Axiomatic Particle Theory

Fermions and Gravitation

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Abstract

An axiomatic proposal for an underlying description of particles and their interactions. The existence of fundamental laws of physics is precluded and only random events exists at the fundamental level. Quarks and Leptons in 3 families are found along with spin 2 massless gravitation. Chiral sector - 3 chiral neutrinos can mix, no charged lepton mixing, quark mixing is allowed. The standard model groups U(1), SU(2) and SU(3) act on vacuum states to generate particles.

1 Introduction

An axiomatic proposal for an underlying description of particles and their interactions. The canonical approach in physics is to deduce equations of motion for the state of a system. The state is not deduced i.e particle content is assumed along with the electrical charges etc. However this approach does not provide a mechanism of how the state changed. As Feynmann noted [1], a particle does not calculate its trajectory in phase space. In addition the reason for the particle content is not addressed. In the Standard Model (SM) [2] of particle physics, the particle content and its interactions are assumed. The fermion content will be addressed in this paper.

In the construction of physics models / theories, assumptions about which mathematical structures are to be used are made. This freedom to choose the mathematical structure is not accessible by Nature. Since Axiomatic Set Theory is a foundation of Mathematics, this will be the foundation for an axiomatic model of nature at the fundamental level.

2 Axioms

A1: No equations of motion for a fundamental system
A2: preclude the creation of same element in sucession
Consider a set S of elements ϵ which represents the state of a system
A1 $\Rightarrow$ no specified relational measure on S
A1 $\Rightarrow$ no functional relations on ϵ
A1 $\Rightarrow$ ϵ are independent
A1 $\Rightarrow$ ϵ are random
independent implies $\Rightarrow$ ϵ are orthogonal
A2 $\Rightarrow$ ϵ form a CAR algebra
A3: minium 2d CAR, $\epsilon \rightarrow \epsilon_i : i = 1, 2$
A3 requires inclusion of all 2d vector spaces V_k
Basis $e_{ki} : e_{ki}^2 = \pm 1$
The basis $e_{ki} : e_{ki}^2 = -1$ act like imaginary units
$signature(V_k) \in \{(1,1), (1,-1), (-1,-1), (-1,1)\}$
Let $V = \cup_k V_k$ is the set of all 2d spaces
ϵ_k are the 2d CAR elements on V_k
Othorgonality of ϵ_k $\Rightarrow$ the 2d vector spaces V_k are orthogonal
All sets form a state S of the system
On V_k the state is $(\epsilon_{k1}, 0)$ or $(0, \epsilon_{k2})$

3 Real and Complex States

The real state $\{(e_{11},0),(0,e_{22}),(e_{31},0),(e_{41},0)\} \in \mathbb{R}^{1,3}$
The real state $\{(e_{11},0),(e_{21},0),(e_{31},0),(0,e_{42})\} \in \mathbb{R}^{3,1}$
These real states are not to be identified as space-time since no co-ordinate chart has been specified
The pair $(e_{11},e_{22}) \in \mathbb{C}$ and similarly other complex states can be found
From [2.19], the states $V_2, V_4 \notin \mathbb{C}$

4 Vacuum States

As particles not defined yet the following $\mathbb{C}^2$ states are consideredto be Vacuum states
$\{(V_1,V_2),(V_3,V_4)\}$
$\{(V_1,V_3),(V_2,V_4)\}$
$\{(V_1,V_4),(V_2,V_3)\}$
as shown below the actions of $U(1)$, $SU(2)$ and $SU(3)$ acts on the vacuum states to generate particles

5 Chirality

Adapting the defintion of chirality [3]
Chirality $c = -\sigma_0\sigma_1\sigma_2\sigma_3, \sigma_i^2 = -1$
On $\mathbb{R}^{1,3}, \sigma_0^2 = 1, c = -1$
On $\mathbb{R}^{3,1}, \sigma_0^2 = -1, c = -1$
as shown below Chiral -1 fermions

6 Scalars

Representations $e^{in\theta} \in U(1)$ can all be constructed from the fundamental representations $n = 0, \pm 1$
Let $\phi = \binom{\phi_1}{\phi_2} \in \mathbb{C}^2$
Action of $U(1)$ on ϕ

$$\binom{\phi^n}{\phi^0} = U(1) \times \phi \to \binom{\phi_1 e^{in\theta}}{\phi_2}$$

Complex scalar doublets $n = \pm 1$ - $\binom{\phi^{\mp}}{\phi^0}$ Higgs doublets when muliplied by dimensionful constant.

7 Chiral -1 Fermions

Rotation of elements of V_2 or V_4 induces $\mathbb{C}^2 \leftrightarrow \mathbb{R}^{1,3}$
The entangled state is

$$\psi_2 = \begin{pmatrix} (V_1, V_2) \\ (V_1, V_3) \\ (V_1, V_4) \end{pmatrix}$$

There are 3 $\mathbb{C}^2$, let $\psi = \binom{\psi_{1i}}{\psi_2} \in \mathbb{C}^2$
For $\psi_2 \notin \mathbb{C}^3$, action of U(1) on ψ

$$\binom{\psi_i^n}{\psi^0} = U(1) \times \psi \to \binom{\psi_{1i} e^{in\theta}}{\psi_2}$$

Complex vectors $n = \pm 1$ - $\binom{\psi_i^{\mp}}{\psi^0}$ - 3 chiral lepton doublets
For $\psi_2 \in \mathbb{C}^3$, $\psi_2 \to \psi_{2j}$, action of U(1) on ψ

$$\binom{\psi_i^n}{\psi_j^m} = U(1) \times \psi \to \binom{\psi_{1i} e^{in\theta}}{\psi_{2j} e^{im\theta}}$$

entangled state ψ_{2j} is a $U(1)$ state $\therefore \sum_j n_j = \pm 1$, therefore the charge on each state ψ_{2j} is $\pm \frac{1}{3}$, the minimum non-zero difference $n - m = -1$, (The charge difference must be a fundamental representationof $U(1)$, thus no further charges), the charges on the quarks are $\mp \frac{1}{3}, \pm \frac{2}{3}$
Note the entangled state is a triplet of quarks
3 chiral quark doublets

8 Chiral +1 Fermions

Let $\phi = \binom{e_1, e_2}{e_3, e_4} \in \mathbb{C}^2$ where $e_1^2 = e_3^2 = 1, e_2^2 = e_4^2 = -1$
In addition $\binom{e_1, e_4}{e_3, e_2} \in \mathbb{C}^2$ thus the following action on the state ϕ

$$su(2, \mathbb{C}) \binom{(e_1, e_2)}{(e_3, e_4)} + su(2, \mathbb{C}) \binom{(e_1, e_4)}{(e_3, e_2)}$$

The chiral state is U(1) invariant to the 2nd $su(2, \mathbb{C})$ thus for example generate electron chiral +1 as below

$$\binom{e_L^-}{\bar{\nu}_{eR}} + \binom{e_L^+}{e_R^-}_{-1}, \binom{e_L^-}{\bar{\nu}_{eR}} + \binom{e_L^-}{e_R^+}_{+1}$$

where the ± 1 indicates the chiralty

9 Gravitation spin 2

The $su(2, \mathbb{C})$ acts on the components and or basis.
Thus 2 $su(2, \mathbb{C})$ act on the basis and 2 $su(2, \mathbb{C})$ act on the components each action generates $s = \pm \frac{1}{2}$

$$(su(2, \mathbb{C}) \times su(2, \mathbb{C}) \times su(2, \mathbb{C}) \times su(2, \mathbb{C}))\mathbb{C}^2 \rightarrow \begin{pmatrix} +2 \\ -2 \end{pmatrix}$$

10 A Probability Distribution

(c, θ) for complex number $ce^{i\theta}$
(c, θ) are random - construct a partition function
$Z = \iint dcd\theta e^{-(c^2+\theta^2)}$
The partition function favours complex states with small amplitude $c$and phase θ For unitary transformations of states, (changes to the phase θ) small $\theta \implies$ also Lie transformations of states

11 Conclusion

From minimal set of axioms, find 3 families of leptons and quarks along with chiral sector - 3 chiral neutrinos can mix, no charged lepton mixing, quark mixing is allowed. The standard model groups U(1), SU(2) and SU(3) act on vacuum states to generate particles. The states can be real or complex states. Future work involves understanding the boson sector of the Standard Model.

References

[1] Richard Feynman's Lectures on Physics (The Messenger Lectures) `https://archive.org` - last accessed Jan 2021

[2] Pham Xuan Yem Quang Ho-Kim.Elementary Particles and Their Interac-tions, Springer-Verlag, 1998

[3] Pham Xuan Yem Quang Ho-Kim.Elementary Particles and Their Interac-tions, 3 Fermions p84, Springer-Verlag, 1998