Toward Advances in Medicine and Interstellar Travel

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Abstract

The motion in a Black Hole spacetime is studied. Several new results are found, in particular about the nature of Dark Matter and Dark Energy. The energy aspect of a matter in curved spacetime is explained. It is understandable why underground detectors for particles of Dark Matter have caught absolutely nothing for so many years of work. Usually, particles have a pretty strong effect on our world. But such small corpuscles as neutrinos have the weakest effect on ordinary matter. I give convincing arguments that Dark Matter acts so weakly on our world that its direct-contact action is equal to zero. That is why Dark Matter passes through the devices that are built for its capture completely without noticing them, completely without labor and friction with these devices. Such Dark Matter is representative for the INVISIBLE world, i.e. the detectors trying to detect it locally are "blind", they see nothing.

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I. MATH-FREE JUSTIFICATION OF THE PAPER

The theoretically possible vanishing of test-particles is investigated using General Relativity methods. This is a promising solution, clear and concise, to the Dark Matter mystery. Four different methods in this paper give the same results for the energy Localization problem pointed to a falling body compression in the first order deviation equation.

In 2020, an Earth-bound observation showed how the gravitational field of one of the large Black Holes stretches one of the stars into a long thin spaghetti. [1] The hole ate its "prey". However, according to my calculations (four different methods gave the same formula, so I'm sure I'm right), half of the way to the surface of the Black Hole the star indeed stretches like spaghetti, but already flying up to the Black Hole event horizon, it begins to shrink into a heap. The fact that the Black Hole can compress falling bodies (and not tear them apart) is proven by me strictly scientifically.

First, there are two conflicting factors.

Factor A. The lower parts of the falling star are closer to the Black Hole, so they should be attracted more strongly than the upper layers of the star. Therefore, according to Newton's theory, the star is being stretched apart.

Factor B. According to observations from Earth, the star will never reach the event horizon of the Black Hole, and if so, then its compression should be seen from the Earth. Factor B turned out to be stronger than factor A.

Another circumstance speaks for this contraction. Black Holes are formed when a cloud of dust (or stellar matter) begins to collapse under its own gravity. If the astronaut is in free fall inside this cloud, he should also be compressed together with the cloud, not stretched.

Let us consider the famous Lemaître coordinates, which have convinced everybody about the stretching.

The Schwarzschild Black Hole in falling coordinates is given by

$$dS^2 = ds^2 - \frac{2M}{r}\, d\rho^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \tag{1}$$

where

$$r = \left(\frac{3}{2}(\rho - s)\right)^{2/3} (2M)^{1/3}\,. \tag{2}$$

Each choice $\rho = \text{const}$ corresponds to the free-falling particle with proper time s. Thus, one would rush to the conclusion that the falling body is stretching because at $s = \text{fixed}$

the proper interval between closely separated free-falling particles $\int dS$ is monotonically increasing. However, I expect that in the co-moving coordinate system the simultaneous events (which are found via infinite fast local motion) have different s.

II. INTRODUCTION

There are physically reasonable solutions in General Relativity with neither event nor Cauchy horizons, but with naked singularities [2]. Let us study the simplest case of a naked singularity in the Reissner-Nordström metric with $M < Q$,

$$dS^2 = -A\,dt^2 + dr^2/A + r^2\,d\Omega^2\,, \quad A = 1 - 2M/r + Q^2/r^2\,. \tag{3}$$

Here and in the following, we use the single-line style like $1 - 2M/r + Q^2/r^2$ for

$$1 - \frac{2M}{r} + \frac{Q^2}{r^2}\,. \tag{4}$$

Here and in the following, s is the parameter of motion, e.g. the proper time. Q, M, $S^{\hat{\nu}}$, s and r are being measured in meters: they are "geometrised". The initial velocity (at $r = R$) is zero, $u_r = 0$. The fall happens along the radial line.

A. Ethical Statement

I agree with the journal Ethical Statement, and follow it in full.

III. EQUATIONS OF GEODETIC MOTION

From the rest state at $r = R$, let us release a small, electrically neutral test body.

The metric is t-independent, so the test-body has a velocity component $u_t = -E = \text{const}$. The falling is radial, so $u^\theta = u^\phi = \text{const} = 0$. Using normalized velocity vector with $u_\nu\,u^\nu = g^{tt}\,u_t\,u_t + g_{rr}\,u^r\,u^r = E^2/(-A) + (u^r)^2/A = -1$, for the radial component of velocity one has

$$(u^r)^2 = E^2 - A\,. \tag{5}$$

Starting at $r = R$ with radial velocity $u^r = 0$, one has

$$E^2 = 1 - 2M/R + Q^2/R^2\,, \tag{6}$$

and

$$(u^r)^2 = Q^2(1/R^2 - 1/r^2) + 2M\,(1/r - 1/R) = (1/r - 1/R)(2M - Q^2\,[1/R + 1/r])\,. \quad (7)$$

Note that if

$$2M - Q^2(1/R + 1/r) < 0 \quad \Leftrightarrow \quad 2M/Q^2 - 1/R < 1/r \quad (8)$$

during the falling $r < R$, one has $(u^r)^2 < 0$, but because of $2M/Q^2 - 1/R > 0$, one obtains $r < 1/(2M/Q^2 - 1/R)$. Thus, the test-body has not reached the singularity at $r = 0$.

IV. VANISHING SIZE

Further research has shown that the proper size of the body shrinks to zero at $r = r_m = 1/(2M/Q^2 - 1/R)$.

Consider a drop of "perfect fluid" falling into a Black Hole. Because the drop is small, the velocity of every part of it is the velocity of the fall. The equation of matter is $T^{\mu\nu}_{;\nu} = 0$, thus $u_\mu\, T^{\mu\nu}_{;\nu} = 0$, where

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p\, g^{\mu\nu}\,, \quad (9)$$

where pressure p and density ρ are the inner characteristics of the drop. Thus,

$$-(\rho + p)_{,\nu}\, u^\nu - (\rho + p)\, u^\nu_{;\nu} + (\rho + p)\, u^\nu\, u^\mu_{;\nu} u_\mu + p_{,\nu}\, u^\nu = 0\,, \quad (10)$$

where $u^\mu_{;\nu} u_\mu = 0$, because $(u^\mu u_\mu)_{;\nu} = (-1)_{;\nu} = 0$. As $u^\nu = dx^\nu/ds$, one has

$$-\frac{d(\rho + p)}{ds} - (\rho + p)\, u^\nu_{;\nu} + \frac{dp}{ds} = 0\,. \quad (11)$$

Here and in the following the index with semicolon means the covariant derivative using Christoffel symbols, while the index with comma means the ordinary derivative with respect to the spacetime coordinate.

This differential equation has no solution, unless the fluid is compressible. Let the equation of state be $p = p(\rho)$. Then

$$\frac{d\rho}{ds} = -(\rho + p(\rho))\, u^\nu_{;\nu}\,. \quad (12)$$

Now the rate (and sign) of the change of the density depends on $D := u^\nu_{;\nu}$, and the formula coincides with the one given in Ref. [3], pages 226–227.

If one inserts the above velocity u^ν into the divergence, one gets to know that $u^\mu_{;\mu} \sim 1/u^r \to -\infty$ at $r = r_m$. It is interesting to note that for a Schwarzschild Black Hole ($M \neq 0$, $Q = 0$) one has

$$D := M \, \frac{4\,r - 3\,R}{\sqrt{2\,M\,R\,r^3\,(R - r)}} \tag{13}$$

With the zero at $r = 3R/4$ being the starting point for the compression. Notably, this happens at an infinite distance from the Black Hole, if R is infinite. Such an unexpected result hardly can be found in Newton's age, even while we still have a weak field at $r = (3/4)R \gg 2M$. The deadly ripping with $D \gg 1$ never begins, but the deadly compression with $D \ll -1$ happens at the singularity $r = 0$. This has been shown by several methods, including the study of the geodetics deviation equation. While the first part of the present note is aimed to raise attention to the problem, the complete study of the problem is found later in the text.

The drop's density at $r \to r_m$ diverges because of

$$\frac{d\rho}{\rho} = \left(-D - D\frac{p(\rho)}{\rho}\right) ds\,. \tag{14}$$

Integration of both sides produces

$$\ln(C\,\rho) = \int \left(-D - D\frac{p(\rho)}{\rho}\right) ds = \int \left(\frac{D}{u^r} + \frac{D}{u^r}\,\frac{p(\rho)}{\rho}\right) dr = \infty\,,$$

where C is a constant of integration.

V. SOLUTION TO THE VANISHING

Because the vanishing seems to go beyond the energy-momentum conservation law and General Relativity, I have endured the known law $T^{\nu\mu}_{;\nu} = 0$ with the tensor of invisible Virtual Matter $X^{\nu\mu}$,

$$(T^{\nu\mu} + X^{\nu\mu})_{;\nu} = 0\,. \tag{15}$$

I call the Virtual Matter "invisible" because it should go through underground "detectors of Dark Matter" without the slightest effort. Why? Because being just a mathematical fix to the vanishing of the test body, Virtual Matter is not a new kind of matter; hence, it does not interact with the visible matter even via the weak interaction. To my understanding, Virtual Matter with $X^{\nu\mu}_{;\nu} = 0$ is called Dark Matter, and Dark Matter with $X^{\nu\mu} = -\Lambda\,g^{\nu\mu}$, where Λ is the cosmological constant, is called Dark Energy.

VI. THE COMPLETE STUDY

A. On the energy localization problem

Recall the demand for an inertial tetrad in the Galilean and Einstein postulates of relativity: in a non-inertial tetrad the laws of physics would be changed, but the latter comes in conflict with Metrology. I have invented the following definition of Nature: Nature is what the Standard Instruments do measure and Instruments are what measure Nature. To measure correctly, the Instruments must be seen as invariants of Metrology, i.e. unchangeable: any places, times, and universes in the multiverse which have alien laws or different fundamental constants are not physical, as the instruments in those places would be changed.

By recalling the basic need to study problems in an inertial coordinate system (tetrad), we found no problem with the local conservation of the most basic laws of physics. But others have faced major problems (cf. e.g. Refs. [5]).

The rate vector in the local (ON) tetrad has

$$\frac{d\,B^{\hat{\nu}}}{ds} = e^{\hat{\nu}}_{\alpha}\,\frac{D\,B^{\alpha}}{ds}\,. \tag{16}$$

Thus, if $B^{\hat{\nu}}$ conserves in inertial tetrad, then

$$\frac{d\,B^{\hat{\nu}}}{ds} = 0\,, \qquad \frac{D\,B^{\alpha}}{ds} = 0\,. \tag{17}$$

But because

$$B^{\alpha} = e^{\alpha}_{\hat{\nu}}\,B^{\hat{\nu}}\,, \tag{18}$$

the inertial tetrad is defined by

$$\frac{D\,e^{\alpha}_{\hat{\nu}}}{ds} = \frac{d\,e^{\alpha}_{\hat{\nu}}}{ds} + \Gamma^{\alpha}_{\beta\,\gamma}\,e^{\beta}_{\hat{\nu}}\,u^{\gamma} = 0\,. \tag{19}$$

In particular, a solution of this describes the yearly fixation of the Earth axis in the polar star area. This also solves the energy localization problem in General Relativity. The known formula

$$T^{\nu\,\mu}_{;\nu} = 0 \tag{20}$$

in an inertial ON tetrad is the needed conservation of energy-momentum

$$T^{\hat{\nu}\,\hat{\mu}}_{,\hat{\nu}} = 0\,, \tag{21}$$

because in inertial ON tetrad all the Christoffel Symbols are zero,

$$\Gamma^{\hat{\alpha}}_{\hat{\nu}\,\hat{\mu}} = 0 \tag{22}$$

due to the strong equivalence principle: physics in a free-moving laboratory is independent of gravity (spacetime position). Eq. (22) follows from Eq. (16) because the left-hand side of the latter equation is a tensor.

B. The model in use

A motion of extended bodies in curved spacetime is a fascinating theme because the point-like particles are way too simple idealization. However, because of the tremendous number of details to be considered, large bodies lose the interest of the reader. What remains is the golden area of study: a small object, but not a microscopic – a drop of "perfect fluid". A drop of fluid is falling along a geodesic line because the drop is small. Drops of fluid are reasonable objects to consider, as there is water in the cosmos [6].

As a background example, the author considers the Schwarzschild metric of the black hole spacetime $g_{\nu\mu} = \mathrm{diag}(-(1-2\,M/r),\, 1/(1-2\,M/r),\, r^2,\, r^2\sin^2\theta)$.

Using the integral of motion $u_t = -E$ and the norm $u^\nu\, u_\nu = -1$, one finds non-zero components of the velocity to be

$$u_t = -E, \qquad u_r = -\frac{\sqrt{E^2 - 1 + (2M/r)}}{1-(2M/r)}\,, \tag{23}$$

where $E = \sqrt{1-(2M/R)}$.

The free-falling ON reference frame (tetrad) has a time-like geodesic vector $e^{\hat{0}}_\nu = u_\nu$ and space-like vectors $e^{\hat{1}}_\nu = (A,\, H,\, 0,\, 0)$, which are radially directed, and $e^{\hat{2}}_\nu = (0,\, 0,\, r,\, 0)$, $e^{\hat{3}}_\nu = (0,\, 0,\, 0,\, r\sin\theta)$ orthogonal to these, with inner product $e^{\hat{q}}_\alpha e^{\hat{u}\,\alpha} = \eta^{\hat{q}\,\hat{u}} = \mathrm{diag}(-1,1,1,1)$.

C. Usefulness of first-order Deviation Equation

We are sure about complicated algorithms, often written with extensive use of the second-order deviation equation (in its higher approximation terms), see e.g. [7]. However, in the present manuscript the author presents an easily accessible way to study any spacetime of interest by employing the first order deviation equation.

Please note that unlike the known deviation equation, the author's first order deviation Eq.(27) includes the property of a bundle of geodesics: a starting point with $E = E(R) \equiv \eta$ with proper time along each geodesic $s \equiv \lambda$. The calculation with the known deviation equation is much more complicated because it includes second-order derivatives.

VII. FIRST METHOD: ALTERNATIVE TO THE KNOWN DEVIATION EQUATION

In this section, the equation of state of the fluid is zero pressure $p = 0$: dust is a particular case of a fluid.

The derivation of deviation equation (see Ref. [3], pages 58, 291) shall be made more clear, because the starting from the bundle of trajectories $x^\alpha = x^\alpha(\lambda, \eta)$ and the definition of a tangent to the geodesic line $u^\alpha = \partial x^\alpha / \partial \lambda$ can lead to the wrong assertion $\mathrm{grad}\, u^\alpha \equiv \partial_u\, u^\alpha \neq 0$. However, here one has

$$\mathrm{grad}\, u^\alpha := \frac{\partial u^\alpha}{\partial x^\nu} = \frac{\partial}{\partial x^\nu}\Big(\frac{\partial x^\alpha}{\partial \lambda}\Big) = \frac{\partial}{\partial \lambda}\Big(\frac{\partial x^\alpha}{\partial x^\nu}\Big) = \frac{\partial}{\partial \lambda}\, \delta^\alpha_\nu = 0\,. \tag{24}$$

One shall rewrite the official derivation using the alternative notations $U^\alpha(\{x^\nu\};\, \lambda, \eta) = U^\alpha(\{x^\nu(\lambda, \eta)\};\, \lambda, \eta) = u^\alpha(\lambda, \eta)$ with

$$U^\alpha_{,\nu} \equiv \frac{\partial U^\alpha(x^0, x^1, x^2, x^3)}{\partial x^\nu} \neq 0. \tag{25}$$

Because mathematically speaking

$$\frac{\partial^2\, x^\alpha}{\partial \eta\, \partial \lambda} = \frac{\partial^2\, x^\alpha}{\partial \lambda\, \partial \eta}\,, \tag{26}$$

obviously holds [9]

$$\frac{\partial n^\alpha}{\partial \lambda} = \frac{\partial u^\alpha}{\partial \eta}\,, \tag{27}$$

where $n^\alpha = \partial x^\alpha / \partial \eta$ with $n^\alpha = n^{\hat{u}}\, e_{\hat{u}}{}^\alpha$, where $n^{\hat{u}}$ is the projection of the vector n^α on the free-falling ON reference frame. The equation turns into

$$\frac{d\, n^{\hat{u}}}{d\lambda}\, e_{\hat{u}}{}^\alpha = \frac{\partial u^\alpha}{\partial \eta} - n^{\hat{u}}\, \frac{\partial e_{\hat{u}}{}^\alpha}{\partial \lambda}\,. \tag{28}$$

Now, because we have realized the necessity of Eqs. (24) and (25), one has

$$\frac{\partial u^\alpha}{\partial \eta} \equiv U^\alpha_{,\nu}\, \frac{\partial x^\nu}{\partial \eta} + \frac{\partial\, U^\alpha}{\partial \eta}\,, \tag{29}$$

where

$$\frac{\partial x^\nu}{\partial \eta} = n^\nu = n^{\hat{u}}\, e^\nu_{\hat{u}} \,. \tag{30}$$

In case of the Schwarzschild metric with proper time $s \equiv \lambda$ one obtains

$$M\, n^{\hat{1}} + \frac{d\, n^{\hat{1}}}{ds}\, r \sqrt{r^2\,(E^2-1) + 2\, M\, r} - r^2 = 0\,, \tag{31}$$

and the s-derivative of the latter (note that $r = r(s)$) results in

$$\frac{d^2\, n^{\hat{1}}}{ds^2} = \frac{2\, M}{r^3}\, n^{\hat{1}} \,. \tag{32}$$

The proper distance is $S^{\hat{\nu}} = \Delta\eta\, n^{\hat{\nu}}$, with a constant $\Delta\eta \ll 1$.

Because this solution has fixed $S^{\hat{0}} = \frac{d\, S^{\hat{0}}}{ds} = 0$, $S^{\hat{1}}$ can be recognized as the distance between the dust particles. The same is stated by the strong equivalence principle [8] as the same time in the locality of the observer, namely $S^{\hat{0}} = 0$.

Amazingly, the radial size of the body can shrink despite the positive acceleration of deviation:

$$f = \frac{d^2\, S^{\hat{1}}}{ds^2} > 0\,, \qquad \frac{d\, S^{\hat{1}}}{ds} < 0\,, \tag{33}$$

if $M\, n^{\hat{1}} > r^2$. The author gives the following explanation to it: The deviation forces (f) are not forces at all. Why? The strong equivalence principle states clearly that the physics of the small laboratory is not affected by the outside curvature of spacetime. Therefore, it is conceptually wrong to introduce alien force in such an oasis.

VIII. SECOND METHOD: THE KNOWN DEVIATION EQUATION AGREES

In this section the pressure is zero, $p = 0$. It is expected that in the (inertial) tetrad

$$\frac{d^n\, h^{\hat{\nu}}}{ds^n} = e^{\hat{\nu}}_\alpha\, \frac{D^n\, h^\alpha}{ds^n}\,, \tag{34}$$

where

$$h^{\hat{\nu}} = e^{\hat{\nu}}_\mu\, h^\mu\,, \quad h^\alpha = e^\alpha_{\hat{\nu}}\, h^{\hat{\nu}}\,, \tag{35}$$

for any tensor h^ν and any n. By the way: the rank of a tensor can take any value. Then the inertial tetrad is defined by

$$\frac{D\, e^\alpha_{\hat{\nu}}}{ds} = \frac{d\, e^\alpha_{\hat{\nu}}}{ds} + \Gamma^\alpha_{\beta\,\gamma}\, e^\beta_{\hat{\nu}}\, u^\gamma = 0\,. \tag{36}$$

It is known that [3]

$$\frac{D^2\, n^\alpha}{ds^2} = -R^\alpha_{\mu\,\rho\,\nu}\, u^\mu\, u^\nu\, n^\rho\,. \tag{37}$$

Thus, for fixed $S^{\hat{0}} = 0$, the radial derivative gives

$$\frac{d^2\, S^{\hat{1}}}{ds^2} = -e^{\hat{1}\,\alpha}\, R_{\alpha\,\mu\,\rho\,\nu}\, u^\mu\, u^\nu\, (e^\rho_{\hat{1}}\, S^{\hat{1}})\,, \tag{38}$$

which in case of Schwarzschild metric gives

$$\frac{d^2\, S^{\hat{1}}}{ds^2} = \frac{2\,M}{r^3}\, S^{\hat{1}}\,, \tag{39}$$

exactly matching Eq. (32).

IX. THIRD METHOD: GEOMETRIC DENSITY CHANGE

In this section, the pressure is again zero, $p = 0$.

Please note that the azimuthal size of the dust cloud shrinks like $1/r$ while approaching the curvature singularity. This azimuthal contraction increases the density of the dust cloud as $1/r^2$, because the geometry shows $\rho \sim 1/(S^{\hat{1}}\, r^2)$. Then

$$\frac{d\rho}{ds} = \frac{d}{ds}\left(\frac{K}{S^{\hat{1}}(s)\, r^2(s)}\right). \tag{40}$$

Here $K = \text{const}$.

From Eqs. (28), (29) and (40) one has

$$\frac{d\rho}{ds} = \rho\, M\, \frac{3\,R - 4\,r}{\sqrt{2\,M\,R\,r^3\,(R-r)}} + \Delta\eta\,\rho\, F(r)\,, \tag{41}$$

where $F(r)$ is certain function. These are exactly Eqs. (12) and (13) for a small falling object (dust cloud) with initial radial size $\Delta R \sim \Delta\eta \approx 0$ and zero inner pressure $p(\rho) = 0$.

X. CONCLUSION

The points in this paper are proven now by four alternative approaches. Therefore, they are true and must be published. One can calculate that the point $r = 0$, where $\rho \to \infty$, becomes the point $r = r_m > 0$ for more general Black Hole metrics, for example in the Kerr metric. But there is no curvature singularity at $r = r_m$. The introduction of virtual matter heals the latter inconsistency.

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