{30}Zn_{36,37}$ ${}^{75}_{33}As_{42}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{97}_{42}Mo_{55}$ ${}^{97}_{43}Tc^*_{54}$ ${}^{121,123}_{51}Sb_{70,72}$
${}^{99}_{44}Ru_{55}$ ${}^{163}_{66}Dy_{97}$ ${}^{185}_{67}Ho_{98}$ ${}^{3\cdot 67}_{80}Hg_{121}$ ${}^{247}_{97}Bk^*_{150}$ ${}^{4\cdot 67}_{105}Db^*_{163}$ ${}^{269,271}_{106}Sg^*_{163,165}$ ${}^{270}_{107}Bh^*_{163}$ ${}^{410}_{163}Ch^{ie}_{247}$

2021/2/1

$$\alpha_1=\frac{1}{137.035999037435}=\frac{1+\dfrac{1}{3\cdot(2\cdot 7\cdot 31-1)}-\dfrac{1}{13\cdot 89\cdot(2\cdot 37\cdot 53+1)-\dfrac{5}{8}}}{4.66920160910299^2\cdot(6+\sum_{n=1}^{3}\dfrac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}})}$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{50,52,53}_{24}Cr_{26,28,29}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{74}_{32}Ge_{42}$ ${}^{89}_{39}Y_{50}$
${}^{95}_{42}Mo_{53}$ ${}^{127}_{53}I_{74}$ ${}^{150,152}_{62}Sm_{88,90}$ ${}^{178}_{72}Hf_{106}$ ${}^{180,186}_{74}W_{106,112}$ ${}^{227}_{89}Ac^*_{138}$ ${}^{262}_{103}Lr^*_{3\cdot 53}$ ${}^{14\cdot 31,435}_{173}Ch^{ie}_{261,262}$

2021/2/1,3

The Fine-structure Constant: $\alpha_1 = 1/137.035999037435$

$\alpha_2 = 1/137.035999111818$

The Speed of Light in Vacuum in Atomic Units:

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = 137.035999074626$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$\alpha = 2.50290787509589$

$$\alpha_1 \delta^2 (2\pi)_{Chen-25\cdot17} = 1 + \frac{1}{128\cdot 9\cdot(2\cdot 3\cdot 173+1) + \frac{7}{2\cdot 5}} \approx 1$$

$$\alpha_1 \delta^2 (2\pi)_{Walllis-9\cdot71} = 1 + \frac{1}{2\cdot 9\cdot(8\cdot 3\cdot 25\cdot 7\cdot 59-1) - \frac{2}{5}} \approx 1$$

$$\alpha_1 \delta^2 (2\pi)_{GL-22\cdot37} = 1 + \frac{1}{163\cdot(6\cdot 11^2\cdot 97+1)} \approx 1$$

$$\alpha_1 \delta^2 (2\pi)_{NC-3} = 1 + \frac{1}{3\cdot(2\cdot 7\cdot 31+1)} - \frac{1}{13\cdot 89\cdot(2\cdot 37\cdot 53+1) - \frac{5}{8}} \approx 1$$

$\alpha_1 \delta^2 (2\pi) \approx 1$

2021/2/1-3

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Appendix I: Research History

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