Approximate formula for zeta function ζ(s) and L function L(s) s=Re

Takamasa Nguchi

2021/01/19

Approximation formulas for zeta function and L function and their evaluation.

1 Introduction

First, this sentence is created by machine translation.[1] There may be some strange sentences.

I created two formulas for each. The range is $1 < x \leqq 2$ *and* $x \geqq 2$.

In both cases,the accuracy increases as the distance from 2 as the starting point increases. There is no mathematical proof.

2 zeta function and L function

2.1 zeta function

(Riemann zeta function)

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad (s > 1)$$

2.2 L-function

(Dirichlet L-function)

$$L(s,\chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} \qquad \chi(n) = \begin{cases} 0 & (n \equiv 0,2 \bmod 4) \\ 1 & (n \equiv 1 \bmod 4) \\ -1 & (n \equiv 3 \bmod 4) \end{cases}$$

(Euler L-function)

$$L(s) = \sum_{n \geqq 1:(Odd)}^{\infty} (-1)^{\frac{n-1}{2}} n^{-s} = 1 - \frac{1}{3^s} + \frac{1}{5^s} - \frac{1}{7^s} + \frac{1}{9^s} - \cdots \quad [2]$$

$$L(s) = \prod_{p;(odd\ \ prime)} (1 - (-1)^{\frac{p-1}{2}} p^{-s})^{-1} = \frac{1}{1+3^{-s}} \times \frac{1}{1-5^{-s}} \times \frac{1}{1+7^{-s}} \times \cdots \quad [2]$$

$$L(2n+1) = \frac{E_{2n}}{(2n)!\ 2^{2n+2}} \pi^{2n+1} \qquad (n = 0,1,2\cdots) \qquad [3]$$

E_{2n} : (Euler number)

3 Approximate formula and error

3.1 zeta function $(1 < x \leqq 2)$

$$f(x) = \coth(x - x^{(\gamma \times \log(x))})$$

3.2 Error

x	$\zeta(x)$	$\mid f(x) \mid$
2	$1.644934066 \cdots$	4.48759×10^{-2}
1.1	$10.58444846 \cdots$	2.00708×10^{-3}
1.01	$100.5779433 \cdots$	1.68233×10^{-4}
1.001	$1000.577288 \cdots$	1.65139×10^{-5}
- -		
1.000001	$1000000.5772157 \cdots$	1.64798×10^{-8}
- -		
1.00000000001	$10 \cdots 00.57721566 \cdots$	1.64797×10^{-13}

3.3 zeta function $(x \geqq 2)$

$$x \longrightarrow \infty \qquad \frac{1}{(1 - a^{-x})} \approx \exp(a^{-x})$$

$$(a = 2\ ,\ x = 10) \qquad \frac{1}{(1 - 2^{-10})} = 1.000977517 \cdots$$

$$\exp(2^{-10}) = 1.000977039 \cdots$$

$$f(x) = \exp\left(f_1(x)^{-x}\right) \qquad \frac{x \quad = \ 1 \longrightarrow \infty}{f_1(x) = \ 0 \longrightarrow 2}$$

$$f(x) = \exp\left\{2^x - \left(\frac{4}{3}\right)^x - f_2(x)\right\}^{-1} \qquad f_2(x) \approx \frac{1}{2} - 10^{-\left(\frac{x}{19.55}\right)}$$

$$f(x) = \exp\left\{2^x - \left(\frac{4}{3}\right)^x\right\}^{-1} \qquad (2 \leqq x < 8) \tag{1}$$

$$f(x) = \exp\left\{2^x - \left(\frac{4}{3}\right)^x - \frac{1}{2}\right\}^{-1} \qquad (x \geqq 8) \tag{2}$$

$$f(x) = \frac{2^x}{(2^x - 1)} \times \frac{3^x}{(3^x - 1)} \times \frac{5^x}{(5^x - 1)} \qquad (x \geqq 2) \tag{3}$$

3.4 Error

x	$\zeta(x)$	$\mid(1)\mid$	$\mid(2)\mid$	$\mid(3)\mid$
2	$1.644934066\cdots$	7.66219×10^{-2}	1.42257×10^{-1}	8.24341×10^{-2}
3	$1.202056903\cdots$	7.67170×10^{-3}	1.31883×10^{-2}	5.67264×10^{-3}
8.5	$1.002859251\cdots$	2.14825×10^{-6}	1.93910×10^{-6}	6.75550×10^{-8}
10	$1.000994575\cdots$	2.84698×10^{-7}	2.09855×10^{-7}	3.59020×10^{-9}
- -				
20	$1.00000095396\cdots$	3.78972×10^{-13}	7.60494×10^{-14}	1.25341×10^{-17}
- -				
40	$1.0\cdots009094948\cdots$	4.06262×10^{-25}	7.32887×10^{-27}	1.57065×10^{-34}

3.5 L function $(x \geqq 2)$

$$f(x)=\exp\left\{-\left(3^x+\left(\frac{3^2}{5}\right)^x-\left(\frac{3^2}{7}\right)^x+\left(\frac{3^3}{5^2}\right)^x+f_1(x)\right)^{-1}\right\}$$

$$f(x)=\exp\left\{-\left(3^x+\left(\frac{3^2}{5}\right)^x-\left(\frac{3^2}{7}\right)^x+\left(\frac{3^3}{5^2}\right)^x\right)^{-1}\right\}\qquad(2\leqq x<10)\qquad(4)$$

$$f(x)=\exp\left\{-\left(3^x+\left(\frac{3^2}{5}\right)^x-\left(\frac{3^2}{7}\right)^x+\left(\frac{3^3}{5^2}\right)^x+\frac{1}{2}\right)^{-1}\right\}\qquad(x\geqq 10)\qquad(5)$$

$$f(x)=\frac{3^x}{(3^x+1)}\times\frac{5^x}{(5^x-1)}\times\frac{7^x}{(7^x+1)}\qquad(x\geqq 2)\qquad(6)$$

3.6 Error

x	$L(x)$	$\mid(4)\mid$	$\mid(5)\mid$	$\mid(6)\mid$
2	$0.915965\cdots$	2.47178×10^{-3}	5.66595×10^{-3}	2.78500×10^{-3}
3	$0.968946146\cdots$	2.55203×10^{-4}	7.22251×10^{-3}	2.90303×10^{-4}
9	$0.999949684\cdots$	5.31971×10^{-10}	7.33865×10^{-10}	3.24964×10^{-10}
10	$0.999983164\cdots$	7.41781×10^{-11}	6.75470×10^{-11}	3.09898×10^{-11}
11	$0.999994375\cdots$	9.65645×10^{-12}	6.16402×10^{-12}	2.92734×10^{-12}
- -				
21	$0.9\cdots9904403\cdots$	4.36101×10^{-22}	2.08376×10^{-22}	1.31070×10^{-22}
- -				
41	$0.9\cdots9972582\cdots$	3.75624×10^{-40}	2.36628×10^{-43}	2.00650×10^{-43}

References

[1] https://translate.google.com google translation

[2] N.Kurokawa 『The quest for Riemann-from ABC to Z-』
Technical Review Company 2012 (54-57)

[3] N.Kurokawa 『Let's solve the Riemann conjecture
～New zeta and approach from factorization～』
Technical Review Company 2014 (212 - 231)

ehime-JAPAN