GPS and the Equivalence Principle

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Abstract

The article "Why there no noon-midnight red shift in GPS" by N. Ashby and M.Weiss is discussed in this paper.

1. Introduction

If the equivalence principle (EP) holds true, then the GPS system should only be influenced by external solar system bodies' tidal forces and the gravitational fields of those tidal forces. It should not be influenced by external solar system bodies' homogenous gravity fields. This is due to the fact that the Earth is in free-fall with respect to the Sun and other Solar System bodies. The article [1] claims that these observations are completely in line with EP and it further attempts to explain the reasoning. We will attempt to show that the arguments used in [1] are somewhat flawed and at the same time hinting that EP itself is flawed. Next, within the context of $\square^{-2}$ proximity, we will use two reference systems - barycentric celestial reference system of solar system (BCRS) and the geocentric celestial reference system (GCRS) and we will assume that the currently used BCRS formulas are correct.

Article [1] further claims that, since the noon-midnight red shift effect is not observed while comparing GPS signal frequency offsets it proves the correctness of EP. Note that the noon-midnight red shift effect ought to be observed if the flow of time within the GPS systems was influenced by the solar gravity field. We think the above claim can be incorrect, since a similar noon-midnight redshift effect is left unobserved within the GPS system. This "other" effect has the same magnitude as the first one, but is negative since GCRS accelerates (experiences an accelerated free-fall) and it should contribute to the aforementioned GPS signal frequency offset.

2. GPS signal frequency offset

The classical first-order doppler shift ($\frac{\square}{\square}$) qualitatively differs from the gravitational frequency shift. The gravitational frequency shift is observed when the signal emitter and receiver experience differing time flows (clocks progress at different rates). On the other hand the classical doppler effect is observed (both in acoustics and optics) when the signal emitter and receiver move at differing speeds. The doppler effect also scales depending on the speed of the signal itself and the medium through which it propagates.

Suppose there is a GPS satellite that is sending a signal to an observer who is situated on the surface of Earth (ground observer). Both the satellite and the observer along with their GCRS frame are in free-fall. We can calculate the relative signal frequency shift in any given reference frame (BCRS included). Let us establish the variables:

$\vec{a}_E$ - acceleration of the GCRS frame,

$\vec{r}$ - position of the ground observer in the BCRS frame,

$\vec{r}_0$ - position of the GPS satellite in the BCRS frame,

$\vec{v}$ - velocity of the grounded observer in the BCRS frame at the moment of receiving the GPS signal,

$\vec{v}_0$ - velocity of the GPS satellite in the BCRS frame at the moment of emitting the GPS signal,

$\vec{n}$ - direction of signal (unit vector),

$\vec{V}_{E0}$ - velocity of the GCRS frame (with respect to the BCRS frame) at the moment of signal emission,

$\vec{V}_E$ - the velocity of the GCRS frame (with respect to the BCRS frame) at the moment of signal reception,

$\vec{V}$ - the velocity of the ground observer (with respect to the GCRS) at the moment of signal reception,

$\vec{V}_0$ - the velocity of the GPS satellite (with respect to the GCRS) at the moment of signal emission.

The first order doppler effect in the BCRS frame can then be described like so:

$$(\frac{\delta f}{f})_{Doppler} \approx \frac{\vec{n}(\vec{v}-\vec{v}_0)}{c} \tag{1}$$

Because GCRS accelerates, it is important to note, that by the time the signal has arrived at the observer the velocity of GCRS has changed, thus the equation:

$$\vec{v}-\vec{v}_0 \approx \vec{V}-\vec{V}_0+\vec{V}_E-\vec{V}_{E0} \approx \vec{V}-\vec{V}_0+\vec{a}_E\frac{|\vec{r}-\vec{r}_0|}{c} \tag{2}$$

and when we plug (2) into (1) we get:

$$(\frac{\delta f}{f})_{Doppler} \approx \frac{\vec{n}(\vec{V}-\vec{V}_0)}{c} + \frac{\vec{n}|\vec{r}-\vec{r}_0|\vec{a}_E}{c^2} = \frac{\vec{n}(\vec{V}-\vec{V}_0)}{c} + \frac{(\vec{r}-\vec{r}_0)\vec{a}_E}{c^2} \tag{3}$$

Previously we have analyzed this in [2] and shown that within the GPS system the contribution of the classical first-order doppler effect (see second term in (3)) compensates for the potential contribution of the homogenous external gravitational field when calculating signal frequency shifts. Which implies that the external homogeneous gravitational field exists not only in BCRS but also in GCRS, which cannot be the case if EP is correct. This is analogous to the Pound-Rebka experiment in which the gravitational frequency shift and the gravitational time dilation (and by extension the gravitational field itself) are proved by the same gravitational signal frequency shift compensation originating from the classical first-order doppler effect.

3. GPS clock/time flow rate

According to EP the flow of time in a free-falling GCRS is not influenced by the Sun's external homogeneous gravitational field, and is instead only influenced by the tidal force potential of Solar system bodies. Also [1] claims that in a free-falling reference frame the acceleration due to free-fall (or the external homogeneous gravitational field) is only expressed as a factor in the relativity of simultaneity effect of Special Relativity (SR). Therefore, if EP is correct, the flow of Geocentric Coordinate Time (TCG) is not influenced by Earth's acceleration due to free-fall (i.e. Sun's external homogeneous gravitational field) and in turn the flow of time in the GPS system is not influenced by Sun's external homogeneous gravitational field. Next we will attempt to show how these assertions are incompatible with SR.

Here is the equation for the Lorentz transformation in its standard form (with c = 1)

$$T = \frac{t - Vx}{\sqrt{1-V^2}} \qquad X = \frac{x - Vt}{\sqrt{1-V^2}} \tag{4}$$

Within the context of SR the Lorentz transformation ought to hold true for its derivatives as well (as implied by Lorentz Covariance)

$$dT = \frac{dt - Vdx}{\sqrt{1-V^2}} \qquad dX = \frac{dx - Vdt}{\sqrt{1-V^2}} \tag{5}$$

If we have two events on the same reference frame, that occur at the same time in two different locations

$$dt = 0 \tag{6}$$

then in a different reference frame those two events will not occur at the same time due to the relativity of simultaneity effect

$$dT = -\frac{Vdx}{\sqrt{1-V^2}} \approx -Vdx \tag{7}$$

On the other hand if we have two events on the same reference frame occuring at the same location

$$dx = 0 \tag{8}$$

(i.e. interval between the proper time of two non-moving clocks in that particular reference frame) then in a different reference frame, due to time dilation, we get

$$dT = \frac{dt}{\sqrt{1-V^2}} \approx (1+\frac{V^2}{2})dt \tag{9}$$

Of course, this time dilation effect is two-fold. In other words if we apply $dX = 0$ (we are now taking the proper time interval of two non-moving clocks in a different reference frame), then from (5) we would get

$$dx = Vdt \tag{10}$$

and in turn we get the following

$$dT = \sqrt{1-V^2}dt \approx (1-\frac{V^2}{2})dt \tag{11}$$

Within the context of special relativity the clocks of one reference frame are not synchronized with the other reference frame (assuming there is a relative velocity between the two frames). Therefore if we wanted to infer the rate of multiple clocks in one reference frame we would need to accurately measure a single clock in the other reference frame. We can see from (9) and (11) that the time dilation effect is two-fold.

We will now apply our previous reasoning while analyzing the flow of time (i.e. clock measurements) in BCRS and GCRS. Within our proximity the transformation of coordinated times between two reference frames can be expressed with the following

$$T = t - \frac{1}{c^2}[\int(\frac{V_E^2}{2} + U_E)dt - \vec{V}_E(\vec{x}_E - \vec{x})] \tag{12}$$

Where

T - Geocentric Coordinate Time (TCG),

t - Barycentric Coordinate Time (TCB),

$\vec{V}_E$ - geocenter velocity in the BCRS frame,

U_E - gravitational potential of solar system bodies at the geocenter,

$\vec{x}_E$ - geocenter position in the BCRS frame,

$\vec{x}$ - position of an arbitrary point in the BCRS frame.

The transformation (12) is derived by integrating the transformation of the differentials. Afterwards an assumption is made that the second term is a contribution from the relativity of simultaneity effect, although if we were to consider the initial transformation we'd see that the term is composed of both the relativity of simultaneity and time dilation effects. We will now attempt to prove this.

The following transformation of the differentials can be obtained from (12)

$$dT = dt - \frac{1}{c^2}(\frac{V_E^2}{2} + U_E)dt + \frac{1}{c^2}\frac{d\vec{V}_E}{dt}(\vec{x}_E - \vec{x})dt + \frac{1}{c^2}\vec{V}_E\frac{d\vec{x}_E}{dt}dt - \frac{1}{c^2}\vec{V}_E d\vec{x} \quad (13)$$

Now we will transform (13) the same way we did (5).

Given

$$dt = 0 \quad (14)$$

we could derive the following from (13)

$$dT = -\frac{1}{c^2}\vec{V}_E d\vec{x} \quad (15)$$

Given

$$d\vec{x} = 0 \quad (16)$$

(13) could be transformed thus

$$dT = (1 + \frac{1}{c^2}\frac{V_E^2}{2} - \frac{1}{c^2}U_E + \frac{1}{c^2}\frac{d\vec{V}_E}{dt}(\vec{x}_E - \vec{x}))dt \quad (17)$$

And given

$$d\vec{x} = \vec{V}_E dt \quad (18)$$

we'd get (again from (13))

$$dT = (1 - \frac{1}{c^2}\frac{V_E^2}{2} - \frac{1}{c^2}U_E dt + \frac{1}{c^2}\frac{d\vec{V}_E}{dt}(\vec{x}_E - \vec{x}))dt \qquad (19)$$

Let's compare and contrast both cases. The contribution of relativity of simultaneity in both cases is the same, same goes for the "two-foldedness" of time dilation. The difference is, that both in BCRS and GCRS the flow of TCG is influenced by Earth's free-fall acceleration, due to it (the free-fall acceleration) not influencing the flow of TCB by definition. If the free-fall acceleration of GCRS (in other words the external homogeneous gravitational field) influences the flow of TCG, then at the same time it ought to influence the proper time of all clocks in GCRS. All of this is evident from (17) and (19).

Note that clock offsets of identical magnitude (lacking any discernible explanation) are observed within GPS, which could be explained by the Sun's external homogeneous gravitational field having an effect on the clocks' flow rate, see., e.g. [3].

[1] N.Ashby,M.Weiss https://arxiv.org/ftp/arxiv/papers/1307/1307.6525.pdf
[2] L.Rimsha, V.Rimsha http://vixra.org/abs/1508.0123
[3] O.Montenbruck et al . http://acc.igs.org/clocks/prn25-period_DLR_gpssoln11.pdf