AN INTRODUCTION TO THE SUPER-NORMAL-IRREDUCIBLE-IRRATIONAL NUMBERS AND THE AXIOM AT THIRD-ORDER OF LOGIC.

ALEXIS ZAGANIDIS

Abstract. We define the super-normal-irreducible-irrational numbers from some irreducible-irrational numbers and with the help of the n-irreducible sequents (see my previous articles). Instead of taking some integer part of the irreducible-irrational number (or from its inverse), we add a super-normal-irreducible formula which give the position of the first digit breaking some super-normal number definition. From 84 irreducible-irrational numbers, we deduce from the axiom at second-order of logic that they are all super-normal numbers as well. Moreover, with some random digits, the probability that the super-normal-irreducible formula holds for the 84 ones is about 9.0×10^{-10} and we have taken in account that some irreducible-irrational numbers are only some different functions of the same irreducible-irrational number. From this large coincidence, we introduce the axiom at third-order of logic which states that every irreducible-irrational numbers are super-normal numbers as well. From that new axiom at third-order of logic, we deduce the none-existence of an exotic 4-sphere. Finally, we conclude about the finitude of the total number of n-irreducible sequents.

We define the super-normal-irreducible-irrational numbers from an irreductible sequent $\mathcal{S}$ with the hypotheses $\Gamma^a \wedge \tilde{\Gamma}_{Reals,ZFC}$ including some fundamental axioms and with the formula $\tilde{\phi}^a$ which is valid only for a unique irrational number a strictly positive and different than 1. Moreover, the subsequents from the hypotheses $\tilde{\Gamma}^a \wedge \tilde{\Gamma}_{Reals,ZFC}$ and the formula $\tilde{\phi}^a$ are not valid. We add the following formulas and hypotheses to convert that irreductible sequent $\mathcal{S}$ into a n-irreducible sequent which is relevant for the super-normality of the irreducible-irrational number a:

$$\Gamma \wedge \tilde{\Gamma}_{Reals,Zermelo} \vdash \exists y \left(\phi_{[y/x]} \wedge 0 < y \wedge \neg y = 1 \wedge y \in \mathbb{N}\right) \wedge$$
$$\neg \exists y \exists z \left(\phi_{[y/x]} \wedge \phi_{[z/x]} \wedge \neg y = z\right)$$

$$\Gamma \equiv \tilde{\Gamma}^a \ \wedge \forall n \forall b \forall d \exists k_0$$

$$a\left(b,0\right) = a \ \wedge$$

$$d\left(b,n\right) b^{k_0-n} < a\left(b,n\right) < \left(d\left(n\right)+1\right) b^{k_0-n} \ \wedge$$

$$a\left(b,n+1\right) = a\left(b,n\right) - d\left(b,n\right) b^{k_0-n} \ \wedge$$

Date: April 9, 2021.

$N_{digit}\left(b,0,d\right)=0\ \wedge$

$d\left(b,n\right)=d\rightarrow N_{digit}\left(b,n,d\right)=N_{digit}\left(b,n-1,d\right)+1\ \wedge$

$\neg d\left(b,n\right)=d\rightarrow N_{digit}\left(b,n,d\right)=N_{digit}\left(b,n-1,d\right)\ \wedge$

$0<x\rightarrow Abs(x)=x\wedge\neg 0<x\rightarrow Abs(x)+x=0\ \wedge$

$\exists\delta\forall\delta' Abs\left(\delta'\right)<\delta\rightarrow Abs\left(Cexp\left(x'+\delta'\right)-Cexp\left(x'\right)-\delta' Cexp\left(x'\right)\right)<Abs\left(\delta'\epsilon\right)\ \wedge$

$n=1\rightarrow Cexp\left(n\right)=n\ \wedge$

$n\in\mathbb{N}\rightarrow\left(f\left(n\right)\in\mathbb{N}\wedge\neg f\left(n\right)<Cexp\left(n\right)\wedge f\left(n\right)<Cexp\left(n\right)+1\right)\ \wedge$

$\phi\equiv\tilde{\phi}^a\wedge\forall x'\forall\epsilon\forall d$

$$\left(x'<x\wedge\neg x'<0\wedge d<x'\wedge\neg d<0\right)\rightarrow\left(x'\right)^{\epsilon} Abs\left(1-x'^{\frac{N_{digit}f\left(x',f\left(x'\right),d\right)}{f\left(x'\right)}}\right)<1\ \wedge$$

$$\neg x^{\epsilon} Abs\left(1-x^{\frac{N_{digit}\left(x,f\left(x\right),d\right)}{f\left(x\right)}}\right)<1$$

where the hypotheses $\Gamma^a\wedge\tilde{\Gamma}_{Reals,ZFC}$ and the formula $\tilde{\phi}^a$ satisfy:

$$\tilde{\Gamma}^a\wedge\tilde{\Gamma}_{Reals,ZFC}\vdash\exists y\left(\tilde{\phi}^a_{[y/x]}\wedge 0<y\wedge\neg y=1\wedge\neg y\in\mathbb{Q}\right)\wedge$$
$$\neg\exists y\exists z\left(\tilde{\phi}^a_{[y/x]}\wedge\tilde{\phi}^a_{[z/x]}\wedge\neg y=z\right)$$

where $Cexp(x)=Exp(x-1)$.

That n-irreducible sequent give the smallest positive integer x such that the distance between the ratio of digits d over the first $Ceiling\left(Exp\left(x-1\right)\right)$ digits in base x and $1/x$ is larger than $1/x^{1+\epsilon}$ for some strictly positive real ϵ.

If we can check numerically that $x>N_Z$, from the axiom at second-order of logic, we can deduce that there is no positive integer x such that the super-normality of a is violated.

From the first 84 irreducible-irrational numbers known (most of them are famous mathematical constants), we found that all of them are super-normal up to $x=5$. Since the number of digits grow up exponentially with x, the meaning of the results is not far from $x=+\infty$. The probability is 9.0×10^{-10} to find the same result with random digits and by taking in account that some irreducible-irrational numbers are only different functions of the same irreducible-irrational number.

Therefore, we postulate at third-order of logic that every irreducible-irrational number is a super-normal-irreducible-irrational number.

We have considered the following 84 mathematical constants:

The Conway constant and its inverse:
1.303577269034296391257099112152551890730702504659404875754861390628550
0.767119850701915359540715997135284064594703512060941496039823831701833

The Feigenbaum constant and its inverse:
4.669201609102990671853203820466201617258185577475768632745651343004134
0.214169377062326492478934818893161783413809015659045443500181457191667

The Niven constant :
0.7052111401

The three Khinchin constants:
2.685452001065306445309714835481795693820382293994462953051152345557218
1.74540566
0.78853056591150896106027632345455466647274966822328164975515640230178

The Gibbs constants with unit jump:
$-1/2+\pi/2\ \int_0^\pi sin(x)/x\ dx$
$+1/2+\pi/2\ \int_0^\pi sin(x)/x\ dx$

The Laplace constant and its inverse:
0.662743419349181580974742097109252907056233549115022417520392534990971853086
1.508879561538319928909884488160578573694278589047769191472078359726460576559

The Golomb-Dickman constant and its inverse:
0.62432998854355087099293638310083724
1.60171707005908755336789757833219300864266450324786224313214921769356604558

The Artin constant and its inverse:
0.37395581361920228805472805434641641511162924860615
2.67411272557002150896041183044548803750239862839769228621522584609442347765

The Landau-Ramanujan inverse square constant:
$(a\ Log\,(x)\,N\,(x)\,N\,(x) = Log\,(x)\,(N\,(x))^2\,/b^2 = x \times x)$
$(\tilde{a}\ \sqrt{Log\,(x)}N\,(x) = \sqrt{Log\,(x)}N\,(x)\,/b = x)$
1.712217963443786953812066012296122346962156854776346981721036801209944169
1.308517467764105529600931476113571198780190163845213947667124446332082976

Euler–Mascheroni constant:
γ

The Levy constant and its inverse:
$e^{\pi^2/12/ln(2)}$
$e^{-\pi^2/12/ln(2)}$

The alternating harmonic series constant:
$ln(2)$

The abscissa of the minimum of the square curvature radius of the exponential:
$ln(2)/2$
The Universal Parabolic constant and its inverse:
$ln\left(1+\sqrt{2}\right)+\sqrt{2}$
$1/\left(ln\left(1+\sqrt{2}\right)+\sqrt{2}\right)$

The half square constants:
$\sqrt{2}, 1/\sqrt{2}$

The equilateral triangle constants:
$\sqrt{3}, 2\sqrt{3}, \sqrt{3}/2, 1/\sqrt{3}, 1/2/\sqrt{3}, 2/\sqrt{3}$

The minimum of the square curvature radius of $1/x$:
$4/\left(3\sqrt{3}\right)$

The Markov constant:
$\sqrt{5}$

The gold number:
$1/2+\sqrt{5}/2$

The exponential constant:
e

The minimum value of x^x:
$1/e$

The minimum of x^x:
$Exp\left(1/e\right)$

The maximum of $1/x^x$:
$Exp\left(-1/e\right)$

The exponential geometric series:
$e/(e-1)$

The negative exponential integral:
$\int_{-1}^{0} Exp(x)dx = (e-1)/e$

The right angle tangent logarithmic spiral:
$a, Exp(a), 1/Exp(a), 2a, Exp(2a)1/Exp(2a)$
0.27441063190284810044017506211094801781885840256703474410204202357688193718860

The integral of x^x:
$\int_0^1 x^x dx$

The Gamma integral 1:
$\int_0^1 1/\Gamma\left(x\right) dx$

The Gamma integral 2:
$\int_0^1 1/\Gamma\left(x+1\right) dx$

The root of LogIntegral:
1.451369234883381050283968485892027449493032283648015863093004

The root of $x\ Exp(x) = 1$:
0.567143290409783872999968662210355549753815787186512508135131079223045793086684

The abscissa of the vertical line that cut the unit disk into a unit piece:
0.567143290409783872999968662210355549753815787186512508135131079223045793086684

The π constant and it inverse:
$\pi, 1/\pi$

The constant from Fourrier transformations:
$2\pi, 1/(2\pi), \sqrt{2\pi}, 1/\sqrt{2\pi}$

The normalized radius of monotone decreasing volumes of n-spheres:
$2/\pi$

The half square and equilateral triangle angles:
$\pi/2, \pi/4, \pi/3$
$\pi/8, \pi/6$

The Cycloide surface constant:
$3\pi, 1/\sqrt{3\pi}$

The Basel problem constant:
$\pi^2/6$

The maximal volume of a n-sphere:
$V\left(4,1\right)$
$\left(V\left(4,1\right)\right)^{-1/5}$

The maximal surface of a n-sphere:
$S\left(6,1\right)$
$\left(S\left(6,1\right)\right)^{-1/6}$

The maximal volume of a none Exotic n-sphere:
$V\left(6,1/2\right), V\left(6,1/\pi\right), V\left(6,1/(2\pi)\right)$

$(V\left(6,1/2\right))^{-1/7}, (V\left(6,1/\pi\right))^{-1/7}, (V\left(6,1/(2\pi)\right))^{-1/7}$

The maximal volume of an exotic n-sphere:
$V\left(7,1/2\right)$
$(V\left(7,1/2\right))^{-1/8}$

The harmonic series times Cosinus:
0.042019

The harmonic series times Cosinus:
1.0707983

The Sinus function at $x=1$:
$Sin(1)$

The Cosinus function at $x=1$:
$Cos(1)$

Where both number are calculated with radian angle and angle defined with unit diameter: $\tilde{\theta}=\theta/2$.

Where the volume and the surface of the n-spheres are defined as:

$V\left(n,r\right)=r^{n+1}\pi^{n/2+1/2}/\Gamma\left(n/2+3/2\right)$
$S\left(n,r\right)=r^{n}2\pi^{n/2+1/2}/\Gamma\left(n/2+1/2\right)$

By defining a unit radius ($r=1$) or a unit diameter ($2\ r=1$) or a unit maximal geodesic length ($\pi\ r=1$) or unit circumference ($2\pi\ r=1$), we can calculate the surface and the volume of the n-spheres.

By defining a unit surface or a unit volume, we can calculate the radius, the diameter, the maximal geodesic length or the circumference of the n-spheres.

With a unit diameter or a unit maximal geodesic length or a unit circumference, the volume and the surface of the n-sphere are decreasing.

With a unit diameter, the volume of the 0-sphere is 1.

We have checked that the volume with a unit diameter or the diameter with a unit volume of a 4-sphere is not a super-normal-irrational number. Therefore, it does not exist an exotic 4-sphere and the corresponding maximal volume with a unit diameter of an exotic sphere is the 7-sphere. Finally, to prove the the none-existence of an exotic 4-sphere is likely impossible without the axiom at third-order of logic.

We have also checked that those numbers are not super-normal-irreducible-irrational numbers:

$e^2, 1/e^4, 11e, 1/(3e), 5\pi, 1/(8\pi), \pi^3, 1/\pi^8, 1/ln\left(3\right), ln\left(6\right), 1/\sqrt{5}, \sqrt{13}, 1/e^{\pi}, e^e$

Therefore, they can not be made with an irreducible sequent.

To conclude this article, we calculate some bound about the total number of n-irreducible sequents. First, we consider a sequence of words (written with l different letters) where each elements of the sequence can not be equal to another element by removing some letters. We focus on a specific symbol for each element of the sequence and the waves formed by it above the background of other letters. If each successive element is strictly larger than the previous one, we have the following maximal number of element:

$N_{increasing}\left(l, n_1, ..., n_l\right) = \prod_{l'=1}^{l} \sum_{k=0}^{n_{l'}} k! \binom{n_{l'}+k}{k}$

where $n_{l'}$ is the number of symbols l' inside the initial word of the sequence.

If the element have the same length after the initial element, the maximal number of elements is:

$N_{equal}\left(n, m, l\right) = \sum_{k=0}^{n} \left(m-k\right)^{l-1} \binom{m}{k}$

where n is the length of the initial word.

The maximal number of n-irreducible sequent made with one word is therefore:

$N_{equal}\left(1030, 10^5, 23\right) \times N_{equal}\left(29, 10^4, 23\right)$

where we have supposed that the theory of everything is the N_Z-irreducible sequent made with two words of maximal length both. Their maximal length are about 10^5 and 10^4 respectively. In general, the formulas inside the sequents are written with 23 different symbols (16 Boolean operations, 4 universal quantifiers, 2 parentheses and 1 vertical bar used repeatedly to index the different parentheses). 1030 and 29 symbols are required to write the smallest n-irreducible sequent $x = 1+1+0$ with some hypotheses among the Reals hypotheses and the ZFC hypotheses.

We remark that the total number of n-irreducible sequents is much smaller than the number N_Z.

Finally, the irreducible-irrational numbers can not be smaller in absolute value than $1/N_Z$ since we can write a n-irreducible sequent with the following formula ϕ :

$\phi \equiv \phi_a \wedge y \times a = 1 \wedge x < y+1 \wedge y < x.$

References

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