The neutron as an electric dipole

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Abstract

This paper completes the analysis of the $1/r$ and a/r^2 Coulomb and nuclear potential functions through a consequent analysis of the mass-energy equivalence using the standard orbital energy formula (Kepler's third law) and ensuring (physical) dimensional consistency. The function yield the desired *crossing* of potentials and the expected combined potential well function.

Further analysis of the q_e/m and g_N/m charge/mass-energy ratios and the different *nature* of the singularities at the center may explain the rather enormous proton/neutron energy/mass when accepting a Yukawa/Schrödinger-type nuclear force and potential.

The model has the advantage of *not* introducing new fundamental constants (except for the nuclear charge g_N), respecting relativity theory (no superluminal speeds), and confirming Planck's quantum of action as the fundamental unit of physical action (h) and angular momentum ($\hbar$) in particle-field exchanges of energy and momentum.

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Introduction

In our previous paper[1], we analyzed the nuclear force between the neutron and the proton inside of the deuteron nucleus (D = n + p) as an electrostatic attractive force between an electric dipole field arising from the (neutral) neutron dipole (n = p + e) and the (positive) electric charge of the proton. Let us think of the dipole concept first, before we go on to think about the neutron. **Figure 1** shows two opposite charges and the equipotential and field lines of the dipole field they create.

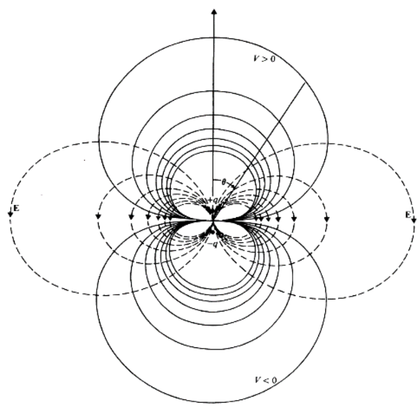

Figure 1: Equipotential (*V*, solid lines) and electric field (*E*, dashed lines) in a dipole field [2]

The limits of this approach are immediately obvious: we have *zero* potential along the midperpendicular (line segment bisector) between the two charges, which implies no work is done when bringing a charge from infinity to the midpoint along this line. Other trajectories from infinity to the zero potential line would involve no *net* work but consist of positive work nullified by negative work. Such trajectories should be analyzed in the context of the minimum or least action principle, which tells us a charge will follow a trajectory which *lowers* its total energy (kinetic *and* potential) by moving along a path which minimizes (physical) action, which we may write as[3]:

$$S = \int_{t_1}^{t_2} (\mathrm{KE} - \mathrm{PE})\,\mathrm{d}t.$$

In plain language, a negative charge will go and sit right on top of the positive charge, while a positive charge will want to join the negative charge.

The electric dipole model also remains silent on what keeps the positive and negative charge separate—not approximately but *exactly*. Indeed, why don't they too just go and sit right on top on each other?

[1] *The electromagnetic deuteron model*, December 2020.
[2] The illustration was taken from a commercial sales site for fiber optic equipment.
[3] For a full development of the least action principle – both from a classical as well as a quantum-mechanical perspective – we refer to Feynman's Lectures, Volume II, Chapter 19 (*The Least Action Principle*). We think its central place (middle of Volume II) is no coincidence.

The nuclear dipole and the *Zitterbewegung* (ring current) hypothesis

The idea is this:

- The positive and negative charge cannot be a matter-antimatter pair because otherwise they would annihilate each other.[4]
- The two charges have energy so they must be in a dance together.[5]
- They are, therefore, spinning and act like two wires with *like currents*, or two spinning charges with a magnetic moment that separates them magnetically.[6]
- Planck's quantum of action ($\hbar$) gives us the angular frequency and the radius of the oscillation (we will calculate this in a minute), and we may assume the oscillation packs one unit ($\hbar$) of physical action.[7]

OK. So how does that work, then?

We may apply a 'mass without mass' – or, to be more precise, an *electromagnetic* mass – model to the neutron-electron so as to calculate the *radius* of the oscillation. The energy difference between the deuteron nucleus (about 1875.613 MeV) and its two constituents (neutron and proton) in their unbound state (939.565 MeV + 938.272 MeV = 1,877.837 MeV) is *negative* and equal to about 2.23 MeV. This shows the proton and neutron are happy to dance together and the oscillation should be of the order of 2.23 MeV *per cycle*. So 2.22 MeV is the sum of the potential and kinetic energy in the oscillation, and it must also pack one unit of $\hbar$, and the charges go at lightspeed, and all of the energy is the oscillation and we may, therefore write its equivalent mass as E/c^2? Yes. The model is summarized below:

$$\left.\begin{array}{c}\left.\begin{array}{c}\mathrm{E}=\mathrm{m}c^2\\ \mathrm{E}=\hbar\omega\end{array}\right\}\Rightarrow \mathrm{m}c^2=\hbar\omega\\ c=a\omega\Leftrightarrow a=\dfrac{c}{\omega}\Leftrightarrow\omega=\dfrac{c}{a}\end{array}\right\}\Rightarrow \mathrm{m}a^2\omega^2=\hbar\omega\Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2=\hbar\frac{c}{a}\Leftrightarrow a=\frac{\hbar}{\mathrm{m}c}$$

The calculation yields this for the charge radius[8]: o calculate the charge radius using the electromagnetic mass model, we must use the mass factor in the $a = \hbar/\mathrm{m}c$ by the (equivalent) orbital energy. We, therefore, get the following *charge* radius[9]:

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[4] At first sight, matter and antimatter differ only by spin. See my paper on issues and gaps in the ring current model of elementary particles.

[5] See Feynman's explanation of the size of an atom.

[6] A positive current in one direction is equivalent to a negative current in the other direction. Assuming the charge has zero rest mass and, therefore, *zitters* around at the speed of light, assigning a magnetic or electric nature to the force depends on the reference frame only.

[7] (*Wirkung* (German) captures the essence of the force)

[8] We equate this to the radius of oscillation but, of course, the usual caveats apply: this will be an average only with elliptical orbitals, and the radius of effective charge-photon radius will be larger because including the electromagnetic field itself.

[9] The $\hbar c$ factor and its dimension can easily be verified from using the 6.582×10^{-16} eV·s value for $\hbar$ and the 299,792,458 m/s value for c and, of course, we should not forget to convert m into fm (10^{-15} m):

$$\hbar c=(6.582119569\times10^{-16})\cdot(299{,}792{,}458)\ \mathrm{eV}\cdot\mathrm{m}$$
$$\approx(1{,}973{,}269{,}804\times10^{-16}\ \mathrm{eV})\cdot(10^{15}\ \mathrm{fm})\approx197.327\ \mathrm{MeV}\cdot\mathrm{fm}$$

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\hbar}{\frac{\mathrm{E}}{c^2}c} = \frac{\hbar c}{\mathrm{E}} \approx \frac{197.327\ \mathrm{MeV} \cdot \mathrm{fm}}{2.224\ \mathrm{MeV}} \approx 88.7\ \mathrm{fm}$$

About 110 times the neutron radius ! This is, clearly, an impossible value, which is why we do not believe the electron and the proton inside of a neutron are held together by the electromagnetic force: we *must* assume some nuclear force. At the same time, we do want to keep thinking of the neutron-proton combination as the electric dipole. So how does that work? Some polar structure because the electron blanket will oscillate back and forth between the two protons, and Schrödinger's *Platzwechsel* (change of place) model of a deuteron nucleus is a model of the oscillation of the electron cloud?[10] And we should think of that as a nuclear force oscillation too, *somehow*? And we should now invent a nuclear charge, isn't it?

Correct. We actually might have a candidate particle here: the neutrino. Could it behave like a *zbw* oscillation when at rest and becoming more photon-like when free?[11] How does this work? We have the mass factor in the denominator of the formula for the Compton or *zbw* (Zitterbewegung) radius, so it must *increase* as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, also *increase* with velocity: we have a simple (inverse) proportionality relation here. The idea is visualized in the illustration below[12]: the *radius* of the circulatory motion must effectively diminish as a linear component gets added to the tangential component of the velocity of the pointlike *zbw* charge.

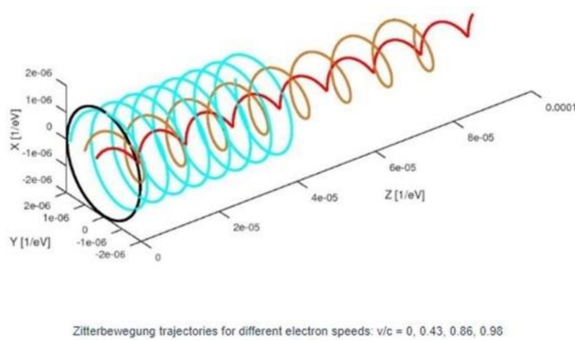

Figure 2: The *Zitterbewegung* radius must decrease with increasing velocity[13]

[10] See the history section in the Wikipedia article on the Yukawa potential.
[11] Interestingly, the latest hypothesis (according to Wikipedia, at least) with regard to neutrinos is that they would have some non-zero rest mass and, therefore, do never quite attain the speed of light. However, it mentions that existing measurements for MeV to GeV neutrinos provided upper limits for deviations of approximately 10^{-9}, or a few parts per billion and that, within the margin of error, this is consistent with no deviation at all. Hence, the question may not have been solved yet.
[12] We thank Prof. Dr. Giorgio Vassallo and his publisher to let us re-use this diagram. It originally appeared in an article by Francesco Celani, Giorgio Vassallo and Antonino Di Tommaso (*Maxwell's equations and Occam's Razor*, November 2017).
[13] The illustration assumes the plane of oscillation is perpendicular to the direction of propagation. Needless to say, this assumption is rather random. The reader may want to imagine that the plane of oscillation rotates or oscillates itself. He should not think of it of being *static* – unless we think of the charge moving in a magnetic field, in which case we should probably think of the plane of oscillation as being *parallel* to the direction of propagation. We will let the reader think through the geometric implications of this.

When the velocity goes to *c*, the circumference of the oscillation must turn into a *linear* wavelength in the process, and we have a photon-like particle![14] This rather remarkable geometric property related our *zbw* electron model with our photon model, which we will not talk about here, however.[15]

All sounds *nuts*. Yes. Is there an alternative? I do not see one. Let us try to organize our thoughts a bit more precisely here.

Binding energies, electromagnetic mass, and charge radii

Of course, we do *not* think of the neutron as a *linear* structure. In fact, the charge radius of a *free* neutron (R_c) is assumed to be zero: it only has a *magnetic* radius (R_m), whose rms value is about 0.8 fm.

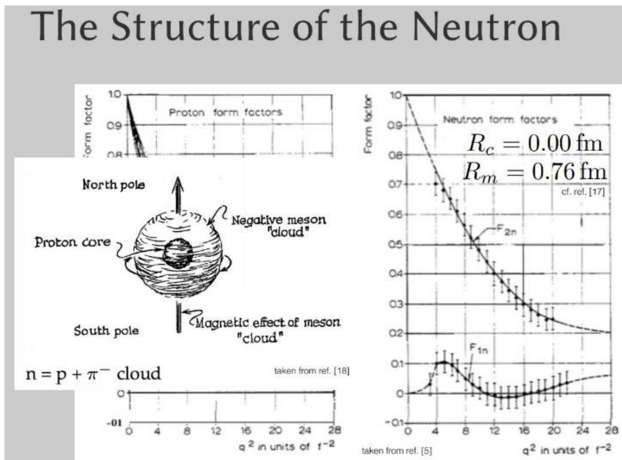

Figure 3: The neutron model and the concepts of charge and magnetic radius (R_c and R_m)[16]

Measuring the magnetic radius of a *free* neutron is difficult because a neutron is stable only inside of a nucleus. Indeed, we already mentioned how the instability of the neutron might be reflected in a *positive* energy difference (~ 1.3 MeV) between the neutron (~ 939.565 MeV) and the proton (~ 938.272 MeV). This energy range (1.3 MeV) is about 2.5 times the energy of a free electron but is only a tiny fraction of the energy of a $\pi^{\pm}$ meson (139.57 MeV) or the muon-electron (105.65 MeV), which is why we find the reference to a π^{-} cloud in the illustration rather misplaced.

Before we continue, let us quickly put this energy value into perspective once more: the energy difference between the deuteron nucleus (about 1875.613 MeV) and its two constituents (neutron and proton) in their unbound state (939.565 MeV + 938.272 MeV = 1,877.837 MeV) is *negative* and about 1.7 times our 1.3 MeV value binding energy of the negative charge in an n = p + e model of the neutron. Erwin Schrödinger, therefore, effectively proposed a *Platzwechsel* model for the deuteron nucleus: the neutron-proton and the proton-proton should continually swap spots.

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[14] We may, therefore, think of the Compton wavelength as a *circular* wavelength: it is the length of a circumference rather than a linear feature!

[15] We may refer the reader to [our paper on *Relativity, Light and Photons*]().

[16] Illustration from Christoph Schweiger's presentation on *the electron-scattering method, and its applications to the structure of nuclei and nucleons* (8 January 2016). Schweiger took these illustrations from Robert Hofstadter's 1961 Nobel Prize Lecture, which has the same title. However, we find Schweiger's added neutron model and the R_c = 0 and R_m = 0.76 fm formulas very didactic.

It is probably good to detail the assumptions once more here:

- All of the energy is in (1) the oscillation of the charge (no rest mass, and orbitals can be circular or elliptical, with $\boldsymbol{v}$ and $\boldsymbol{c}$ as tangential velocity *vectors*[17]: E = m$\boldsymbol{c}^2$), and (2) the relativistic or *effective* mass of the charge.
- The model does not incorporate any spin angular momentum of the charge itself: all angular momentum is orbital. The Planck-Einstein relation then expresses the quantization of the orbital angular momentum and can, therefore, be written as a *vector* equation as well: E = $\boldsymbol{\hbar}\cdot\boldsymbol{\omega}$. The angular velocity – for circular as well as elliptical orbitals – is given by $\omega = d\theta/dt = v_{\perp}/r$ or, in vector notation, $\boldsymbol{\omega} = \boldsymbol{r}\times v/r^2$.
- The radial force is the electrostatic Coulomb force and the energy in the orbital is, therefore, an energy *per unit charge*, and the energy equation for the orbital must, therefore, also be written in terms of E/q. For the (physical) dimensions to make sense, we must also write the (orbital) kinetic energy as energy *per unit charge*. The orbital energy equation (and its physical dimensions) can then be written as:

$$\frac{\mathrm{E}}{\mathrm{q}} = \frac{\mathrm{KE}+\mathrm{PE}}{\mathrm{q}} = \frac{1}{2}\frac{\mathrm{m}v^2}{\mathrm{q}} - \frac{\mu}{r} = \frac{1}{2}\frac{\mathrm{m}v^2}{\mathrm{q}} - \frac{\mathrm{kq_e}}{r} = \frac{1}{2}\frac{\mathrm{m}v^2}{\mathrm{q}} + \frac{\mathrm{q_e}}{4\pi\varepsilon_0 r}$$

$$\left[\frac{\mathrm{E}}{\mathrm{q}}\right] = \left[\frac{1}{2}\frac{\mathrm{m}v^2}{\mathrm{q}} + \frac{\mathrm{q_e}}{4\pi\varepsilon_0 r}\right] = \frac{\mathrm{kg}\dfrac{\mathrm{m}^2}{\mathrm{s}^2}}{\mathrm{C}} + \frac{\mathrm{C}}{\dfrac{\mathrm{C}^2}{\mathrm{Nm}^2}\mathrm{m}} = \frac{\mathrm{N}\dfrac{\mathrm{s}^2}{\mathrm{m}}\dfrac{\mathrm{m}^2}{\mathrm{s}^2}}{\mathrm{C}} + \frac{\mathrm{Nm}^2}{\mathrm{Cm}} = \frac{\mathrm{Nm}}{\mathrm{C}} + \frac{\mathrm{Nm}}{\mathrm{C}} = \frac{\mathrm{Nm}}{\mathrm{C}}$$

We believe the energy per unit charge formula is relativistically correct because the kinetic energy uses the velocity v along the orbital (which is denoted as escape velocity v_e[18]). We may also already not the m/q factor, which is the *inverse* of Bohr's magneton q/m. Indeed, we think of elementary particles as spin-1/2 particles. Hence, their gyromagnetic ratio[19] is ½ because of the following *identity*[20]:

$$\frac{\mu}{\mathrm{L}} = \mathrm{g}\cdot\frac{\mathrm{q}}{\mathrm{m}} = \frac{1}{2}\cdot\frac{\mathrm{q}}{\mathrm{m}} \Leftrightarrow \frac{\mathrm{m}\cdot\mu}{\mathrm{q}\cdot\mathrm{L}} = \frac{1}{2}$$

[17] The tangential velocity $\boldsymbol{v}$ is equal to $\boldsymbol{c}$ for circular orbits. When considering elliptical orbitals, lightspeed is reached only when the charge passes the center (zero potential energy: all energy is kinetic). *When treating both rest as well as relativistic mass* (m_0 and $m = \gamma m_0$) *as electrostatic mass only*, the Lorentz factor (γ) can be expanded to yield the following series expansion of the total energy:

$$\mathrm{m}c^2 = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\left(1+\frac{1}{2}\beta^2+\frac{3}{8}\beta^4+\cdots\right)$$

Dimensions are easily checked when not confusing the *mass* symbol m (expressed in kg or N·m/s² units) with the distance unit (*meter*):

$$[\mathrm{m}] = \left[\frac{\mathrm{E}}{c^2}\right] = \mathrm{Nm}\frac{\mathrm{s}^2}{\mathrm{m}^2} = \mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}} = \mathrm{kg} = \left[\frac{\mathrm{U}(r)}{c^2}\right] = \left[\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\frac{1}{c^2}\right] = \frac{\mathrm{C}^2}{\dfrac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}}\frac{\mathrm{s}^2}{\mathrm{m}^3} = \mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}} = \mathrm{kg}$$

[18] See the MIT OCW reference course on central force motion.

[19] The gyromagnetic ratio is usually denoted as g but symbols become quite confusing, especially when considering the μ in the g = μ/L equation below is the *magnetic moment* (not the standard electromagnetic parameter).

[20] See our paper on a speculative but realist interpretation of quantum physics based on the ring current model.

$$electron: \frac{\mathrm{m_e} \cdot \mu_\mathrm{e}}{\mathrm{q_e} \cdot \mathrm{L_e}} = \frac{\mathrm{m_e} \cdot \frac{\mathrm{q_e}\hbar}{2\mathrm{m_e}}}{\mathrm{q_e} \cdot \hbar} = \frac{1}{2}$$

$$muon: \frac{\mathrm{m_\mu} \cdot \mu_\mu}{\mathrm{q_e} \cdot \mathrm{L_\mu}} = \frac{\mathrm{m_\mu} \cdot \frac{\mathrm{q_e}\hbar}{2\mathrm{m_\mu}}}{\mathrm{q_e} \cdot \hbar} = \frac{1}{2}$$

$$proton: \frac{\mathrm{m_p} \cdot \mu_\mathrm{p}}{\mathrm{q_e} \cdot \mathrm{L_p}} = \frac{\mathrm{m_p} \cdot \frac{2\mathrm{q_e}\hbar}{\mathrm{m_p}}}{\mathrm{q_e} \cdot 4\hbar} = \frac{1}{2}$$

The importance thing to note here is that we can write the angular momentum *L* as:

$$L = \frac{1}{2}\frac{\mathrm{q}}{\mathrm{m}} \Leftrightarrow \frac{\mathrm{m}}{2\mathrm{q}} = \frac{1}{4L}$$

How do we reconcile this with the view that the *total* angular momentum of such orbitals packs one $\hbar$—not a half-unit $\hbar/2$!

Hmm… That is complicated. We must assume the angular momentum captures half of the energy only – the kinetic energy – and that the other half must be in the electromagnetic field. Logical? You tell me. We will need this later, so let us write it down and see where we get:

$$L = \frac{\mathrm{q}}{\mathrm{m}} \Leftrightarrow \frac{\mathrm{m}}{2\mathrm{q}} = \frac{1}{2L}$$

OK. Let us now move on the calculation, which should yield yet another radius value.

To calculate the charge radius using the electromagnetic mass model, we must use the mass factor in the $a = \hbar/mc$ by the (equivalent) orbital energy. We, therefore, get the following *charge* radius[21]:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\hbar}{\frac{\mathrm{E}}{c^2}c} = \frac{\hbar c}{\mathrm{E}} \approx \frac{197.327\ \mathrm{MeV \cdot fm}}{1.293\ \mathrm{MeV}} \approx 152\ \mathrm{fm}$$

About 71.716 times the deuteron charge radius ! This is, clearly, yes another impossible value, which is why we do not believe the electron and the proton inside of a neutron are held together by the electromagnetic force: we *must* assume some nuclear force.

The neutron oscillation and elliptical orbitals

The (electric) dipole field of a single charge from a binomial expansion of the terms of the potential in d, which is the distance between the two opposite charges +q and –q, which we will want to equate with a pointlike but massive proton and a *Zitterbewegung* (zbw) electron consisting of a pointlike negative charge moving in and out.

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[21] The $\hbar c$ factor and its dimension can easily be verified from using the 6.582×10^{-16} eV·s value for $\hbar$ and the 299,792,458 m/s value for c and, of course, we should not forget to convert m into fm (10^{-15} m):

$$\hbar c = (6.582119569 \times 10^{-16}) \cdot (299{,}792{,}458)\ \mathrm{eV \cdot m}$$
$$\approx (1{,}973{,}269{,}804 \times 10^{-16}\ \mathrm{eV}) \cdot (10^{15}\ \mathrm{fm}) \approx 197.327\ \mathrm{MeV \cdot fm}$$

The pointlike *zbw* charge reaches maximum velocity (c) when passing through the center of the radial field and overshooting it. Its motion, therefore, combines a radial and a tangential component. The image of a 3D polar rose comes to mind here (**Figure 4**).

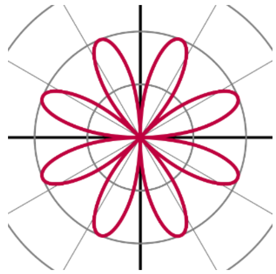

Figure 4: Polar rose : $r(\varphi) = a_0 \cdot \cos(k_0\varphi + \gamma_0)$

The *phase* $\varphi = \omega \cdot t$ is given by the (angular) frequency $\omega = E/\hbar$, in which E must represent the *total* energy of the oscillation—kinetic + potential which, using the binomial theorem, can be written as[22]:

$$\mathrm{m}c^2 = \mathrm{m}_0c^2 + \frac{1}{2}\mathrm{m}_0v^2 + \frac{3}{8}\mathrm{m}_0\frac{v^4}{c^2} + \cdots = \mathrm{m}_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right)$$
$$= \mathrm{m}_0c^2\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

The relativistically correct formula for kinetic energy defines kinetic energy as the difference between the *total* energy and the potential energy: KE = E − PE. The potential energy must, therefore, be given by the m_0c^2 term. This term is zero for $r = 0$ but non-zero because of the potential energy in the radial field at distances $r > 0$. The *total* energy of a charge in a (static) Coulomb field is given by[23]:

$$\mathrm{U}(r) = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}$$

The potential itself is equal to $\mathrm{V}(r) = \mathrm{U}(r)/\mathrm{q_e}$:

$$\mathrm{V}(r) = \frac{\mathrm{U}(r)}{\mathrm{q_e}} = \frac{\mathrm{q_e}}{4\pi\varepsilon_0}\frac{1}{r}$$

[22] The total energy is given by $E = \mathrm{m}c^2 = \gamma\mathrm{m}_0c^2$ which can be expanded into a power series using the binomial theorem (Feynman's *Lectures*, I-15-8 and I-15-9 (relativistic dynamics). He does so by first expanding $\gamma\mathrm{m}_0$:

$$\mathrm{m} = \frac{\mathrm{m}_0}{\sqrt{1 + \frac{v^2}{c^2}}} = \mathrm{m}_0(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots)$$

This is multiplied with c^2 again to obtain the series in the text.

[23] $\mathrm{U}(r) = \mathrm{V(r)}\cdot\mathrm{q_e} = \mathrm{V(r)}\cdot\mathrm{q_e} = (\mathrm{k_e}\cdot\mathrm{q_e}/r)\cdot\mathrm{q_e} = \mathrm{k_e}\cdot\mathrm{q_e}^2/r$ with $\mathrm{k_e} \approx 9\times10^9$ N·m²/C². Potential *energy* (U) is, therefore, expressed in *joule* (1 J = 1 N·m), while potential (V) is expressed in *joule*/*Coulomb* (J/C). Since the 2019 revision of the SI units, the electric, magnetic, and fine-structure constants have been co-defined as $\varepsilon_0 = 1/\mu_0c^2 = \mathrm{q_e}^2/2\alpha hc$. The CODATA/NIST value for the standard error on the value ε_0, μ_0, and α is currently set at 1.5×10^{10} F/m, 1.5×10^{10} H/m, and 1.5×10^{10} (mathematical dimension only), respectively.

We could define the kg (mass) in terms of newton (force) and acceleration (m/s^2). Can we do the same for the *coulomb*? Rewriting the energy equation as a function of the *relative* velocity and the *radial* distance *r* does the trick:

$$\mathrm{m}c^2 = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

We may say this *defines* the mass of the pointlike charge as *electromagnetic* mass only, which now consists of a kinetic and potential piece. The energy in the oscillation, therefore, defines the total mass m = E/c^2 of the neutron electron (n = p + e_n). The kinetic energy is thus given by[24]:

$$\mathrm{KE} = \mathrm{m}c^2 - \mathrm{U}(r) = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right) - \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r} = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\left(1 - 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$
$$= \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\left(\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots\right)$$

The first term in the series gives us the non-relativistic kinetic energy $\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{\beta^2}{2}\frac{1}{r}$. In line with the usual convention for measuring potential energy, we will now set the reference point for potential energy at zero at infinity, and the potential energy will, therefore, be defined as *negative*, going from 0 for r $\rightarrow \infty$ to $-\infty$ for $r \rightarrow 0$. This makes for a *negative* total energy which is in line with the concept of a *negative* ionization energy for an electron in an atomic orbital which, for a one-proton atom (hydrogen), is given by the Rydberg formula.

Analogy between the gravitational radial field and the Coulomb field

We established an analogy between gravitational masses and electromagnetic mass above. We will soon introduce Kepler's third law, which gives us the cycle time of a mass of 1 kg (the mass unit) in orbital around a mass M[25]:

$$\mathrm{T} = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{a^3}{\mathrm{GM}}}$$

The formula uses the concept of the *standard gravitational parameter* μ = GM. The physical dimension of G and M cancel out nicely when writing the gravitational constant G as (approximately) 6.674×10−11 m^3·kg^{-1}·s^{-2}, which is what most textbooks (and Wikipedia) do. So if we are going to describe a charge in orbit, we should use the electric constant k = 1/4πε_0, whose physical dimension is equal to [k] = [1/4πε_0] = N·m^2/C^2: N·m^2 divided by the square of the charge. How comes m^3/kg·s^2 does not look like that? The answer is: it amounts to the same. We can relate the unit of mass (expressed in kg) to a force (expressed in newton) through Newton's second law: a force of 1 N will give a mass of 1 kg

[24] In line with the usual convention for measuring potential energy, we will set the reference point for potential energy at zero at infinity.

[25] We gratefully acknowledge the MIT OCW course on central force motion for the formulas. We consider orbital angular momentum only and we are, therefore, essentially modelling spin-zero particles neglecting the spin angular momentum (*S*) in the (vector) spin-coupling equation ***J*** = ***L*** + ***S***. We assume orbital angular momentum respects the Planck-Einstein relation: *L* will, therefore, be an integer multiple of Planck's quantum of action *ħ*: *L* = *n*·*ħ* = *n*·E/ω. We note the IAEA logo shows elliptical orbitals (closed trajectories) too !

an acceleration of 1 m/s^2. Hence, 1 kg = 1 N·s^2/m. Hence, a squared kg is equivalent to [1 N·s^2/m]^2 Inserting this into the structure of the proportionality constants for all inverse-square force laws, and then converting N back to kg so as to get that we get that m^3·kg^{-1}·s^{-2} expression, we get a consistent dimensional equation:

$$\frac{\mathrm{N}\cdot\mathrm{m}^2}{(\mathrm{kg})^2}=\frac{\mathrm{N}\cdot\mathrm{m}^2}{(\mathrm{N}\cdot\mathrm{s}^2/\mathrm{m})^2}=\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{N}^2\cdot\mathrm{s}^4/\mathrm{m}^2}=\frac{\mathrm{m}^4}{\mathrm{N}\cdot\mathrm{s}^4}=\frac{\mathrm{m}^4}{\mathrm{kg}\cdot(\mathrm{m}/\mathrm{s}^2)\cdot\mathrm{s}^4}=\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}=\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$$

The formula for the cycle time then works out nicely—dimensionally speaking:

$$\left[2\pi\sqrt{\frac{a^3}{\mathrm{GM}}}\right]=\sqrt{\frac{\mathrm{m}^3}{\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}\cdot\mathrm{kg}}}=\sqrt{\mathrm{s}^2}=[\mathrm{T}]$$

So how does that work out when we have electromagnetic mass? We just need to think differently about the force: a force is that what changes the state of motion by changing the *energy* of the charge. Hence, we just use Einstein's mass-energy equivalence relation: m = E/c^2. Hence, 1 kg is 1 J·s^2/m^2 = 1 N. Hence, we think of the *inertia* of the energy rather than the inertia of the mass. Putting the same squared mass or energy factor in the denominator of the denominator 😊, and just mass/energy in the numerator of the denominator of that square root function for the cycle time, we get:

$$[\mathrm{T}]=\sqrt{\frac{\mathrm{m}^3}{\left(\frac{\mathrm{Nm}^2}{\left[\frac{\mathrm{E}}{c^2}\right]^2}\right)\cdot\left[\frac{\mathrm{E}}{c^2}\right]}}=\sqrt{\frac{\mathrm{m}^3}{\mathrm{J}\cdot\mathrm{m}\cdot\frac{[c^2]}{[\mathrm{E}]}}}=\sqrt{\frac{\mathrm{m}^2\cdot\frac{\mathrm{s}^2}{\mathrm{m}^2}}{\mathrm{J}\cdot\frac{1}{\mathrm{J}}}}=\sqrt{\mathrm{s}^2}$$

Now, that was not too difficult, was it? The question is now: how are we going to write our *standard electromagnetic parameter* μ_k? Look at our formulas with the series expansion of energies: the mc^2 factor gets replaced by $\frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{r}$ and that works for both kinetic and potential energy (the radial force changes both). Remarkably simple. Hence, we can just write μ_k as $\mu_k = k_e q_e$, just like we wrote μ_G = GM and that is it.[26]

So let us summarize what we have so far. We substituted an energy per unit mass for an energy per unit charge concept. We need the total energy, which depends both on the velocity *v* as well as distance *r*.

We believe this formula is relativistically correct because the kinetic energy uses the velocity v along the orbital (which is denoted as escape velocity v_e[27]). The kinetic energy is, therefore, equal to KE = mv_e^2/2 = m(ωr)^2/2 = m$\omega^2 r^2$/2. So let us think about this. We use the mass concept as a measure of the inertia to a change of motion as per the relativistically correct expression of Newton's second law F = dp/dt = d(m·v/dt for a centripetal force, whose magnitude can also be written as the following function of the acceleration *a*:

[26] The difference between the m for mass and the m for meter is obvious from the context.
[27] See the referenced MIT OCW reference course.

$$F = \frac{dp}{dt} = \frac{d(\gamma m_0 v)}{dt} = \frac{d(\gamma)}{dt} m_0 v + \gamma m_0 \frac{dv}{dt} = \gamma^3 m_0 a = \gamma^2 m_v a$$

But so we will think of a force as that what changes *energy*. We will show we need to choose our reference point carefully so as to make sense of μ_k.

Orbital geometries

We will now get into the promised nitty-gritty of elliptical orbitals. The general formulas for orbital motion in a non-zero potential assuming closed and, therefore, elliptical orbitals with negative energy (E < 0) and an eccentricity less than 1 (0 < *e* < 0), are the following[28]:

$$r = \frac{L^2/\mu}{1 + e \cdot \cos\theta}$$

$$L = r^2\theta = |\boldsymbol{r} \times \boldsymbol{v}| = n \cdot \hbar = n \cdot \frac{E}{\omega}$$

$$\mu = \left(\frac{2\pi}{T}\right)^2 a^3 = \omega^2 a^3$$

We should a *base state* for *n* = 1 in the E = *n*·*ħ* equation and excited states for *n* > 1. As for the $\mu = \left(\frac{2\pi}{T}\right)^2 \cdot a^3 = \omega^2 a^3$ law, this is just Kepler's third law which calculates the frequency of any orbit around a large mass as a function of (i) the so-called *standard gravitational parameter*[29] μ = G·M and (ii) *a*, which is the length of the orbit's semi-major axis:

$$T = 2\pi\sqrt{\frac{a^3}{\mu_G}} = 2\pi\sqrt{\frac{a^3}{GM}}$$

Hence, for an orbital assuming *all mass is electromagnetic*, we get

$$T = 2\pi\sqrt{\frac{a^3}{\mu_k}} = 2\pi\sqrt{\frac{a^3}{k_e q_e}}$$

We already showed the dimensional analysis of this works out OK[30] so let us now use the formula. We should use a trick here. We have, in effect, a very useful point to evaluate potential and kinetic energy is at the *periapsis*, where the distance between the charge and the center of the radial field is closed. The θ angle is there set at 0, which allows us to define *r* = 0 and *v* = *c* as complementary limits:

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[28] We refer, once again, to the MIT OCW course on central force motion for the formulas. We consider orbital angular momentum only and we are, therefore, essentially modelling spin-zero particles neglecting the spin angular momentum (*S*) in the (vector) spin-coupling equation ***J*** = ***L*** + ***S***. We assume orbital angular momentum respects the Planck-Einstein relation: *L* will, therefore, be an integer multiple of Planck's quantum of action *ħ*: *L* = *n*·*ħ* = *n*·E/ω. We note the IAEA logo shows elliptical orbitals (closed trajectories) too !

[29] When *googling* this, you will find plenty of references, but the Wikipedia article on elliptical orbits is OK.

[30] $\left[2\pi\sqrt{\frac{a^3}{GM}}\right] = \sqrt{\frac{m^3}{\frac{m^3}{kg \cdot s^2} \cdot kg}} = \sqrt{s^2} = [T]$

$$\lim_{r_\pi \to 0} v_\pi = c$$

$$\lim_{v_\pi \to c} r_\pi = 0$$

However, to avoid the division by zero, non-limit values for r_π and r_π are used, which we can obtain from the general orbital formulas:

$$r_\pi = \frac{L^2/\mu}{1 + e \cdot \cos(\theta = 0)} = \frac{L^2}{\mu(1+e)}$$

$$L = r^2\theta = |\boldsymbol{r_\pi} \times \boldsymbol{v_\pi}| = r_\pi v_\pi \sin(\theta = 0) = r_\pi v_\pi$$

$$\Leftrightarrow v_\pi^2 = \frac{L^2}{r_\pi^2} \Leftrightarrow v_\pi^2 = \frac{L^2\mu(1+e)}{r_\pi L^2} = \frac{\mu(1+e)}{r_\pi} = \frac{\mu(1+e)}{r_\pi} = \frac{\mu^2(1+e)^2}{L^2}$$

Using the *L* = m/q formula, we can then do calculate the eccentricity *e* from the orbital energy formula[31]:

$$\frac{\mathrm{E}}{\mathrm{m}} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{v_\pi^2}{2} - \frac{\mu \mathrm{q_e}}{\mathrm{m} r_\pi} = \frac{\mu^2(1+e)^2}{L^2} - \frac{\mu^2(1+e)}{L^3} = \frac{\mu^2(1+e)^2}{L^2} - \frac{\mu(1+e)}{L^3}$$

$$= \frac{L\mu^2(1+e)^2 - \mu(1+e)}{L^3}$$

This differs rather substantially from the formula for the orbital energy in a gravitational field[32]:

$$\frac{\mathrm{E}}{\mathrm{m}} = \frac{v_\pi^2}{2} - \frac{\mu_\mathrm{G}}{r_\pi} = \frac{\mu_\mathrm{G}^2(1+e)^2}{2L^2} - \frac{\mu_\mathrm{G}^2(1+e)}{L^2} = \frac{\mu_\mathrm{G}^2(1+2e+e^2-2-2e)}{2L^2} = \frac{\mu_\mathrm{G}^2(e^2-1)}{2L^2}$$

Why are these so formulas so different? Not sure, but the use of the E/q and E/m factors in these orbital equations makes a difference. While it complicates the structure of the formulas, it is obvious that this mass/charge ratio must, therefore, come into play *somehow*. Also, the structure of the two radial forces (gravitational and electrostatic) are not same. The electromagnetic force is two-dimensional, as the electrostatic (time-independent) field *and* magnetic (time-dependent) field always go hand-in-hand.

We believe this formula establishes an equivalence between gravitational and electromagnetic mass through the mass-energy equivalence relation, which we will write as c^2 = E/m in the next section.

[31] The eccentricity e will be smaller than 1 ($0 < e < 1$) for a closed trajectory and we, therefore, prefer to write $1 - e^2$ rather than $e^2 - 1$. The expression only makes sense if the total energy is *negative*, which is the assumption (or convention, we should say) that we started out with.

[32] See the referenced MIT OCW reference course.

The nuclear force field and potential

The mass as inertia and the energy per unit mass hold the key to writing the orbital energy *per unit mass* as follows for the three forces and charges we have been considering—gravitational, electrostatic, and nuclear:

1. Gravitational force, with the *masses* (M and m) as charge and G as physical proportionality constant:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{v^2}{2} - \frac{\mathrm{GMm}}{\mathrm{m}r}$$

One can easily the dimensions work out to the required m^2/s^2 dimension:

$$\left[\frac{v^2}{2} - \frac{\mathrm{GMm}}{\mathrm{m}r}\right] = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}\mathrm{kg}^2}{\mathrm{kg}\cdot\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}\mathrm{kg}^2}{\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

2. Coulomb force, with the *electric charges* (q_e) as charge and k_e as physical proportionality constant:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{v^2}{2} - \frac{\mathrm{k_e q_e^2}}{\mathrm{m}r}$$

We can check the physical dimensions once more:

$$\left[\frac{v^2}{2} - \frac{\mathrm{GMm}}{\mathrm{m}r}\right] = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\mathrm{C}^2}{\mathrm{kg}\cdot\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\mathrm{C}^2}{\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

3. Nuclear force, with the *nuclear charges* (g_N) as charge and k_N as physical proportionality constant and *a* as the range parameter to ensure dimensional consistency:

$$\frac{\mathrm{E}}{\mathrm{m}} = c^2 = \frac{v^2}{2} - \frac{\mathrm{k_N g_N^2}}{\mathrm{m}r^n}\cdot a^{n-1} = \frac{v^2}{2} - \frac{\mathrm{k_N g_N^2}}{\mathrm{m}r}\cdot\frac{a^n}{r^n} = \frac{v^2}{2} - \frac{\mathrm{k_N g_N^2}}{\mathrm{m}r}\cdot\left(\frac{r}{a}\right)^{-n)}$$

One can easily ascertain a nuclear range or *distance scale* parameter (a_N) has to be introduced so as to ensure the physical dimensions come out OK:

$$\left[\frac{v^2}{2} - \frac{\mathrm{k_N g_N^2}}{\mathrm{m}r^{n+1}}\cdot a^n\right] = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\mathrm{C}^2 a^n}{\mathrm{kg}\cdot r^{n+1}} = \frac{\mathrm{m}^2}{\mathrm{s}^2} - \frac{\mathrm{Nm}^2\cdot\mathrm{m}^n}{\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}\cdot\mathrm{m}^{n+1}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

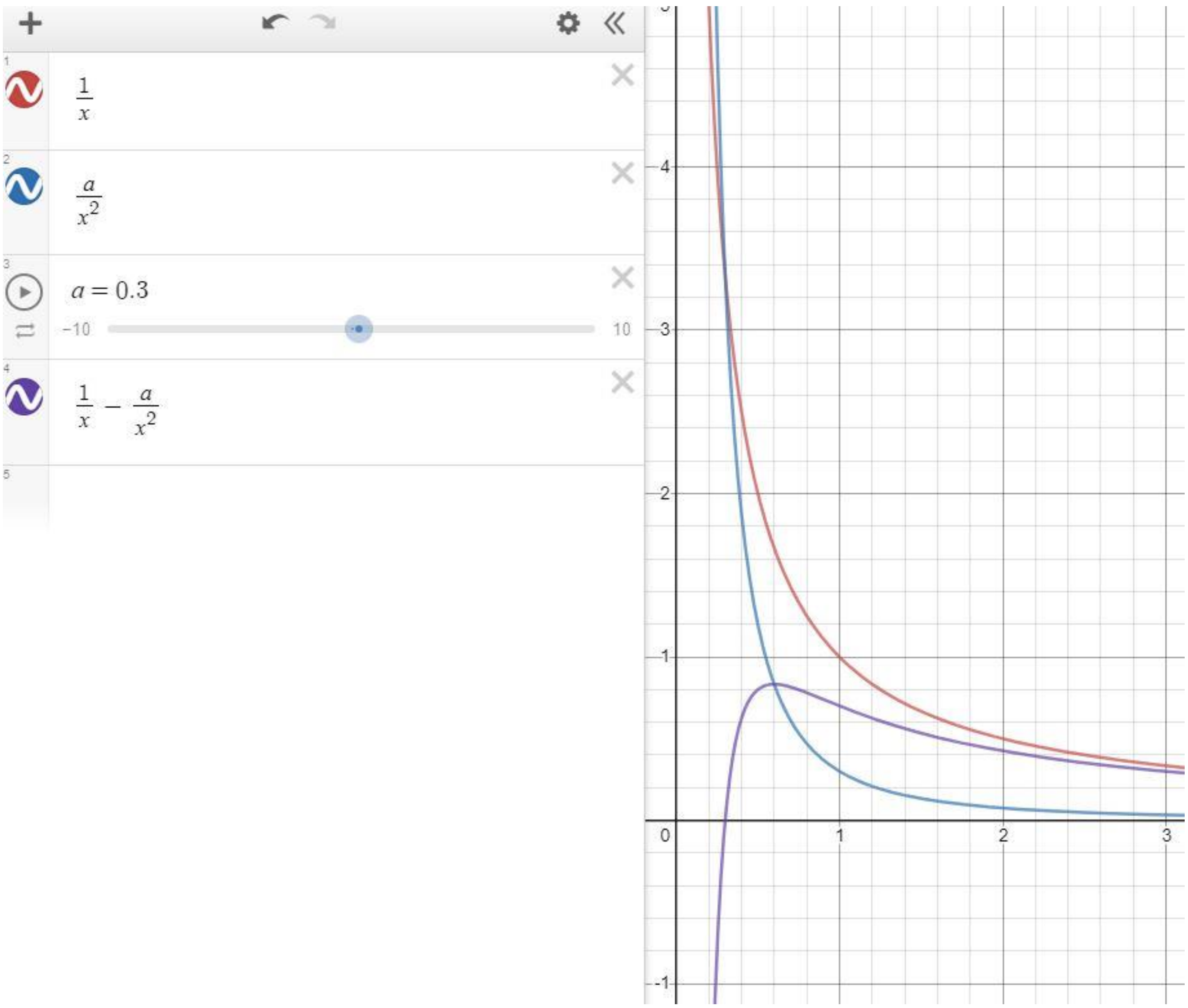
For *n* = 1, we get a nuclear force field based on a Yukawa-like nuclear potential:

$$\mathrm{F_N} = \frac{\mathrm{k_N g_N^2}}{\mathrm{m}}\cdot\frac{a}{r^3}$$

If we give the same numerical value to both q_e and g_N – about 1.6×10^{-19} C and Y respectively[33] – we , we might say the *nuclear* permittivity or standard nuclear parameter is determined by the scaling parameter *a*.

Combining the $1/r$ and a/r^2 potentials, we now get the desired *crossing* of potentials and the expected combined potential well function[34]:

$$U(r) = U_C(r) - U_N(r) = \frac{1}{r} - \frac{a}{r^2}$$



An inverse-*cube* law for a force implies the potential in 3D cannot be symmetric, which is fine as we are modeling plane- or disc-like orbitals/oscillations. Further analysis may usefully focus on:

— An analysis of the q_e/m and g_N/m charge/mass-energy ratios.

[33] For a new force, one should propose a new charge and a new unit to measure it. We suggest the Dirac, which we abbreviate as Y so as to honor Yukawa.

[34] We leave it to the reader to play with the sign conventions.

— An analysis of the different *nature* of the singularities at the center: limits ($r \to 0$) for $1/r$ and a/r^2 functions yield infinity (∞) but these two infinities are, clearly, not of the same nature. Indeed, the different type of singularity one gets at the center from an a/r^2 function (a more 'massive' infinite potential energy, so to speak) may explain the different (Coulomb versus nuclear) charge/mass ratios and the rather enormous proton/neutron energies/masses.

— Linking our simplified model with the interesting work analyzing non-integer or higher-powers of distance functions and effective nuclear field theory, which all rely heavily on expansion into power series, taking into account the energy conservation law must put geometric constraints on them.

— Developing the associated vector potential functions ($\boldsymbol{A}_\mathrm{C}$ and $\boldsymbol{A}_\mathrm{N}$) using the electromagnetic Lorentz gauge:

$$\nabla \cdot \boldsymbol{A} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}$$

For a time-independent scalar potential, which is what we have been modeling, the Lorentz gauge is ($\nabla \cdot \boldsymbol{A} = 0$) because the time derivative is zero: $\partial\phi/\partial t = 0 \Leftrightarrow \nabla \cdot \boldsymbol{A} = 0$.[35] The magnetic field, therefore, vanishes. The time-dependent magnetic field should absorb half the energy in accordance with relativity theory[36] and it should then be easy to develop the equivalent of Maxwell's equations for the nuclear force field using the theorems of Gauss and Stokes.

The model has the advantage of *not* introducing new fundamental constants (except for the nuclear charge g_N), respecting relativity theory (no superluminal speeds), and confirming Planck's quantum of action as the fundamental unit of physical action (h) and angular momentum ($\hbar$) in particle-field exchanges of energy and momentum.

Brussels, 14 January 2021

[35] The Lorentz gauge does not refer to the Dutch physicist H.A. Lorentz but to the Danish physicist Ludvig Valentin Lorenz. The reader should not think we have a *choice* here: the Lorentz gauge is one and the same for time-dependent and time-independent fields, but it vanishes with time-independent fields (electromagnetostatics). . See our remarks on the vector potential and the Lorentz gauge in our paper on the electromagnetic deuteron model.

[36] When using natural units ($c = 1$), the relativity of electric and magnetic fields becomes more obvious.