Mathematical Constraints and Rearrangements of Schrodinger's Equation

[Aryamoy Mitra]

Abstract;
This short paper comprises a set of differential and integral reformulations to a one-dimensional, nonrelativistic position-space variant of Schrodinger's equation, in a fashion that interchanges Planck's constant with De Broglie's wavelengths for massive particles. As far as interpretative discussions are concerned, this merely constitutes a mathematically abstract, arbitrary approach to the equation and its constituents. Consequently, it bears almost no *A Priori*, conceptual or intuitive substance in the context of quantum mechanics.

Consider a nonrelativistic, massive particle oscillating over a one-dimensional positional space, whose state was described by:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{h^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right)\psi(x,t)$$

or, equivalently:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{h^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)$$

wherein V denotes the particle's scalar potential, and ψ its apparent wave-function.

Differentiating (with respect to time-dependency), on both sides results in:

$$\frac{\partial}{\partial t}i\hbar\frac{\partial}{\partial t}\psi(x,t) = \frac{\partial}{\partial t}\left(-\frac{h^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)\right)$$

$$i\hbar\frac{\partial^2}{\partial t^2}\psi(x,t) = -\frac{h^2}{2m}\frac{\partial^3}{\partial t\partial x^2}\psi(x,t) + \frac{\partial}{\partial t}V(x,t)\psi(x,t)$$

If the particle in question is subject to exhibiting a De Broglie wavelength, h may be reformulated using:

$$\lambda = \frac{h}{p}$$

$$h = \lambda p$$

$$i\hbar\frac{\partial^2}{\partial t^2}\psi(x,t) = -\frac{\lambda^2 p^2}{2m}\frac{\partial^3}{\partial t\partial x^2}\psi(x,t) + \frac{\partial}{\partial t}V(x,t)\psi(x,t)$$

Whilst quantum-mechanical systems are unamenable to conventions sourced from classical observations, they may be appreciative of the canonical relation:

$$\frac{p^2}{2m} = \frac{mv^2}{2} = \mathrm{E}_k$$

$$i\hbar\frac{\partial^2}{\partial t^2}\psi(x,t) = -\lambda^2\mathrm{E}_k\frac{\partial^3}{\partial t\partial x^2}\psi(x,t) + \frac{\partial}{\partial t}V(x,t)\psi(x,t)$$

$$i\hbar\frac{\partial^2}{\partial t^2}\psi(x,t) + \lambda^2\mathrm{E}_k\frac{\partial^3}{\partial t\partial x^2}\psi(x,t) = \frac{\partial}{\partial t}V(x,t)\psi(x,t)$$

$$\lambda^2\mathrm{E}_k\frac{\partial^3}{\partial t\partial x^2}\psi(x,t) = \frac{\partial}{\partial t}V(x,t)\psi(x,t) - i\hbar\frac{\partial^2}{\partial t^2}\psi(x,t)$$

$\frac{\partial}{\partial t}V(x,t)\psi(x,t)$ refers to the partial derivative of the product of a particle's wave-function and its scalar potential.

If progression through time is held to be invariant, then one can repurpose the *product rule* for ordinary derivatives, in the form:

$$\frac{\partial}{\partial t}V(x,t)\psi(x,t) = \frac{\partial V}{\partial t}\psi + \frac{\partial \psi}{\partial t}V$$

Therefore:

$$\lambda^2 \mathrm{E}_k \frac{\partial^3}{\partial t \partial x^2}\psi(x,t) = \frac{\partial V}{\partial t}\psi + \frac{\partial \psi}{\partial t}V - i\hbar \frac{\partial^2}{\partial t^2}\psi(x,t)$$

Abbreviating variable notations for both ψ and V, and rearranging the above, one may determine -

$$\lambda^2 \mathrm{E}_k = \frac{\frac{\partial V}{\partial t}\psi + \frac{\partial \psi}{\partial t}V - i\hbar \frac{\partial^2 \psi}{\partial t^2}}{\frac{\partial^3 \psi}{\partial t \partial x^2}}$$

Since $\lambda^2 \mathrm{E}_k$ will necessarily be real,

$$\frac{\frac{\partial V}{\partial t}\psi + \frac{\partial \psi}{\partial t}V - i\hbar \frac{\partial^2 \psi}{\partial t^2}}{\frac{\partial^3 \psi}{\partial t \partial x^2}} \epsilon\ R^+$$

despite the complex values assigned by $\psi(x,t)$.

Similarly,

$$\mathrm{E}_k = \frac{\frac{\partial V}{\partial t}\psi + \frac{\partial \psi}{\partial t}V - i\hbar \frac{\partial^2 \psi}{\partial t^2}}{\frac{\partial^3 \psi}{\partial t \partial x^2}\lambda^2} \ and\ \lambda = \sqrt{\frac{\frac{\partial V}{\partial t}\psi + \frac{\partial \psi}{\partial t}V - i\hbar \frac{\partial^2 \psi}{\partial t^2}}{\frac{\partial^3 \psi}{\partial t \partial x^2}\mathrm{E}_k}}$$

Schrodinger's wave equation ascribes the kinetic energy of a particle by virtue of a differential operator (that evolves into a Laplacian for three-dimensional vector spaces).

One can also integrate (with respect to time-dependency) on both sides of the function:

$$\int i\hbar \frac{\partial}{\partial t}\psi(x,t)\,dt = \int -\frac{h^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)\,dt$$

or, equivalently:

$$\int i\hbar \frac{\partial}{\partial t}\psi(x,t)\,dt = \int -\lambda^2 \mathrm{E}_k \frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)\,dt$$

$$i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt = -\lambda^2 \mathrm{E}_k \int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt + \int V(x,t)\psi(x,t)\,dt \qquad (E\,*)$$

$$\int V(x,t)\psi(x,t)\,dt\ \ can't\ successfully\ be\ integrated\ by\ parts;$$

$$let\ V = u, and\ \psi = dv/dx$$

Adjusting roles for both functions obtains:

$$\int V\psi\,dt = V\int \psi\,dt - \int \left(\int \psi\,dt\right)\frac{\partial}{\partial t}V\,dt$$

Consequently, returning to $(E\,*)$

$$i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt + \lambda^2 \mathrm{E}_k \int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt = \int V(x,t)\psi(x,t)\,dt$$

$$\lambda^2 \mathrm{E}_k \int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt = \int V(x,t)\psi(x,t)\,dt - \ i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt$$

$$\lambda^2 \mathrm{E}_k = \frac{\int V(x,t)\psi(x,t)\,dt - \ i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt}{\int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt}$$

$\int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt$ can't be rewritten as $\frac{\partial}{\partial x}\psi(x,t)$, since ψ is multivariate in character.

Finally, abbreviating variable notations for ψ and V, one can derive the equivalent formulations:

$$\mathrm{E}_k = \frac{\int V(x,t)\psi(x,t)\,dt - \ i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt}{\lambda^2 \int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt}\ \ and\ \lambda = \sqrt{\frac{\int V(x,t)\psi(x,t)\,dt - \ i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt}{\mathrm{E}_k \int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt}}$$

wherein

$$\frac{\int V(x,t)\psi(x,t)\,dt - \ i\hbar \int \frac{\partial}{\partial t}\psi(x,t)\,dt}{\int \frac{\partial^2}{\partial x^2}\psi(x,t)\,dt}\ \epsilon\ R^+$$

despite any complex values assigned in the integrals of the expression.

A Note on the Copenhagen School of Quantum Mechanics

In order for a wavefunction to collapse in a manner that isn't deterministic, there must necessarily exist a substantive argument as to the credence of its projections. If its mathematical implications are solely probabilistic, one may rightly question how precisely its probabilistic distribution is engendered, and whether it is a manifestation of empirical truths (by virtue of measurement), more so than it is one of intrinsic, phenomenal truths (by virtue of objective existence). In any event, its divergence from the strictures of classical mechanics should neither be abnegated as unreal, nor neglected as being an arbitrary fact of the universe. Any conceptual unifications in this regard, must necessarily be characterized as being both rationalistic and substantiated empirically. Most meaningful interpretations that have been promulgated thus far, have scarcely invoked any scientific consensuses in either modern or historical academia.

Abstract mathematics can underpin a physical concept, only insofar as it facilitates an explication of the mechanisms that guide it. When considering an intuitive analogy (or concrete paradigm), abstractions in the form of probability amplitudes are futile in describing the trajectory that an electron undertakes.