Multi categories analytic method using Continuous Bernoulli distribution and conditional distribution

Kuan-Shian Wang
General Manager, Ji-Tong Co., Ltd. (Taiwan)
Mei-Yu Lee
Assistant Professor, Minghsin University of Science and Technology

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Abstract
This book provides four model designs to discuss how continuous Bernoulli distribution extends to the analysis of K categories. By contrast to the discrete polynomial distribution which is extended from Bernoulli distribution depending on the additive property, the random variable of continuous Bernoulli should be tested the pdf, cdf, distribution, and checked if maintain the characteristics of CB distribution or not. Model 1 is from random variable method(variable-added), Model 2 and 3 are from the probability model-building and suitable for the parameter-added or the conditional relationship of variables, respectively. Model 4 is from the continuous trinomial distribution and suitable for the joint relationship of variables.

Context

Chapter 1 Bernoulli distribution to Trinomial distribution using tree diagram

The Bernoulli distribution can convert to Trinomial distribution. There are three categories and two random variables,

X_1	X_2	$1-X_1-X_2$
p_1	p_2	$1-p_1-p_2$

1.Explained Trinomial distribution from X_1 to X_2 using Bernoulli distribution,

X_1	$1-X_1$
p_1	$1-p_1$

$X_1 = 0$

X_2	$1-X_1-X_2$
$\dfrac{p_2}{1-p_1}$	$1-\dfrac{p_2}{1-p_1}$

$X_1 \sim Bernoulli(p_1), X_2|x_1 = 0 \sim Bernoulli\left(\dfrac{p_2}{1-p_1}\right),$ the tree diagram,

$P(X_1 = 1) = p_1$

$$P(X_2 = 1|X_1 = 0) = \frac{p_2}{1-p_1}$$

$P(X_1 = 0) = 1 - p_1$ ---

$$P(X_2 = 0|X_1 = 0) = 1 - \frac{p_2}{1-p_1}$$

$P(X_1 = 1) = p_1, P(X_1 = 0, X_2 = 1) = p_2, P(X_1 = 0, X_2 = 0) = 1 - p_1 - p_2,$

$$f(x_1, x_2) = f(x_1)f(x_2|x_1 = 0) = (p_1)^{x_1}(1-p_1)^{1-x_1}\left(\frac{p_2}{1-p_1}\right)^{x_2}\left(1-\frac{p_2}{1-p_1}\right)^{1-x_1-x_2}$$

$= (p_1)^{x_1}(p_2)^{x_2}(1-p_1-p_2)^{1-x_1-x_2},$

$x_i = 0,1, 0 < p_i < 1, i = 1,2, x_1 + x_2 = 0,1, 0 < p_1 + p_2 < 1,$

$f(x_2) = \sum_{x_1} f(x_1, x_2) = (p_2)^{x_2}(1-p_2)^{1-x_2}, x_2 = 0,1, 0 < p_2 < 1,$

$X_2 \sim Bernoulli(p_2),$

2.Explained Trinomial distribution from X_2 to X_1 using Bernoulli distribution,

X_2	$1-X_2$
p_2	$1-p_2$

$X_2=0$

X_1	$1-X_1-X_2$
$\frac{p_1}{1-p_2}$	$1-\frac{p_1}{1-p_2}$

The posterior probability,

$P(X_2=1)=p_2$

$$P(X_2=0)=1-p_2 --- \begin{cases} P(X_1=1|X_2=0)=\frac{p_1}{1-p_2} \\ P(X_1=0|X_2=0)=1-\frac{p_1}{1-p_2} \end{cases}$$

$P(X_2=1)=p_2, P(X_2=0,X_1=1)=p_1, P(X_1=0,X_2=0)=1-p_1-p_2,$

$$X_1|x_2=0 \sim Bernoulli\left(\frac{p_1}{1-p_2}\right)$$

$$f(x_1,x_2)=f(x_1)f(x_2|x_1=0)=f(x_2)f(x_1|x_2=0).$$

3.Merge X_1 and X_2 to one random variable,

X_1+X_2	$1-X_1-X_2$
p_1+p_2	$1-p_1-p_2$

$X_1+X_2 \sim Bernoulli(p_1+p_2).$

Chapter 2 K categories and the random variable of each category being Continuous Bernoulli distribution

There are four models to construct k categories and the random variable of each category is the Continuous Bernoulli distribution.

Continuous Bernoulli distribution, $X \sim CB(\lambda)$,
The probability density function,
$f_X(x;\lambda) = C(\lambda)\lambda^x(1-\lambda)^{1-x}, 0 \le x \le 1, 0 < \lambda < 1,$

$$C(\lambda) = \begin{cases} \frac{2tanh^{-1}(1-2\lambda)}{1-2\lambda}, \lambda \neq \frac{1}{2} \\ 2, \lambda = \frac{1}{2} \end{cases}$$

Section 1. Model 1

The setting random variables method,
There are k categories, $X_1, X_2, \ldots, X_k$ are continuous random variables,

λ_1	λ_2	…..	$\lambda_k = 1 - \sum_{i=1}^{k-1} \lambda_i$

$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,\ldots,k$
Because $X_i + X_j$ is not $CB(\lambda_i + \lambda_j), i \neq j$, the difference of $\lambda_i + \lambda_j$ and λ_i will use the other method. This method is setting a new random variable Y which probability distribution is $CB(\lambda_i + \lambda_j)$.

1.Merge λ_1 and λ_2

New X	$1-X$
$\lambda_1 + \lambda_2$	$1 - \lambda_1 - \lambda_2$

$X \sim Bernoulli(\lambda_1 + \lambda_2)$,

$$E(X) = \begin{cases} \frac{\lambda_1 + \lambda_2}{2(\lambda_1 + \lambda_2) - 1} + \frac{1}{2tan^{-1}(1 - (\lambda_1 + \lambda_2)\lambda)} & if\ \lambda_1 + \lambda_2 \neq \frac{1}{2} \\ \frac{1}{2} & if\ \lambda_1 + \lambda_2 = \frac{1}{2} \end{cases}$$

$X \neq X_1 + X_2$.

$X_1 \sim Bernoulli(\lambda_1)$,

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$E(X_1)=\begin{cases}\dfrac{\lambda_1}{2\lambda_1-1}+\dfrac{1}{2\tan^{-1}(1-\lambda_1)} & \text{if } \lambda_1 \neq \dfrac{1}{2}\\ \dfrac{1}{2} & \text{if } \lambda_1 = \dfrac{1}{2}\end{cases}$$

$X_2 \sim Bernoulli(\lambda_2)$,

X_2	$1-X_2$
λ_2	$1-\lambda_2$

$$E(X_2)=\begin{cases}\dfrac{\lambda_2}{2\lambda_2-1}+\dfrac{1}{2\tan^{-1}(1-\lambda_2)} & \text{if } \lambda_2 \neq \dfrac{1}{2}\\ \dfrac{1}{2} & \text{if } \lambda_2 = \dfrac{1}{2}\end{cases}$$

$E(X_1+X_2)\neq E(X_1)+E(X_2)$.

2. The statistical analysis method,
This model can do the following testing,

(1) $H_0 : \lambda_i = \lambda_0$, λ_0 is constant, $i = 1,2,....,k$,
$(1-\alpha)\times 100\%$ C.I. for λ_i

(2) $H_0 : \lambda_i + \lambda_j = \lambda_0, \lambda_0$ is constant, $i,j = 1,2,....,k, i \neq j$,
$(1-\alpha)\times 100\%$ C.I. for $\lambda_i + \lambda_j$

(3) $H_0 : \lambda_i = \lambda_{i,0}, \lambda_j = \lambda_{j,0}$ $\lambda_{i,0}, \lambda_{j,0}$ is constant, $i,j = 1,2,....,k, i \neq j$,
this testing contains two steps,
1st step, $H_0 : \lambda_i = \lambda_{i,0}$,
2nd step. $H_0 : \lambda_i + \lambda_j = \lambda_{i,0} + \lambda_{j,0}$,

(4) $H_0 : \lambda_i = \lambda_j$,

Please see model 1--- chapter 3,

Section 2. Model 2

Continuous Bernoulli distribution and conditional Continuous Bernoulli distribution to build the analysis probability model,

There are k categories, $X_1, X_2, \ldots, X_k$ are continuous random variables,

λ_1	λ_2	…..	$\lambda_k = 1 - \sum_{i=1}^{k-1} \lambda_i$

$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,\ldots,k$

For easy to explain, k=3,

There are 3 categories, X_1 and X_2 are continuous random variables,

λ_1	λ_2	$1-\lambda_1-\lambda_2$

the first step, selecting one random variable, X_1,

the second step selecting one random variable, $X_2|x_1$,

$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, \frac{X_2}{1-x_1} \sim CB\left(\lambda^* = \frac{\lambda_2}{1-\lambda_1}\right), 0 \le x_2 \le 1 - x_1,$$

$$f_{X_1}(x_1;\lambda_1) = C(\lambda_1)(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1}, 0 \le x_1 \le 1, 0 < \lambda_1 < 1,$$

$$C(\lambda_1) = \begin{cases} \dfrac{ln(1-\lambda_1) - ln(\lambda_1)}{1-2\lambda_1}, \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda_1 \ne \dfrac{1}{2} \end{cases},$$

$$f_{X_2|x_1}(x_2|x_1) = C(\lambda^*)(\lambda^*)^{\frac{x_2}{1-x_1}}(1-\lambda^*)^{1-\frac{x_2}{1-x_1}}, 0 \le \frac{x_2}{1-x_1} \le 1, 0 < \lambda^* = \frac{\lambda_2}{1-\lambda_1} < 1,$$

$$C(\lambda^*) = \begin{cases} \dfrac{ln(1-\lambda^*) - ln(\lambda^*)}{1-2\lambda^*}, \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda^* \ne \dfrac{1}{2} \end{cases},$$

1. $X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, \frac{X_2}{1-x_1} \sim CB\left(\lambda^* = \frac{\lambda_2}{1-\lambda_1}\right), 0 \le x_2 \le 1-x_1$,

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$1-X_1 = X_2 + (1-X_1-X_2)$$

$\frac{X_2}{1-x_1}$	$1-\frac{X_2}{1-x_1}$
$\frac{\lambda_2}{1-\lambda_1}$	$1-\frac{\lambda_2}{1-\lambda_1}$

$$f_{X_1}(x_1;\lambda_1) f_{X_2|x_1}(x_2|x_1) = C(\lambda_1)(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1} C(\lambda^*)(\lambda^*)^{\frac{x_2}{1-x_1}}(1-\lambda^*)^{1-\frac{x_2}{1-x_1}}$$

$0 \le x_1 \le 1, 0 \le x_2 \le 1-x_1$,

But $f_{X_2}(x_2) = \int_0^{1-x_2} f_{X_1}(x_1;\lambda_1) f_{X_2|x_1}(x_2|x_1) dx_1$, it is not $CB(\lambda_2)$,

2. $X_2 \sim CB(\lambda_2), 0 \le x_2 \le 1, \frac{X_1}{1-x_2} \sim CB\left(\lambda^* = \frac{\lambda_1}{1-\lambda_2}\right), 0 \le x_1 \le 1-x_2$,

X_2	$1-X_2$
λ_2	$1-\lambda_2$

$$1-X_2 = X_1 + (1-X_1-X_2)$$

$\frac{X_1}{1-x_2}$	$1-\frac{X_1}{1-x_2}$
$\frac{\lambda_1}{1-\lambda_2}$	$1-\frac{\lambda_1}{1-\lambda_2}$

$$f_{X_2}(x_2;\lambda_2) f_{X_1|x_2}(x_1|x_2) = C(\lambda_2)(\lambda_2)^{x_2}(1-\lambda_2)^{1-x_2} C(\lambda^*)^{\frac{x_1}{1-x_2}}(1-\lambda^*)^{1-\frac{x_1}{1-x_2}}$$

$0 \le x_2 \le 1, 0 \le x_1 \le 1-x_2$,

But $f_{X_1}(x_1) = \int_0^{1-x_1} f_{X_2}(x_2;\lambda_2) f_{X_1|x_2}(x_1|x_2) dx_2$, it is not $CB(\lambda_1)$,

$f_{X_1}(x_1;\lambda_1) f_{X_2}(x_2|x_1) \ne f_{X_2}(x_2;\lambda_2) f_{X_1|x_2}(x_1|x_2)$, the Bayesian cannot be applied.

Please see model 2--- chapter 5,

3. The statistical analysis method,

The k categories can divide to the following

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,..., k$

There are k-1 categories, $\frac{X_2}{1-x_1}, \frac{X_3}{1-x_1}, \ldots, \frac{X_k}{1-x_1}$ are continuous random variables given X_1 and λ_1,

$\frac{\lambda_2}{1-\lambda_1}$	$\frac{\lambda_3}{1-\lambda_1}$	…..	$\frac{\lambda_k}{1-\lambda_1} = 1 - \sum_{i=2}^{k-1} \frac{\lambda_i}{1-\lambda_1}$

$$\frac{X_i}{1-x_1} \sim CB\left(\lambda^* = \frac{\lambda_i}{1-\lambda_1}\right), 0 \le \frac{x_i}{1-x_1} \le 1, i = 2,3,..., k$$

The testing,

(1) $H_0 : \lambda^* = \lambda_0$, λ_0 is constant, $i = 2,3,....,k,$

$(1-\alpha)\times 100\%$ C.I. for λ^*

(2) $H_0 : \frac{\lambda_i + \lambda_j}{1-\lambda_1} = \lambda_0, \lambda_0$ is constant, $i, j = 2,3,....,k, i \ne j,$

$(1-\alpha)\times 100\%$ C.I. for $\frac{\lambda_i + \lambda_j}{1-\lambda_1}$

(3) $H_0 : \frac{\lambda_i}{1-\lambda_1} = \lambda_{i,0}, \frac{\lambda_j}{1-\lambda_1} = \lambda_{j,0}$ $\lambda_{i,0}, \lambda_{j,0}$ is constant, $i, j = 2,3,....,k, i \ne j,$

this testing contains two steps,

1st step, $H_0 : \frac{\lambda_i}{1-\lambda_1} = \lambda_{i,0}$,

2nd step. $H_0 : \frac{\lambda_i + \lambda_j}{1-\lambda_1} = \lambda_{i,0} + \lambda_{j,0}$,

(4) $H_0 : \frac{\lambda_i}{1-\lambda_1} = \frac{\lambda_j}{1-\lambda_2}$, $i, j = 2,3,....,k, i \ne j,$

Please see model 2--- chapter 4,

Section 3. Model 3

Continuous Bernoulli distribution and new conditional Continuous Bernoulli distribution to construct the analysis probability model,

There are k categories, $X_1, X_2, \ldots, X_k$ are continuous random variables,

λ_1	λ_2	…..	$\lambda_k = 1 - \sum_{i=1}^{k-1} \lambda_i$

$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,\ldots,k$

For easy to explain, k=3,

There are 3 categories, X_1 and X_2 are continuous random variables,

λ_1	λ_2	$1-\lambda_1-\lambda_2$

the first step, selecting one random variable, X_1,

the second step selecting one random variable, $X_2|x_1$,

$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1,$$

$$f_{X_1}(x_1;\lambda_1) = C_1(\lambda_1)(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1}, 0 \le x_1 \le 1, 0 < \lambda_1 < 1,$$

$$C_1(\lambda_1) = \begin{cases} \dfrac{ln(1-\lambda_1) - ln(\lambda_1)}{1-2\lambda_1}, \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda_1 \ne \dfrac{1}{2} \end{cases},$$

$$f_{X_2|x_1}(x_2|x_1) = C_2(\lambda_1, \lambda_2, x_1)\left(\frac{\lambda_2}{1-\lambda_1}\right)^{x_2}\left(1 - \frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1-x_2}, 0 \le x_2 \le 1 - x_1, 0 < \lambda_2 < 1 - \lambda_1,$$

$$C_2(\lambda_1, \lambda_2, x_1) = \frac{ln(\lambda_2) - ln(1-\lambda_1-\lambda_2)}{\left(\dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} - \left(1 - \dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}},$$

1. $X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1,$

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$1 - X_1 = X_2 + (1 - X_1 - X_2)$$

X_2	$1-X_1-X_2$
$\dfrac{\lambda_2}{1-\lambda_1}$	$1-\dfrac{\lambda_2}{1-\lambda_1}$

$$f_{X_1}(x_1;\lambda_1) f_{X_2|x_1}(x_2|x_1) = C_1(\lambda_1) C_2(\lambda_1, \lambda_2, x_1)(\lambda_1)^{x_1}(\lambda_2)^{x_2}(1-\lambda_1-\lambda_2)^{1-x_1-x_2}$$

$0 \le x_1 \le 1, 0 \le x_2 \le 1 - x_1,$

But $f_{X_2}(x_2) = \int_0^{1-x_2} f_{X_1}(x_1;\lambda_1) f_{X_2|x_1}(x_2|x_1) dx_1$, it is not $CB(\lambda_2)$,

2. $X_2 \sim CB(\lambda_2), 0 \le x_2 \le 1, X_1|x_2 \sim CB(\lambda_2, \lambda_1, x_2), 0 \le x_1 \le 1 - x_2$,

X_2	$1 - X_2$
λ_2	$1 - \lambda_2$

$$1 - X_2 = X_1 + (1 - X_1 - X_2)$$

X_1	$1 - X_1 - X_2$
$\dfrac{\lambda_1}{1-\lambda_2}$	$1 - \dfrac{\lambda_1}{1-\lambda_2}$

$f_{X_2}(x_2;\lambda_2) f_{X_1|x_2}(x_1|x_2) = C_1(\lambda_2) C_2(\lambda_1, \lambda_2, x_2)(\lambda_1)^{x_1}(\lambda_2)^{x_2}(1 - \lambda_1 - \lambda_2)^{1-x_1-x_2}$,
$0 \le x_2 \le 1, 0 \le x_1 \le 1 - x_2$,
But $f_{X_1}(x_1) = \int_0^{1-x_1} f_{X_2}(x_2;\lambda_2) f_{X_1|x_2}(x_1|x_2) dx_2$, it is not $CB(\lambda_1)$,
$f_{X_1}(x_1;\lambda_1) f_{X_2}(x_2|x_1) \neq f_{X_2}(x_2;\lambda_2) f_{X_1|x_2}(x_1|x_2)$, the Bayesian cannot be applied.

3. The statistical analysis method,
(1) The $X_i = \beta_{0,i} + \beta_{1.i} H_i(X_1) + \varepsilon_i, i = 2,3,...,k$ $H_i(X_1)$ is the function of X_1,
(2) $X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_i|x_1 \sim CB(\lambda_1, \lambda_i, x_1), 0 \le x_i \le 1 - x_1$,

$$X_{j\backslash}|x_1 \sim CB(\lambda_1, \lambda_j, x_1), 0 \le x_j \le 1 - x_1$$

$H_0 : \dfrac{\lambda_i}{1-\lambda_1} = \dfrac{\lambda_j}{1-\lambda_2}$, $i, j = 2,3,....,k, i \neq j$,

Please see model 3--- chapter 5, chapter 6, chapter 7.

Section 4. Model 4

For easy to explain, k=3,

There are 3 categories, X_1 and X_2 are continuous random variables,

X_1	X_2	$1-X_1-X_2$
λ_1	λ_2	$1-\lambda_1-\lambda_2$

The Continuous Trinomial distribution and trial number=1,
$f_{X_1,X_2}(x_1,x_2;\lambda_1,\lambda_2)=C(\lambda_1,\lambda_2)\lambda_1^{x_1}\lambda_2^{x_2}(1-\lambda_1-\lambda_2)^{1-x_1-x_2}$,
$0<x_1<1, 0<x_2<1, 0<x_1+x_2<1, 0<\lambda_1<1, 0<\lambda_2<1, 0<\lambda_1+\lambda_2<1$,

Please refer chapter 9 of book, “Continuous Bernoulli --- simulator and test statistic”, this is free book, this book can download from http://vixra.org/abs/2012.0088.

Chapter 3 Continuous Bernoulli distribution to manage K categories --- Model 1

There are k categories, $X_1, X_2, \ldots, X_k$ are continuous random variables,

λ_1	λ_2	…..	$\lambda_k = 1 - \sum_{i=1}^{k-1} \lambda_i$

$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,\ldots,k$

Because $X_i + X_j$ is not $CB(\lambda_i + \lambda_j), i \ne j$, the difference of $\lambda_i + \lambda_j$ and λ_i will use the other method. This method is setting a new random variable Y which probability distribution is $CB(\lambda_i + \lambda_j)$.

Section 1.The marginal probability of Y and X*i*

$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,3,\ Y \sim CB(\lambda_i + \lambda_j), i \ne j$ and $Y \ge X_1$, but the joint probability density function cannot be converted, there are only marginal probability density and the relationship of Y and X_i.

(1)The pdf and df of X_i,

$$f_{X_i}(x_i;\lambda_i) = C(\lambda_i)(\lambda_i)^{x}(1-\lambda_i)^{1-x_i}, 0 \le x_i \le 1, 0 < \lambda_i < 1,$$

$$C(\lambda_i) = \begin{cases} \dfrac{2\,tanh^{-1}(1-2\lambda_i)}{1-2\lambda_i}, \lambda_i \ne \dfrac{1}{2} \\ 2, \lambda_i = \dfrac{1}{2} \end{cases}$$

$$F_{X_i}(x_i;\lambda_i) = \begin{cases} \dfrac{(\lambda_i)^{x}(1-\lambda_i)^{1-x_i} + \lambda_i - 1}{2\lambda_i - 1}, \lambda_i \ne \dfrac{1}{2} \\ x_i, \lambda_i = \dfrac{1}{2} \end{cases}, 0 < x_i < 1$$

$$E(X_i) = \begin{cases} \dfrac{\lambda_i}{2\lambda_i - 1} + \dfrac{1}{2\,tan^{-1}(1-2\lambda_i)} & if\ \lambda_i \ne \dfrac{1}{2} \\ \dfrac{1}{2} & if\ \lambda_i = \dfrac{1}{2} \end{cases}$$

$$RND = F_{X_i}(x_i;\lambda_i), x_i = \begin{cases} \dfrac{log_e(RND \times (2\lambda_i - 1) - (\lambda_i - 1)) - log_e(1-\lambda_i)}{log_e\left(\dfrac{\lambda_i}{1-\lambda_i}\right)}, \lambda_i \ne \dfrac{1}{2} \\ RND, \lambda_i = \dfrac{1}{2} \end{cases}$$

(2)The *pdf* and *df* of Y

$$f_Y(y;\lambda_i+\lambda_j)=C(\lambda_i+\lambda_j)(\lambda_i+\lambda_j)^y(1-(\lambda_i+\lambda_j))^{1-y},0\le y\le 1,0<\lambda_i+\lambda_j<1,$$

$$C(\lambda_i+\lambda_j)=\begin{cases}\dfrac{2\,tanh^{-1}(1-2(\lambda_i+\lambda_j))}{1-2(\lambda_i+\lambda_j)},\lambda_i+\lambda_j\ne\dfrac{1}{2}\\ 2,\lambda_i+\lambda_j=\dfrac{1}{2}\end{cases}$$

$$F_Y(y;\lambda_i+\lambda_j)=\begin{cases}\dfrac{(\lambda_i+\lambda_j)^y(1-\lambda_i+\lambda_j)^{1-y}+(\lambda_i+\lambda_j)-1}{2(\lambda_i+\lambda_j)-1},\lambda_i+\lambda_j\ne\dfrac{1}{2}\\ y,\lambda_i+\lambda_j=\dfrac{1}{2}\end{cases},0<y<1$$

$$E(Y)=\begin{cases}\dfrac{\lambda_i+\lambda_j}{2(\lambda_i+\lambda_j)-1}+\dfrac{1}{2\,tan^{-1}(1-2(\lambda_i+\lambda_j))} & if\ \lambda_i+\lambda_j\ne\dfrac{1}{2}\\ \dfrac{1}{2} & if\ \lambda_i+\lambda_j=\dfrac{1}{2}\end{cases}$$

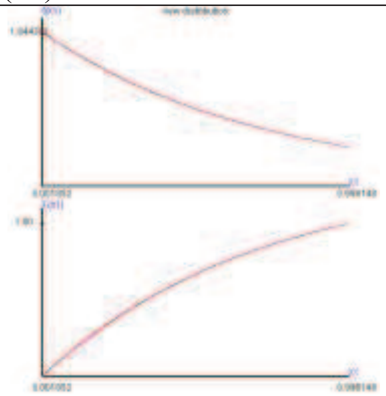
$$RND=F_Y(y;\lambda_i+\lambda_j),$$

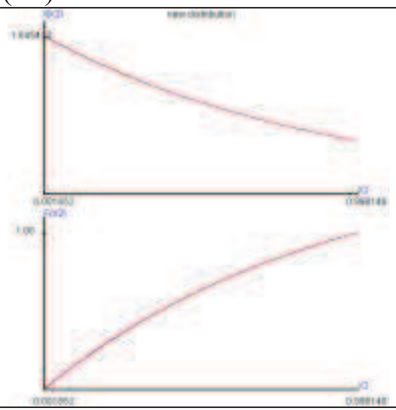
$$y=\begin{cases}\dfrac{log_e(RND\times(2(\lambda_i+\lambda_j)-1)-((\lambda_i+\lambda_j)-1))-log_e(1-(\lambda_i+\lambda_j))}{log_e\left(\dfrac{(\lambda_i+\lambda_j)}{1-(\lambda_i+\lambda_j)}\right)},\lambda_i+\lambda_j\ne\dfrac{1}{2}\\ RND,\lambda_i+\lambda_j=\dfrac{1}{2}\end{cases}$$

When $RND=F_Y(y;\lambda_i+\lambda_j)=F_{X_i}(x_i;\lambda_i),y\ge x_1.$

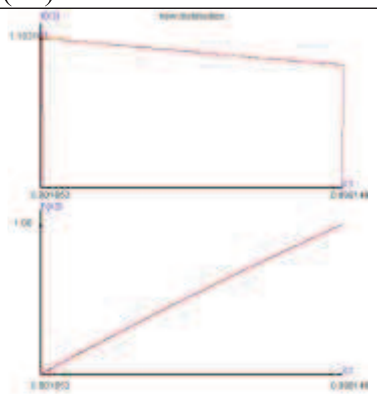
(3) $X_1 + X_2$ is not $X \sim CB(\lambda_1 + \lambda_2), 0 \le x_1 + x_2 \le 1$,

(1)X1~CB(lamda1=0.2),X2~CB(lamda2=0.25),

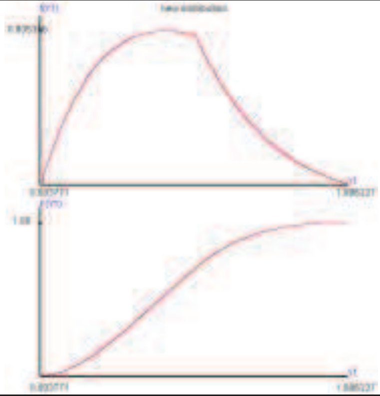
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.38801
	Geometrical Mean :	0.25581
	Harmonic Mean :	0.03130
	Variance :	0.07589
	S.D. :	0.27549
	Skewed Coef. :	0.47615
	Kurtosis Coef. :	2.11585
	MAD :	0.23443
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.33901
	Q1 :	0.14978
	Q2 :	0.33901
	Q3 :	0.59631
	IQR :	0.44653
	C.V. :	0.71001

f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.41022
	Geometrical Mean :	0.27619
	Harmonic Mean :	0.00970
	Variance :	0.07854
	S.D. :	0.28025
	Skewed Coef. :	0.37856
	Kurtosis Coef. :	1.99898
	MAD :	0.23998
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.36904
	Q1 :	0.16592
	Q2 :	0.36904
	Q3 :	0.63091
	IQR :	0.46500
	C.V. :	0.68316

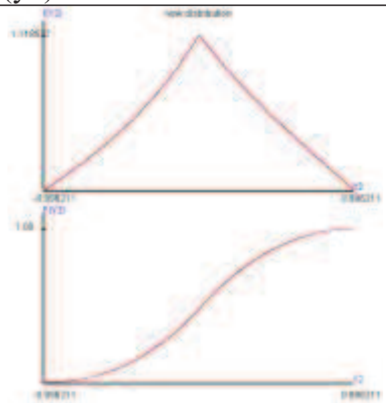
X3~ CB(lamda1+lamda2=0.45)

f(x3),F(x3)	Coefficient	
	Mathematical Mean:	0.48329
	Geometrical Mean :	0.34971
	Harmonic Mean :	0.04740
	Variance :	0.08316
	S.D. :	0.28838
	Skewed Coef. :	0.06955
	Kurtosis Coef. :	1.80671
	MAD :	0.24964
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.47495
	Q1 :	0.23183
	Q2 :	0.47495
	Q3 :	0.73058
	IQR :	0.49875
	C.V. :	0.59670

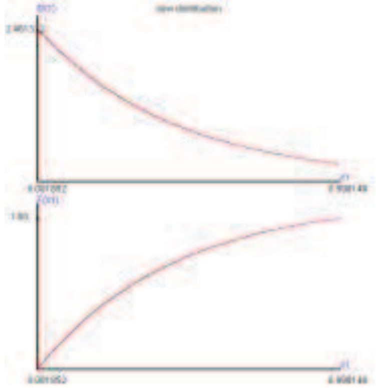
Y1=X1+X2,

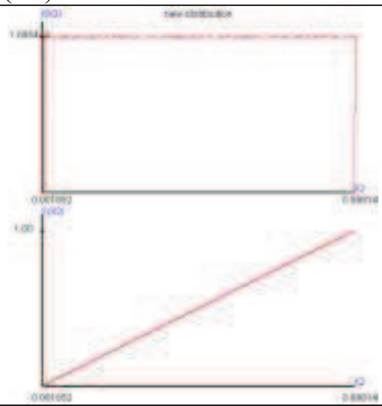
f(y1),F(y1)	Coefficient	
	Mathematical Mean:	0.79822
	Geometrical Mean :	0.67848
	Harmonic Mean :	0.50239
	Variance :	0.15442
	S.D. :	0.39297
	Skewed Coef. :	0.30110
	Kurtosis Coef. :	2.52713
	MAD :	0.32252
	Range :	1.99986
	Mid_range :	1.00000
	Median :	0.78065
	Q1 :	0.49440
	Q2 :	0.78065
	Q3 :	1.06479
	IQR :	0.57038
	C.V. :	0.49230

Y2=X2-X1,

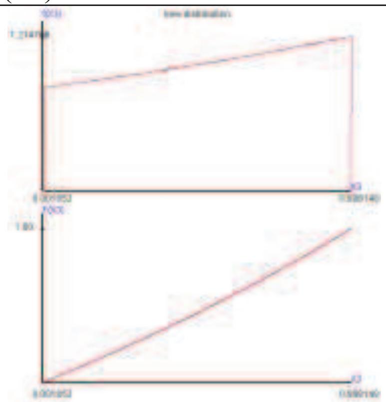
f(y2),F(y2)	Coefficient	
	Mathematical Mean:	0.02221
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	0.15444
	S.D. :	0.39299
	Skewed Coef. :	-0.02675
	Kurtosis Coef. :	2.52738
	MAD :	0.31707
	Range :	1.99983
	Mid_range :	-0.00000
	Median :	0.02057
	Q1 :	-0.24679
	Q2 :	0.02057
	Q3 :	0.29656
	IQR :	0.54334
	C.V. :	17.69069

(2)X1~CB(lamda1=0.1),X2~CB(lamda2=0.5),

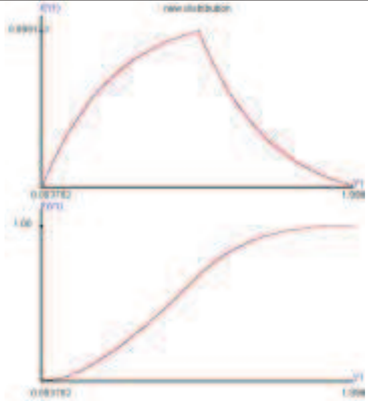
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33011 Geometrical Mean : 0.20661 Harmonic Mean : 0.02353 Variance : 0.06650 S.D. : 0.25788 Skewed Coef. : 0.74400 Kurtosis Coef. : 2.58171 MAD : 0.21453 Range : 1.00000 Mid_range : 0.50000 Median : 0.26749 Q1 : 0.11438 Q2 : 0.26749 Q3 : 0.49999 IQR : 0.38561 C.V. : 0.78121

f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.49998 Geometrical Mean : 0.36782 Harmonic Mean : 0.01589 Variance : 0.08334 S.D. : 0.28869 Skewed Coef. : 0.00005 Kurtosis Coef. : 1.79995 MAD : 0.25001 Range : 1.00000 Mid_range : 0.50000 Median : 0.49997 Q1 : 0.24994 Q2 : 0.49997 Q3 : 0.74999 IQR : 0.50004 C.V. : 0.57740

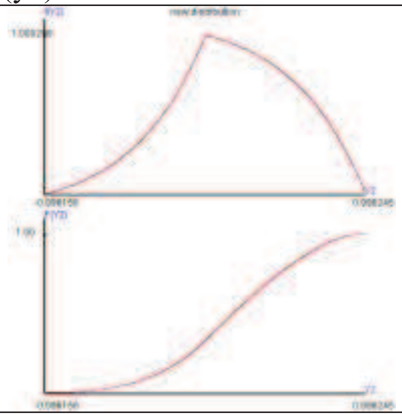
X3~ CB(lamda1+lamda2=0.6)

f(x3),F(x3)	Coefficient
	Mathematical Mean: 0.53370 Geometrical Mean : 0.40612 Harmonic Mean : 0.06341 Variance : 0.08265 S.D. : 0.28748 Skewed Coef. : -0.14031 Kurtosis Coef. : 1.82726 MAD : 0.24857 Range : 1.00000 Mid_range : 0.50000 Median : 0.55033 Q1 : 0.29049 Q2 : 0.55033 Q3 : 0.78541 IQR : 0.49492 C.V. : 0.53867

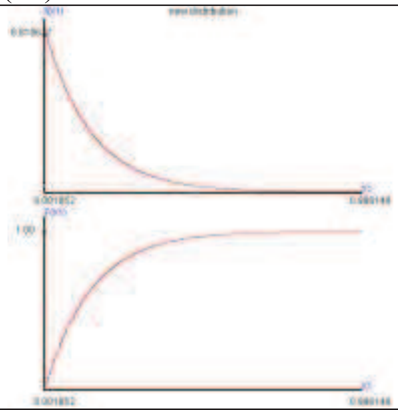
Y1=X1+X2,

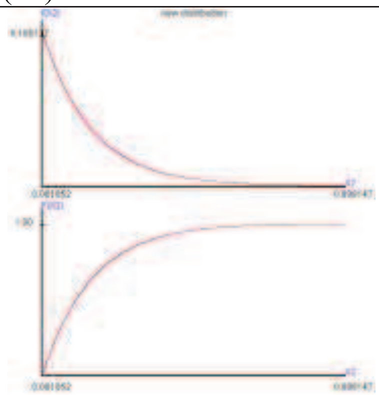
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.83009 Geometrical Mean : 0.71614 Harmonic Mean : 0.54206 Variance : 0.14984 S.D. : 0.38709 Skewed Coef. : 0.21975 Kurtosis Coef. : 2.54594 MAD : 0.31602 Range : 1.99984 Mid_range : 1.00000 Median : 0.82534 Q1 : 0.53763 Q2 : 0.82534 Q3 : 1.08938 IQR : 0.55174 C.V. : 0.46632

Y2=X2-X1,

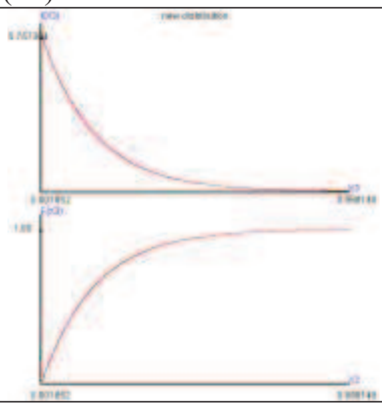
f(y2),F(y2)	Coefficient
	Mathematical Mean: 0.16987 Geometrical Mean : none Harmonic Mean : none Variance : 0.14985 S.D. : 0.38711 Skewed Coef. : -0.21994 Kurtosis Coef. : 2.54612 MAD : 0.31604 Range : 1.99981 Mid_range : 0.00004 Median : 0.17469 Q1 : -0.08944 Q2 : 0.17469 Q3 : 0.46229 IQR : 0.55173 C.V. : 2.27888

(3)X1~CB(lamda1=0.001),X2~CB(lamda2=0.002),

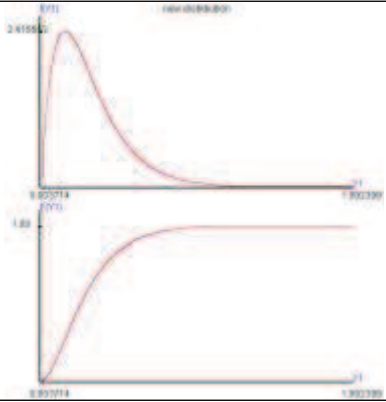
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.14378
	Geometrical Mean :	0.08108
	Harmonic Mean :	0.00845
	Variance :	0.01996
	S.D. :	0.14127
	Skewed Coef. :	1.79638
	Kurtosis Coef. :	7.08620
	MAD :	0.10536
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.10021
	Q1 :	0.04160
	Q2 :	0.10021
	Q3 :	0.20028
	IQR :	0.15867
	C.V. :	0.98259

f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.15895
	Geometrical Mean :	0.08990
	Harmonic Mean :	0.00258
	Variance :	0.02390
	S.D. :	0.15459
	Skewed Coef. :	1.71138
	Kurtosis Coef. :	6.48962
	MAD :	0.11614
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.11124
	Q1 :	0.04619
	Q2 :	0.11124
	Q3 :	0.22217
	IQR :	0.17598
	C.V. :	0.97260

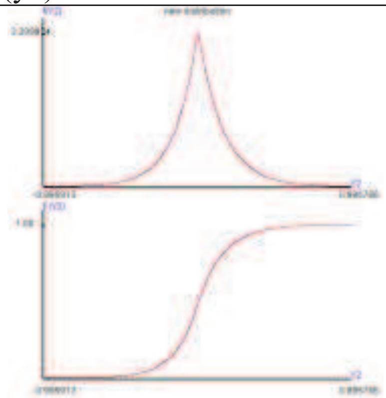
X3~ CB(lamda1+lamda2=0.003)

f(x3),F(x3)	Coefficient	
	Mathematical Mean:	0.16921
	Geometrical Mean :	0.09598
	Harmonic Mean :	0.00922
	Variance :	0.02663
	S.D. :	0.16319
	Skewed Coef. :	1.64984
	Kurtosis Coef. :	6.09710
	MAD :	0.12329
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.11886
	Q1 :	0.04938
	Q2 :	0.11886
	Q3 :	0.23722
	IQR :	0.18785
	C.V. :	0.96446

Y1=X1+X2,

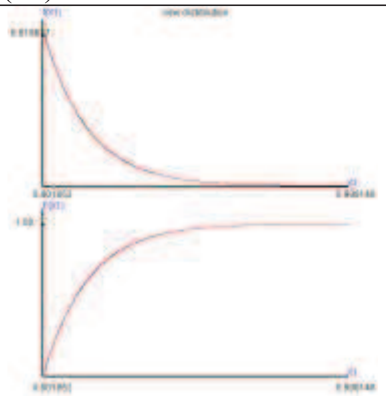
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.30273 Geometrical Mean : 0.23200 Harmonic Mean : 0.15230 Variance : 0.04385 S.D. : 0.20941 Skewed Coef. : 1.23968 Kurtosis Coef. : 4.88221 MAD : 0.16259 Range : 1.99608 Mid_range : 0.99806 Median : 0.25569 Q1 : 0.14649 Q2 : 0.25569 Q3 : 0.40969 IQR : 0.26320 C.V. : 0.69175

Y2=X2-X1,

f(y2),F(y2)	Coefficient
	Mathematical Mean: 0.01517 Geometrical Mean : none Harmonic Mean : none Variance : 0.04386 S.D. : 0.20943 Skewed Coef. : 0.13678 Kurtosis Coef. : 4.88151 MAD : 0.15068 Range : 1.99910 Mid_range : -0.00006 Median : 0.00814 Q1 : -0.09235 Q2 : 0.00814 Q3 : 0.11907 IQR : 0.21142 C.V. : 13.80541

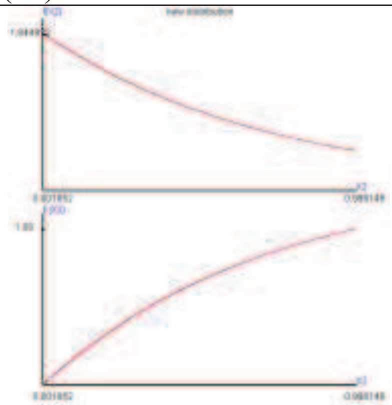
(4)X1~CB(lamda1=0.001),X2~CB(lamda2=0.2),

f(x1),F(x1)



Coefficient	
Mathematical Mean:	0.14378
Geometrical Mean :	0.08108
Harmonic Mean :	0.00845
Variance :	0.01996
S.D. :	0.14127
Skewed Coef. :	1.79638
Kurtosis Coef. :	7.08620
MAD :	0.10536
Range :	1.00000
Mid_range :	0.50000
Median :	0.10021
Q1 :	0.04160
Q2 :	0.10021
Q3 :	0.20028
IQR :	0.15867
C.V. :	0.98259

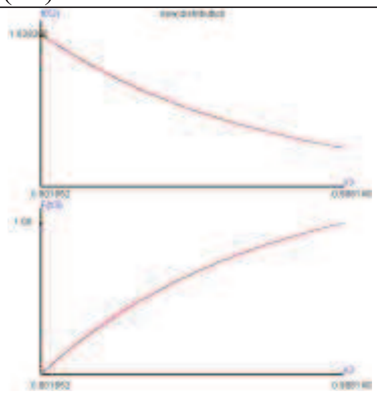
f(x2),F(x2)



Coefficient	
Mathematical Mean:	0.38800
Geometrical Mean :	0.25576
Harmonic Mean :	0.00865
Variance :	0.07590
S.D. :	0.27550
Skewed Coef. :	0.47611
Kurtosis Coef. :	2.11581
MAD :	0.23444
Range :	1.00000
Mid_range :	0.50000
Median :	0.33901
Q1 :	0.14974
Q2 :	0.33901
Q3 :	0.59631
IQR :	0.44656
C.V. :	0.71006

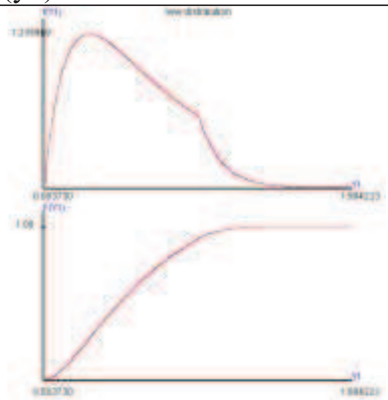
X3~ CB(lamda1+lamda2=0.2001)

f(x3),F(x3)

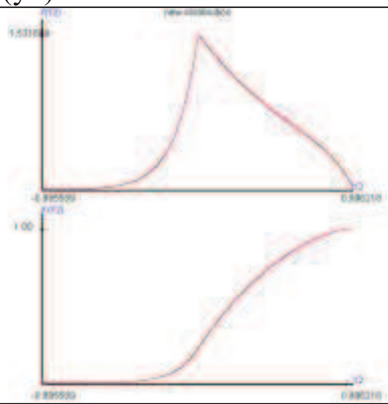


Coefficient	
Mathematical Mean:	0.38848
Geometrical Mean :	0.25624
Harmonic Mean :	0.02885
Variance :	0.07596
S.D. :	0.27560
Skewed Coef. :	0.47401
Kurtosis Coef. :	2.11303
MAD :	0.23456
Range :	1.00000
Mid_range :	0.50000
Median :	0.33965
Q1 :	0.15011
Q2 :	0.33965
Q3 :	0.59709
IQR :	0.44698
C.V. :	0.70942

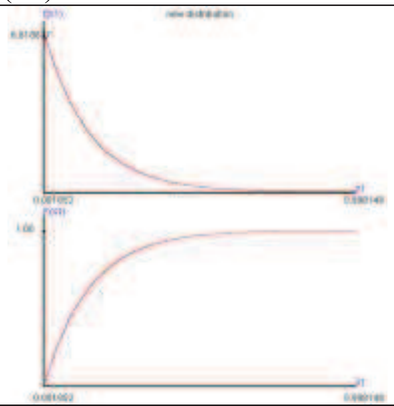
Y1=X1+X2,

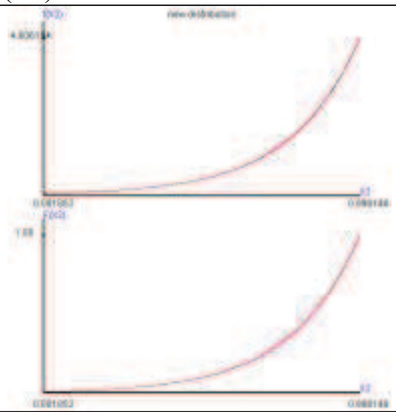
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.53177 Geometrical Mean : 0.42486 Harmonic Mean : 0.28434 Variance : 0.09585 S.D. : 0.30960 Skewed Coef. : 0.50586 Kurtosis Coef. : 2.62237 MAD : 0.25767 Range : 1.99789 Mid_range : 0.99898 Median : 0.49047 Q1 : 0.27958 Q2 : 0.49047 Q3 : 0.75474 IQR : 0.47516 C.V. : 0.58220

Y2=X2-X1,

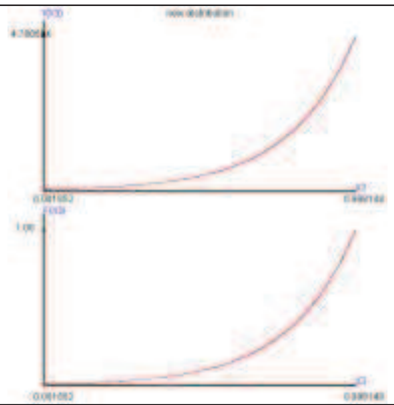
f(y2),F(y2)	Coefficient
	Mathematical Mean: 0.24422 Geometrical Mean : none Harmonic Mean : none Variance : 0.09587 S.D. : 0.30962 Skewed Coef. : 0.16479 Kurtosis Coef. : 2.62263 MAD : 0.25387 Range : 1.99913 Mid_range : 0.00035 Median : 0.20771 Q1 : 0.01836 Q2 : 0.20771 Q3 : 0.46641 IQR : 0.44806 C.V. : 1.26782

(5)X1~CB(lamda1=0.001),X2~CB(lamda2=0.99),

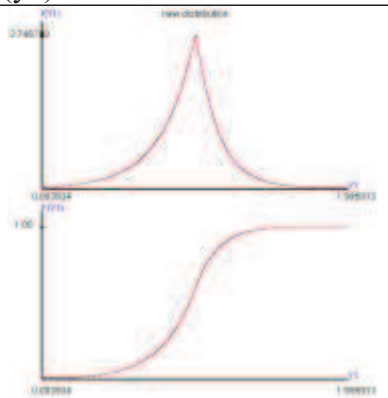
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.14378 Geometrical Mean : 0.08108 Harmonic Mean : 0.00845 Variance : 0.01996 S.D. : 0.14127 Skewed Coef. : 1.79638 Kurtosis Coef. : 7.08620 MAD : 0.10536 Range : 1.00000 Mid_range : 0.50000 Median : 0.10021 Q1 : 0.04160 Q2 : 0.10021 Q3 : 0.20028 IQR : 0.15867 C.V. : 0.98259

f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.79256 Geometrical Mean : 0.75290 Harmonic Mean : 0.24327 Variance : 0.03707 S.D. : 0.19252 Skewed Coef. : -1.41563 Kurtosis Coef. : 4.83045 MAD : 0.14892 Range : 1.00000 Mid_range : 0.50000 Median : 0.85133 Q1 : 0.70476 Q2 : 0.85133 Q3 : 0.93812 IQR : 0.23336 C.V. : 0.24291

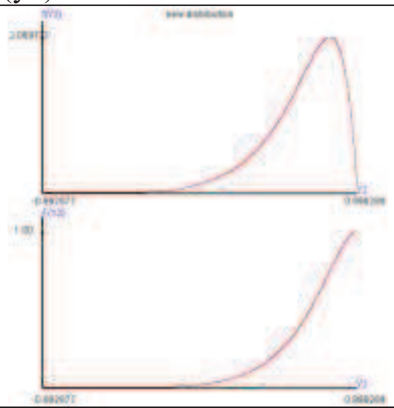
X3~ CB(lamda1+lamda2=0.9901)

f(x3),F(x3)	Coefficient
	Mathematical Mean: 0.79647 Geometrical Mean : 0.75842 Harmonic Mean : 0.50196 Variance : 0.03599 S.D. : 0.18970 Skewed Coef. : -1.43914 Kurtosis Coef. : 4.94415 MAD : 0.14637 Range : 1.00000 Mid_range : 0.50000 Median : 0.85449 Q1 : 0.71086 Q2 : 0.85449 Q3 : 0.93946 IQR : 0.22860 C.V. : 0.23817

Y1=X1+X2,

f(y1),F(y1)	Coefficient	
	Mathematical Mean:	0.93633
	Geometrical Mean :	0.89651
	Harmonic Mean :	0.82529
	Variance :	0.05702
	S.D. :	0.23879
	Skewed Coef. :	-0.37012
	Kurtosis Coef. :	4.27366
	MAD :	0.17669
	Range :	1.99938
	Mid_range :	0.99992
	Median :	0.96212
	Q1 :	0.81555
	Q2 :	0.96212
	Q3 :	1.06893
	IQR :	0.25337
	C.V. :	0.25503

Y2=X2-X1,

f(y2),F(y2)	Coefficient	
	Mathematical Mean:	0.64878
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	0.05702
	S.D. :	0.23880
	Skewed Coef. :	-1.11376
	Kurtosis Coef. :	4.27383
	MAD :	0.18775
	Range :	1.99666
	Mid_range :	0.00166
	Median :	0.70178
	Q1 :	0.52128
	Q2 :	0.70178
	Q3 :	0.82956
	IQR :	0.30828
	C.V. :	0.36807

Section 2. The test statistic of λ

$X_1, X_2, ..., X_n \overset{iid}{\sim} CB(\lambda)$, n random samples from $CB(\lambda)$.
The Z test statistic for large sample,

$n \geq 6+250 \times |\lambda$ -0.5|, if $0.1 \leq \lambda \leq 0.9$,
$n \geq 100+2000 \times (\lambda$ -0.1), if λ <0.1,
$n \geq 100+2000 \times (\lambda$ -0.9),if λ >0.9,

$$\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{\sigma} \longrightarrow Normal(0,1), \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}, ,$$

$H_0 : \lambda = c \quad H_0 : \lambda = c,$

$$Z^* = \frac{\sqrt{n}\left(\overline{X}-\mu_0\right)}{\sigma_0} \rightarrow Z \sim Normal(0,1), \left|Z^*\right| > Z_{\alpha/2} \text{ rejected } H_0 \text{ and } P\left(Z > Z_{\alpha/2}\right) = \frac{\alpha}{2}.$$

$$\mu_0 = \begin{cases} \dfrac{c}{2c-1} + \dfrac{1}{2 tan^{-1}(1-2c)} & if\ c \neq \dfrac{1}{2} \\ \dfrac{1}{2} & if\ \lambda = \dfrac{1}{2} \end{cases}$$

$$\sigma_0^2 = \begin{cases} \dfrac{(1-c)c}{(1-2c)^2} + \dfrac{1}{\left(2 tan^{-1}(1-2c)\right)^2} & if\ c \neq \dfrac{1}{2} \\ \dfrac{1}{12} & if\ c = \dfrac{1}{2} \end{cases}$$

Please refer book, “Continuous Bernoulli - simulator and test statistic”, this is free book, this book can download from http://vixra.org/abs/2012.0088.

Section 3. The joint probability distribution of Y and X*ᵢ*

The simulator can compute the joint probability distribution of Y and X_i,

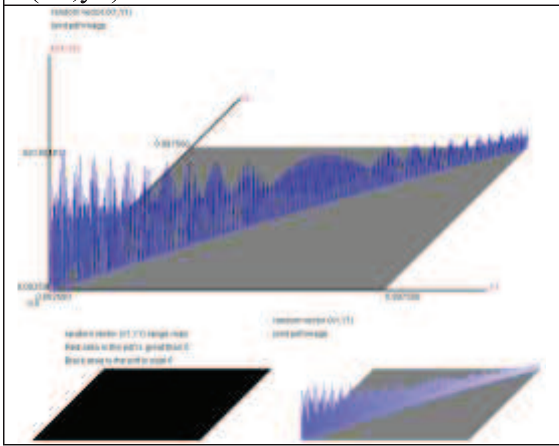
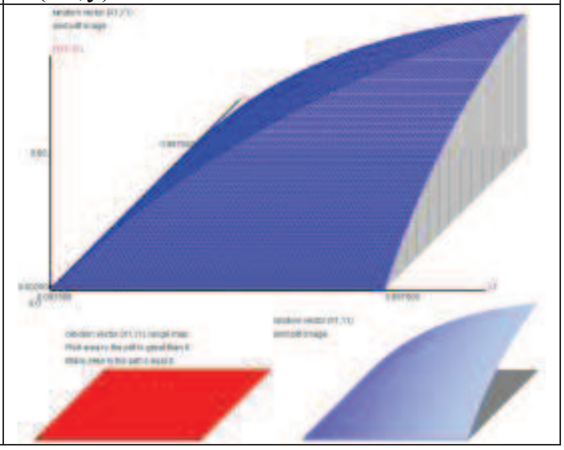
$RND = F_Y(y;\lambda_i + \lambda_j) = F_{X_i}(x_i;\lambda_i), y \geq x_1$.

There are 3 categories, X_1 and Y are continuous random variables,

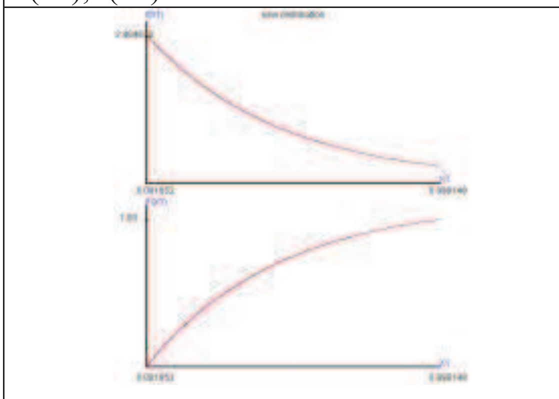
λ_1	λ_2	$1-\lambda_1-\lambda_2$

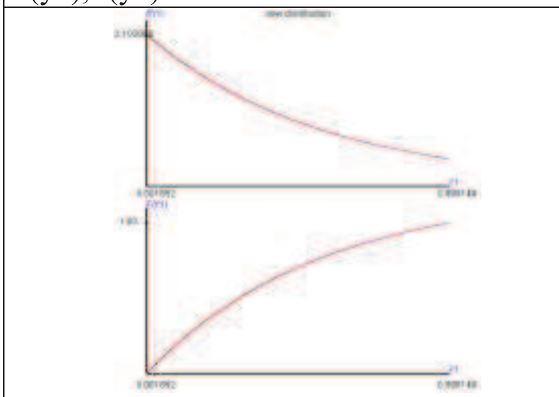
$X_1 \sim CB(\lambda_1), Y_1 \sim CB(\lambda_1+\lambda_2), f_{X_1,Y_1}(x_1,y_1) = ?$

(3-1) λ_1=0.1, λ_2=0.05, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

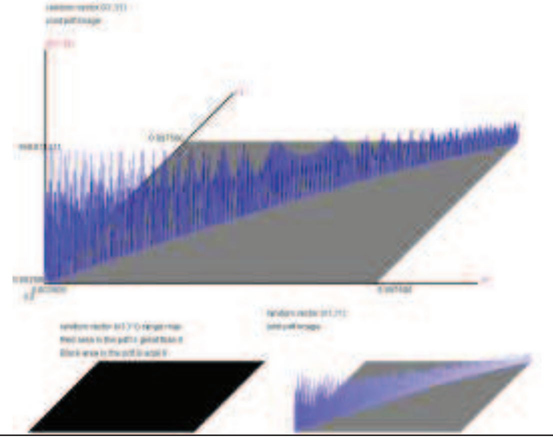
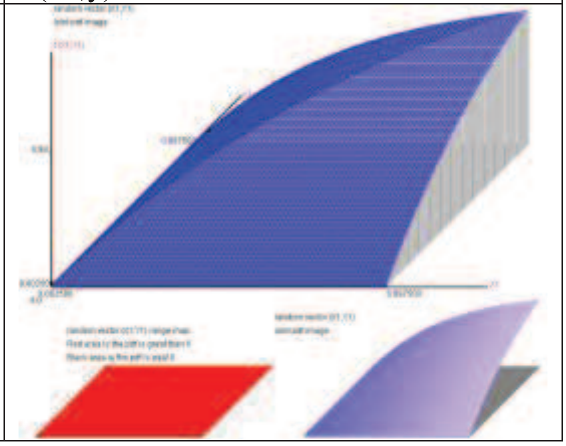
f(x1,y1)	F(x1,y)
	

E(X1)= 0.3302, Var(X1)= 0.0665, E(Y1)= 0.3623, Var(Y1)= 0.0722, Cov(X1,Y1)= 0.0692, X1 and Y1 correlation coefficient=0.9987.

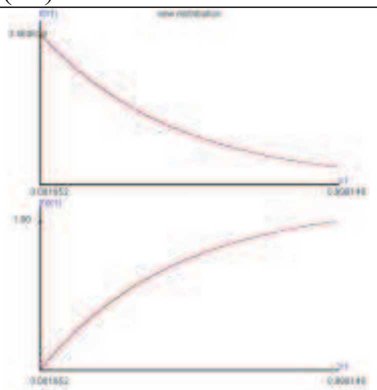
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33015 Geometrical Mean : 0.20662 Harmonic Mean : 0.02220 Variance : 0.06651 S.D. : 0.25790 Skewed Coef. : 0.74369 Kurtosis Coef. : 2.58117 MAD : 0.21454 Range : 1.00000 Mid_range : 0.50000 Median : 0.26756 Q1 : 0.11440 Q2 : 0.26756 Q3 : 0.50008 IQR : 0.38568 C.V. : 0.78115

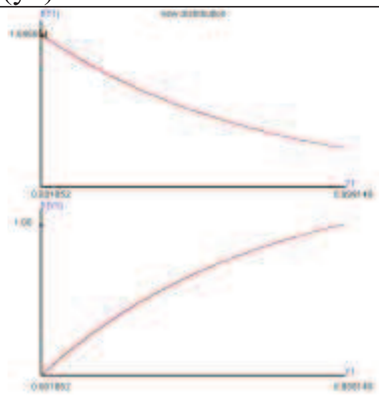
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.36225 Geometrical Mean : 0.23320 Harmonic Mean : 0.02599 Variance : 0.07215 S.D. : 0.26861 Skewed Coef. : 0.59238 Kurtosis Coef. : 2.29150 MAD : 0.22654 Range : 1.00000 Mid_range : 0.50000 Median : 0.30596 Q1 : 0.13292 Q2 : 0.30596 Q3 : 0.55433 IQR : 0.42142 C.V. : 0.74151

(3-2) λ_1=0.1, λ_2=0.1, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

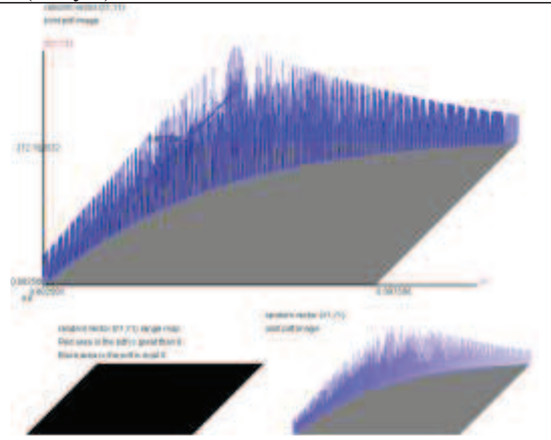
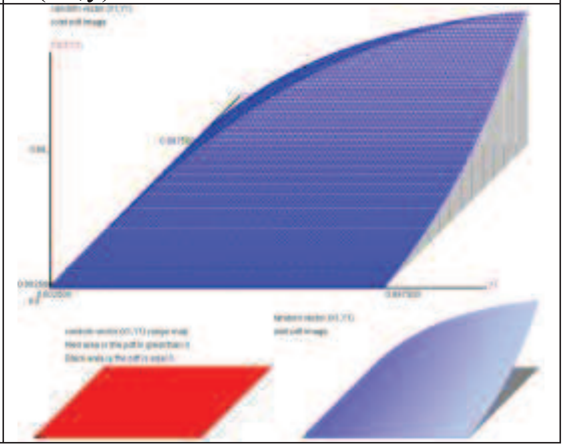
f(x1,y1)	F(x1,y)
	

E(X1)= 0.3302, Var(X1)= 0.0665, E(Y1)= 0.3881, Var(Y1)= 0.0759, Cov(X1,Y1)= 0.0707, X1 and Y1 correlation coefficient=0.9957.

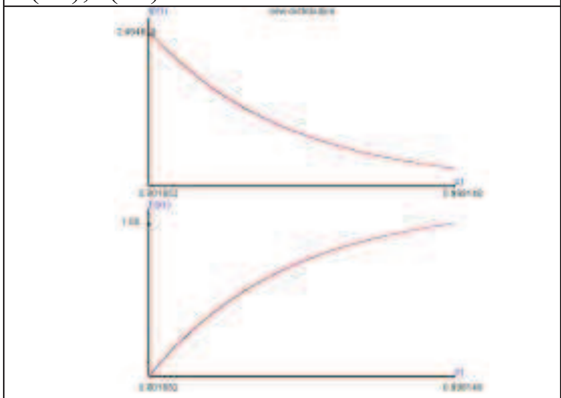
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33015 Geometrical Mean : 0.20662 Harmonic Mean : 0.02220 Variance : 0.06651 S.D. : 0.25790 Skewed Coef. : 0.74369 Kurtosis Coef. : 2.58117 MAD : 0.21454 Range : 1.00000 Mid_range : 0.50000 Median : 0.26756 Q1 : 0.11440 Q2 : 0.26756 Q3 : 0.50008 IQR : 0.38568 C.V. : 0.78115

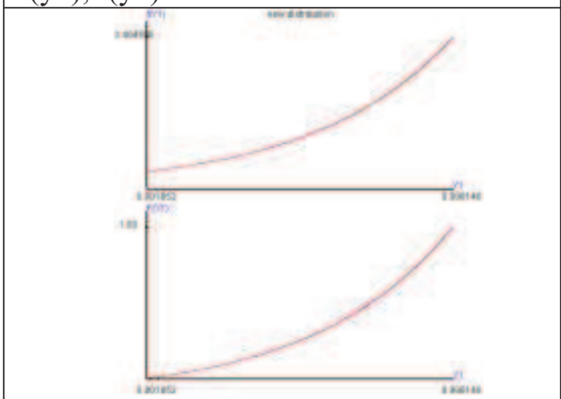
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25582 Harmonic Mean : 0.02954 Variance : 0.07590 S.D. : 0.27550 Skewed Coef. : 0.47584 Kurtosis Coef. : 2.11554 MAD : 0.23444 Range : 1.00000 Mid_range : 0.50000 Median : 0.33910 Q1 : 0.14980 Q2 : 0.33910 Q3 : 0.59640 IQR : 0.44660 C.V. : 0.70996

(3-3) λ_1=0.1, λ_2=0.8, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

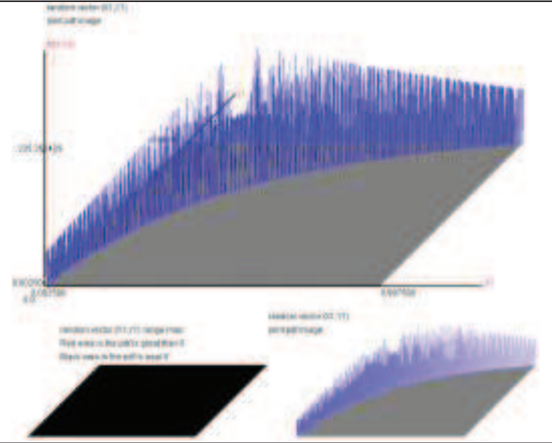
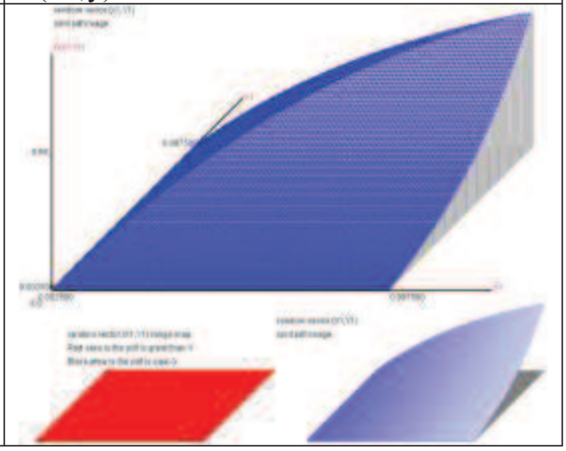
f(x1,y1)	F(x1,y)
	

E(X1)= 0.3302, Var(X1)= 0.0665, E(Y1)= 0.6699, Var(Y1)= 0.0665, Cov(X1,Y1)= 0.0585, X1 and Y1 correlation coefficient=0.8796.

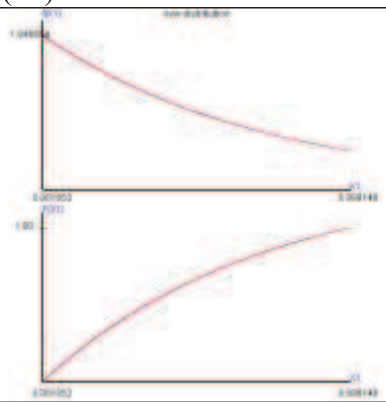
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33015 Geometrical Mean : 0.20662 Harmonic Mean : 0.02220 Variance : 0.06651 S.D. : 0.25790 Skewed Coef. : 0.74369 Kurtosis Coef. : 2.58117 MAD : 0.21454 Range : 1.00000 Mid_range : 0.50000 Median : 0.26756 Q1 : 0.11440 Q2 : 0.26756 Q3 : 0.50008 IQR : 0.38568 C.V. : 0.78115

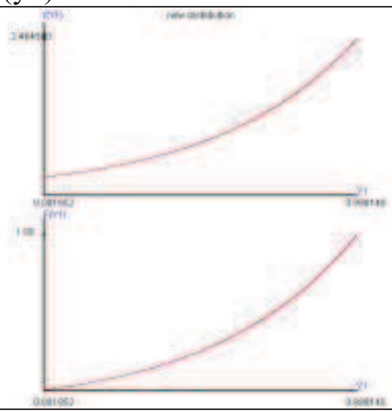
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.66991 Geometrical Mean : 0.58008 Harmonic Mean : 0.16713 Variance : 0.06651 S.D. : 0.25789 Skewed Coef. : -0.74423 Kurtosis Coef. : 2.58218 MAD : 0.21453 Range : 1.00000 Mid_range : 0.50000 Median : 0.73254 Q1 : 0.50004 Q2 : 0.73254 Q3 : 0.88566 IQR : 0.38561 C.V. : 0.38497

(3-4) λ_1=0.2, λ_2=0.7, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

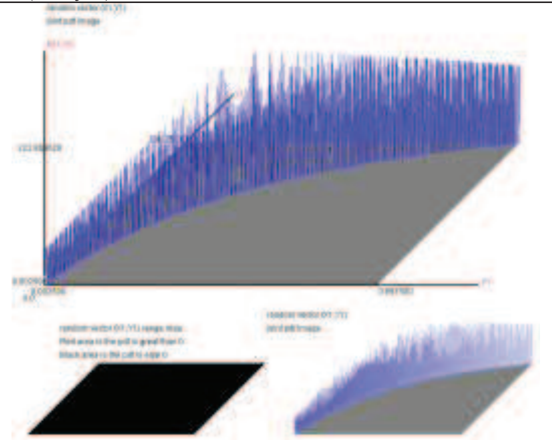
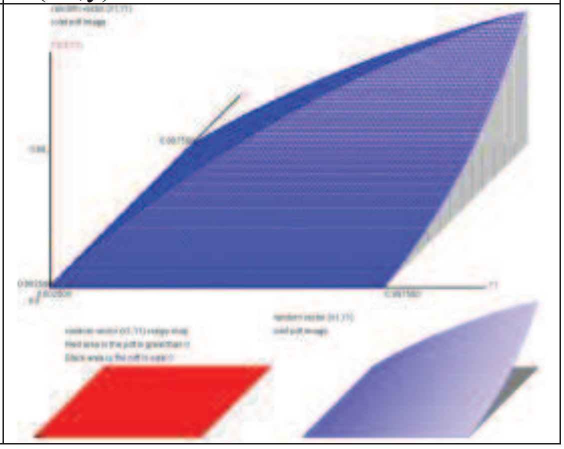
f(x1,y1)	F(x1,y)
	

E(X1)= 0.3881, Var(X1)= 0.0759, E(Y1)= 0.6699, Var(Y1)= 0.0665,
Cov(X1,Y1)= 0.0649, X1 and Y1 correlation coefficient=0.9134.

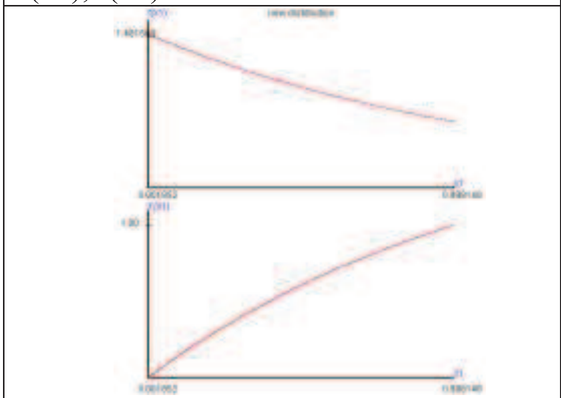
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25582 Harmonic Mean : 0.02954 Variance : 0.07590 S.D. : 0.27550 Skewed Coef. : 0.47584 Kurtosis Coef. : 2.11554 MAD : 0.23444 Range : 1.00000 Mid_range : 0.50000 Median : 0.33910 Q1 : 0.14980 Q2 : 0.33910 Q3 : 0.59640 IQR : 0.44660 C.V. : 0.70996

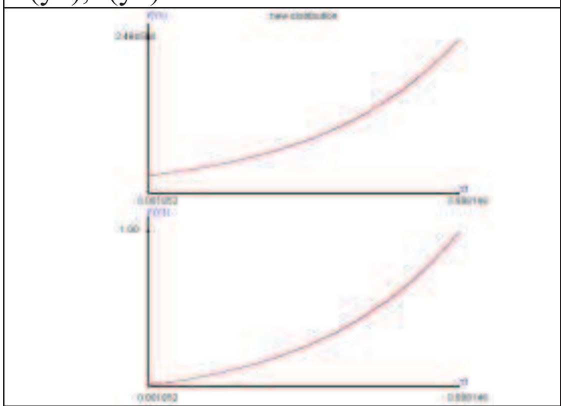
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.66991 Geometrical Mean : 0.58008 Harmonic Mean : 0.16713 Variance : 0.06651 S.D. : 0.25789 Skewed Coef. : -0.74423 Kurtosis Coef. : 2.58218 MAD : 0.21453 Range : 1.00000 Mid_range : 0.50000 Median : 0.73254 Q1 : 0.50004 Q2 : 0.73254 Q3 : 0.88566 IQR : 0.38561 C.V. : 0.38497

(3-5) λ_1=0.3, λ_2=0.6, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

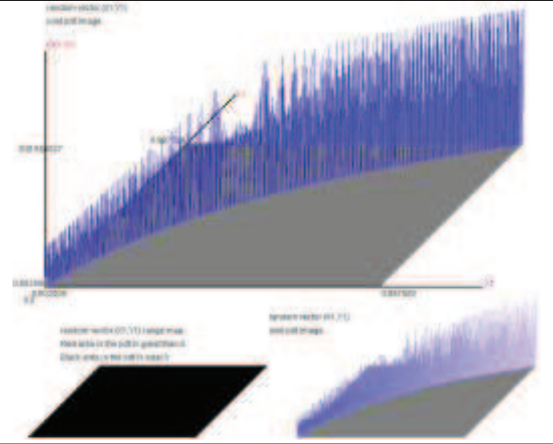
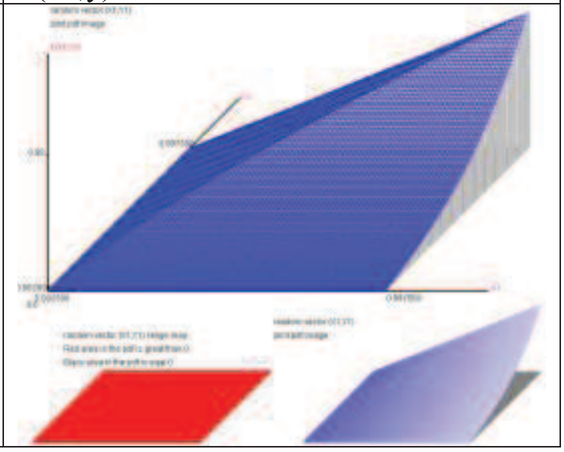
f(x1,y1)	F(x1,y)
	

E(X1)= 0.4303, Var(X1)= 0.0804, E(Y1)= 0.6699, Var(Y1)= 0.0665,
Cov(X1,Y1)= 0.0684, X1 and Y1 correlation coefficient=0.9355.

f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.43026
	Geometrical Mean :	0.29538
	Harmonic Mean :	0.03662
	Variance :	0.08043
	S.D. :	0.28360
	Skewed Coef. :	0.29236
	Kurtosis Coef. :	1.91845
	MAD :	0.24393
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.39718
	Q1 :	0.18196
	Q2 :	0.39718
	Q3 :	0.66055
	IQR :	0.47859
	C.V. :	0.65913

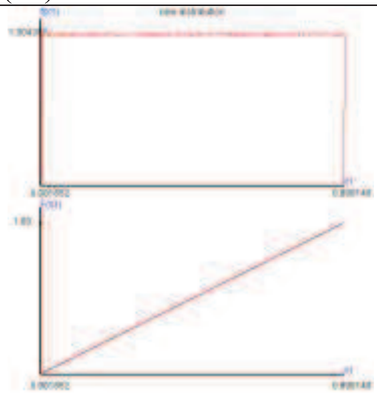
f(y1),F(y1)	Coefficient	
	Mathematical Mean:	0.66991
	Geometrical Mean :	0.58008
	Harmonic Mean :	0.16713
	Variance :	0.06651
	S.D. :	0.25789
	Skewed Coef. :	-0.74423
	Kurtosis Coef. :	2.58218
	MAD :	0.21453
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.73254
	Q1 :	0.50004
	Q2 :	0.73254
	Q3 :	0.88566
	IQR :	0.38561
	C.V. :	0.38497

(3-6) λ_1=0.5, λ_2=0.4, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

f(x1,y1)	F(x1,y)
	

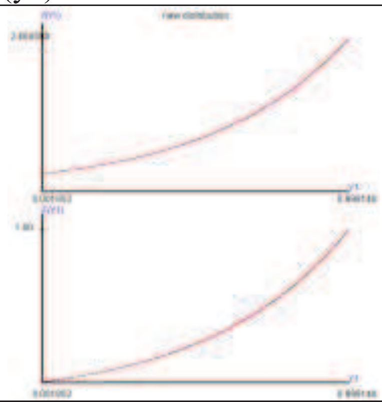
E(X1)= 0.5000, Var(X1)= 0.0833, E(Y1)= 0.6699, Var(Y1)= 0.0665,
Cov(X1,Y1)= 0.0719, X1 and Y1 correlation coefficient=0.9659.

f(x1),F(x1)



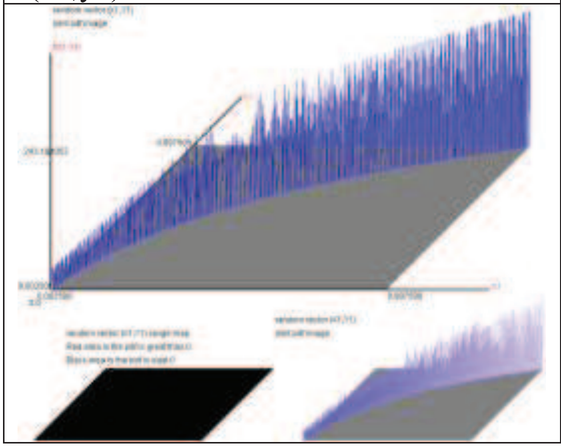
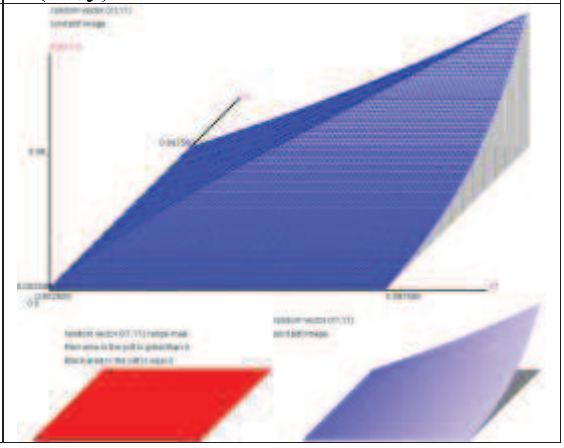
Coefficient	
Mathematical Mean:	0.50004
Geometrical Mean :	0.36790
Harmonic Mean :	0.05342
Variance :	0.08333
S.D. :	0.28868
Skewed Coef. :	-0.00022
Kurtosis Coef. :	1.80001
MAD :	0.25000
Range :	1.00000
Mid_range :	0.50000
Median :	0.50007
Q1 :	0.25003
Q2 :	0.50007
Q3 :	0.75006
IQR :	0.50003
C.V. :	0.57731

f(y1),F(y1)



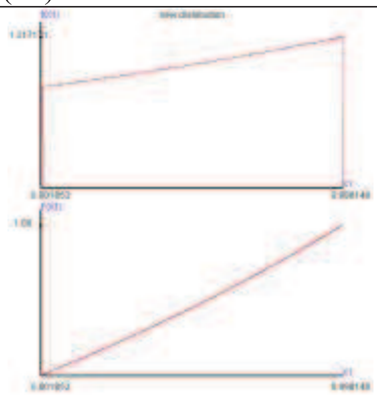
Coefficient	
Mathematical Mean:	0.66991
Geometrical Mean :	0.58008
Harmonic Mean :	0.16713
Variance :	0.06651
S.D. :	0.25789
Skewed Coef. :	-0.74423
Kurtosis Coef. :	2.58218
MAD :	0.21453
Range :	1.00000
Mid_range :	0.50000
Median :	0.73254
Q1 :	0.50004
Q2 :	0.73254
Q3 :	0.88566
IQR :	0.38561
C.V. :	0.38497

(3-7) λ_1=0.6, λ_2=0.35, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

f(x1,y1)	F(x1,y)
	

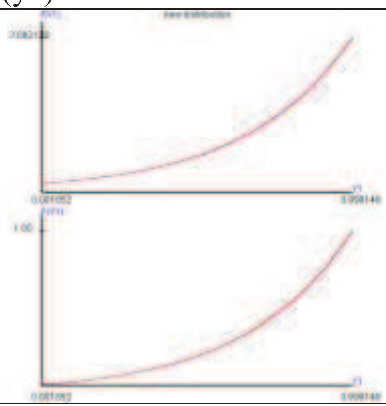
E(X1)= 0.5337, Var(X1)= 0.0827, E(Y1)= 0.7160, Var(Y1)= 0.0567, Cov(X1,Y1)= 0.0657, X1 and Y1 correlation coefficient=0.9603.

f(x1),F(x1)



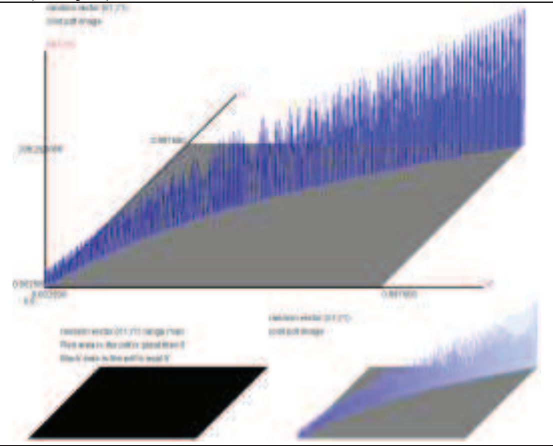
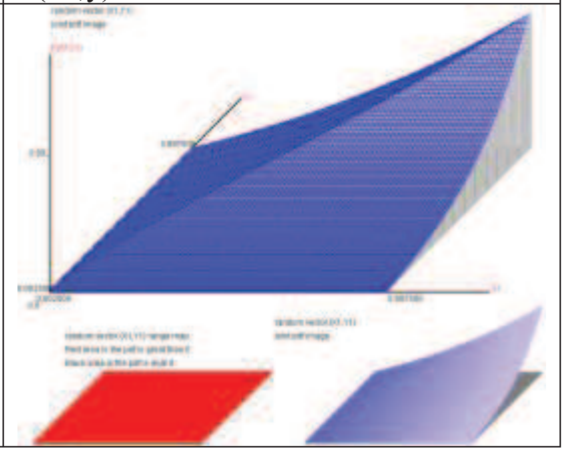
Coefficient	
Mathematical Mean:	0.53374
Geometrical Mean :	0.40613
Harmonic Mean :	0.06503
Variance :	0.08265
S.D. :	0.28750
Skewed Coef. :	-0.14058
Kurtosis Coef. :	1.82732
MAD :	0.24858
Range :	1.00000
Mid_range :	0.50000
Median :	0.55041
Q1 :	0.29053
Q2 :	0.55041
Q3 :	0.78546
IQR :	0.49493
C.V. :	0.53865

f(y1),F(y1)

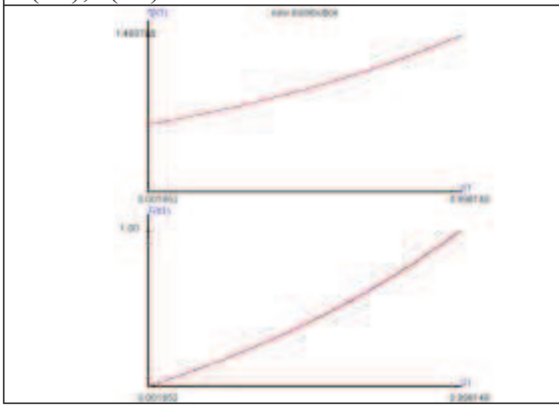


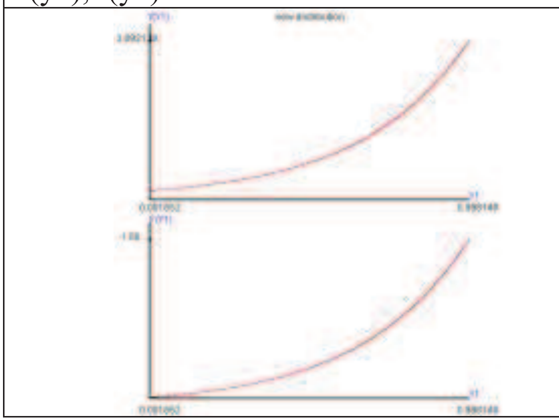
Coefficient	
Mathematical Mean:	0.71596
Geometrical Mean :	0.64435
Harmonic Mean :	0.24628
Variance :	0.05670
S.D. :	0.23813
Skewed Coef. :	-0.97717
Kurtosis Coef. :	3.17194
MAD :	0.19335
Range :	1.00000
Mid_range :	0.50000
Median :	0.78205
Q1 :	0.57901
Q2 :	0.78205
Q3 :	0.90823
IQR :	0.32922
C.V. :	0.33260

(3-8) λ_1=0.7, λ_2=0.25, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

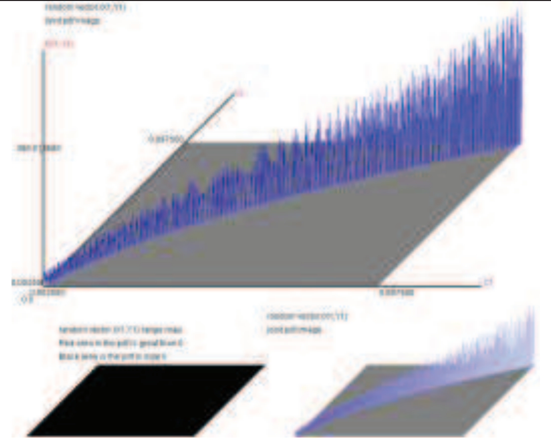
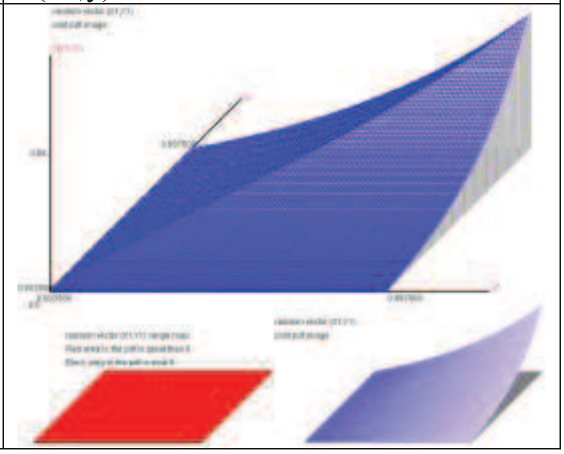
f(x1,y1)	F(x1,y)
	

E(X1)= 0.5698, Var(X1)= 0.0804, E(Y1)= 0.7160, Var(Y1)= 0.0567, Cov(X1,Y1)= 0.0658, X1 and Y1 correlation coefficient=0.9737.

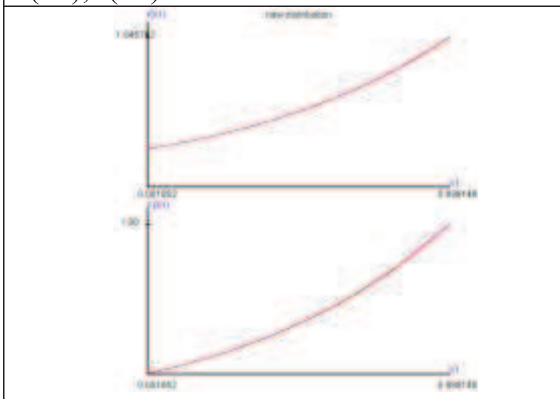
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.56982 Geometrical Mean : 0.44932 Harmonic Mean : 0.08139 Variance : 0.08043 S.D. : 0.28359 Skewed Coef. : -0.29282 Kurtosis Coef. : 1.91881 MAD : 0.24393 Range : 1.00000 Mid_range : 0.50000 Median : 0.60295 Q1 : 0.33957 Q2 : 0.60295 Q3 : 0.81812 IQR : 0.47855 C.V. : 0.49769

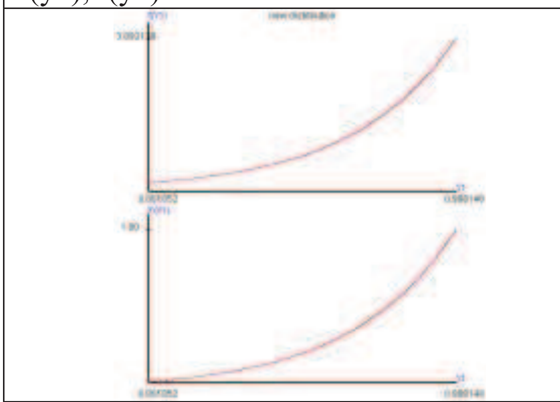
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.71596 Geometrical Mean : 0.64435 Harmonic Mean : 0.24628 Variance : 0.05670 S.D. : 0.23813 Skewed Coef. : -0.97717 Kurtosis Coef. : 3.17194 MAD : 0.19335 Range : 1.00000 Mid_range : 0.50000 Median : 0.78205 Q1 : 0.57901 Q2 : 0.78205 Q3 : 0.90823 IQR : 0.32922 C.V. : 0.33260

(3-9) λ_1=0.8, λ_2=0.15, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

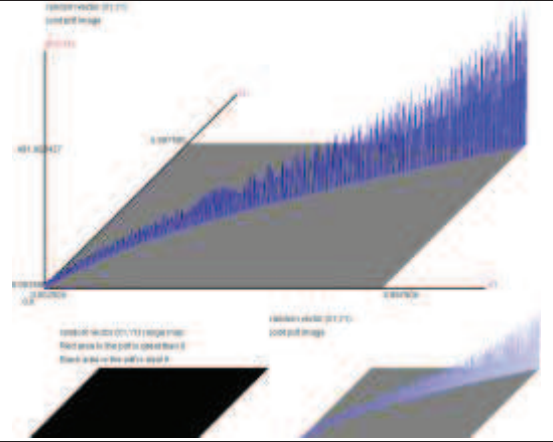
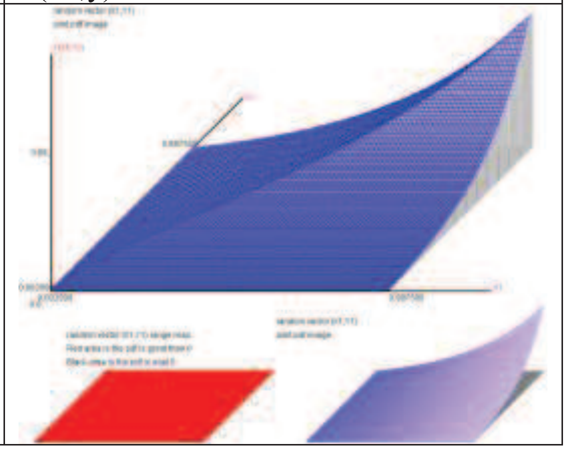
f(x1,y1)	F(x1,y)
	

E(X1)= 0.6120, Var(X1)= 0.0759, E(Y1)= 0.7160, Var(Y1)= 0.0567, Cov(X1,Y1)= 0.0647, X1 and Y1 correlation coefficient=0.9862.

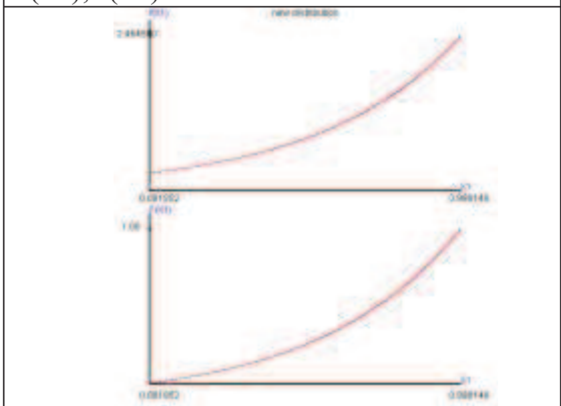
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.61202 Geometrical Mean : 0.50264 Harmonic Mean : 0.10813 Variance : 0.07590 S.D. : 0.27550 Skewed Coef. : -0.47632 Kurtosis Coef. : 2.11615 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.66102 Q1 : 0.40372 Q2 : 0.66102 Q3 : 0.85026 IQR : 0.44654 C.V. : 0.45014

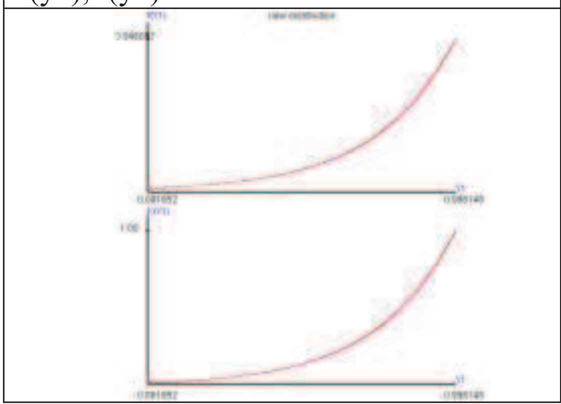
f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.71596 Geometrical Mean : 0.64435 Harmonic Mean : 0.24628 Variance : 0.05670 S.D. : 0.23813 Skewed Coef. : -0.97717 Kurtosis Coef. : 3.17194 MAD : 0.19335 Range : 1.00000 Mid_range : 0.50000 Median : 0.78205 Q1 : 0.57901 Q2 : 0.78205 Q3 : 0.90823 IQR : 0.32922 C.V. : 0.33260

(3-10) λ_1=0.9, λ_2=0.08, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

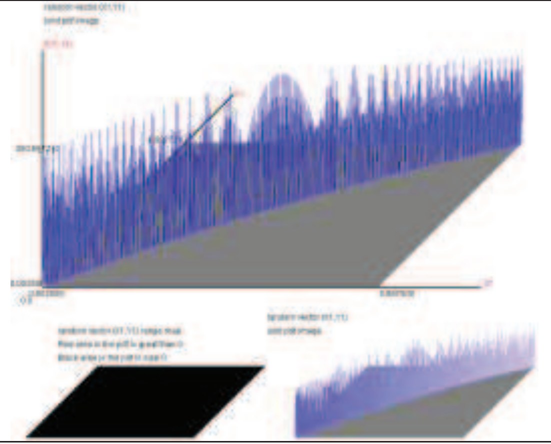
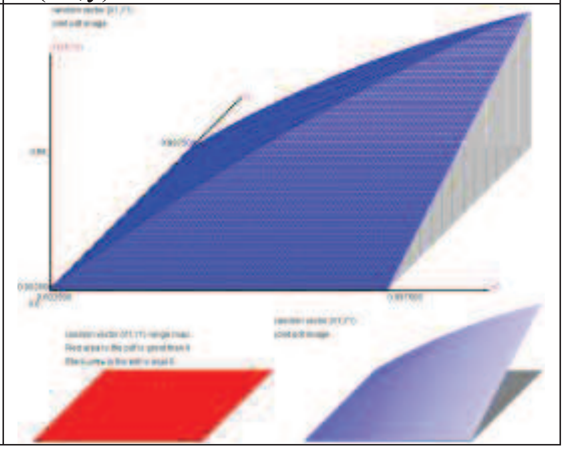
f(x1,y1)	F(x1,y)
	

E(X1)= 0.6699, Var(X1)= 0.0665, E(Y1)= 0.7639, Var(Y1)= 0.0448,
Cov(X1,Y1)= 0.0539, X1 and Y1 correlation coefficient=0.9883.

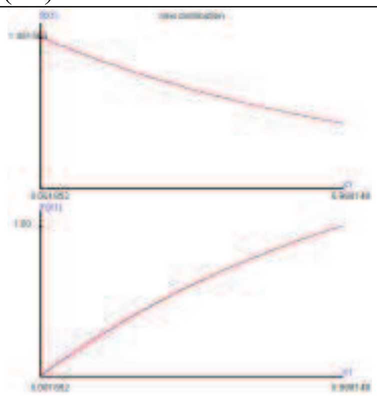
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.66991 Geometrical Mean : 0.58008 Harmonic Mean : 0.16713 Variance : 0.06651 S.D. : 0.25789 Skewed Coef. : -0.74423 Kurtosis Coef. : 2.58218 MAD : 0.21453 Range : 1.00000 Mid_range : 0.50000 Median : 0.73254 Q1 : 0.50004 Q2 : 0.73254 Q3 : 0.88566 IQR : 0.38561 C.V. : 0.38497

f(y1),F(y1)	Coefficient
	Mathematical Mean: 0.76390 Geometrical Mean : 0.71242 Harmonic Mean : 0.37967 Variance : 0.04476 S.D. : 0.21156 Skewed Coef. : -1.24410 Kurtosis Coef. : 4.08608 MAD : 0.16675 Range : 1.00000 Mid_range : 0.50000 Median : 0.82712 Q1 : 0.65909 Q2 : 0.82712 Q3 : 0.92784 IQR : 0.26875 C.V. : 0.27695

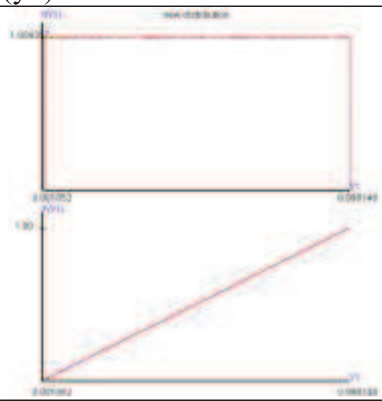
(3-11) λ_1=0.3, λ_2=0.2, $f_{X_1,Y_1}(x_1,y_1), f_{X_1}(x_1), f_{Y_1}(y_1)$,

f(x1,y1)	F(x1,y)
	

E(X1)= 0.4303, Var(X1)= 0.0804, E(Y1)= 0.5000, Var(Y1)= 0.0833,
Cov(X1,Y1)= 0.0814, X1 and Y1 correlation coefficient=0.9942.

f(x1),F(x1)	Coefficient
	

Coefficient	
Mathematical Mean:	0.43026
Geometrical Mean :	0.29538
Harmonic Mean :	0.03662
Variance :	0.08043
S.D. :	0.28360
Skewed Coef. :	0.29236
Kurtosis Coef. :	1.91845
MAD :	0.24393
Range :	1.00000
Mid_range :	0.50000
Median :	0.39718
Q1 :	0.18196
Q2 :	0.39718
Q3 :	0.66055
IQR :	0.47859
C.V. :	0.65913

f(y1),F(y1)	Coefficient
	

Coefficient	
Mathematical Mean:	0.50004
Geometrical Mean :	0.36790
Harmonic Mean :	0.05342
Variance :	0.08333
S.D. :	0.28868
Skewed Coef. :	-0.00022
Kurtosis Coef. :	1.80001
MAD :	0.25000
Range :	1.00000
Mid_range :	0.50000
Median :	0.50007
Q1 :	0.25003
Q2 :	0.50007
Q3 :	0.75006
IQR :	0.50003
C.V. :	0.57731

Section 4. How to analyze three categories' parameters

There are 3 categories, X_1 and Y_1 are continuous random variables,

λ_1	λ_2	$1-\lambda_1-\lambda_2$

$X_1 \sim CB(\lambda_1), Y_1 \sim CB(\lambda_1+\lambda_2), f_{X_1,Y_1}(x_1,y_1)=?$

The regression analysis can get the non-linear model Y1=b0+b1*H(X1),
the λ_2 is not 0 when rejected H0:b1=0.

The $\hat{\lambda}_1+\hat{\lambda}_2$ is from Y1 sample mean($\overline{Y}_1$) and $\hat{\lambda}_1$ is from X1($\overline{X}_1$), $\hat{\lambda}_2=\hat{\lambda}_1+\hat{\lambda}_2-\hat{\lambda}_1$ could be computed.

The simulated data is using $RND=F_{Y_1}(y_1;\lambda_1+\lambda_2)=F_{X_1}(x_1;\lambda_1), y_1 \ge x_1$ to get (X1,Y1) paired samples and X1<=Y1. The non-linear model Y1=b0+b1*H(X1) will be computed.

(1) λ_1=0.3, λ_2=0.2, $\lambda_1+\lambda_2$=0.5,
(i)paired sample size=100,
The part of paired samples,

X1	Y1
0.9607445715	0.9746347202
0.1203721277	0.1696841349
0.0282774687	0.0414307076
0.7939963964	0.8569704964
0.6754964044	0.7626501755
0.2037241222	0.2774423673
0.3246340487	0.4208308430
0.1207959625	0.1702515470
0.1740023265	0.2398877452
0.4780280820	0.5828281308
0.1232156947	0.1734870830
0.8437279387	0.8938186684

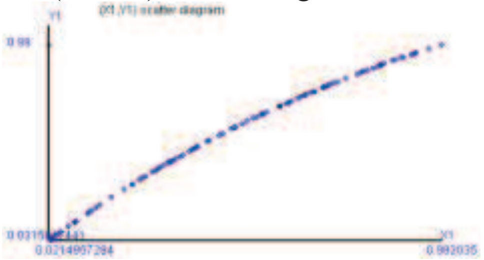
The analysis result,

Y1 estimated=1.5700653633+-1.5834087029*exp(-X1),
ANOVA

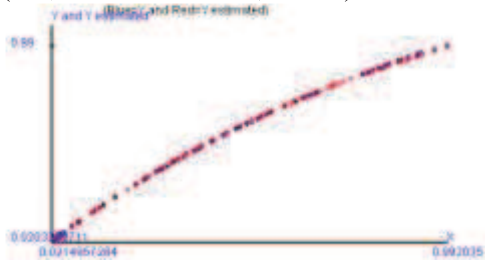
Source	df	SS	MS
Regression	1	7.5415492370	7.5415492370
Error	98	0.0027105346	0.0000276585
Total	99	7.5442597716	

H0:slope=0, test statistic=272666.441124 , p value=0.000000
R2=0.999641, R2(adj)=0.999637,MSE=0.000028,
H0:residual population~Gumbel(mu=-0.001817,sigma=0.004845)
chi square test statistic=12.140000, p value=0.016142

(X1,Y1) scatter diagram



(X1,R=Y estimated ,B=Y1) scatter diagram



H0: λ_1=0.3,

$\overline{X}_1$=0.4836358027, , n=100,

$\hat{\lambda}_1$=0.4508649848,

Z test=1.8822623277, p value=0.060064>0.05, failed to reject H0: λ_1=0.3.

H0: $\lambda_1+\lambda_2$=0.5,

$\overline{Y}_1$=0.5568830393, n=100,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.6653759430,

Z test=1.96828253548, p value=0.049180>0.04,

failed to reject H0: $\lambda_1+\lambda_2$=0.5 when significant level=0.04.

$\hat{\lambda}_2$=0.2145109582.

(ii)paired sample size=1,000,

Y1 estimated=1.5680768009+-1.5797169993*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	82.9220718863	82.9220718863
Error	998	0.0299611077	0.0000300211
Total	999	82.9520329939	

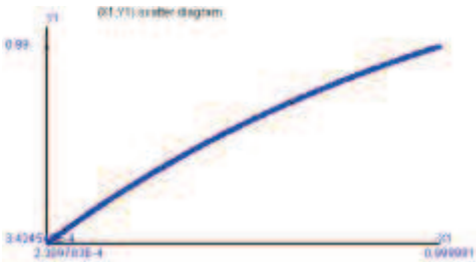
H0:slope=0, test statistic=2762121.770793 , p value=0.000000

R2=0.999639, R2(adj)=0.999638,MSE=0.000030,

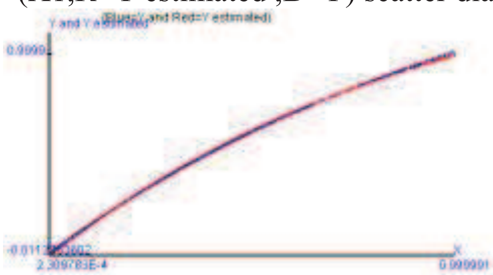
H0:residual population~Gumbel(mu=-0.001970,sigma=0.004448)

chi square test statistic=171.600000, p value=0.000000,

(X1,Y) scatter diagram



(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.3,

$\overline{X}_1$=0.4222394055, n=1000,

$\hat{\lambda}_1$=**0.2793598894**,

Z test=-0.8932517803, p value=0.371756>0.05, failed to reject H0: λ_1=0.3.

H0: $\lambda_1+\lambda_2$=0.5,

$\overline{Y}_1$=0.4916717847, n=1000,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.4748661796,

Z test=-0.9186616699, p value=0.358150>0.05,

failed to reject H0: $\lambda_1+\lambda_2$=0.5.

$\hat{\lambda}_2$=0.1955062902.

(iii)paired sample size=10,000,

Y1 estimated= 1.5675966143+-1.5791670133*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	v 817.9131296704	817.9131296704
Error	9998	0.3003792537	0.0000300439
Total	9999	818.2135089240	

H0:slope=0, test statistic=27223902.352315 , p value=0.000000

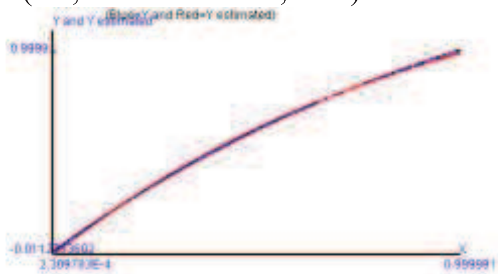
R2=0.999633, R2(adj)=0.999633,MSE=0.000030,

H0:residual population~Semi circle(mu=0.001586,R=0.009631)

chi square test statistic=3497.537200, p value=0.000000

(X1,Y) scatter diagram

(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.3,

$\overline{X}_1$=0.4333491513, n=10000,

$\hat{\lambda}_1$=0.3080661185,

Z test=1.0923987272, p value=0.275138>0.05, failed to reject H0: λ_1=0.3.

H0: $\lambda_1+\lambda_2$=0.5,

$\overline{Y}_1$=0.5038055535, n=10000,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.512534403,

Z test=1.29778394807, p value=0.194784>0.04,

failed to reject H0: $\lambda_1+\lambda_2$=0.5 when significant level=0.04.

$\hat{\lambda}_2$=0.204468284500.

(2) λ_1=0.5, λ_2=0.2,

(i)paired sample size=100,

Y1 estimated=1.5643036403+-1.5708371599*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	9.5842941585	9.5842941585
Error	98	0.0035574157	0.0000363002
Total	99	9.5878515742	

H0:slope=0, test statistic=264028.976445 , p value=0.000000

R2=0.999629, R2(adj)=0.999625,MSE=0.000036,

H0:residual population~Semi circle(mu=0.000761,R=0.012995)

chi square test statistic=3.600000, p value=0.462222

(X1,Y) scatter diagram

(X1,R=Y estimated ,B=Y) scatter diagram

H0: λ_1=0.5,

$\overline{X}_1$=0.4966609953, n=100,

$\hat{\lambda}_1$=0.4898199658,

Z test=-0.1176897237, p value=0.907334>0.05,

failed to reject H0: λ_1=0.5.

H0: $\lambda_1+\lambda_2$=0.7,

$\overline{Y}_1$=0.5599253663, , n=100,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.6736573090,

Z test=-0.3499402787, p value=0.726838>0.05,

failed to reject H0: $\lambda_1+\lambda_2$=0.7.

$\hat{\lambda}_2$=0.1838373432.

(ii)paired sample size=1000,

Y1 estimated=1.5598643966+-1.5663011884*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	80.5415912429	80.5415912429
Error	998	0.0356525667	0.0000357240
Total	999	80.5772438096	

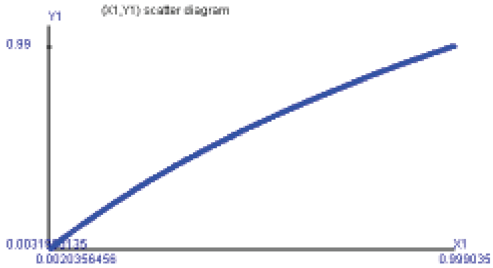
H0:slope=0, test statistic=2254550.390353 , p value=0.000000

R2=0.999558, R2(adj)=0.999557,MSE=0.000036,

H0:residual population~Gumbel(mu=-0.002167,sigma=0.005260)

chi square test statistic=126.700000, p value=0.000000

(X1,Y) scatter diagram



(X1,R=Y estimated ,B=Y) scatter diagram

H0: λ_1=0.5,

$\overline{X}_1$=0.4977575724, n=1000,

$\hat{\lambda}_1$=0.4931093294,

Z test=-0.2520538196, p value=0.801234>0.05, failed to reject H0: λ_1=0.5.

H0: $\lambda_1+\lambda_2$=0.7,

$\overline{Y}_1$=0.5675733998, n=100,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.6940930635,

Z test=-0.2538413030, p value=0.799874>0.05,

failed to reject H0: $\lambda_1+\lambda_2$=0.7.

$\hat{\lambda}_2$=0.200983734100.

(iii)paired sample size=10,000,

Y1 estimated=1.5600051956+-1.5665302976*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	802.7853778523	802.7853778523
Error	9998	0.3577325569	0.0000357804
Total	9999	803.1431104091	

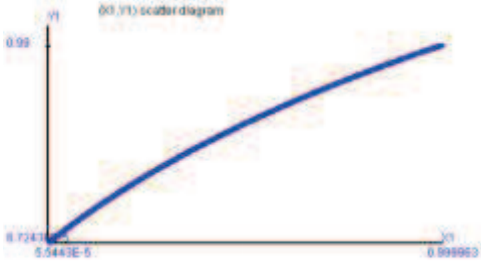
H0:slope=0, test statistic=22436448.831478 , p value=0.000000

R2=0.999555, R2(adj)=0.999555,MSE=0.000036,

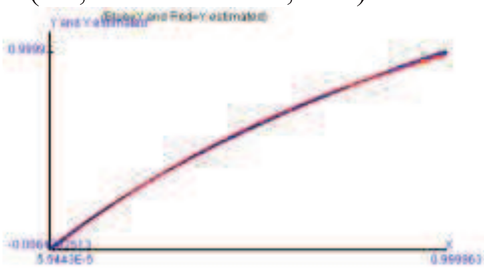
H0:residual population~Normal(mu=-0.000239,sigma*sigma=0.000036)

chi square test statistic=2877.572400, p value=0.000000

(X1,Y) scatter diagram



(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.5,

$\overline{X}_1$=0.4959252131, n=10000,

$\hat{\lambda}_1$=0.4876132409,

Z test=-1.4317579646, p value=0.152648>0.05, failed to reject H0: λ_1=0.5.

H0: $\lambda_1+\lambda_2$=0.7,

$\overline{Y}_1$=0.5659034879, n=10000,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.6896794360,

Z test=-1.3915253096, p value=0.164258>0.05,

failed to reject H0: $\lambda_1+\lambda_2$=0.7.

$\hat{\lambda}_2$=0.202066195100.

(3) λ_1=0.01, λ_2=0.98,

(i)paired sample size=100,

Y1 estimated=1.0572509484+0.1390947589*exp(-X1)*log(X1),

ANOVA

Source	df	SS	MS
Regression	1	5.0156265772	5.0156265772
Error	98	0.0915677673	0.0009343650
Total	99	5.1071943445	

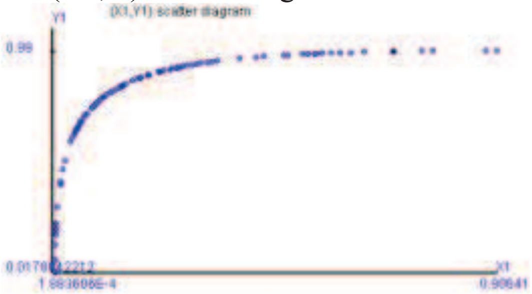
H0:slope=0, test statistic=5367.952271 , p value=0.000000

R2=0.982071, R2(adj)=0.981888,MSE=0.000934,

H0:residual population~Double exponential(lamda=47.630911,mu=0.009036)

chi square test statistic=30.340000, p value=0.000007

(X1,Y) scatter diagram



(X1,R=Y estimated ,B=Y) scatter diagram

H0: λ_1=0.01,

$\overline{X}_1$=0.2230699670, n=100,

$\hat{\lambda}_1$=0.0148193727,

Z test=0.8077556252, p value=0.413078>0.05, failed to reject H0: λ_1=0.01.

H0: $\lambda_1 + \lambda_2$=0.99,

$\overline{Y}_1$=0.7825839288, n=100,

$\hat{\lambda}_1 + \hat{\lambda}_2$=0.9870542189,

Z test=-0.5376696864, p value=0.591840>0.05,

failed to reject H0: $\lambda_1 + \lambda_2$=0.99.

$\hat{\lambda}_2$=0.9722348462.

(ii)paired sample size=1,000,

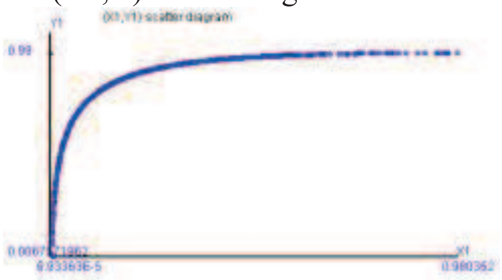
Y1 estimated=1.0645073391+0.1424359428*exp(-X1)*log(X1),

ANOVA

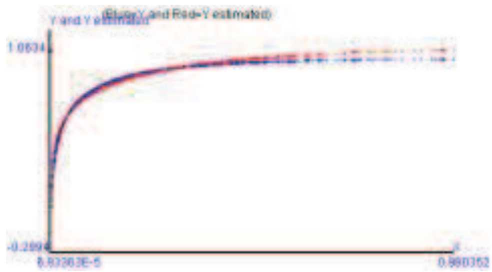
Source	df	SS	MS
Regression	1	32.4557462834	32.4557462834
Error	998	0.5966475550	0.0005978432
Total	999	33.0523938383	

H0:slope=0, test statistic=54288.054182 , p value=0.000000
R2=0.981948, R2(adj)=0.981930,MSE=0.000598,
H0:residual population~Normal(mu=0.000880,sigma*sigma=0.000525)
chi square test statistic=389.000000, p value=0.000000

(X1,Y) scatter diagram



(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.01,
$\overline{X}_1$=0.2098841803, n=1000,
$\hat{\lambda}_1$=0.0106635072,
Z test=0.3891665369, p value=0.696918>0.05, failed to reject H0: λ_1=0.01.

H0: $\lambda_1+\lambda_2$=0.99,
$\overline{Y}_1$=0.7995297839, n=1000,
$\hat{\lambda}_1+\hat{\lambda}_2$=0.9917362444,
Z test=1.0850764725, p value=0.278406>0.05,
failed to reject H0: $\lambda_1+\lambda_2$=0.99.

$\hat{\lambda}_2$=0.9810727374.

(iii)paired sample size=10,000,

Y1 estimated=1.0640708236+0.1421598888*exp(-X1)*log(X1),

ANOVA

Source	df	SS	MS
Regression	1	368.3220651463	368.3220651463
Error	9998	6.5814993966	0.0006582816
Total	9999	374.9035645430	

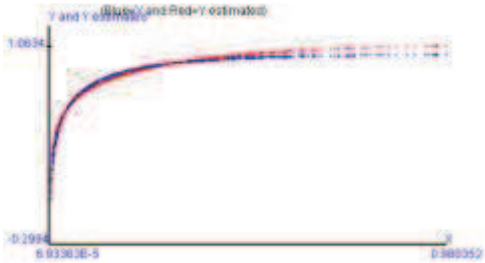
H0:slope=0, test statistic=559520.526464 , p value=0.000000

R2=0.982445, R2(adj)=0.982443,MSE=0.000658,

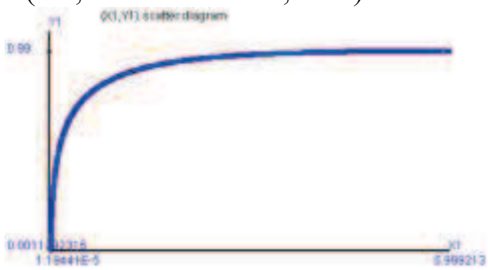
H0:residual population~Normal(mu=0.002463,sigma*sigma=0.000523)

chi square test statistic=5458.338000, p value=0.000000

(X1,Y) scatter diagram



(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.01,

$\overline{X}_1$=0.2047714215, n=10000,

$\hat{\lambda}_1$=0.0093036295,

Z test=-1.4242186620, p value=0.154708>0.05, failed to reject H0: λ_1=0.01.

H0: $\lambda_1+\lambda_2$=0.99,

$\overline{Y}_1$=0.7895459071, n=10000,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.989165227,

Z test=-1.7561206993, p value=0.079484>0.05,

failed to reject H0: $\lambda_1+\lambda_2$=0.99.

$\hat{\lambda}_2$=0.9798615975.

(4) λ_1=0.2, λ_2=0.2,

(i)paired sample size=100,

Y1 estimated=1.5846042984+-1.5899168911*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	8.7794388060	8.7794388060
Error	98	0.0003866507	0.0000039454
Total	99	8.7798254567	

H0:slope=0, test statistic=2225225.752166 , p value=0.000000

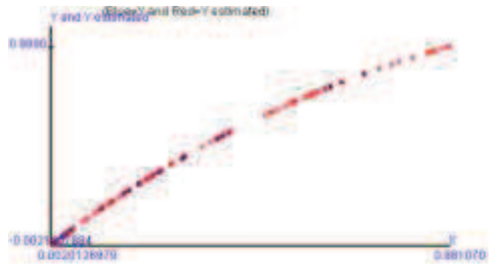
R2=0.999956, R2(adj)=0.999956,MSE=0.000004,

H0:residual population~Raised cosine(mu=-0.000000,s=0.005469)

chi square test statistic=78.080000, p value=0.000000

(X1,Y) scatter diagram

(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.2,

$\overline{X}_1$=0.3787710273, n=100,

$\hat{\lambda}_1$=0.1810004845,

Z test=-0.3289117389, p value=0.742840>0.05, failed to reject H0: λ_1=0.2.

H0: $\lambda_1+\lambda_2$=0.4,

$\overline{Y}_1$=0.4533066720, n=100,

$\hat{\lambda}_1+\hat{\lambda}_2$=0.3626232977,

Z test=-0.4601555264, p value=0.645778>0.05,

failed to reject H0: $\lambda_1+\lambda_2$=0.4.

$\hat{\lambda}_2$=0.1816228137.

H0: $\lambda_1=\lambda_2$,

1st step computing $\hat{\lambda}_1$=0.1810004845,

H0: $\lambda_1+\lambda_2=2\times\hat{\lambda}_1$=0.362000968,

Z test=0.0077258695, p value=0.993596 >0.05,

ailed to reject H0: $\lambda_1=\lambda_2$

(ii)paired sample size=10,000,

Y1 estimated=1.5848649614+-1.5902900415*exp(-X1),

ANOVA

Source	df	SS	MS
Regression	1	833.2451334080	833.2451334080
Error	9998	0.0360056758	0.0000036013
Total	9999	833.2811390838	

H0:slope=0, test statistic=231374211.476341 , p value=0.000000

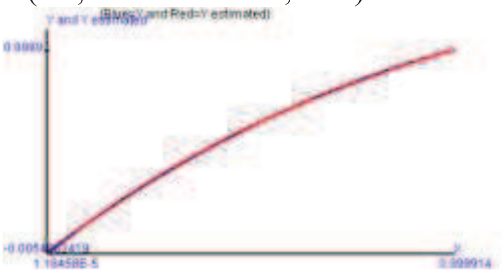
R2=0.999957, R2(adj)=0.999957,MSE=0.000004,

H0:residual population~Raised cosine(mu=0.000000,s=0.005251)

chi square test statistic=3069.865200, p value=0.000000,

(X1,Y) scatter diagram

(X1,R=Y estimated ,B=Y) scatter diagram



H0: λ_1=0.2,

$\overline{X}_1$=0.3921348247, n=10000,

$\hat{\lambda}_1$=0.2087112131,

Z test=1.5621369793, p value=0.118592>0.05, failed to reject H0: λ_1=0.2.

H0: $\lambda_1 + \lambda_2$=0.4,

$\overline{Y}_1$=0.4705344897, n=10000,

$\hat{\lambda}_1 + \hat{\lambda}_2$=0.4121634134,

Z test=1.3899724184, p value=0.164946>0.05,

failed to reject H0: $\lambda_1 + \lambda_2$=0.4.

$\hat{\lambda}_2$=0.2034522003.

H0: $\lambda_1 = \lambda_2$,

1st step computing $\hat{\lambda}_1$=0.2087112131,

H0: $\lambda_1 + \lambda_2 = 2 \times \hat{\lambda}_1$=0.417422426,

Z test=-0.6565827242, p value=0.511600>0.05,

ailed to reject H0: $\lambda_1 = \lambda_2$

Chapter 4 Bernoulli distribution and conditional Bernoulli distribution--- Model 2

Section 1. The joint probability density function and marginal probability density function

1. The marginal probability density function and conditional probability density function,

(1)X1 marginal probability density function,

$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, \frac{X_2}{1-x_1} \sim CB\left(\lambda^* = \frac{\lambda_2}{1-\lambda_1}\right), 0 \le x_2 \le 1-x_1,$$

$$f_{X_1}(x_1;\lambda_1) = C(\lambda_1)(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1}, 0 \le x_1 \le 1, 0 < \lambda_1 < 1,$$

$$C(\lambda_1) = \begin{cases} \dfrac{ln(1-\lambda_1) - ln(\lambda_1)}{1-2\lambda_1}, \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda_1 \ne \dfrac{1}{2} \end{cases},$$

$$E(X_1) = \begin{cases} \dfrac{\lambda_1}{2\lambda_1 - 1} + \dfrac{1}{2\,tan^{-1}(1-2\lambda_1)} & if\ \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2} & if\ \lambda_1 = \dfrac{1}{2} \end{cases}$$

$$Var(X_1) = \begin{cases} \dfrac{(1-\lambda_1)\lambda_1}{(1-2\lambda_1)^2} + \dfrac{1}{(2\,tan^{-1}(1-2\lambda_1))^2} & if\ \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{12} & if\ \lambda_1 = \dfrac{1}{2} \end{cases}$$

(2) Conditional probability density function

$$f_{X_2|x_1}(x_2|x_1) = C(\lambda^*)(\lambda^*)^{\frac{x_2}{1-x_1}}(1-\lambda^*)^{1-\frac{x_2}{1-x_1}}, 0 \le \frac{x_2}{1-x_1} \le 1, 0 < \lambda^* = \frac{\lambda_2}{1-\lambda_1} < 1,$$

$$C(\lambda^*) = \begin{cases} \dfrac{ln(1-\lambda^*) - ln(\lambda^*)}{1-2\lambda^*}, \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda^* \ne \dfrac{1}{2} \end{cases},$$

$$E(X_2|x_1) = \begin{cases} \left(\dfrac{\lambda^*}{2\lambda^* - 1} + \dfrac{1}{2\,tan^{-1}(1-2\lambda_1)}\right)(1-x_1) & if\ \lambda^* \ne \dfrac{1}{2} \\ \dfrac{1}{2}(1-x_1) & if\ \lambda^* = \dfrac{1}{2} \end{cases}$$

$$Var(X_2|x_1)=\begin{cases}\left(\frac{(1-\lambda^*)\lambda^*}{(1-2\lambda^*)^2}+\frac{1}{(2\,tan^{-1}(1-2\lambda^*))^2}\right)(1-x_1)^2 & if\lambda^*\neq\frac{1}{2}\\ \frac{1}{12}(1-x_1)^2 & if\ \lambda^*=\frac{1}{2}\end{cases}$$

(3)E(X2),Var(X2),
X_2 is not CB(λ_2).

$$E(X_2)=EE(X_2|x_1)$$

$$E(X_2)=\begin{cases}\left(\frac{\lambda^*}{2\lambda^*-1}+\frac{1}{2\,tan^{-1}(1-2\lambda_1)}\right)(1-E(X_1)) & if\ \lambda^*\neq\frac{1}{2}\\ \frac{1}{2}(1-E(X_1)) & if\ \lambda^*=\frac{1}{2}\end{cases}$$

$$Var(X_2)=EVar(X_2|x_1)+VaE(X_2|x_1)$$

$$E\left((1-X_1)^2\right)=E(X_1^2)-2E(X_1)+1=Var(X_1)+(E(X_1))^2-2E(X_1)+1,$$

$$EVar(X_2|x_1)=\begin{cases}\left(\frac{(1-\lambda^*)\lambda^*}{(1-2\lambda^*)^2}+\frac{1}{(2\,tan^{-1}(1-2\lambda^*))^2}\right)E\left((1-X_1)^2\right) & if\lambda^*\neq\frac{1}{2}\\ \frac{1}{12}E\left((1-X_1)^2\right) & if\ \lambda^*=\frac{1}{2}\end{cases}$$

$$Var(1-X_1)=Var(X_1),$$

$$VarE(X_2|x_1)=\begin{cases}\left(\frac{\lambda^*}{2\lambda^*-1}+\frac{1}{2\,tan^{-1}(1-2\lambda_1)}\right)Var(X_1) & if\ \lambda^*\neq\frac{1}{2}\\ \frac{1}{2}Var(X_1) & if\ \lambda^*=\frac{1}{2}\end{cases}$$

(4)The joint probability density function,

$$f_{X_2}(x_2;\lambda_2)f_{X_1|x_2}(x_1|x_2)=C(\lambda_2)(\lambda_2)^{x_2}(1-\lambda_2)^{1-x_2}C(\lambda^*)^{\frac{x_1}{1-x_2}}(1-\lambda^*)^{1-\frac{x_1}{1-x_2}}$$

$0\leq x_2\leq 1, 0\leq x_1\leq 1-x_2$,

$$E(X_1X_2)=E(X_1E(X_2|x_1))$$

$$E(X_1X_2)=\begin{cases}\left(\frac{\lambda^*}{2\lambda^*-1}+\frac{1}{2\,tan^{-1}(1-2\lambda_1)}\right)\left(E(X_1)-E(X_1^2)\right) & if\ \lambda^*\neq\frac{1}{2}\\ \frac{1}{2}E(X_1)-E(X_1^2) & if\ \lambda^*=\frac{1}{2}\end{cases}$$

$$Cov(X_1,X_2)=E(X_1X_2)-E(X_1)E(X_2),$$

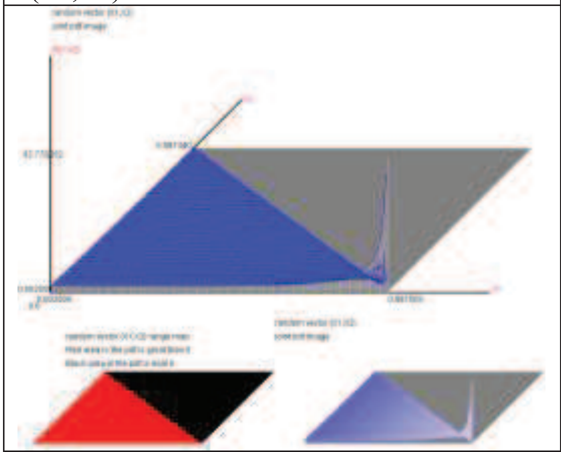
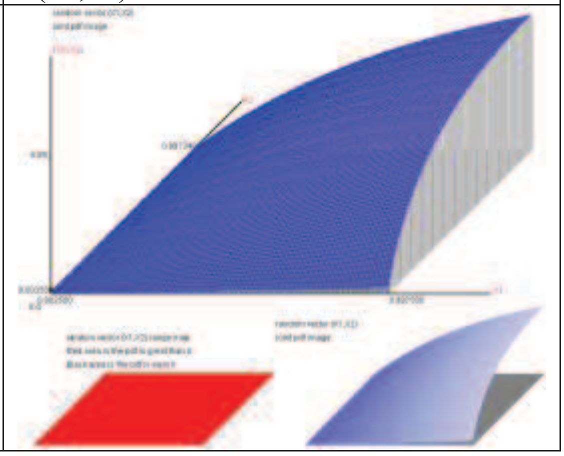
$$\rho(X_1,X_2)=\frac{Cov(X_1,X_2)}{\sqrt{Var(X_1)Var(X_2)}}.$$

Section 2. The joint probability density function image

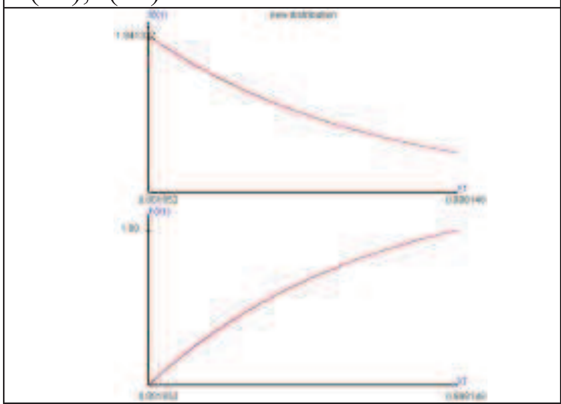
$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, \frac{X_2}{1-x_1} \sim CB\left(\lambda^* = \frac{\lambda_2}{1-\lambda_1}\right), 0 \le x_2 \le 1-x_1,$$

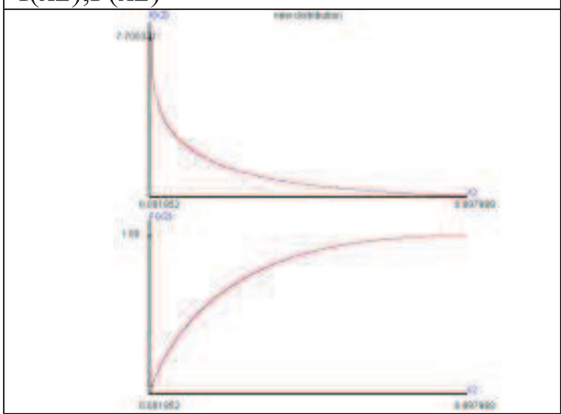
(1) 0.2,

(1-1) $\lambda_1 = 0.2, \quad \lambda_2 = 0.1,$

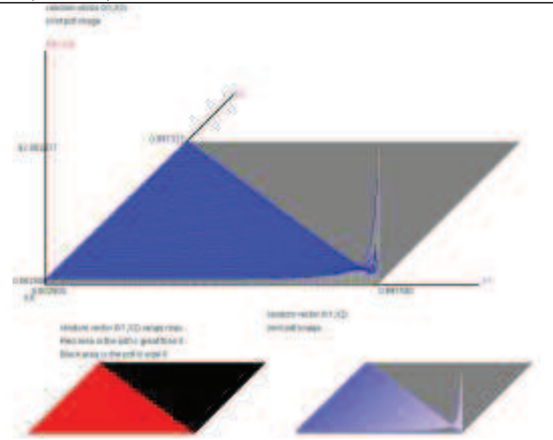
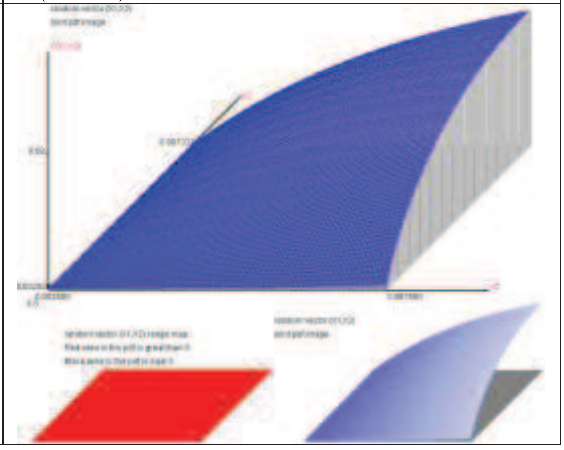
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3881, Var(X1)= 0.0759, E(X2)= 0.2125, Var(X2)= 0.0405, Cov(X1,X2)= -0.0264, X1 and X2 correlation coefficient=-0.4753.

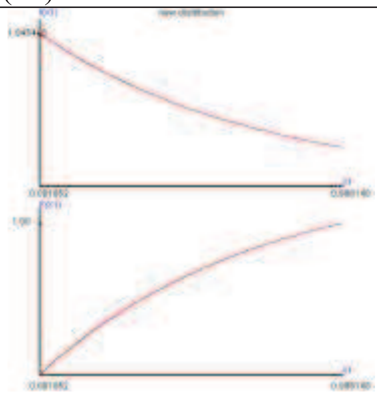
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38806 Geometrical Mean : 0.25582 Harmonic Mean : 0.02852 Variance : 0.07592 S.D. : 0.27554 Skewed Coef. : 0.47607 Kurtosis Coef. : 2.11580 MAD : 0.23447 Range : 1.00000 Mid_range : 0.50000 Median : 0.33907 Q1 : 0.14980 Q2 : 0.33907 Q3 : 0.59633 IQR : 0.44653 C.V. : 0.71003

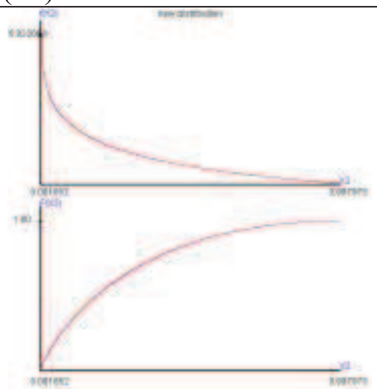
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.21251 Geometrical Mean : 0.11084 Harmonic Mean : 0.00239 Variance : 0.04052 S.D. : 0.20131 Skewed Coef. : 1.22119 Kurtosis Coef. : 3.92552 MAD : 0.16033 Range : 0.99984 Mid_range : 0.49992 Median : 0.14794 Q1 : 0.05345 Q2 : 0.14794 Q3 : 0.31631 IQR : 0.26286 C.V. : 0.94728

(1-2) $\lambda_1 = 0.2, \quad \lambda_2 = 0.2,$

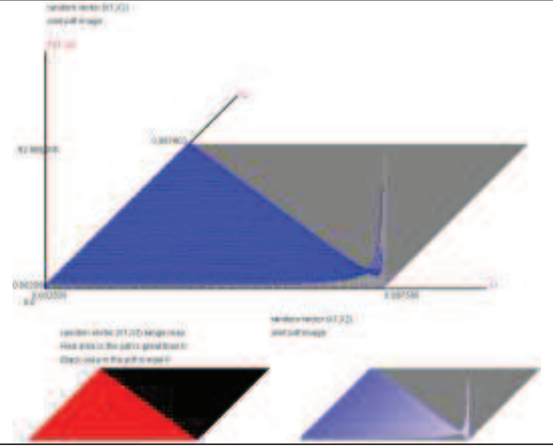
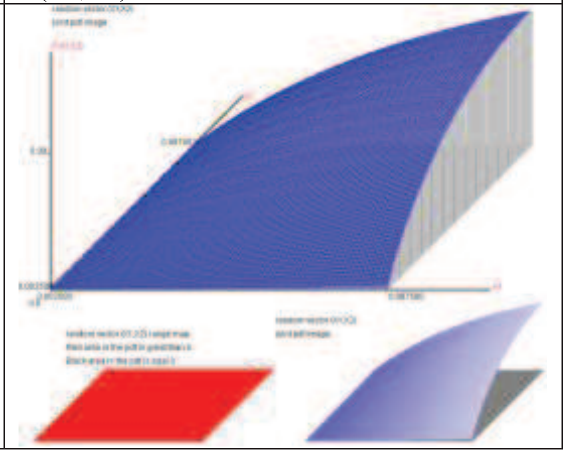
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.2511, Var(X2)= 0.0482, Cov(X1,X2)= -0.0311, X1 and X2 correlation coefficient=-0.5150.

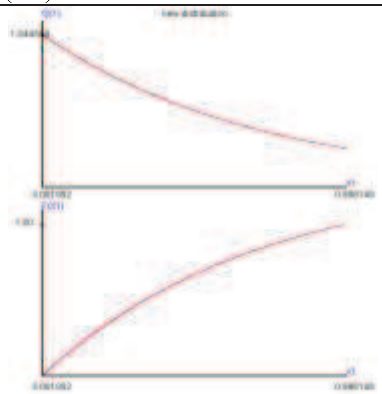
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38801 Geometrical Mean : 0.25580 Harmonic Mean : 0.02700 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : 0.47603 Kurtosis Coef. : 2.11583 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.33905 Q1 : 0.14980 Q2 : 0.33905 Q3 : 0.59629 IQR : 0.44649 C.V. : 0.71000

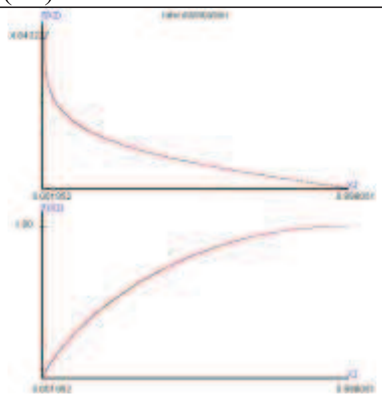
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.25110 Geometrical Mean : 0.13888 Harmonic Mean : 0.00569 Variance : 0.04816 S.D. : 0.21945 Skewed Coef. : 0.97658 Kurtosis Coef. : 3.17705 MAD : 0.17924 Range : 0.99983 Mid_range : 0.49991 Median : 0.18856 Q1 : 0.07052 Q2 : 0.18856 Q3 : 0.38184 IQR : 0.31132 C.V. : 0.87396

(1-3) $\lambda_1 = 0.2$, $\lambda_2 = 0.3$,

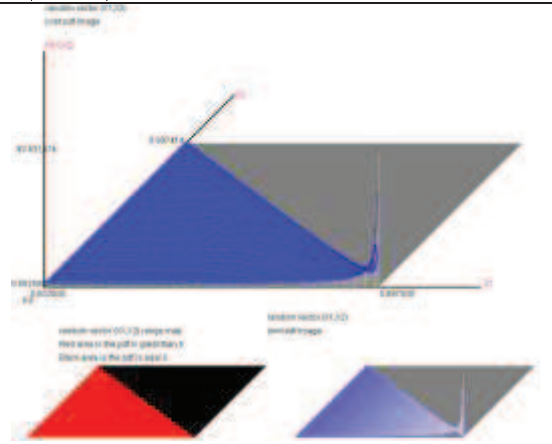
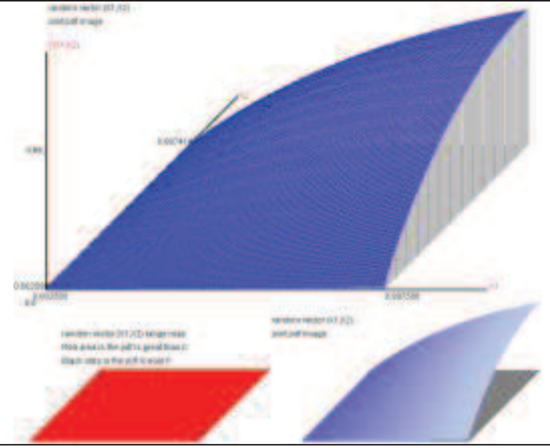
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3881, Var(X1)= 0.0759, E(X2)= 0.2801, Var(X2)= 0.0530, Cov(X1,X2)= -0.0347, X1 and X2 correlation coefficient=-0.5480.

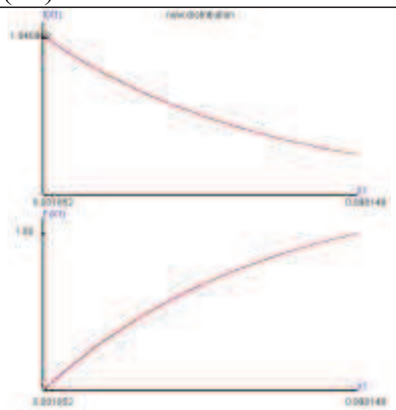
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.38806
	Geometrical Mean :	0.25580
	Harmonic Mean :	0.02828
	Variance :	0.07592
	S.D. :	0.27554
	Skewed Coef. :	0.47607
	Kurtosis Coef. :	2.11571
	MAD :	0.23448
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.33905
	Q1 :	0.14980
	Q2 :	0.33905
	Q3 :	0.59637
	IQR :	0.44657
	C.V. :	0.71005

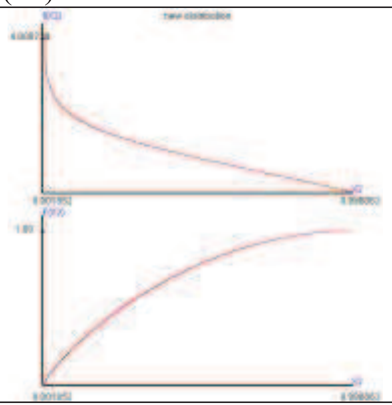
f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.28008
	Geometrical Mean :	0.16221
	Harmonic Mean :	0.00419
	Variance :	0.05296
	S.D. :	0.23012
	Skewed Coef. :	0.81285
	Kurtosis Coef. :	2.78196
	MAD :	0.19072
	Range :	0.99990
	Mid_range :	0.49995
	Median :	0.22256
	Q1 :	0.08617
	Q2 :	0.22256
	Q3 :	0.42956
	IQR :	0.34338
	C.V. :	0.82164

(1-4) $\lambda_1 = 0.2, \quad \lambda_2 = 0.4,$

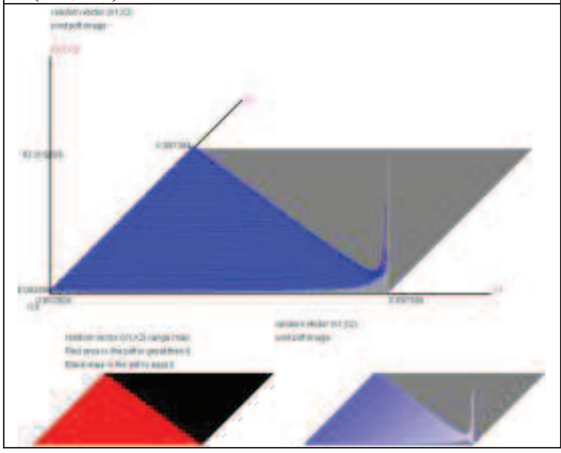
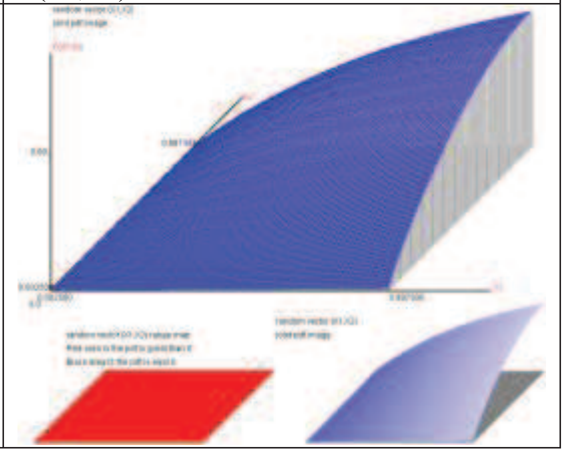
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3881, Var(X1)= 0.0759, E(X2)= 0.3060, Var(X2)= 0.0565,
Cov(X1,X2)= -0.0379, X1 and X2 correlation coefficient=-0.5794.

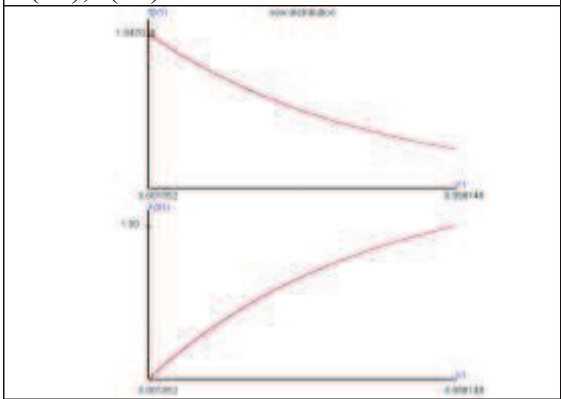
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25585 Harmonic Mean : 0.03196 Variance : 0.07590 S.D. : 0.27550 Skewed Coef. : 0.47582 Kurtosis Coef. : 2.11564 MAD : 0.23444 Range : 1.00000 Mid_range : 0.50000 Median : 0.33913 Q1 : 0.14981 Q2 : 0.33913 Q3 : 0.59633 IQR : 0.44652 C.V. : 0.70995

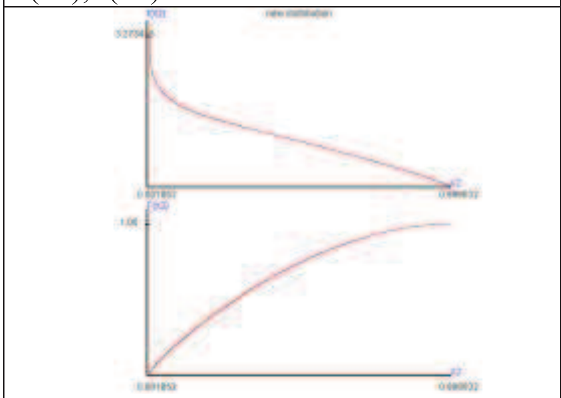
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.30596 Geometrical Mean : 0.18489 Harmonic Mean : 0.00858 Variance : 0.05650 S.D. : 0.23769 Skewed Coef. : 0.67939 Kurtosis Coef. : 2.52106 MAD : 0.19894 Range : 0.99991 Mid_range : 0.49996 Median : 0.25501 Q1 : 0.10271 Q2 : 0.25501 Q3 : 0.46986 IQR : 0.36715 C.V. : 0.77689

(1-5) $\lambda_1 = 0.2, \quad \lambda_2 = 0.5,$

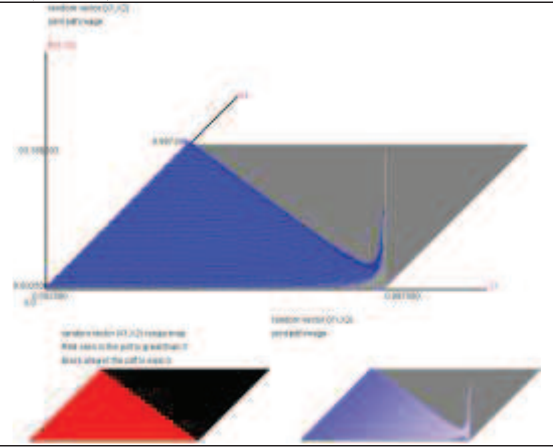
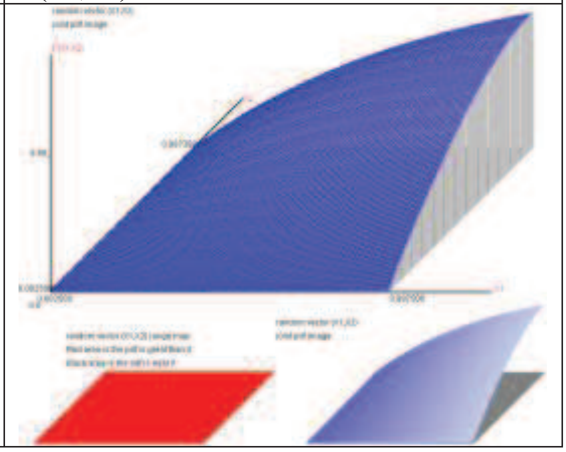
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.3320, Var(X2)= 0.0594, Cov(X1,X2)= -0.0412, X1 and X2 correlation coefficient=-0.6132.

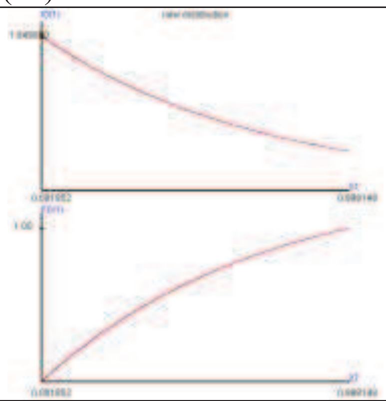
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.38803
	Geometrical Mean :	0.25580
	Harmonic Mean :	0.02888
	Variance :	0.07590
	S.D. :	0.27550
	Skewed Coef. :	0.47593
	Kurtosis Coef. :	2.11560
	MAD :	0.23445
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.33906
	Q1 :	0.14981
	Q2 :	0.33906
	Q3 :	0.59636
	IQR :	0.44656
	C.V. :	0.71000

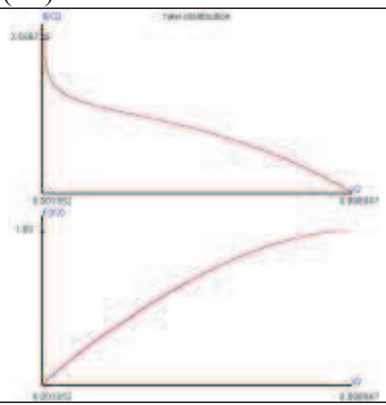
f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.33196
	Geometrical Mean :	0.20928
	Harmonic Mean :	0.01100
	Variance :	0.05939
	S.D. :	0.24369
	Skewed Coef. :	0.55426
	Kurtosis Coef. :	2.32337
	MAD :	0.20548
	Range :	0.99988
	Mid_range :	0.49994
	Median :	0.28902
	Q1 :	0.12184
	Q2 :	0.28902
	Q3 :	0.50816
	IQR :	0.38632
	C.V. :	0.73410

(1-6) $\lambda_1 = 0.2$, $\lambda_2 = 0.6$,

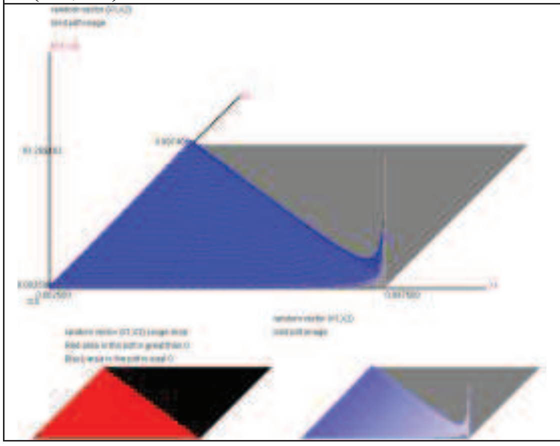
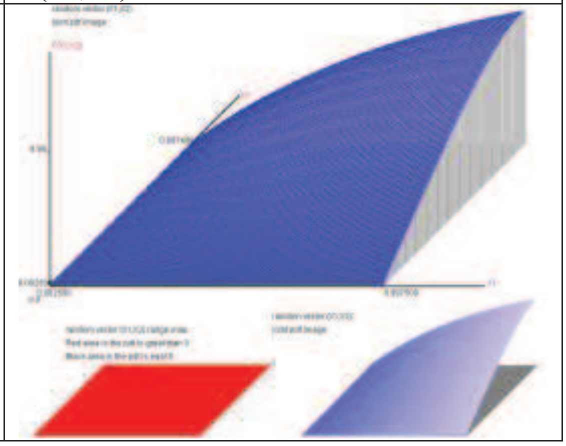
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3881, Var(X1)= 0.0759, E(X2)= 0.3609, Var(X2)= 0.0618, Cov(X1,X2)= -0.0448, X1 and X2 correlation coefficient=-0.6538.

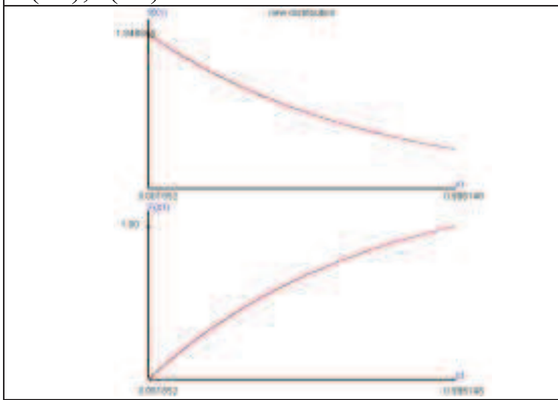
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.38811
	Geometrical Mean :	0.25586
	Harmonic Mean :	0.03088
	Variance :	0.07592
	S.D. :	0.27554
	Skewed Coef. :	0.47564
	Kurtosis Coef. :	2.11507
	MAD :	0.23448
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.33914
	Q1 :	0.14982
	Q2 :	0.33914
	Q3 :	0.59653
	IQR :	0.44671
	C.V. :	0.70994

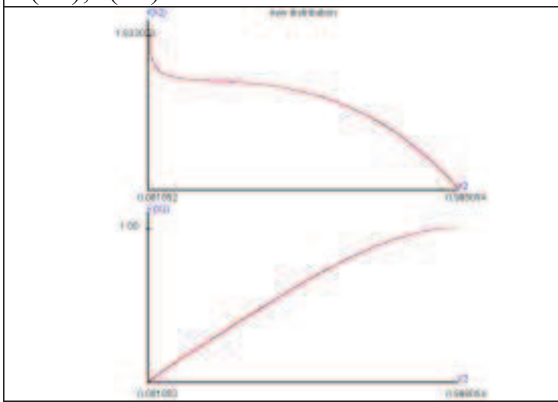
f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.36090
	Geometrical Mean :	0.23828
	Harmonic Mean :	0.01256
	Variance :	0.06178
	S.D. :	0.24856
	Skewed Coef. :	0.42337
	Kurtosis Coef. :	2.16510
	MAD :	0.21076
	Range :	0.99990
	Mid_range :	0.49995
	Median :	0.32798
	Q1 :	0.14617
	Q2 :	0.32798
	Q3 :	0.54790
	IQR :	0.40173
	C.V. :	0.68872

(1-7) $\lambda_1 = 0.2, \quad \lambda_2 = 0.7,$

f(x1,x2)	F(x1,x2)
	

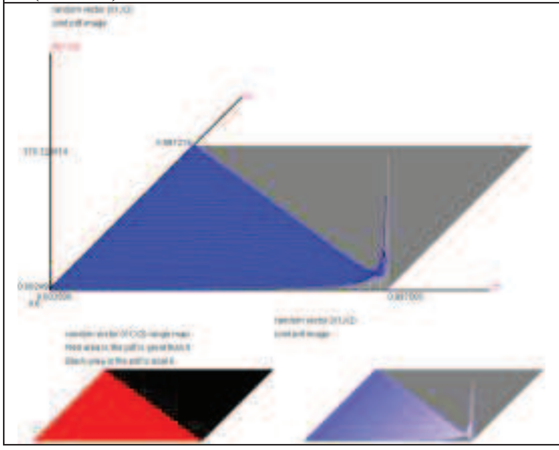
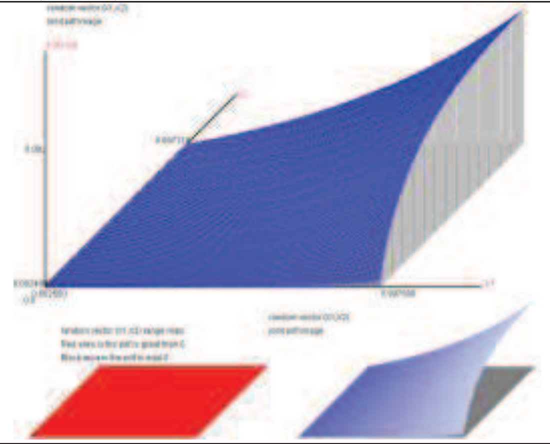
E(X1)= 0.3881, Var(X1)= 0.0759, E(X2)= 0.3995, Var(X2)= 0.0637, Cov(X1,X2)= -0.0496, X1 and X2 correlation coefficient=-0.7125.

f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38809 Geometrical Mean : 0.25584 Harmonic Mean : 0.02726 Variance : 0.07591 S.D. : 0.27553 Skewed Coef. : 0.47582 Kurtosis Coef. : 2.11539 MAD : 0.23447 Range : 1.00000 Mid_range : 0.50000 Median : 0.33911 Q1 : 0.14981 Q2 : 0.33911 Q3 : 0.59643 IQR : 0.44662 C.V. : 0.70996

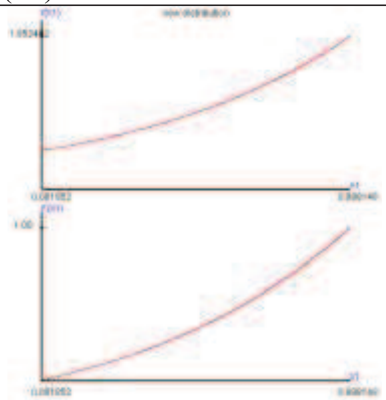
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.39947 Geometrical Mean : 0.27977 Harmonic Mean : 0.01745 Variance : 0.06372 S.D. : 0.25243 Skewed Coef. : 0.25994 Kurtosis Coef. : 2.03479 MAD : 0.21486 Range : 0.99991 Mid_range : 0.49995 Median : 0.38044 Q1 : 0.18350 Q2 : 0.38044 Q3 : 0.59650 IQR : 0.41301 C.V. : 0.63192

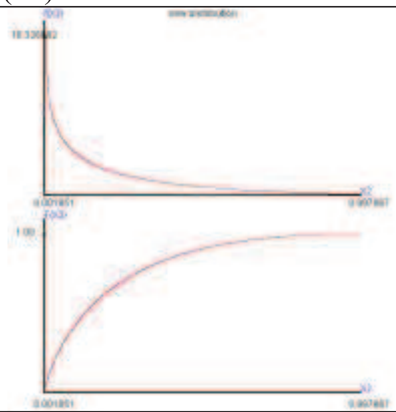
(2) $\lambda_1 = 0.8$,

(2-1) $\lambda_1 = 0.8$, $\lambda_2 = 0.1$,

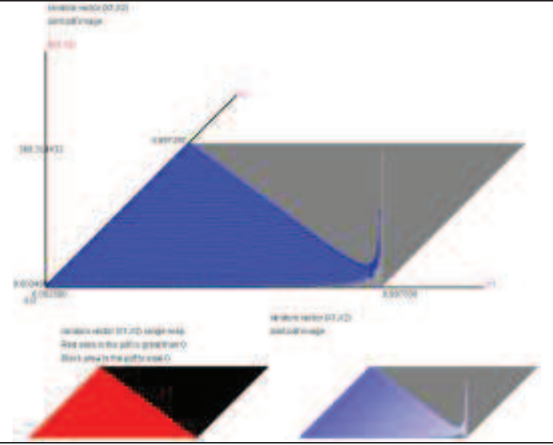
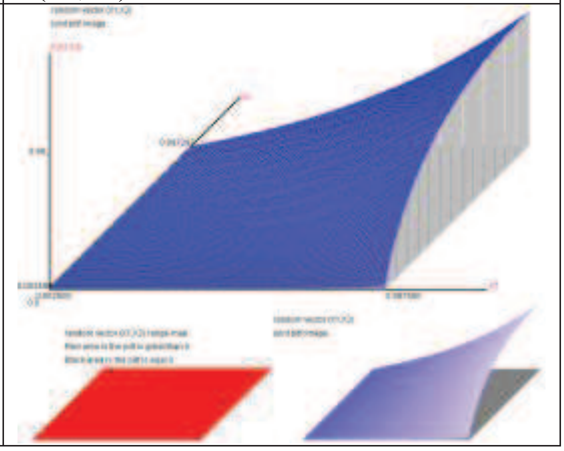
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.6121, Var(X1)= 0.0759, E(X2)= 0.1940, Var(X2)= 0.0379, Cov(X1,X2)= -0.0380, X1 and X2 correlation coefficient=-0.7082.

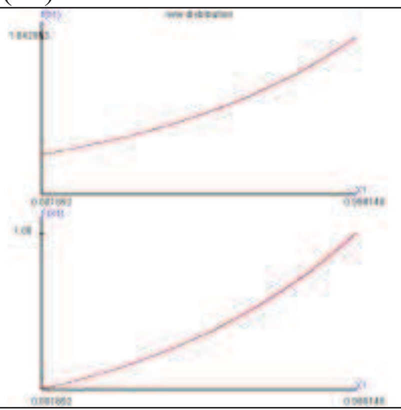
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.61210 Geometrical Mean : 0.50273 Harmonic Mean : 0.10001 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : -0.47646 Kurtosis Coef. : 2.11635 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.66110 Q1 : 0.40374 Q2 : 0.66110 Q3 : 0.85033 IQR : 0.44659 C.V. : 0.45007

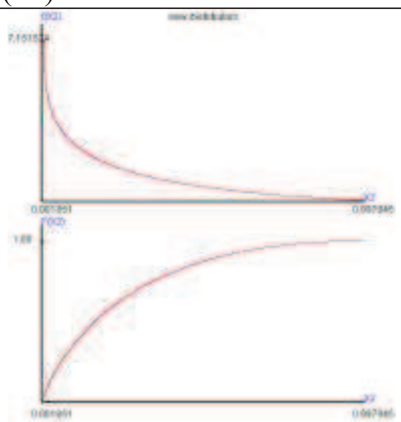
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.19397 Geometrical Mean : 0.09405 Harmonic Mean : 0.00162 Variance : 0.03785 S.D. : 0.19455 Skewed Coef. : 1.33877 Kurtosis Coef. : 4.30771 MAD : 0.15334 Range : 0.99972 Mid_range : 0.49986 Median : 0.12732 Q1 : 0.04279 Q2 : 0.12732 Q3 : 0.28766 IQR : 0.24487 C.V. : 1.00300

(2-2) $\lambda_1 = 0.8, \quad \lambda_2 = 0.15,$

f(x1,x2)	F(x1,x2)
	

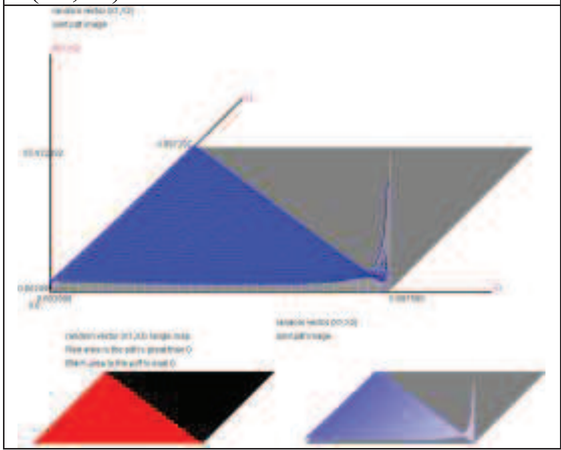
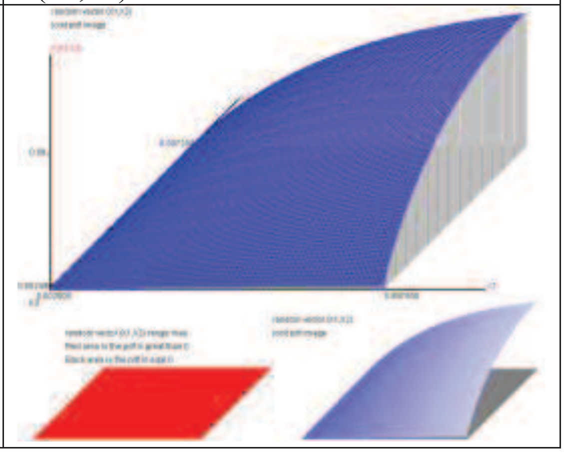
E(X1)= 0.6120, Var(X1)= 0.0759, E(X2)= 0.2289, Var(X2)= 0.0442,
Cov(X1,X2)= -0.0448, X1 and X2 correlation coefficient=-0.7729.

f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.61199 Geometrical Mean : 0.50262 Harmonic Mean : 0.09955 Variance : 0.07590 S.D. : 0.27550 Skewed Coef. : -0.47614 Kurtosis Coef. : 2.11588 MAD : 0.23444 Range : 1.00000 Mid_range : 0.50000 Median : 0.66098 Q1 : 0.40371 Q2 : 0.66098 Q3 : 0.85020 IQR : 0.44649 C.V. : 0.45017

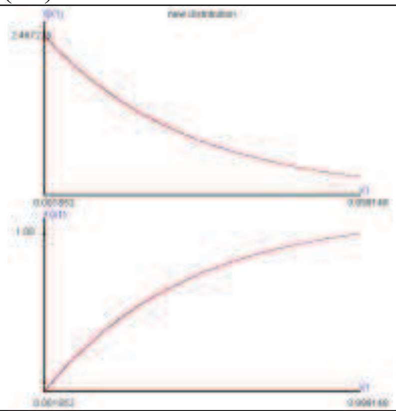
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.22885 Geometrical Mean : 0.12132 Harmonic Mean : 0.00419 Variance : 0.04419 S.D. : 0.21022 Skewed Coef. : 1.10978 Kurtosis Coef. : 3.55361 MAD : 0.16950 Range : 0.99980 Mid_range : 0.49990 Median : 0.16405 Q1 : 0.05937 Q2 : 0.16405 Q3 : 0.34496 IQR : 0.28560 C.V. : 0.91858

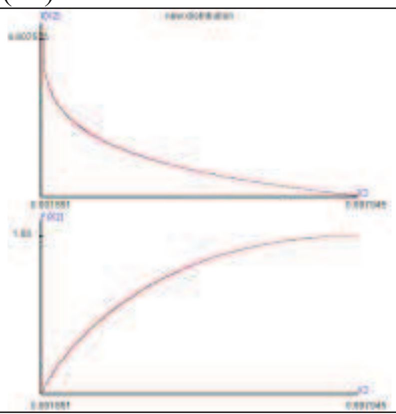
(3) $\lambda_1 = 0.1$,

(3-1) $\lambda_1 = 0.1$, $\lambda_2 = 0.2$,

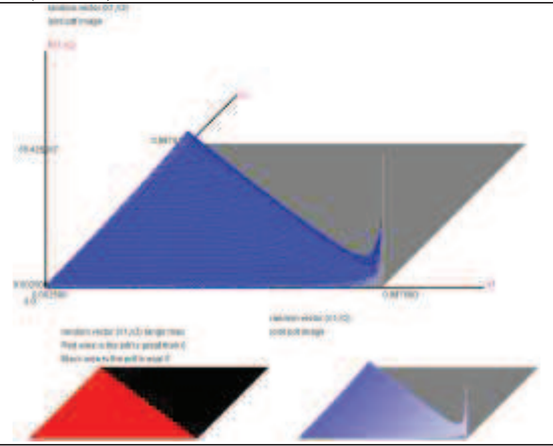
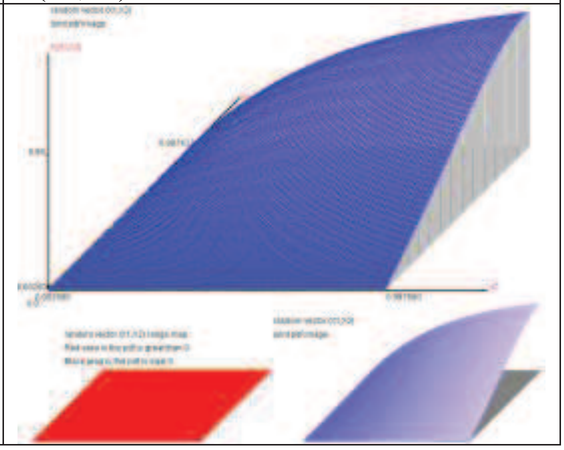
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3302, Var(X1)= 0.0665, E(X2)= 0.2668, Var(X2)= 0.0503,
Cov(X1,X2)= -0.0265, X1 and X2 correlation coefficient=-0.4580.

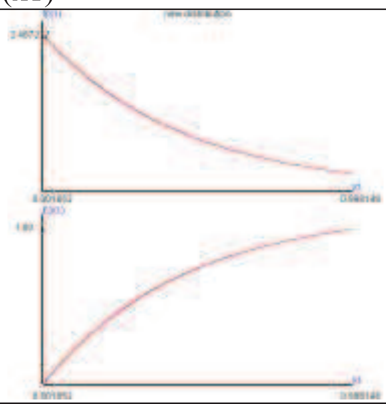
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33020 Geometrical Mean : 0.20663 Harmonic Mean : 0.02272 Variance : 0.06654 S.D. : 0.25795 Skewed Coef. : 0.74365 Kurtosis Coef. : 2.58080 MAD : 0.21459 Range : 1.00000 Mid_range : 0.50000 Median : 0.26756 Q1 : 0.11440 Q2 : 0.26756 Q3 : 0.50016 IQR : 0.38576 C.V. : 0.78120

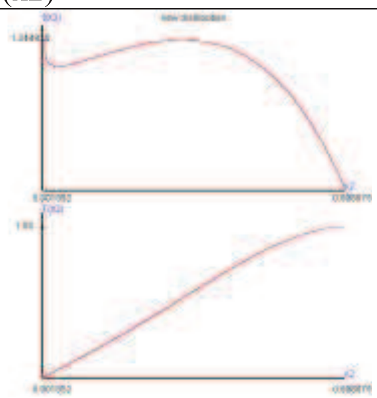
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.26680 Geometrical Mean : 0.15377 Harmonic Mean : 0.00584 Variance : 0.05033 S.D. : 0.22434 Skewed Coef. : 0.89733 Kurtosis Coef. : 2.98416 MAD : 0.18438 Range : 0.99980 Mid_range : 0.49990 Median : 0.20688 Q1 : 0.08068 Q2 : 0.20688 Q3 : 0.40579 IQR : 0.32512 C.V. : 0.84088

(3-2) $\lambda_1 = 0.1, \quad \lambda_2 = 0.8,$

f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3302, Var(X1)= 0.0665, E(X2)= 0.4434, Var(X2)= 0.0642, Cov(X1,X2)= -0.0440, X1 and X2 correlation coefficient=-0.6740.

f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33020 Geometrical Mean : 0.20663 Harmonic Mean : 0.02272 Variance : 0.06654 S.D. : 0.25795 Skewed Coef. : 0.74365 Kurtosis Coef. : 2.58080 MAD : 0.21459 Range : 1.00000 Mid_range : 0.50000 Median : 0.26756 Q1 : 0.11440 Q2 : 0.26756 Q3 : 0.50016 IQR : 0.38576 C.V. : 0.78120

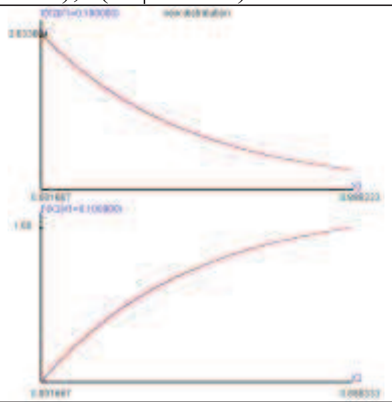
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.44342 Geometrical Mean : 0.33013 Harmonic Mean : 0.02829 Variance : 0.06419 S.D. : 0.25337 Skewed Coef. : 0.08203 Kurtosis Coef. : 1.97997 MAD : 0.21562 Range : 0.99993 Mid_range : 0.49996 Median : 0.44002 Q1 : 0.23255 Q2 : 0.44002 Q3 : 0.64605 IQR : 0.41349 C.V. : 0.57139

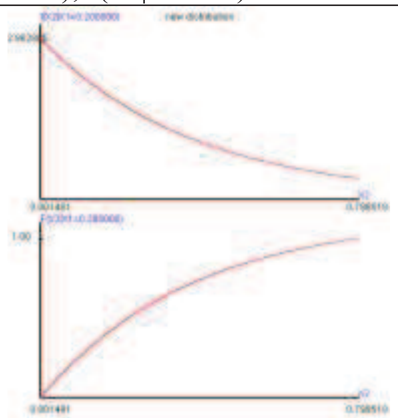
Section 3. The conditional probability density function

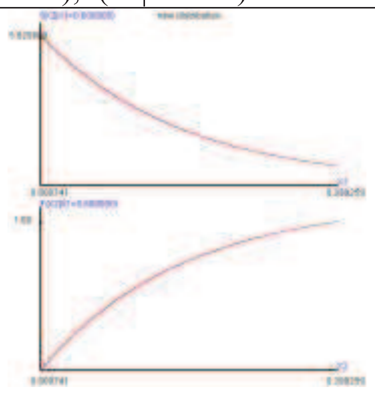
$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, \dfrac{X_2}{1-x_1}|x_1 \sim CB\left(\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}\right), 0 \le x_2 \le 1-x_1$, f(x2|x1)=?

(1) $\lambda_1 = 0.1$,

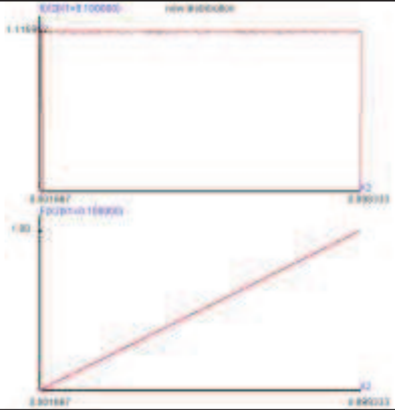
(1-1) $\lambda_2 = 0.1, \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{1}{9}$,

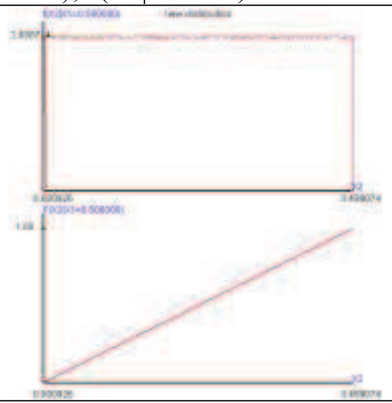
f(x2|x1=0.1),F(x2|x1=0.1)	Coefficient
	Mathematical Mean: 0.30427 Geometrical Mean : 0.19172 Harmonic Mean : 0.02077 Variance : 0.05508 S.D. : 0.23469 Skewed Coef. : 0.70561 Kurtosis Coef. : 2.50158 MAD : 0.19595 Range : 0.90000 Mid_range : 0.45000 Median : 0.24907 Q1 : 0.10686 Q2 : 0.24907 Q3 : 0.46224 IQR : 0.35538 C.V. : 0.77133

f(x2|x1=0.2),F(x2|x1=0.2)	Coefficient
	Mathematical Mean: 0.27046 Geometrical Mean : 0.17042 Harmonic Mean : 0.01846 Variance : 0.04352 S.D. : 0.20862 Skewed Coef. : 0.70561 Kurtosis Coef. : 2.50158 MAD : 0.17418 Range : 0.80000 Mid_range : 0.40000 Median : 0.22140 Q1 : 0.09499 Q2 : 0.22140 Q3 : 0.41088 IQR : 0.31589 C.V. : 0.77133

f(x2|x1=0.6),F(x2|x1=0.6)	Coefficient
	Mathematical Mean: 0.13523 Geometrical Mean : 0.08521 Harmonic Mean : 0.00923 Variance : 0.01088 S.D. : 0.10431 Skewed Coef. : 0.70561 Kurtosis Coef. : 2.50158 MAD : 0.08709 Range : 0.40000 Mid_range : 0.20000 Median : 0.11070 Q1 : 0.04749 Q2 : 0.11070 Q3 : 0.20544 IQR : 0.15795 C.V. : 0.77133

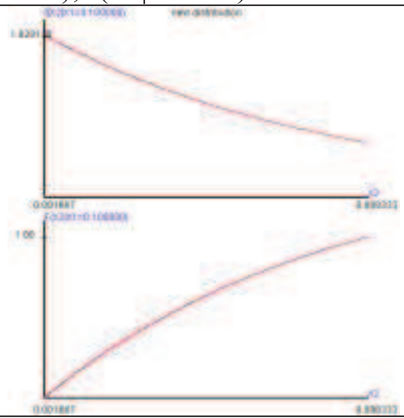
(1-2) $\lambda_2 = 0.45, \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{1}{2}$,

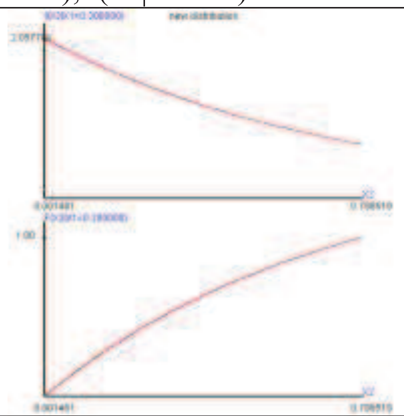
f(x2|x1=0.1),F(x2|x1=0.1)	Coefficient
	Mathematical Mean: 0.45004 Geometrical Mean : 0.33111 Harmonic Mean : 0.04807 Variance : 0.06750 S.D. : 0.25981 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.22500 Range : 0.90000 Mid_range : 0.45000 Median : 0.45006 Q1 : 0.22503 Q2 : 0.45006 Q3 : 0.67506 IQR : 0.45003 C.V. : 0.57731

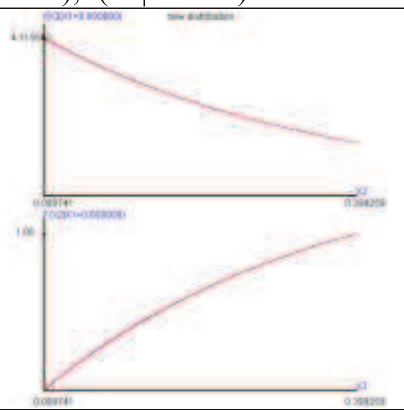
f(x2|x1=0.5),F(x2|x1=0.5)	Coefficient
	Mathematical Mean: 0.25002 Geometrical Mean : 0.18395 Harmonic Mean : 0.02671 Variance : 0.02083 S.D. : 0.14434 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.12500 Range : 0.50000 Mid_range : 0.25000 Median : 0.25004 Q1 : 0.12502 Q2 : 0.25004 Q3 : 0.37503 IQR : 0.25002 C.V. : 0.57731

(2) $\lambda_1 = 0.8$,

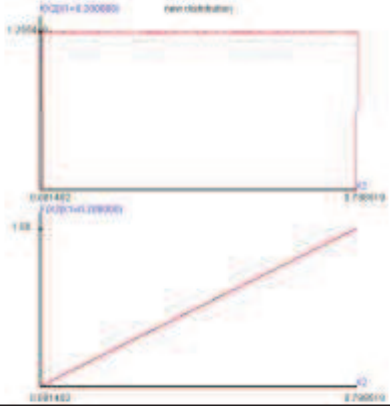
(2-1) $\lambda_2 = 0.05, \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{0.05}{2} = 0.4$,

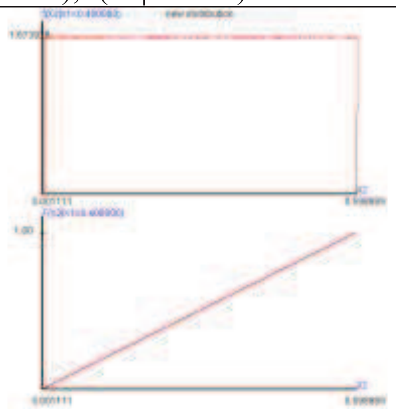
$f(x2 \mid x1=0.1),F(x2 \mid x1=0.1)$	Coefficient	
	Mathematical Mean:	0.36925
	Geometrical Mean :	0.24863
	Harmonic Mean :	0.02974
	Variance :	0.06362
	S.D. :	0.25222
	Skewed Coef. :	0.37830
	Kurtosis Coef. :	1.99877
	MAD :	0.21598
	Range :	0.90000
	Mid_range :	0.45000
	Median :	0.33222
	Q1 :	0.14938
	Q2 :	0.33222
	Q3 :	0.56791
	IQR :	0.41852
	C.V. :	0.68306

$f(x2 \mid x1=0.2),F(x2 \mid x1=0.2)$	Coefficient	
	Mathematical Mean:	0.32822
	Geometrical Mean :	0.22100
	Harmonic Mean :	0.02644
	Variance :	0.05026
	S.D. :	0.22420
	Skewed Coef. :	0.37830
	Kurtosis Coef. :	1.99877
	MAD :	0.19198
	Range :	0.80000
	Mid_range :	0.40000
	Median :	0.29531
	Q1 :	0.13278
	Q2 :	0.29531
	Q3 :	0.50481
	IQR :	0.37202
	C.V. :	0.68306

$f(x2 \mid x1=0.6),F(x2 \mid x1=0.6)$	Coefficient	
	Mathematical Mean:	0.16411
	Geometrical Mean :	0.11050
	Harmonic Mean :	0.01322
	Variance :	0.01257
	S.D. :	0.11210
	Skewed Coef. :	0.37830
	Kurtosis Coef. :	1.99877
	MAD :	0.09599
	Range :	0.40000
	Mid_range :	0.20000
	Median :	0.14765
	Q1 :	0.06639
	Q2 :	0.14765
	Q3 :	0.25240
	IQR :	0.18601
	C.V. :	0.68306

(2-2) $\lambda_2 = 0.1,\ \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{1}{2} = 0.5$,

f(x2|x1=0.2),F(x2|x1=0.2)	Coefficient
	Mathematical Mean: 0.40003 Geometrical Mean : 0.29432 Harmonic Mean : 0.04273 Variance : 0.05333 S.D. : 0.23094 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.20000 Range : 0.80000 Mid_range : 0.40000 Median : 0.40006 Q1 : 0.20003 Q2 : 0.40006 Q3 : 0.60005 IQR : 0.40002 C.V. : 0.57731

f(x2|x1=0.4),F(x2|x1=0.4)	Coefficient
	Mathematical Mean: 0.30002 Geometrical Mean : 0.22074 Harmonic Mean : 0.03205 Variance : 0.03000 S.D. : 0.17321 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.15000 Range : 0.60000 Mid_range : 0.30000 Median : 0.30004 Q1 : 0.15002 Q2 : 0.30004 Q3 : 0.45004 IQR : 0.30002 C.V. : 0.57731

Section 4. Statistical analysis of conditional Continuous Bernoulli

The statistical analysis method is same as chapter 3.
There are 4 categories,

λ_1	λ_2	λ_3	λ_4

$\lambda_1+\lambda_2+\lambda_3+\lambda_4=1$,
X_2 and Y_1 are continuous random variables,

X_1	$1-X_1$
λ_1	$1-\lambda_1$

λ_1, X_1 are known,

$\frac{\lambda_2}{1-\lambda_1}$	$\frac{\lambda_3}{1-\lambda_1}$	$\frac{\lambda_4}{1-\lambda_1}$

$$\frac{X_2}{1-x_1}|x_1 \sim CB\left(\frac{\lambda_2}{1-\lambda_1}\right), \frac{Y_1}{1-x_1}|x_1 \sim CB\left(\frac{\lambda_2+\lambda_3}{1-\lambda_1}\right),$$

The regression analysis can get the non-linear model Y1=b0+b1*H(X2) when X_1 are known,

the λ_3 is not 0 when rejected H0:b1=0.
The $\frac{\hat{\lambda}_2+\hat{\lambda}_3}{1-\hat{\lambda}_1}$ is from $\frac{Y_1}{1-x_1}$ sample mean($\frac{\overline{Y}_1}{1-x_1}$) and $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$ is from $\frac{X_2}{1-x_1}$ sample mean ($\frac{\overline{X}_2}{1-x_1}$), $\frac{\hat{\lambda}_3}{1-\hat{\lambda}_1}=\frac{\hat{\lambda}_2+\hat{\lambda}_3}{1-\hat{\lambda}_1}-\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$ could be computed.

The simulated data is using $RND = F_{Y_1}(y_1;\lambda_2+\lambda_3|x_1) = F_{X_2}(x_2;\lambda|x_1), y_1 \ge x_2$ to get ($\frac{X_2}{1-x_1}, \frac{Y_1}{1-x_1}$) paired samples and $X_2 \le Y_1$ given x_1 is known. The non-linear model Y1=b0+b1*H(X2) will be computed.

(1) λ_1=0.2, x1=0.2, λ_2=0.2, λ_3=0.2, $\frac{\lambda_2}{1-\lambda_1}$=0.25, $\frac{\lambda_3}{1-\lambda_1}$=0.25, $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.5,

(i)paired sample size=100,

The part of paired samples,

X2/(1-x1)	Y1/(1-x1)
0.7121367906	0.8140133563
0.0973397180	0.1521287477
0.3517404050	0.4807788301
0.1663853013	0.2505890860
0.1840159879	0.2745564225
0.5185633413	0.6514573676
0.0451328898	0.0725615264
0.8177912238	0.8891906390
0.6453787693	0.7618114053
0.2286630121	0.3332138217

X2	Y1
0.5697094325	0.6512106850
0.0778717744	0.1217029982
0.2813923240	0.3846230640
0.1331082411	0.2004712688
0.1472127903	0.2196451380
0.4148506731	0.5211658941
0.0361063118	0.0580492211
0.6542329790	0.7113525112
0.5163030154	0.6094491242
0.1829304097	0.2665710574

The analysis result,

Y1/(1-x1) estimated=1.5908946143+-1.5833866584*exp(-X2/(1-x1)),

ANOVA

Source	df	SS	MS
Regression	1	8.2474789105	8.2474789105
Error	98	0.0010908889	0.0000111315
Total	99	8.2485697994	

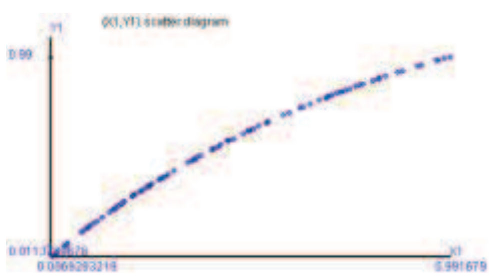
H0:slope=0, test statistic=740912.237432 , p value=0.000000,

R2=0.999868, R2(adj)=0.999866,MSE=0.000011,

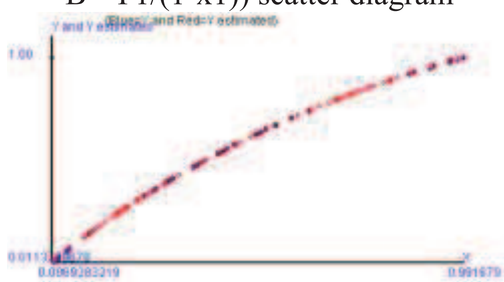
H0:residual population~Semi circle(mu=0.000048,R=0.005657)

chi square test statistic=6.540000, p value=0.162001

(X2/(1-x1),Y1/(1-x1)) scatter diagram



(X2/(1-x1), ,R= Y1/(1-x1)estimated , B= Y1/(1-x1)) scatter diagram



H0: $\frac{\lambda_2}{1-\lambda_1}$ =0.25,

$\frac{\overline{X}_2}{1-x_1}$ =0.4121582172, n=100,

$\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$ =0.2544882677,

Z test=0.061813567, p value=0.950666>0.05, failed to reject H0: $\frac{\lambda_2}{1-\lambda_1}$ =0.25.

H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$ =0.5,

$\frac{\overline{Y}_1}{1-x_1}$ =0.5020391912, n=100,

$\frac{\hat{\lambda}_2+\hat{\lambda}_3}{1-\hat{\lambda}_1}$ =0.50000586945,

Z test=0.0686005751, p value=0.945350 >0.05,

failed to reject H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$ =0.5

$\frac{\hat{\lambda}_3}{1-\hat{\lambda}_1}$ =0.245517601750.

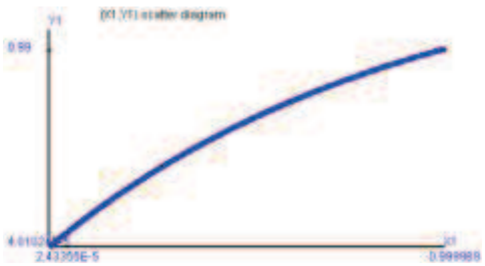
(ii)paired sample size=10,000,
The analysis result,

Y2/(1-x1) estimated= 1.5919609756+-1.5849456142*exp(-X2/(1-x1)),
ANOVA

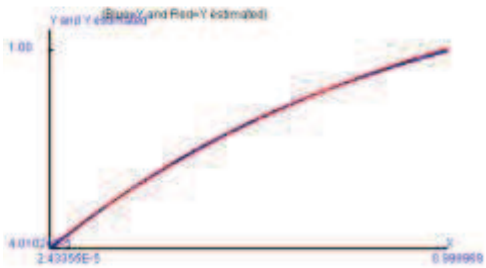
Source	df	SS	MS
Regression	1	831.4335500097	831.4335500097
Error	9998	0.1219374154	0.0000121962
Total	9999	831.5554874251	

H0:slope=0, test statistic=68171632.185628 , p value=0.000000,
R2=0.999853, R2(adj)=0.999853,MSE=0.000012,
H0:residual population~Normal(mu=-0.000237,sigma*sigma=0.000013)
chi square test statistic=4050.383200, p value=0.000000,

(X2/(1-x1),Y1/(1-x1)) scatter diagram



(X2/(1-x1), ,R= Y1/(1-x1)estimated , B= Y1/(1-x1)) scatter diagram



H0: $\frac{\lambda_2}{1-\lambda_1}$=0.25,

$\frac{\overline{X}_2}{1-x_1}$=0.4085459379, n=10000, $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.2458652355,

Z test=-0.6705912913, p value=0.502618>0.05, failed to reject H0: $\frac{\lambda_2}{1-\lambda_1}$=0.25.

H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.5,

$\frac{\overline{Y}_1}{1-x_1}$=0.4984102676, n=10000, $\frac{\hat{\lambda}_2+\hat{\lambda}_3}{1-\hat{\lambda}_1}$=0.4950674321,

Z test=-0.5709831659, p value=0.567936 >0.05,

failed to reject H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.5 or H0: $\frac{\lambda_2}{1-\lambda_1}=\frac{\lambda_3}{1-\lambda_1}$,

$\frac{\hat{\lambda}_3}{1-\hat{\lambda}_1}$=0.2492021966.

H0: $\frac{\lambda_2}{1-\lambda_1}=\frac{\lambda_3}{1-\lambda_1}$, $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.2458652355,

H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=2 $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.4917304710,

Z test=0.4065375370, p value=0.683960>0.05, failed to reject H0.

(2) λ_1=0.5, x1=0.7, λ_2=0.1, λ_3=0.2, $\frac{\lambda_2}{1-\lambda_1}$=0.2, $\frac{\lambda_3}{1-\lambda_1}$=0.4, $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.6,

(i)paired sample size=100,

The analysis result,

Y1/(1-x1) estimated=-0.1423442919+1.1822783237*|X2/(1-x1)|^0.5,

ANOVA

Source	df	SS	MS
Regression	1	7.4732906755	7.4732906755
Error	98	0.0390855520	0.0003988322
Total	99	7.5123762275	

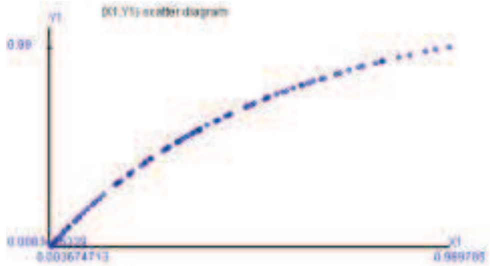
H0:slope=0, test statistic=18737.933831 , p value=0.000000

R2=0.994797, R2(adj)=0.994744,MSE=0.000399,

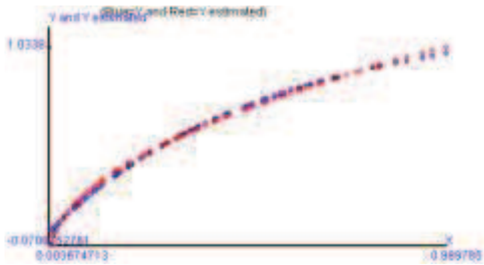
H0:residual population~Raised cosine(mu=0.000000,s=0.054982)

chi square test statistic=23.760000, p value=0.000094,

(X2/(1-x1),Y1/(1-x1)) scatter diagram



(X2/(1-x1), ,R= Y1/(1-x1)estimated , B= Y1/(1-x1)) scatter diagram



H0: $\frac{\lambda_2}{1-\lambda_1}$=0.2,

$\frac{\overline{X}_2}{1-x_1}$=0.3713241342, n=100, $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.1665759961,

Z test=-0.5992448747, p value=0.549188>0.05, failed to reject H0: $\frac{\lambda_2}{1-\lambda_1}$=0.2.

H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.6,

$\frac{\overline{Y}_1}{1-x_1}$=0.5242120051, n=100, $\frac{\hat{\lambda}_2+\hat{\lambda}_3}{1-\hat{\lambda}_1}$=0.5720745565,

Z test=-0.3253646135, p value=0.745488 >0.05,

failed to reject H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.6.

$\frac{\hat{\lambda}_3}{1-\hat{\lambda}_1}$=0.4054985605.

H0: $\frac{\lambda_2}{1-\lambda_1}=\frac{\lambda_3}{1-\lambda_1}$, $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.1665759961,

H0: $\frac{\lambda_2+\lambda_3}{1-\lambda_1}$=3 $\frac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.4997279883,

Z test=0.8369433650, p value=0.402636>0.05, failed to reject H0.

(ii)paired sample size=10,000,

The analysis result,

Y1/(1-x1) estimated=0.1459467246+1.1846262220*|X2/(1-x1)|^0.5

ANOVA

Source	df	SS	MS
Regression	1	826.2866479488	826.2866479488
Error	9998	3.8940314439	0.0003894810
Total	9999	830.1806793926	

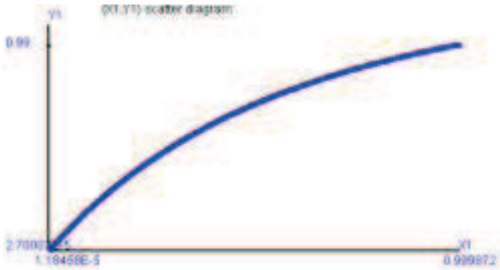
H0:slope=0, test statistic=2121506.727740 , p value=0.000000

R2=0.995309, R2(adj)=0.995309,MSE=0.000389,

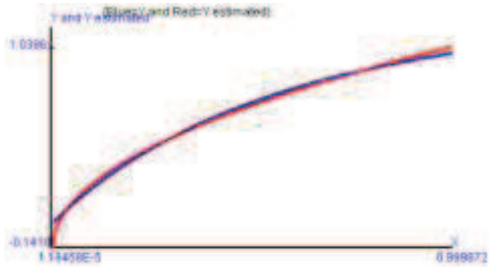
H0:residual population~Raised cosine(mu=-0.000000,s=0.054607)

chi square test statistic=2746.014400, p value=0.000000

(X2/(1-x1),Y1/(1-x1)) scatter diagram



(X2/(1-x1), ,R= Y1/(1-x1)estimated , B= Y1/(1-x1)) scatter diagram



H0: $\dfrac{\lambda_2}{1-\lambda_1}$=0.2,

$\dfrac{\overline{X}_2}{1-x_1}$=0.3928642953, n=10000, $\dfrac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.2102905116,

Z test=1.8269455011, p value=0.067932>0.05, failed to reject H0: $\dfrac{\lambda_2}{1-\lambda_1}$=0.2.

H0: $\dfrac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.6,

$\dfrac{\overline{Y}_1}{1-x_1}$=0.5386650155, n=10000, $\dfrac{\hat{\lambda}_2+\hat{\lambda}_3}{1-\hat{\lambda}_1}$=0.6142051864,

Z test=1.7729692595, p value=0.076598>0.05,

failed to reject H0: $\dfrac{\lambda_2+\lambda_3}{1-\lambda_1}$=0.6.

$\dfrac{\hat{\lambda}_3}{1-\hat{\lambda}_1}$=0.4039146748.

H0: $\dfrac{\lambda_2}{1-\lambda_1}=\dfrac{\lambda_3}{1-\lambda_1}$, $\dfrac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.2102905116,

H0: $\dfrac{\lambda_2+\lambda_3}{1-\lambda_1}$=3 $\dfrac{\hat{\lambda}_2}{1-\hat{\lambda}_1}$=0.6308715348,

Z test=-2.0163156861, p value=0.044072>0.04, failed to reject H0 when significant level=0.04.

Chapter 5 Bernoulli distribution and new conditional Continuous Bernoulli distribution to analyze data

Section 1. The joint pdf, cumulative probability distribution function, simulator and expected value and variance

1.The probability density function

The Continuous Bernoulli distribution is marginal probability distribution,

$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1,$$

$$f_{X_1}(x_1; \lambda_1) = C_1(\lambda_1)(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1}, 0 \le x_1 \le 1, 0 < \lambda_1 < 1,$$

$$C_1(\lambda_1) = \begin{cases} \dfrac{ln(1-\lambda_1) - ln(\lambda_1)}{1-2\lambda_1}, \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda_1 \ne \dfrac{1}{2} \end{cases},$$

The new conditional Bernoulli distribution is

$$f_{X_2|x_1}(x_2|x_1) = C_2(\lambda_1, \lambda_2, x_1)\left(\frac{\lambda_2}{1-\lambda_1}\right)^{x_2}\left(1 - \frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1-x_2}, 0 \le x_2 \le 1 - x_1, 0 < \lambda_2 < 1 - \lambda_1,$$

which is affected by λ_1, λ_2 and x_1.

$$\int_0^{1-x_1} f_{X_2|x_1}(x_2|x_1)dx_2 = C_2(\lambda_1, \lambda_2, x_1)\left(\frac{1-\lambda_1-\lambda_2}{1-\lambda_1}\right)^{1-x_1}\int_0^{1-x_1}\left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{x_2}dx_2 --- (1.1),$$

$$(i) 1 - \lambda_1 \ne 2\lambda_2, (1.1) = C_2(\lambda_1, \lambda_2, x_1)\left(\frac{1-\lambda_1-\lambda_2}{1-\lambda_1}\right)^{1-x_1}\frac{\left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{x_2}}{ln\left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)}\Bigg|_0^{1-x_1}$$

$$= C_2(\lambda_1, \lambda_2, x_1)\left(\frac{1-\lambda_1-\lambda_2}{1-\lambda_1}\right)^{1-x_1}\frac{\left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{1-x_1} - 1}{ln\left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)}$$

$$C_2(\lambda_1,\lambda_2,x_1)\left(\frac{1}{1-\lambda_1}\right)^{1-x_1}\frac{\left((\lambda_2)^{1-x_1}-(1-\lambda_1-\lambda_2)^{1-x_1}\right)}{ln(\lambda_2)-ln(1-\lambda_1-\lambda_2)}=1,$$

$$C_2(\lambda_1,\lambda_2,x_1)=\frac{ln(\lambda_2)-ln(1-\lambda_1-\lambda_2)}{\left(\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}-\left(1-\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}},$$

$$(ii)1-\lambda_1=2\lambda_2,(1.1)=C_2(\lambda_1,\lambda_2,x_1)\left(\frac{1}{2}\right)^{1-x_1}(1-x_1)=1,C_2(\lambda_1,\lambda_2,x_1)=\frac{2^{1-x_1}}{1-x_1},$$

$$C_2(\lambda_1,\lambda_2,x_1)=\begin{cases}\dfrac{(ln(\lambda_2)-ln(1-\lambda_1-\lambda_2))}{\left(\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}-\left(1-\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}},1-\lambda_1\neq 2\lambda_2,\\ \dfrac{2^{1-x_1}}{1-x_1},1-\lambda_1\neq 2\lambda_2,\end{cases},$$

$$f_{X_1}(x_1;\lambda_1)f_{X_2|x_1}(x_2|x_1)=C_1(\lambda_1)(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1}C_2(\lambda_1,\lambda_2,x_1)\left(\frac{\lambda_2}{1-\lambda_1}\right)^{x_2}\left(1-\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1-x_2}$$

$$=C_1(\lambda_1)C_2(\lambda_1,\lambda_2,x_1)(\lambda_1)^{x_1}(\lambda_2)^{x_2}(1-\lambda_1-\lambda_2)^{1-x_1-x_2},0\le x_1\le 1,0\le x_2\le 1-x_1,$$

But $f_{X_2}(x_2)=\int_0^{1-x_2}f_{X_1}(x_1;\lambda_1)f_{X_2|x_1}(x_2|x_1)dx_1$, it is not $CB(\lambda_2)$,

2. The cumulative probability distribution function

$X_1\sim CB(\lambda_1),0\le x_1\le 1, X_2|x_1\sim CB(\lambda_1,\lambda_2,x_1),0\le x_2\le 1-x_1,$

(1) $X_1\sim CB(\lambda_1),0\le x_1\le 1,$

$$F_{X_1}(x_1;\lambda_1)=\begin{cases}\dfrac{(\lambda_1)^{x_1}(1-\lambda_1)^{1-x_1}+\lambda_1-1}{2\lambda_1-1},\lambda_1\neq\dfrac{1}{2}\\ x_1,\lambda_1=\dfrac{1}{2}\end{cases},0<x_1<1$$

(2) $X_2|x_1\sim CB\left(\frac{\lambda_2}{1-\lambda_1}\right),0\le x_2\le 1-x_1,$

$(i)1-\lambda_1\neq 2\lambda_2$

$$F_{X_2|x_1}(x_2|x_1)=\frac{(1-\lambda_1-\lambda_2)^{1-x_1-x_2}(\lambda_2)^{x_2}-(1-\lambda_1-\lambda_2)^{1-x_1}}{(\lambda_2)^{1-x_1}-(1-\lambda_1-\lambda_2)^{1-x_1}},0<x_2\le 1-x_1,$$

$(ii)1-\lambda_1=2\lambda_2,$

$$F_{X_2|x_1}(x_2|x_1)=\frac{x_2}{1-x_1},0<x_2\le 1-x_1,$$

3. The simulator

The simulator,

The random number= $RND_1 = F_{X_1}\left(x_1;\lambda_1\right) \sim Uniform(0,1)$,

$$x_1 \text{ simulated value} = \begin{cases} \dfrac{log_e\left(RND_1 \times \left(2\lambda_1 - 1\right) - \left(\lambda_1 - 1\right)\right) - log_e\left(1-\lambda_1\right)}{log_e\left(\dfrac{\lambda_1}{1-\lambda_1}\right)}, \lambda_1 \neq \dfrac{1}{2} \\ RND_1, \lambda_1 = \dfrac{1}{2} \end{cases}$$

The random number= $RND_2 = F_{X_2|x_1}\left(x_2|x_1\right) \sim Uniform(0,1)$,

x_2 simulated value=

$$\begin{cases} \dfrac{log_e\left(\left(1-\lambda_1-\lambda_2\right)^{1-x_1} + RND_2\left(\left(\lambda_2\right)^{1-x_1} - \left(1-\lambda_1-\lambda_2\right)^{1-x_1}\right)\right) - \left(1-x_1\right)log_e\left(1-\lambda_1-\lambda_2\right)}{log_e\left(\lambda_2\right) - log_e\left(1-\lambda_1-\lambda_2\right)}, 1-\lambda_1 \neq 2\lambda_2 \\ RND_2\left(1-x_1\right), 1-\lambda_1 = 2\lambda_2 \end{cases}$$

Please see the appendix 1.

4. Expected value

$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1,$$

$$\mu_1 = E(X_1) = \begin{cases} \dfrac{\lambda_1}{2\lambda_1 - 1} + \dfrac{1}{2\,tan^{-1}(1-2\lambda_1)} & if\ \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{2} & if\ \lambda_1 = \dfrac{1}{2} \end{cases},$$

$$\sigma_1^2 = Var(X_1) = \begin{cases} \dfrac{(1-\lambda_1)\lambda_1}{(1-2\lambda_1)^2} + \dfrac{1}{\left(2\,tan^{-1}(1-2\lambda_1)\right)^2} & if\ \lambda_1 \ne \dfrac{1}{2} \\ \dfrac{1}{12} & if\ \lambda_1 = \dfrac{1}{2} \end{cases}$$

$$X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1,$$

$(i)\ 1 - \lambda_1 \ne 2\lambda_2$

$$f_{X_2|x_1}(x_2|x_1) = \frac{\left(ln(\lambda_2) - ln(1-\lambda_1-\lambda_2)\right)}{\left(\dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} - \left(1 - \dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}} \left(\frac{\lambda_2}{1-\lambda_1}\right)^{x_2} \left(1 - \frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1-x_2},$$

$$E(X_2|x_1) = \frac{\left(ln(\lambda_2) - ln(1-\lambda_1-\lambda_2)\right)}{\left(\dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} - \left(1 - \dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}} \left(1 - \frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} \int_0^{1-x_1} x_2 \left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{x_2} dx_2$$

$$= \frac{\left(ln(\lambda_2) - ln(1-\lambda_1-\lambda_2)\right)}{\left(\dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} - \left(1 - \dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}} \left(1 - \frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} \times$$

$$\left(x_2 \frac{\left(\dfrac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{x_2}}{ln\left(\dfrac{\lambda_2}{1-\lambda_1-\lambda_2}\right)} \Bigg|_0^{1-x_1} - \frac{1}{ln\left(\dfrac{\lambda_2}{1-\lambda_1-\lambda_2}\right)} \int_0^{1-x_1} \left(\frac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{x_2} dx_2 \right)$$

$$= \frac{\left(ln(\lambda_2) - ln(1-\lambda_1-\lambda_2)\right)}{\left(\dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1} - \left(1 - \dfrac{\lambda_2}{1-\lambda_1}\right)^{1-x_1}} \left(1 - \frac{\lambda_2}{1-\lambda_1}\right) \times \left(1 - x_1 - \frac{\left(\dfrac{\lambda_2}{1-\lambda_1-\lambda_2}\right)^{1-x_1} - 1}{\left(ln\left(\dfrac{\lambda_2}{1-\lambda_1-\lambda_2}\right)\right)^2} \right)$$

$(ii)\ 1 - \lambda_1 = 2\lambda_2,$

$$E(X_2|x_1) = \frac{1}{1-x_1} \int_0^{1-x_1} x_2\, dx_2 = \left(\frac{1+x_1}{2}\right).$$

Please see the appendix 1 about Variance.

Section 2. The joint pdf, cumulative probability distribution function, simulator and expected value

1.The probability density function

$$X_2 \sim CB(\lambda_2), 0 \le x_2 \le 1, X_1|x_2 \sim CB(\lambda_2, \lambda_1, x_2), 0 \le x_1 \le 1 - x_2,$$
$$f_{X_2}(x_2; \lambda_2) = C_1(\lambda_2)(\lambda_2)^{x_2}(1-\lambda_2)^{1-x_2}, 0 \le x_2 \le 1, 0 < \lambda_2 < 1,$$
$$C_2(\lambda_2) = \begin{cases} \dfrac{ln(1-\lambda_2) - ln(\lambda_2)}{1-2\lambda_2}, \lambda_2 \ne \dfrac{1}{2} \\ \dfrac{1}{2}, \lambda_2 \ne \dfrac{1}{2} \end{cases},$$

$$f_{X_1|x_2}(x_1|x_2) = C_2(\lambda_1, \lambda_2, x_2)\left(\frac{\lambda_1}{1-\lambda_2}\right)^{x_1}\left(1 - \frac{\lambda_1}{1-\lambda_2}\right)^{1-x_1-x_2}, 0 \le x_1 \le 1 - x_2, 0 < \lambda_1 < 1 - \lambda_2,$$

$$\int_0^{1-x_2} f_{X_1|x_2}(x_1|x_2)dx_1 = C_2(\lambda_1, \lambda_2, x_2)\left(\frac{1-\lambda_1-\lambda_2}{1-\lambda_2}\right)^{1-x_2}\int_0^{1-x_2}\left(\frac{\lambda_1}{1-\lambda_1-\lambda_2}\right)^{x_1}dx_1 --- (1.2),$$

$$(i) 1-\lambda_2 \ne 2\lambda_1, (1.2) = C_2(\lambda_1, \lambda_2, x_2)\left(\frac{1-\lambda_1-\lambda_2}{1-\lambda_2}\right)^{1-x_2}\frac{\left(\frac{\lambda_1}{1-\lambda_1-\lambda_2}\right)^{x_1}}{ln\left(\frac{\lambda_1}{1-\lambda_1-\lambda_2}\right)}\Bigg|_0^{1-x_2}$$

$$= C_2(\lambda_1, \lambda_2, x_2)\left(\frac{1-\lambda_1-\lambda_2}{1-\lambda_2}\right)^{1-x_2}\frac{\left(\frac{\lambda_1}{1-\lambda_1-\lambda_2}\right)^{1-x_2} - 1}{ln\left(\frac{\lambda_1}{1-\lambda_1-\lambda_2}\right)}$$

$$C_2(\lambda_1, \lambda_2, x_2)\left(\frac{1}{1-\lambda_2}\right)^{1-x_2}\frac{\left((\lambda_2)^{1-x_2} - (1-\lambda_1-\lambda_2)^{1-x_2}\right)}{ln(\lambda_1) - ln(1-\lambda_1-\lambda_2)} = 1,$$
$$C_2(\lambda_1, \lambda_2, x_2) = \frac{ln(\lambda_1) - ln(1-\lambda_1-\lambda_2)}{\left(\frac{\lambda_1}{1-\lambda_2}\right)^{1-x_2} - \left(1 - \frac{\lambda_1}{1-\lambda_2}\right)^{1-x_2}},$$

$$(ii)1-\lambda_2=2\lambda_1,(1.1)=C_2(\lambda_1,\lambda_2,x_2)\left(\frac{1}{2}\right)^{1-x_2}(1-x_2)=1,C_2(\lambda_1,\lambda_2,x_1)=\frac{2^{1-x_2}}{1-x_2},$$

$$C_2(\lambda_1,\lambda_2,x_2)=\begin{cases}\dfrac{ln(\lambda_1)-ln(1-\lambda_1-\lambda_2)}{\left(\dfrac{\lambda_1}{1-\lambda_2}\right)^{1-x_2}-\left(1-\dfrac{\lambda_1}{1-\lambda_2}\right)^{1-x_2}},1-\lambda_2\neq 2\lambda_1,\\[2ex] \dfrac{2^{1-x_2}}{1-x_2},1-\lambda_2\neq 2\lambda_1,\end{cases},$$

$$f_{X_2}(x_2;\lambda_2)f_{X_1|x_2}(x_1|x_2)=C_1(\lambda_2)(\lambda_2)^{x_2}(1-\lambda_2)^{1-x_2}C_2(\lambda_1,\lambda_2,x_2)\left(\frac{\lambda_1}{1-\lambda_2}\right)^{x_1}\left(1-\frac{\lambda_1}{1-\lambda_2}\right)^{1-x_1-x_2}$$
$$=C_1(\lambda_2)C_2(\lambda_1,\lambda_2,x_2)(\lambda_1)^{x_1}(\lambda_2)^{x_2}(1-\lambda_1-\lambda_2)^{1-x_1-x_2},0\le x_2\le 1,0\le x_1\le 1-x_2,$$

But $f_{X_1}(x_1)=\int_0^{1-x_1}f_{X_2}(x_2;\lambda_2)f_{X_1|x_2}(x_1|x_2)dx_2$, it is not $CB(\lambda_1)$,

2. The cumulative probability distribution function

$X_2\sim CB(\lambda_2),0\le x_2\le 1,X_1|x_2\sim CB(\lambda_2,\lambda_1,x_2),0\le x_1\le 1-x_2,$

(1) $X_2\sim CB(\lambda_2),0\le x_2\le 1,$

$$F_{X_2}(x_2;\lambda_2)=\begin{cases}\dfrac{(\lambda_2)^{x_2}(1-\lambda_2)^{1-x_2}+\lambda_2-1}{2\lambda_2-1},\lambda_2\neq\dfrac{1}{2}\\[2ex] x_2,\lambda_2=\dfrac{1}{2}\end{cases},0<x_2<1$$

(2) $X_1|x_2\sim CB\left(\frac{\lambda_1}{1-\lambda_2}\right),0\le x_1\le 1-x_2,$

$(i)1-\lambda_2\neq 2\lambda_1$

$$F_{X_1|x_2}(x_1|x_2)=\frac{(1-\lambda_1-\lambda_2)^{1-x_2-x_1}(\lambda_1)^{x_1}-(1-\lambda_1-\lambda_2)^{1-x_2}}{(\lambda_1)^{1-x_2}-(1-\lambda_1-\lambda_2)^{1-x_2}},0<x_1\le 1-x_2,$$

$(ii)1-\lambda_2=2\lambda_1,$

$$F_{X_1|x_2}(x_1|x_2)=\frac{x_1}{1-x_2},0<x_1\le 1-x_2,$$

3. The simulator

The simulator,

The random number= $RND_2 = F_{X_2}(x_2; \lambda_2) \sim Uniform(0,1)$,

$$x_2 \text{ simulated value} = \begin{cases} \dfrac{log_e(RND_2 \times (2\lambda_2 - 1) - (\lambda_2 - 1)) - log_e(1-\lambda_2)}{log_e\left(\dfrac{\lambda_2}{1-\lambda_2}\right)}, \lambda_2 \neq \dfrac{1}{2} \\ RND_2, \lambda_2 = \dfrac{1}{2} \end{cases}$$

The simulator,

The random number= $RND_1 = F_{X_1|x_2}(x_1|x_2) \sim Uniform(0,1)$,

x_1 simulated value

=

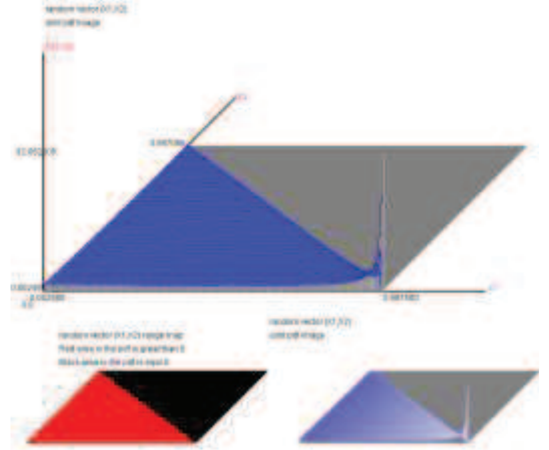
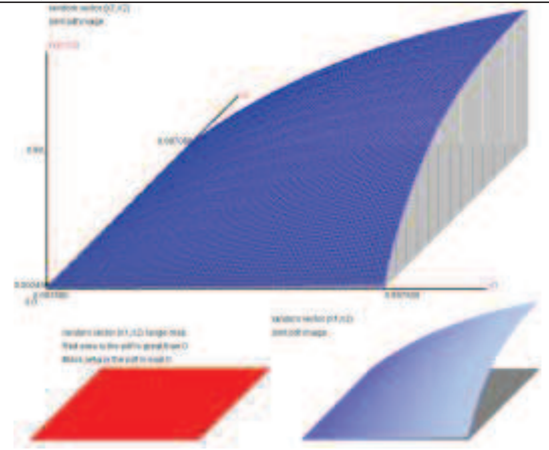
$$\begin{cases} \dfrac{log_e\left((1-\lambda_1-\lambda_2)^{1-x_2} + RND_1\left((\lambda_1)^{1-x_2} - (1-\lambda_1-\lambda_2)^{1-x_2}\right)\right) - (1-x_2)log_e(1-\lambda_1-\lambda_2)}{log_e(\lambda_1) - log_e(1-\lambda_1-\lambda_2)}, 1-\lambda_2 \neq 2\lambda_1 \\ RND_2(1-x_2), 1-\lambda_2 = 2\lambda_1 \end{cases}$$

Section 3. The image of joint probability density function and marginal probability density function

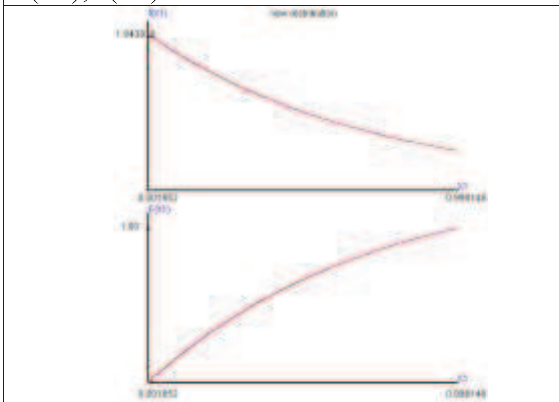
$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2 | x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1,$

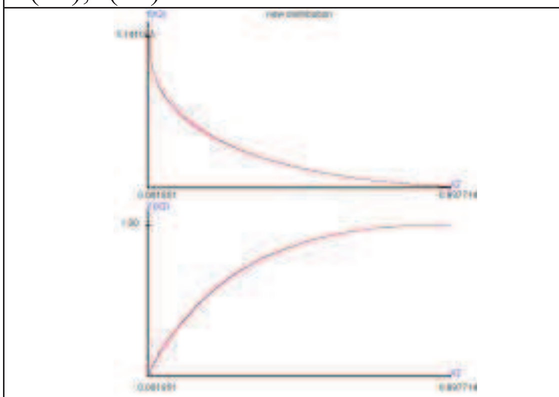
(1) $\lambda_1 = 0.2,$

(1-1) $\lambda_1 = 0.2, \quad \lambda_2 = 0.1,$

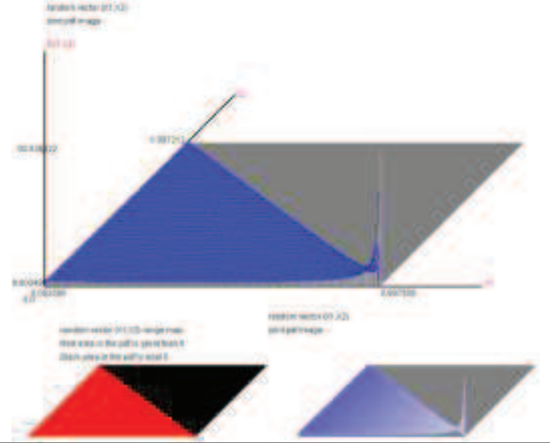
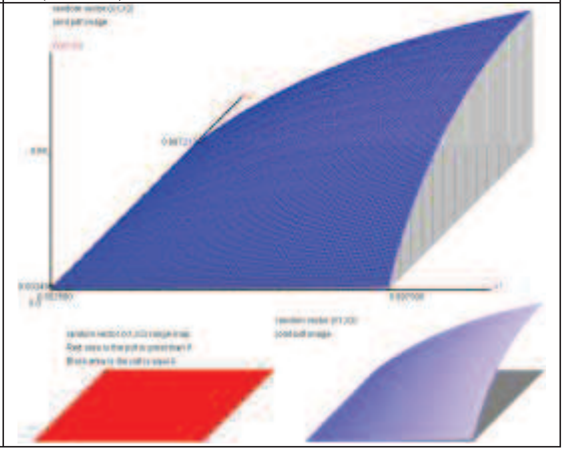
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.2358, Var(X2)= 0.0418, Cov(X1,X2)= -0.0252, X1 and X2 correlation coefficient=-0.4475.

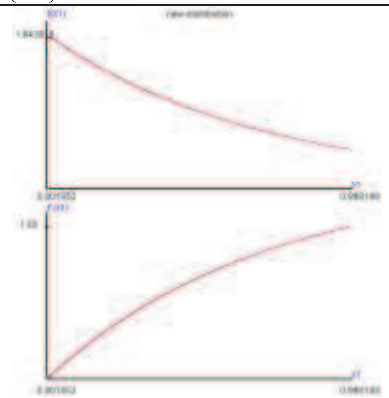
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.38805
	Geometrical Mean :	0.25584
	Harmonic Mean :	0.02875
	Variance :	0.07589
	S.D. :	0.27549
	Skewed Coef. :	0.47583
	Kurtosis Coef. :	2.11562
	MAD :	0.23443
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.33911
	Q1 :	0.14979
	Q2 :	0.33911
	Q3 :	0.59636
	IQR :	0.44657
	C.V. :	0.70994

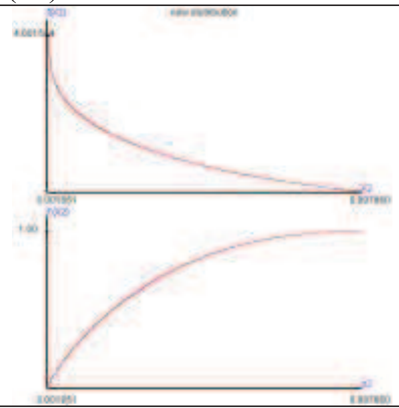
f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.23579
	Geometrical Mean :	0.13539
	Harmonic Mean :	0.00372
	Variance :	0.04177
	S.D. :	0.20439
	Skewed Coef. :	1.05569
	Kurtosis Coef. :	3.49719
	MAD :	0.16490
	Range :	0.99956
	Mid_range :	0.49978
	Median :	0.17858
	Q1 :	0.07088
	Q2 :	0.17858
	Q3 :	0.35137
	IQR :	0.28048
	C.V. :	0.86682

(1-2) $\lambda_1 = 0.2, \quad \lambda_2 = 0.2,$

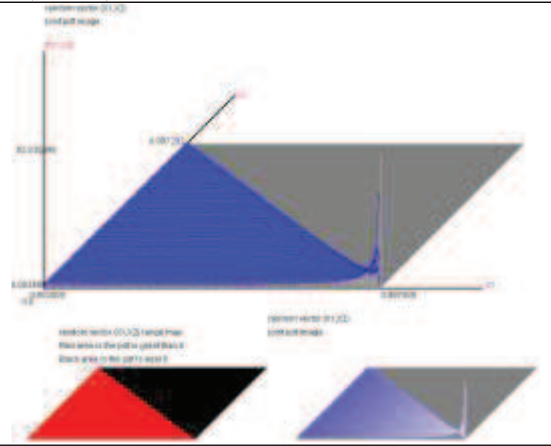
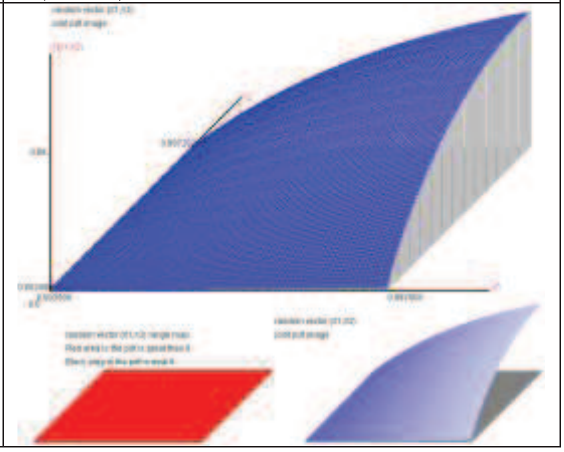
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.2653, Var(X2)= 0.0484, Cov(X1,X2)= -0.0305, X1 and X2 correlation coefficient=-0.5031.

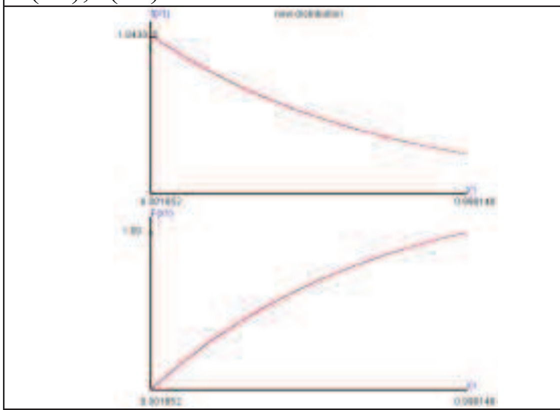
f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.38805
	Geometrical Mean :	0.25584
	Harmonic Mean :	0.02875
	Variance :	0.07589
	S.D. :	0.27549
	Skewed Coef. :	0.47583
	Kurtosis Coef. :	2.11562
	MAD :	0.23443
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.33911
	Q1 :	0.14979
	Q2 :	0.33911
	Q3 :	0.59636
	IQR :	0.44657
	C.V. :	0.70994

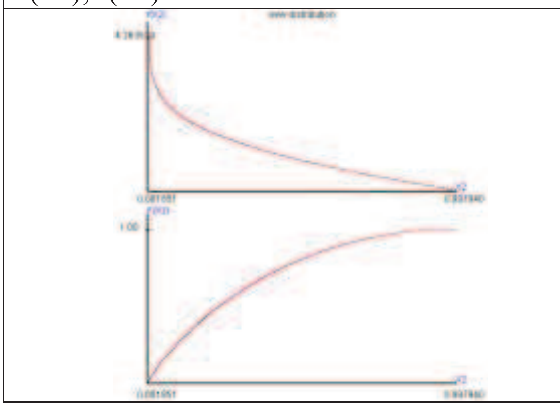
f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.26526
	Geometrical Mean :	0.15537
	Harmonic Mean :	0.00386
	Variance :	0.04837
	S.D. :	0.21993
	Skewed Coef. :	0.89126
	Kurtosis Coef. :	3.01149
	MAD :	0.18049
	Range :	0.99971
	Mid_range :	0.49986
	Median :	0.20868
	Q1 :	0.08306
	Q2 :	0.20868
	Q3 :	0.40136
	IQR :	0.31830
	C.V. :	0.82910

(1-3) $\lambda_1 = 0.2, \quad \lambda_2 = 0.3,$

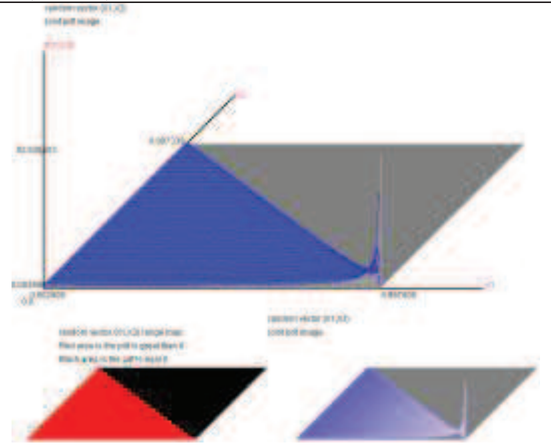
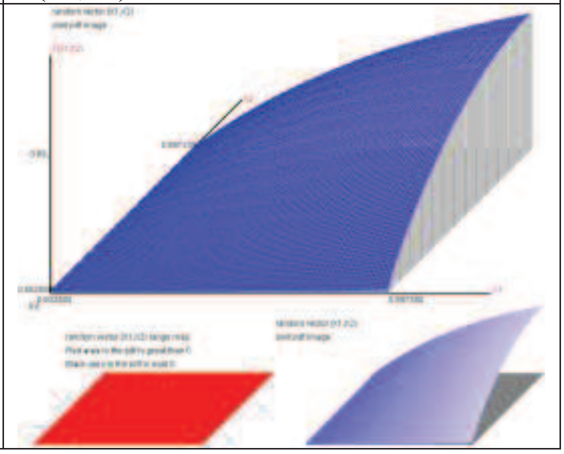
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.2868, Var(X2)= 0.0528,
Cov(X1,X2)= -0.0344, X1 and X2 correlation coefficient=-0.5437.

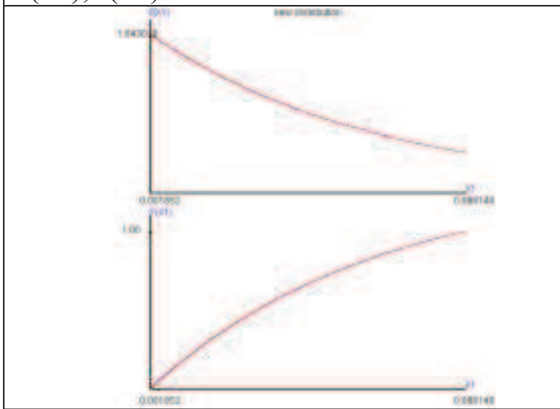
<table>
<tr><th>f(x1),F(x1)</th><th colspan="3">Coefficient</th></tr>
<tr><td rowspan="16"></td><td>Mathematical Mean</td><td>:</td><td>0.38805</td></tr>
<tr><td>Geometrical Mean</td><td>:</td><td>0.25584</td></tr>
<tr><td>Harmonic Mean</td><td>:</td><td>0.02875</td></tr>
<tr><td>Variance</td><td>:</td><td>0.07589</td></tr>
<tr><td>S.D.</td><td>:</td><td>0.27549</td></tr>
<tr><td>Skewed Coef.</td><td>:</td><td>0.47583</td></tr>
<tr><td>Kurtosis Coef.</td><td>:</td><td>2.11562</td></tr>
<tr><td>MAD</td><td>:</td><td>0.23443</td></tr>
<tr><td>Range</td><td>:</td><td>1.00000</td></tr>
<tr><td>Mid_range</td><td>:</td><td>0.50000</td></tr>
<tr><td>Median</td><td>:</td><td>0.33911</td></tr>
<tr><td>Q1</td><td>:</td><td>0.14979</td></tr>
<tr><td>Q2</td><td>:</td><td>0.33911</td></tr>
<tr><td>Q3</td><td>:</td><td>0.59636</td></tr>
<tr><td>IQR</td><td>:</td><td>0.44657</td></tr>
<tr><td>C.V.</td><td>:</td><td>0.70994</td></tr>
</table>

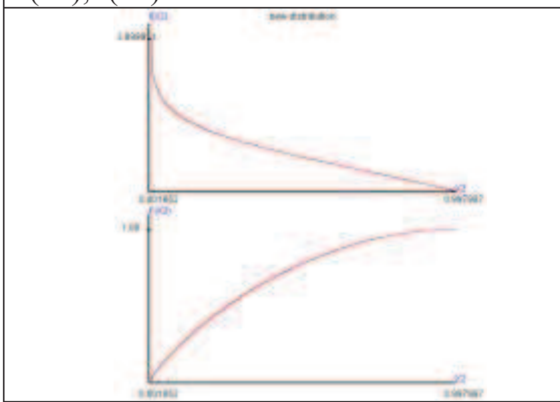
<table>
<tr><th>f(x2),F(x2)</th><th colspan="3">Coefficient</th></tr>
<tr><td rowspan="16"></td><td>Mathematical Mean</td><td>:</td><td>0.28684</td></tr>
<tr><td>Geometrical Mean</td><td>:</td><td>0.17073</td></tr>
<tr><td>Harmonic Mean</td><td>:</td><td>0.00395</td></tr>
<tr><td>Variance</td><td>:</td><td>0.05284</td></tr>
<tr><td>S.D.</td><td>:</td><td>0.22987</td></tr>
<tr><td>Skewed Coef.</td><td>:</td><td>0.77700</td></tr>
<tr><td>Kurtosis Coef.</td><td>:</td><td>2.72876</td></tr>
<tr><td>MAD</td><td>:</td><td>0.19073</td></tr>
<tr><td>Range</td><td>:</td><td>0.99979</td></tr>
<tr><td>Mid_range</td><td>:</td><td>0.49990</td></tr>
<tr><td>Median</td><td>:</td><td>0.23255</td></tr>
<tr><td>Q1</td><td>:</td><td>0.09302</td></tr>
<tr><td>Q2</td><td>:</td><td>0.23255</td></tr>
<tr><td>Q3</td><td>:</td><td>0.43790</td></tr>
<tr><td>IQR</td><td>:</td><td>0.34488</td></tr>
<tr><td>C.V.</td><td>:</td><td>0.80138</td></tr>
</table>

(1-4) $\lambda_1 = 0.2,\quad \lambda_2 = 0.4,$

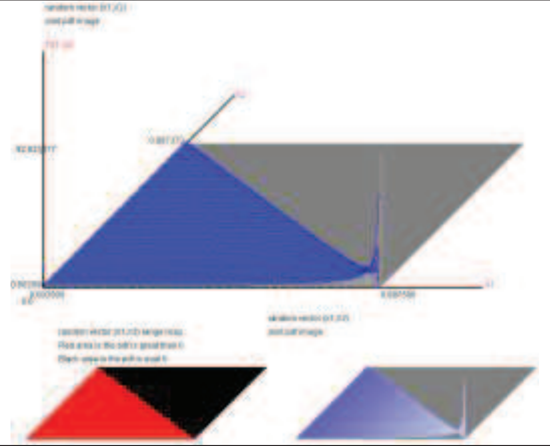
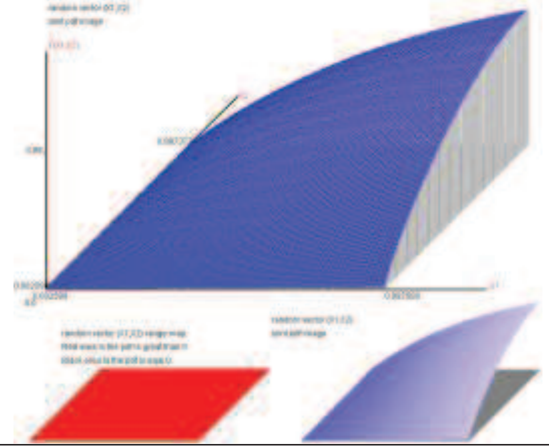
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.3060, Var(X2)= 0.0565,
Cov(X1,X2)= -0.0379, X1 and X2 correlation coefficient=-0.5794.

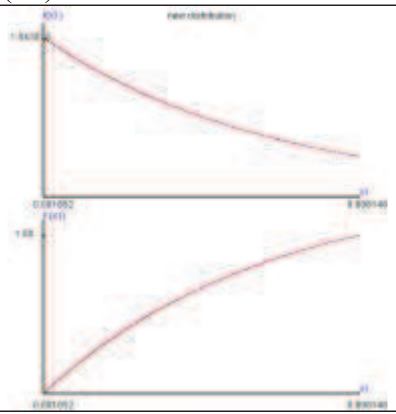
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25584 Harmonic Mean : 0.02875 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : 0.47583 Kurtosis Coef. : 2.11562 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.33911 Q1 : 0.14979 Q2 : 0.33911 Q3 : 0.59636 IQR : 0.44657 C.V. : 0.70994

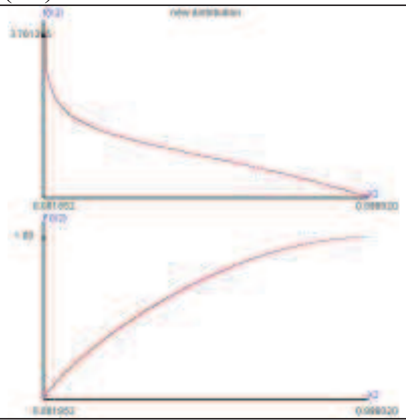
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.30596 Geometrical Mean : 0.18488 Harmonic Mean : 0.00402 Variance : 0.05651 S.D. : 0.23771 Skewed Coef. : 0.67955 Kurtosis Coef. : 2.52113 MAD : 0.19895 Range : 0.99984 Mid_range : 0.49992 Median : 0.25502 Q1 : 0.10271 Q2 : 0.25502 Q3 : 0.46986 IQR : 0.36715 C.V. : 0.77695

(1-5) $\lambda_1 = 0.2, \quad \lambda_2 = 0.5,$

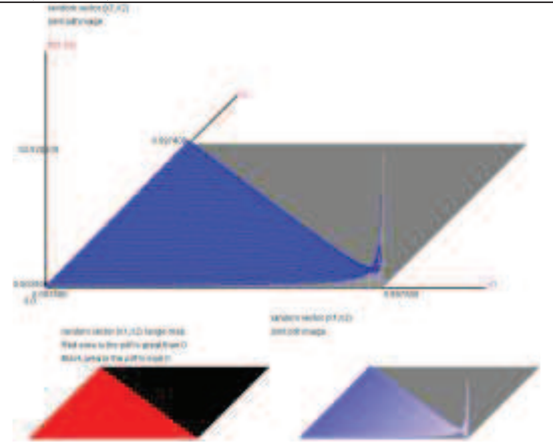
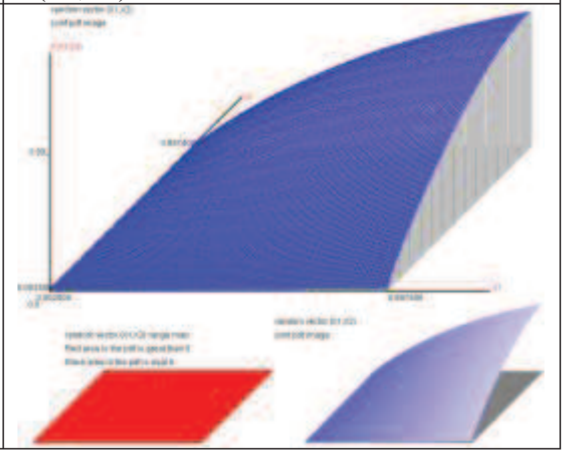
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.3251, Var(X2)= 0.0599,
Cov(X1,X2)= -0.0415, X1 and X2 correlation coefficient=-0.6151.

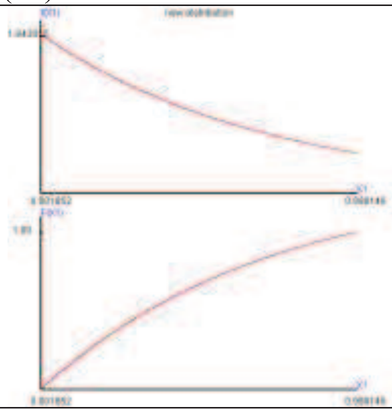
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25584 Harmonic Mean : 0.02875 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : 0.47583 Kurtosis Coef. : 2.11562 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.33911 Q1 : 0.14979 Q2 : 0.33911 Q3 : 0.59636 IQR : 0.44657 C.V. : 0.70994

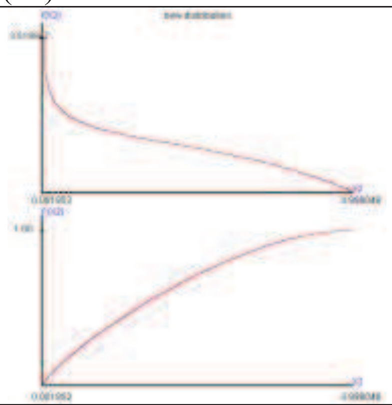
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.32507 Geometrical Mean : 0.19955 Harmonic Mean : 0.00409 Variance : 0.05987 S.D. : 0.24469 Skewed Coef. : 0.58520 Kurtosis Coef. : 2.34831 MAD : 0.20634 Range : 0.99987 Mid_range : 0.49994 Median : 0.27871 Q1 : 0.11332 Q2 : 0.27871 Q3 : 0.50115 IQR : 0.38782 C.V. : 0.75273

(1-6) $\lambda_1 = 0.2, \quad \lambda_2 = 0.6,$

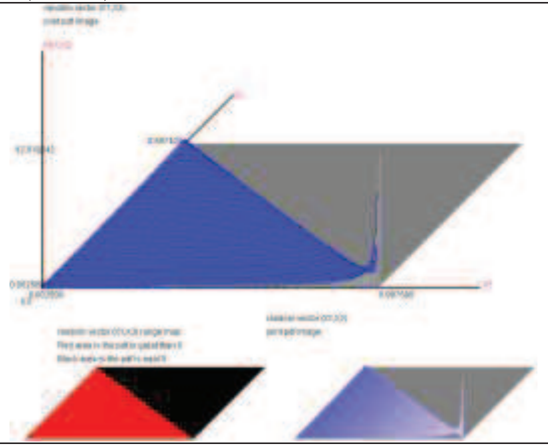
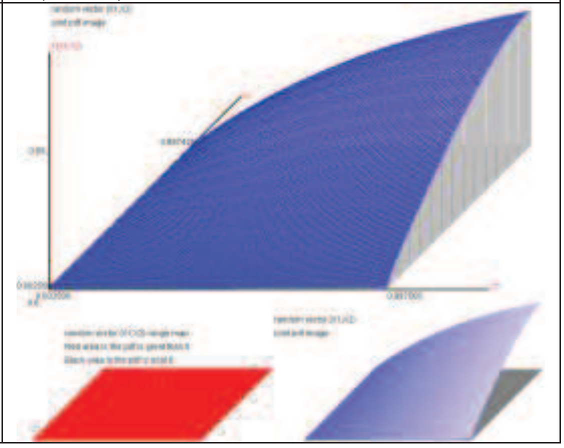
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.3466, Var(X2)= 0.0633,
Cov(X1,X2)= -0.0454, X1 and X2 correlation coefficient=-0.6551.

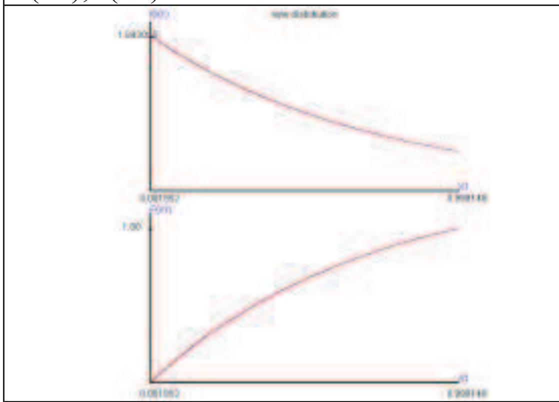
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25584 Harmonic Mean : 0.02875 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : 0.47583 Kurtosis Coef. : 2.11562 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.33911 Q1 : 0.14979 Q2 : 0.33911 Q3 : 0.59636 IQR : 0.44657 C.V. : 0.70994

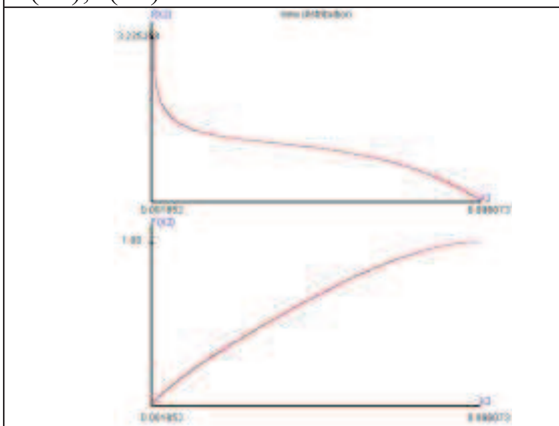
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.34665 Geometrical Mean : 0.21676 Harmonic Mean : 0.00416 Variance : 0.06329 S.D. : 0.25158 Skewed Coef. : 0.48190 Kurtosis Coef. : 2.18989 MAD : 0.21367 Range : 0.99990 Mid_range : 0.49995 Median : 0.30687 Q1 : 0.12655 Q2 : 0.30687 Q3 : 0.53534 IQR : 0.40880 C.V. : 0.72574

(1-7) $\lambda_1 = 0.2$, $\lambda_2 = 0.7$,

f(x1,x2)	F(x1,x2)
	

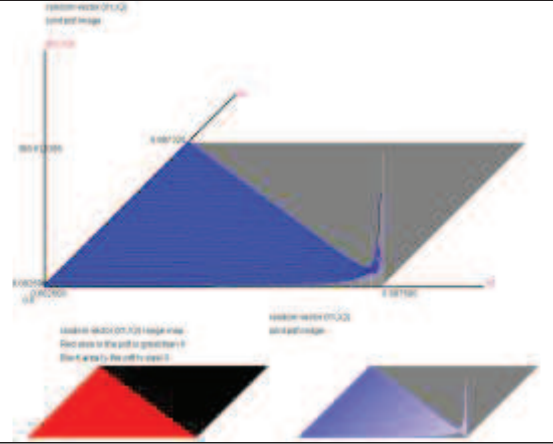
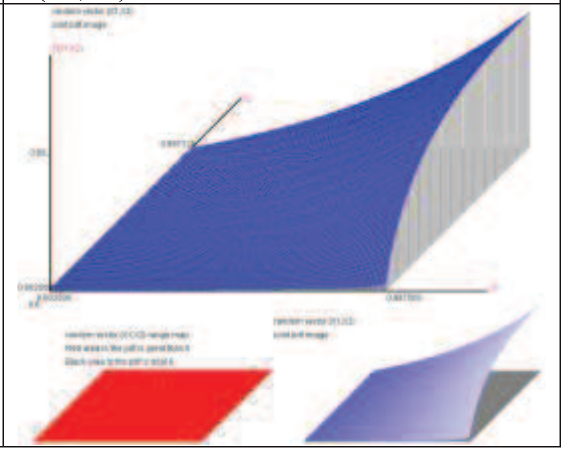
E(X1)= 0.3880, Var(X1)= 0.0759, E(X2)= 0.3761, Var(X2)= 0.0673, Cov(X1,X2)= -0.0507, X1 and X2 correlation coefficient=-0.7095.

f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.38805 Geometrical Mean : 0.25584 Harmonic Mean : 0.02875 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : 0.47583 Kurtosis Coef. : 2.11562 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.33911 Q1 : 0.14979 Q2 : 0.33911 Q3 : 0.59636 IQR : 0.44657 C.V. : 0.70994

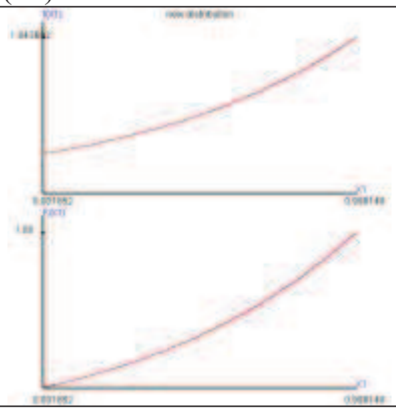
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.37613 Geometrical Mean : 0.24137 Harmonic Mean : 0.00425 Variance : 0.06728 S.D. : 0.25938 Skewed Coef. : 0.34525 Kurtosis Coef. : 2.02820 MAD : 0.22193 Range : 0.99993 Mid_range : 0.49996 Median : 0.34740 Q1 : 0.14708 Q2 : 0.34740 Q3 : 0.57964 IQR : 0.43255 C.V. : 0.68960

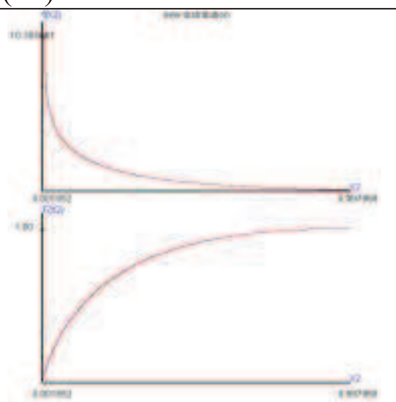
(2) $\lambda_1 = 0.8$,

(2-1) $\lambda_1 = 0.8$, $\lambda_2 = 0.1$,

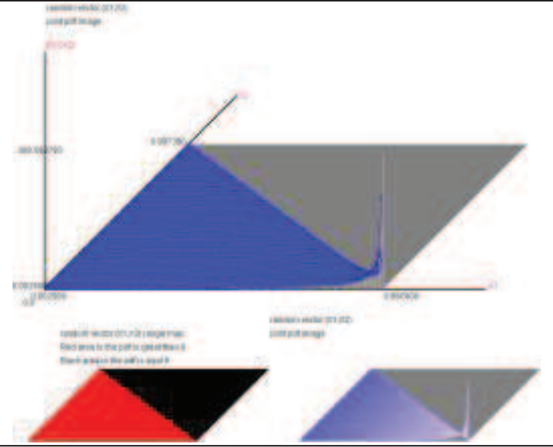
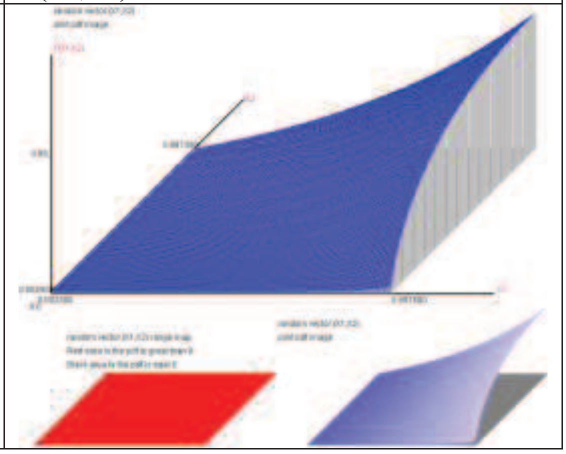
f(x1,x2)	F(x1,x2)
	

E(X1)= 0.6120, Var(X1)= 0.0759, E(X2)= 0.1940, Var(X2)= 0.0378,
Cov(X1,X2)= -0.0379, X1 and X2 correlation coefficient=-0.7081.

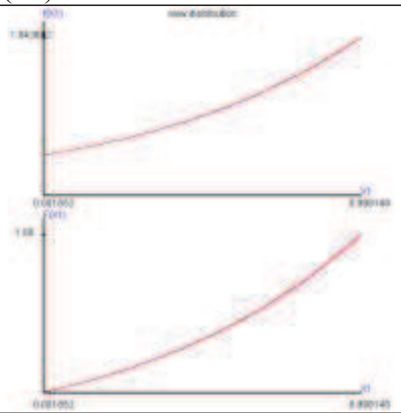
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.61202 Geometrical Mean : 0.50267 Harmonic Mean : 0.10548 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : -0.47630 Kurtosis Coef. : 2.11604 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.66104 Q1 : 0.40370 Q2 : 0.66104 Q3 : 0.85024 IQR : 0.44654 C.V. : 0.45012

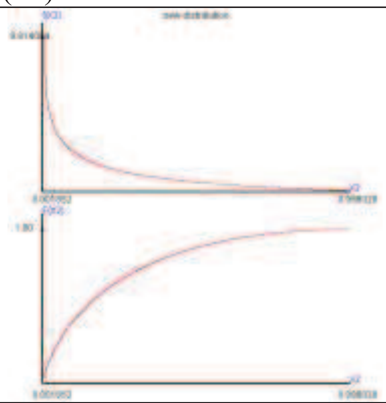
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.19398 Geometrical Mean : 0.09409 Harmonic Mean : 0.00107 Variance : 0.03785 S.D. : 0.19454 Skewed Coef. : 1.33850 Kurtosis Coef. : 4.30585 MAD : 0.15333 Range : 0.99982 Mid_range : 0.49991 Median : 0.12731 Q1 : 0.04283 Q2 : 0.12731 Q3 : 0.28763 IQR : 0.24480 C.V. : 1.00289

(2-2) $\lambda_1 = 0.8$, $\lambda_2 = 0.15$,

f(x1,x2)	F(x1,x2)
	

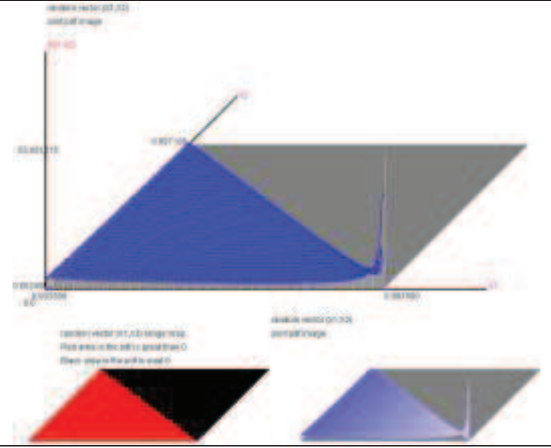
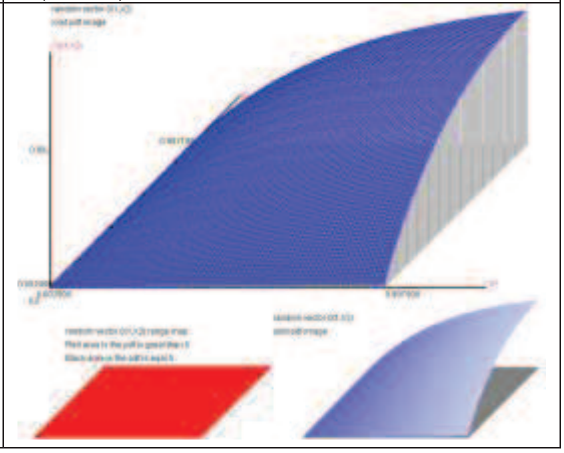
E(X1)= 0.6120, Var(X1)= 0.0759, E(X2)= 0.2145, Var(X2)= 0.0440,
Cov(X1,X2)= -0.0442, X1 and X2 correlation coefficient=-0.7640.

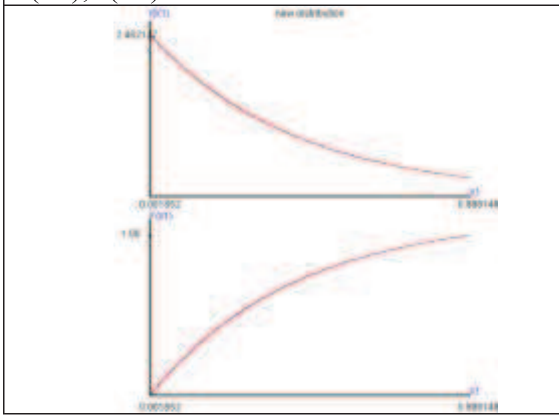
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.61202 Geometrical Mean : 0.50267 Harmonic Mean : 0.10548 Variance : 0.07589 S.D. : 0.27549 Skewed Coef. : -0.47630 Kurtosis Coef. : 2.11604 MAD : 0.23443 Range : 1.00000 Mid_range : 0.50000 Median : 0.66104 Q1 : 0.40370 Q2 : 0.66104 Q3 : 0.85024 IQR : 0.44654 C.V. : 0.45012

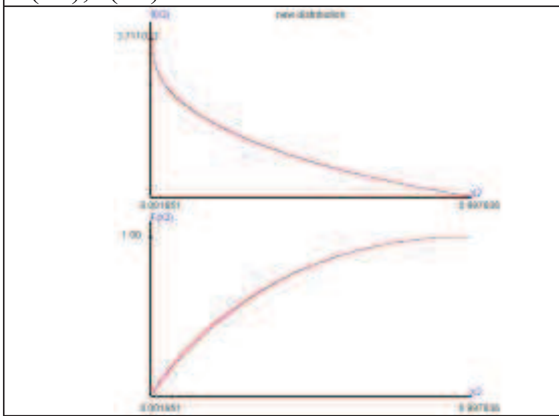
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.21449 Geometrical Mean : 0.10420 Harmonic Mean : 0.00108 Variance : 0.04403 S.D. : 0.20983 Skewed Coef. : 1.20316 Kurtosis Coef. : 3.77074 MAD : 0.16818 Range : 0.99988 Mid_range : 0.49994 Median : 0.14351 Q1 : 0.04730 Q2 : 0.14351 Q3 : 0.32456 IQR : 0.27726 C.V. : 0.97828

(3) $\lambda_1 = 0.1$,

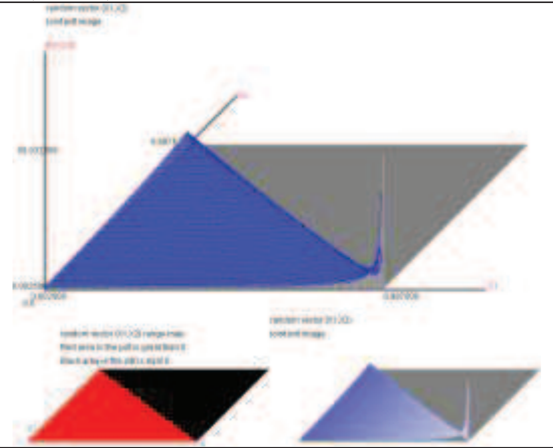
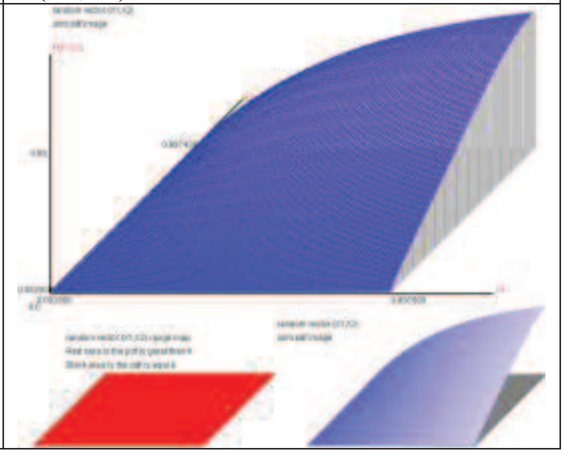
(3-1) $\lambda_1 = 0.1$, $\lambda_2 = 0.2$,

f(x1,x2)	F(x1,x2)
	

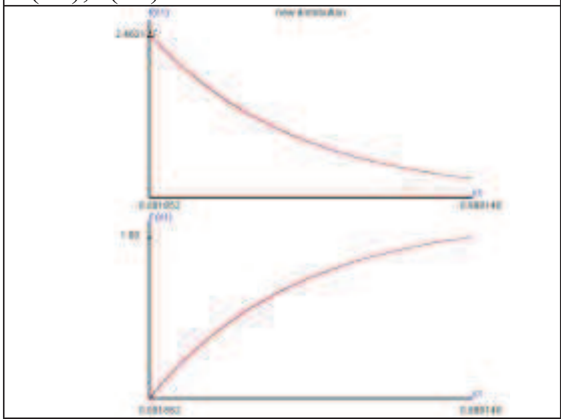
f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33015 Geometrical Mean : 0.20663 Harmonic Mean : 0.02160 Variance : 0.06650 S.D. : 0.25788 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.21453 Range : 1.00000 Mid_range : 0.50000 Median : 0.26758 Q1 : 0.11439 Q2 : 0.26758 Q3 : 0.50004 IQR : 0.38565 C.V. : 0.78112

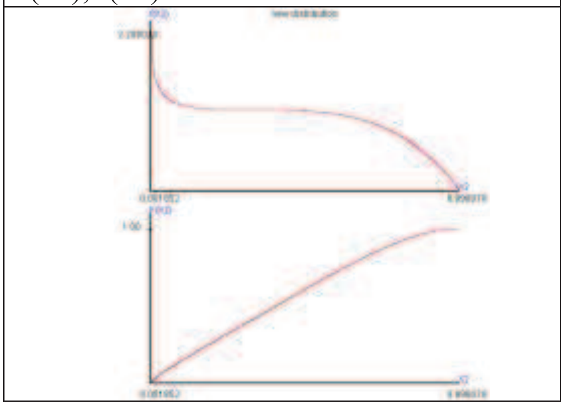
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.28207 Geometrical Mean : 0.17158 Harmonic Mean : 0.00592 Variance : 0.05050 S.D. : 0.22473 Skewed Coef. : 0.80888 Kurtosis Coef. : 2.82808 MAD : 0.18555 Range : 0.99969 Mid_range : 0.49984 Median : 0.22860 Q1 : 0.09460 Q2 : 0.22860 Q3 : 0.42639 IQR : 0.33180 C.V. : 0.79672

(3-2) $\lambda_1 = 0.1$, $\lambda_2 = 0.8$,

f(x1,x2)	F(x1,x2)
	

E(X1)= 0.3301, Var(X1)= 0.0665, E(X2)= 0.4200, Var(X2)= 0.0687,
Cov(X1,X2)= -0.0457, X1 and X2 correlation coefficient=-0.6755.

f(x1),F(x1)	Coefficient
	Mathematical Mean: 0.33015 Geometrical Mean : 0.20663 Harmonic Mean : 0.02160 Variance : 0.06650 S.D. : 0.25788 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.21453 Range : 1.00000 Mid_range : 0.50000 Median : 0.26758 Q1 : 0.11439 Q2 : 0.26758 Q3 : 0.50004 IQR : 0.38565 C.V. : 0.78112

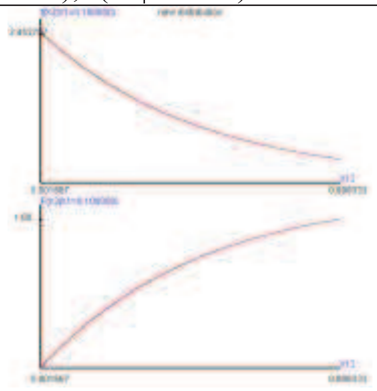
f(x2),F(x2)	Coefficient
	Mathematical Mean: 0.42004 Geometrical Mean : 0.29002 Harmonic Mean : 0.00700 Variance : 0.06870 S.D. : 0.26212 Skewed Coef. : 0.16096 Kurtosis Coef. : 1.92988 MAD : 0.22469 Range : 0.99993 Mid_range : 0.49996 Median : 0.40951 Q1 : 0.19270 Q2 : 0.40951 Q3 : 0.63207 IQR : 0.43937 C.V. : 0.62402

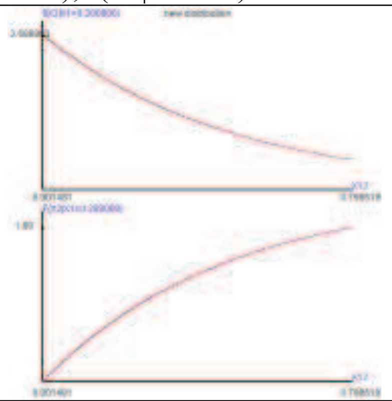
Section 4. The conditional probability density function image

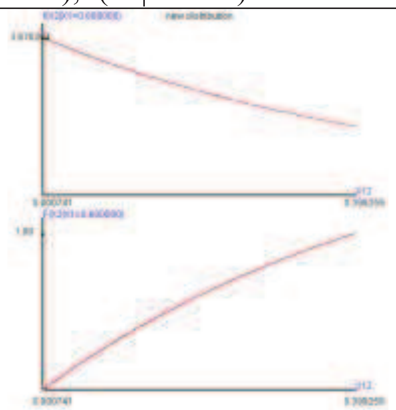
$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1$, f(x2|x1)=?

(1) $\lambda_1 = 0.1$,

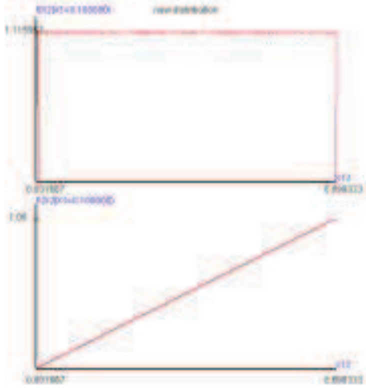
(1-1) $\lambda_2 = 0.1, \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{1}{9}$,

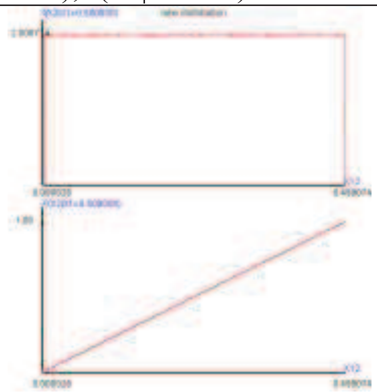
f(x2|x1=0.1),F(x2|x1=0.1)	Coefficient	
	Mathematical Mean:	0.31723
	Geometrical Mean :	0.20243
	Harmonic Mean :	0.02229
	Variance :	0.05714
	S.D. :	0.23905
	Skewed Coef. :	0.63762
	Kurtosis Coef. :	2.37073
	MAD :	0.20082
	Range :	0.90000
	Mid_range :	0.45000
	Median :	0.26455
	Q1 :	0.11431
	Q2 :	0.26455
	Q3 :	0.48420
	IQR :	0.36989
	C.V. :	0.75353

f(x2|x1=0.2),F(x2|x1=0.2)	Coefficient	
	Mathematical Mean:	0.29393
	Geometrical Mean :	0.19010
	Harmonic Mean :	0.02133
	Variance :	0.04669
	S.D. :	0.21609
	Skewed Coef. :	0.56877
	Kurtosis Coef. :	2.25259
	MAD :	0.18259
	Range :	0.80000
	Mid_range :	0.40000
	Median :	0.24994
	Q1 :	0.10891
	Q2 :	0.24994
	Q3 :	0.45030
	IQR :	0.34139
	C.V. :	0.73518

f(x2|x1=0.6),F(x2|x1=0.6)	Coefficient	
	Mathematical Mean:	0.17260
	Geometrical Mean :	0.11864
	Harmonic Mean :	0.01474
	Variance :	0.01288
	S.D. :	0.11351
	Skewed Coef. :	0.28704
	Kurtosis Coef. :	1.91416
	MAD :	0.09766
	Range :	0.40000
	Mid_range :	0.20000
	Median :	0.15958
	Q1 :	0.07320
	Q2 :	0.15958
	Q3 :	0.26494
	IQR :	0.19173
	C.V. :	0.65764

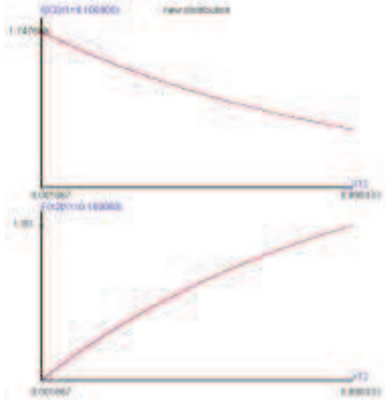
(1-2) $\lambda_2 = 0.45, \frac{\lambda_2}{1-\lambda_1} = \frac{1}{2}$,

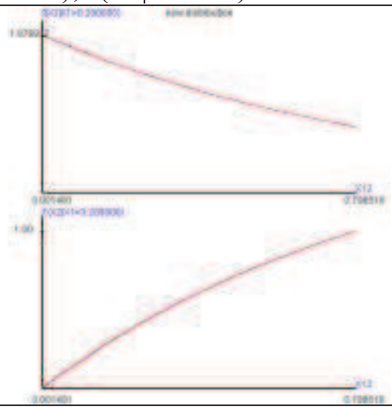
f(x2|x1=0.1),F(x2|x1=0.1)	Coefficient
	Mathematical Mean: 0.45004 Geometrical Mean : 0.33111 Harmonic Mean : 0.04807 Variance : 0.06750 S.D. : 0.25981 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.22500 Range : 0.90000 Mid_range : 0.45000 Median : 0.45006 Q1 : 0.22503 Q2 : 0.45006 Q3 : 0.67506 IQR : 0.45003 C.V. : 0.57731

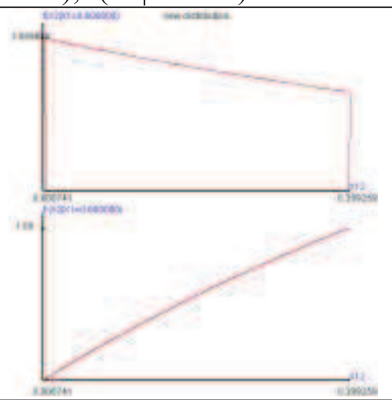
f(x2|x1=0.5),F(x2|x1=0.5)	Coefficient
	Mathematical Mean: 0.25002 Geometrical Mean : 0.18395 Harmonic Mean : 0.02671 Variance : 0.02083 S.D. : 0.14434 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.12500 Range : 0.50000 Mid_range : 0.25000 Median : 0.25004 Q1 : 0.12502 Q2 : 0.25004 Q3 : 0.37503 IQR : 0.25002 C.V. : 0.57731

(2) $\lambda_1 = 0.8$,

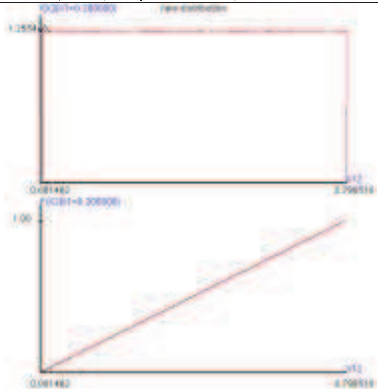
(2-1) $\lambda_2 = 0.05, \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{0.05}{2} = 0.4$,

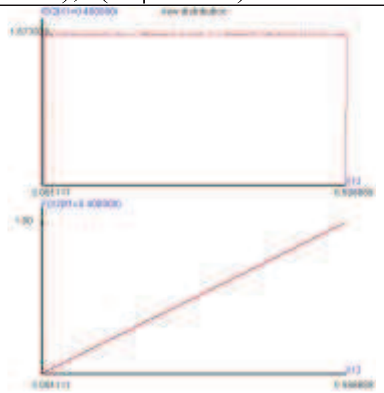
f(x2|x1=0.1),F(x2|x1=0.1)	Coefficient		
	Mathematical Mean:		0.37706
	Geometrical Mean	:	0.25602
	Harmonic Mean	:	0.03109
	Variance	:	0.06433
	S.D.	:	0.25363
	Skewed Coef.	:	0.34081
	Kurtosis Coef.	:	1.96115
	MAD	:	0.21763
	Range	:	0.90000
	Mid_range	:	0.45000
	Median	:	0.34308
	Q1	:	0.15547
	Q2	:	0.34308
	Q3	:	0.57963
	IQR	:	0.42416
	C.V.	:	0.67264

f(x2|x1=0.2),F(x2|x1=0.2)	Coefficient		
	Mathematical Mean:		0.34218
	Geometrical Mean	:	0.23433
	Harmonic Mean	:	0.02891
	Variance	:	0.05134
	S.D.	:	0.22658
	Skewed Coef.	:	0.30320
	Kurtosis Coef.	:	1.92742
	MAD	:	0.19479
	Range	:	0.80000
	Mid_range	:	0.40000
	Median	:	0.31485
	Q1	:	0.14388
	Q2	:	0.31485
	Q3	:	0.52551
	IQR	:	0.38163
	C.V.	:	0.66215

f(x2|x1=0.6),F(x2|x1=0.6)	Coefficient		
	Mathematical Mean:		0.18541
	Geometrical Mean	:	0.13154
	Harmonic Mean	:	0.01746
	Variance	:	0.01321
	S.D.	:	0.11492
	Skewed Coef.	:	0.15188
	Kurtosis Coef.	:	1.83190
	MAD	:	0.09934
	Range	:	0.40000
	Mid_range	:	0.20000
	Median	:	0.17823
	Q1	:	0.08476
	Q2	:	0.17823
	Q3	:	0.28239
	IQR	:	0.19763
	C.V.	:	0.61978

(2-2) $\lambda_2 = 0.1,\ \dfrac{\lambda_2}{1-\lambda_1} = \dfrac{1}{2} = 0.5$,

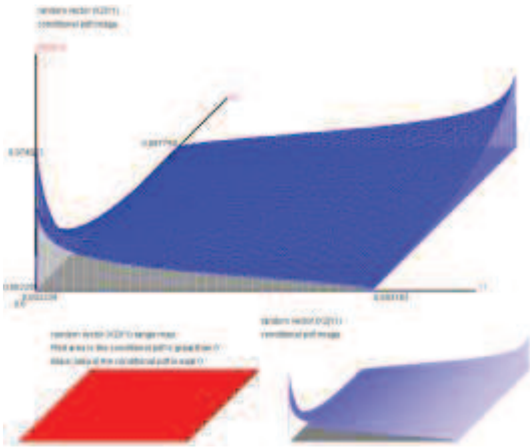
f(x2|x1=0.2),F(x2|x1=0.2)	Coefficient
	Mathematical Mean: 0.40003 Geometrical Mean : 0.29432 Harmonic Mean : 0.04273 Variance : 0.05333 S.D. : 0.23094 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.20000 Range : 0.80000 Mid_range : 0.40000 Median : 0.40006 Q1 : 0.20003 Q2 : 0.40006 Q3 : 0.60005 IQR : 0.40002 C.V. : 0.57731

f(x2|x1=0.4),F(x2|x1=0.4)	Coefficient
	Mathematical Mean: 0.30002 Geometrical Mean : 0.22074 Harmonic Mean : 0.03205 Variance : 0.03000 S.D. : 0.17321 Skewed Coef. : -0.00022 Kurtosis Coef. : 1.80001 MAD : 0.15000 Range : 0.60000 Mid_range : 0.30000 Median : 0.30004 Q1 : 0.15002 Q2 : 0.30004 Q3 : 0.45004 IQR : 0.30002 C.V. : 0.57731

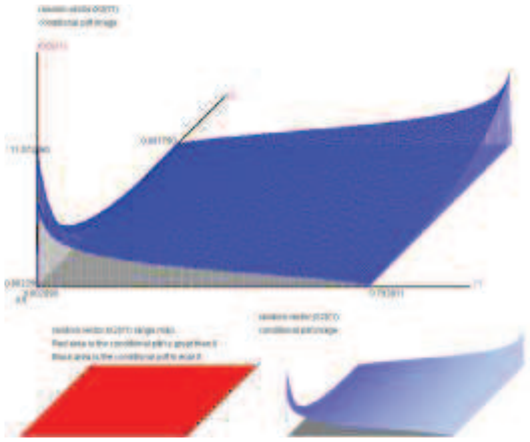
(3) 3D image of $X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1)$ given x_1 and λ_1 are known,

(3-1)f(X2|Y1= λ_2) when x1=0.1,

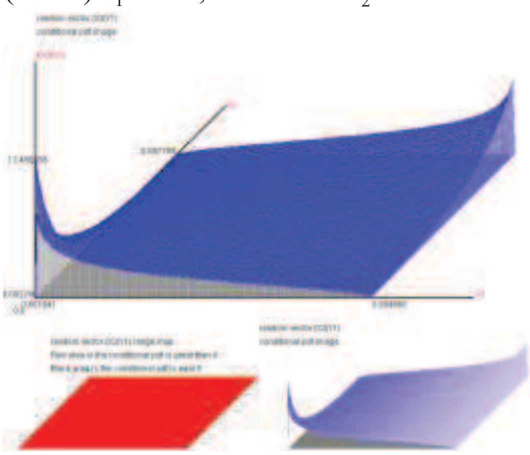
(3-1-1) $\lambda_1 = 0.1$, 0.0001<= λ_2 <=0.8999



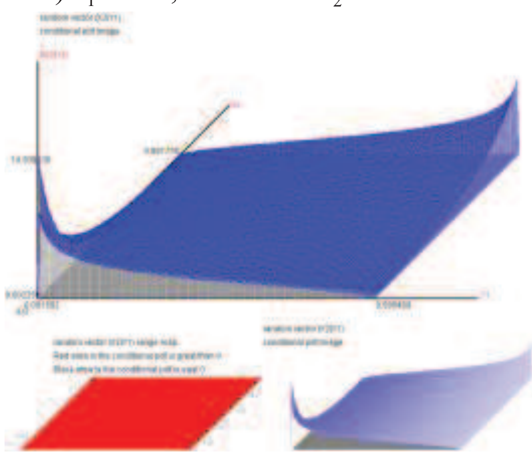
(3-1-2) $\lambda_1 = 0.2$, 0.0001<= λ_2 <=0.7999



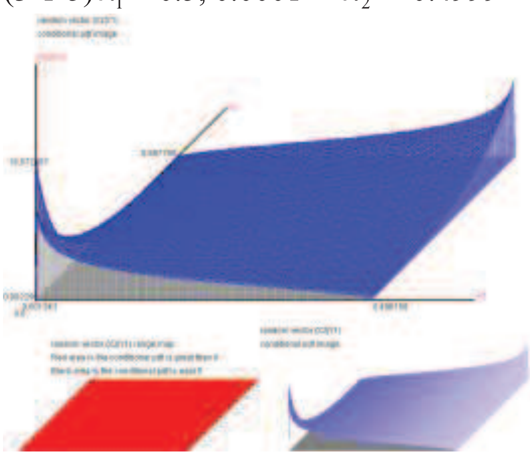
(3-1-3) $\lambda_1 = 0.3$, 0.0001<= λ_2 <=0.6999



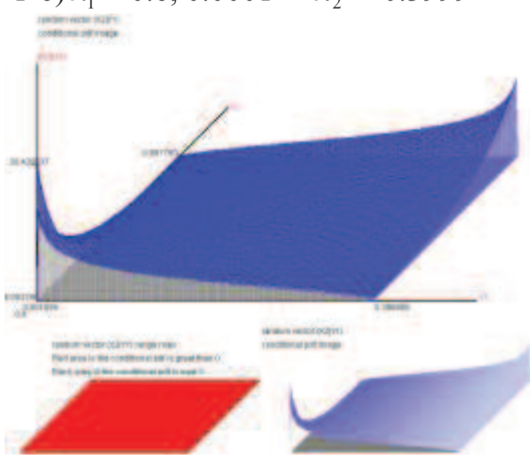
(3-1-4) $\lambda_1 = 0.4$, 0.0001<= λ_2 <=0.5999



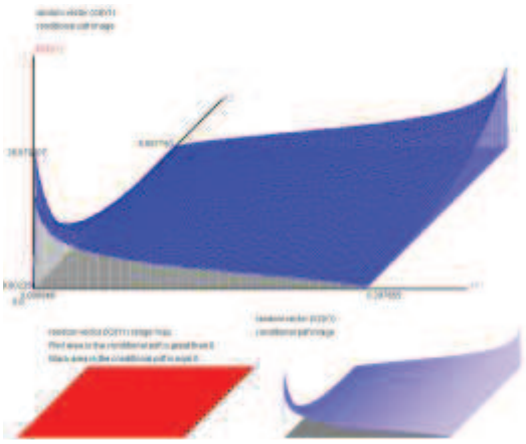
(3-1-5) $\lambda_1 = 0.5$, 0.0001<= λ_2 <=0.4999



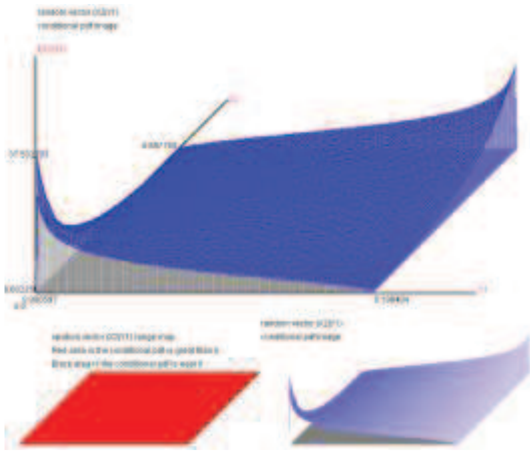
(3-1-6) $\lambda_1 = 0.6$, 0.0001<= λ_2 <=0.3999



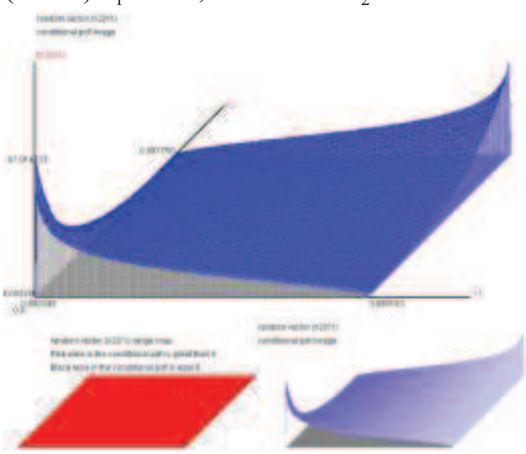
(3-1-7) $\lambda_1 = 0.7$, $0.0001 <= \lambda_2 <= 0.2999$



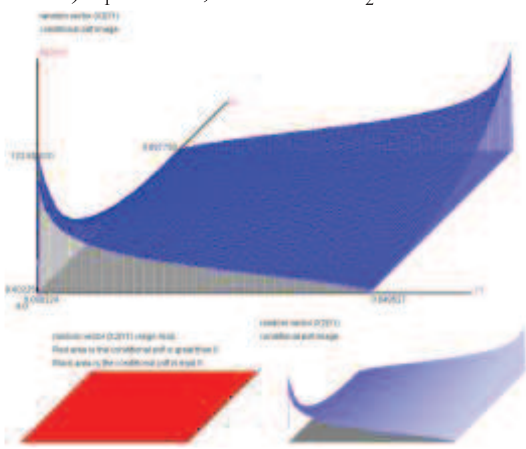
(3-1-8) $\lambda_1 = 0.8$, $0.0001 <= \lambda_2 <= 0.1999$



(3-1-9) $\lambda_1 = 0.9$, $0.0001 <= \lambda_2 <= 0.0999$

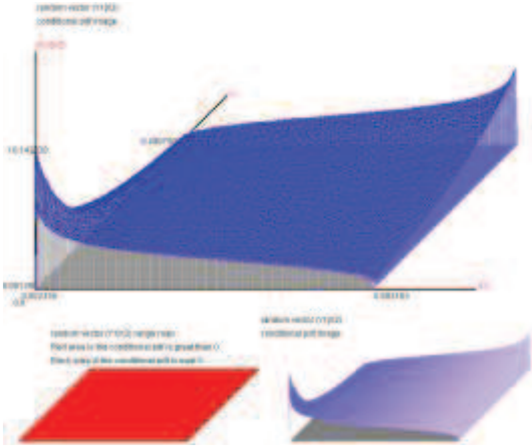


(3-1-10) $\lambda_1 = 0.95$, $0.0001 <= \lambda_2 <= 0.0499$

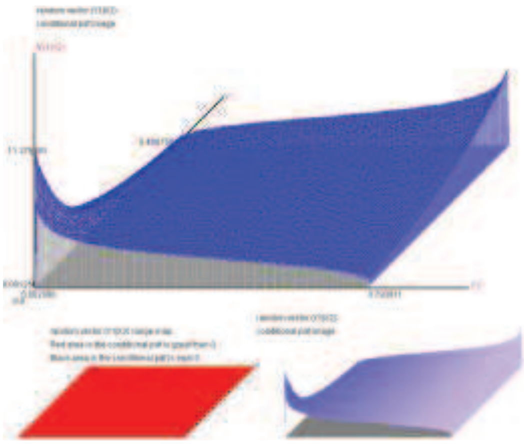


(3-2)f(X2|Y1= λ_2) when x1=0.5,

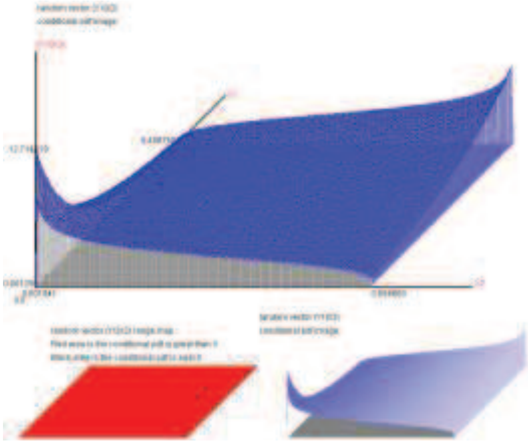
(3-2-1) $\lambda_1 = 0.1$, 0.0001<= λ_2 <=0.8999



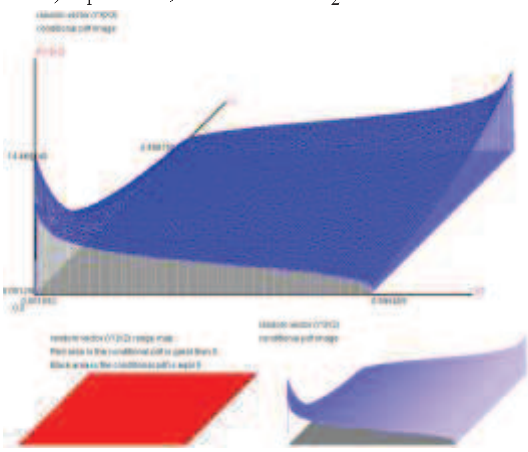
(3-2-2) $\lambda_1 = 0.2$, 0.0001<= λ_2 <=0.7999



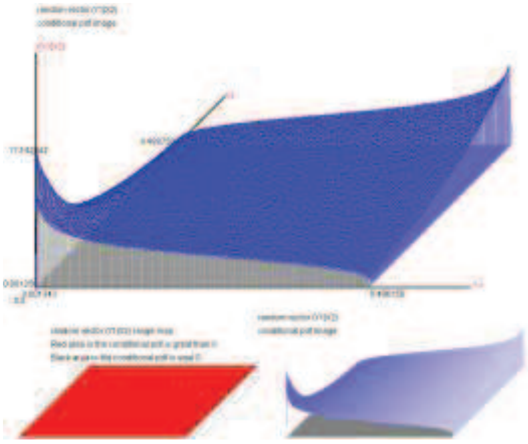
(3-2-3) $\lambda_1 = 0.3$, 0.0001<= λ_2 <=0.6999



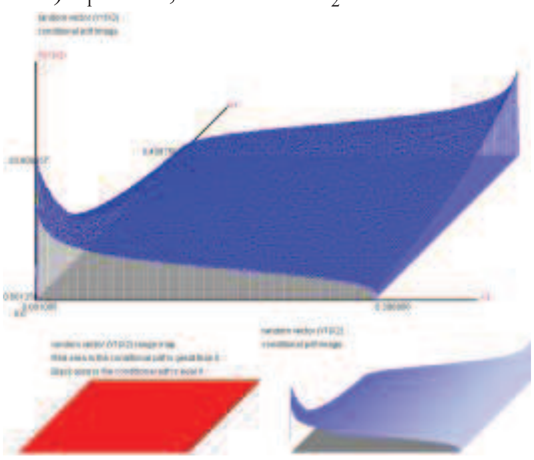
(3-2-4) $\lambda_1 = 0.4$, 0.0001<= λ_2 <=0.5999



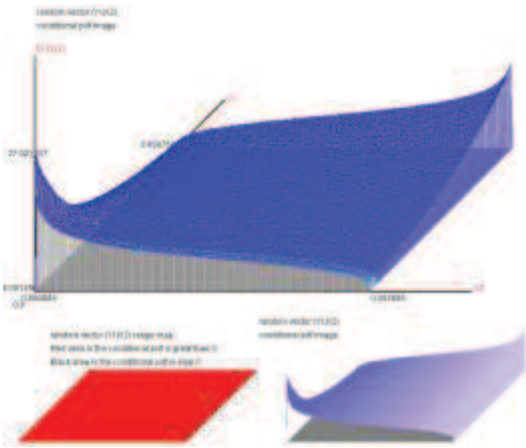
(3-2-5) $\lambda_1 = 0.5$, 0.0001<= λ_2 <=0.4999



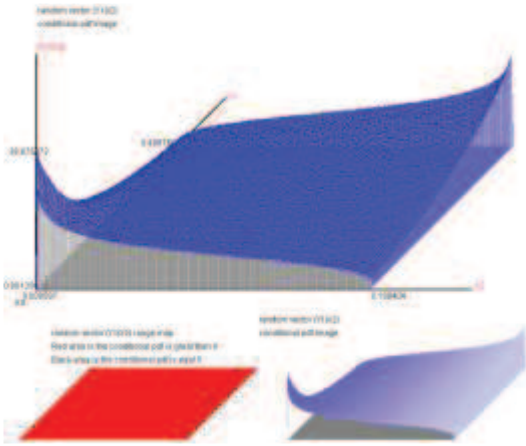
(3-2-6) $\lambda_1 = 0.6$, 0.0001<= λ_2 <=0.3999



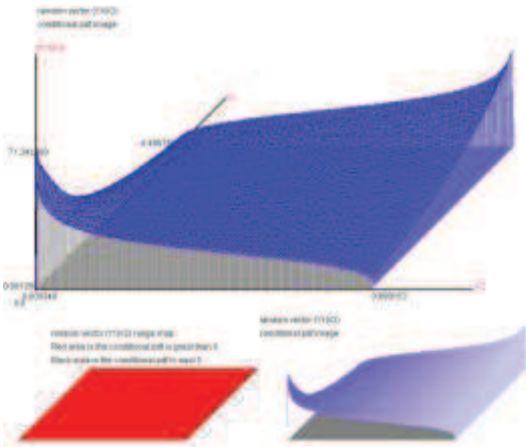
(3-2-7) $\lambda_1 = 0.7$, $0.0001 <= \lambda_2 <= 0.2999$



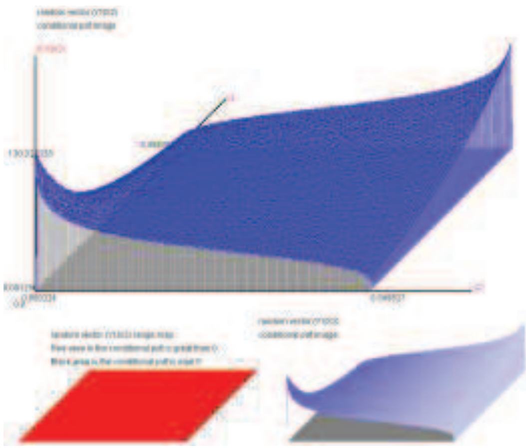
(3-2-8) $\lambda_1 = 0.8$, $0.0001 <= \lambda_2 <= 0.1999$



(3-2-9) $\lambda_1 = 0.9$, $0.0001 <= \lambda_2 <= 0.0999$

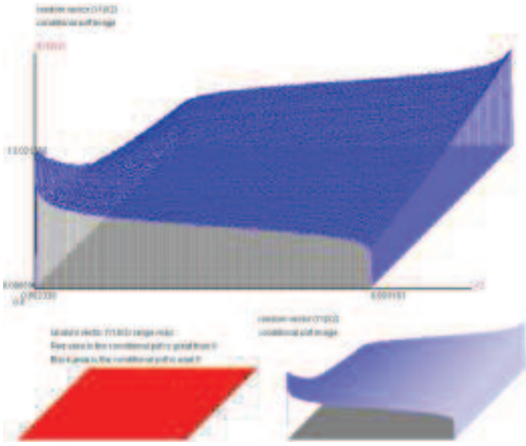


(3-2-10) $\lambda_1 = 0.95$, $0.0001 <= \lambda_2 <= 0.0499$

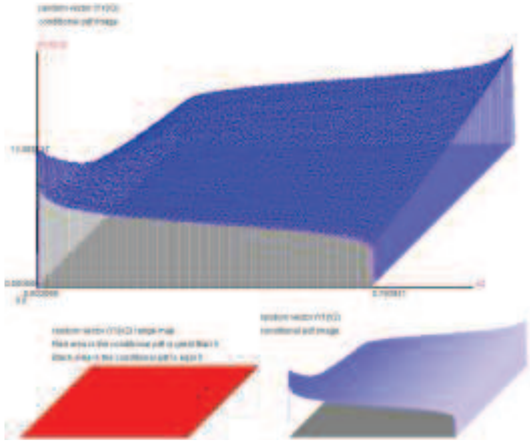


(3-3)f(X2|Y1= λ_2) when x1=0.8,

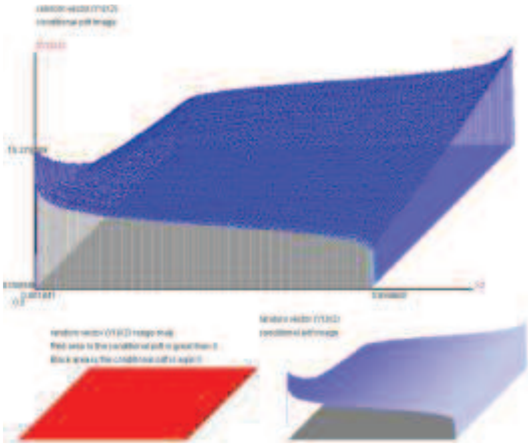
(3-3-1) $\lambda_1 = 0.1$, 0.0001<= λ_2 <=0.8999



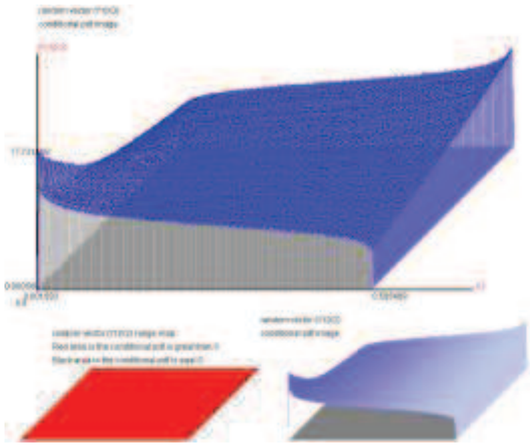
(3-3-2) $\lambda_1 = 0.2$, 0.0001<= λ_2 <=0.7999



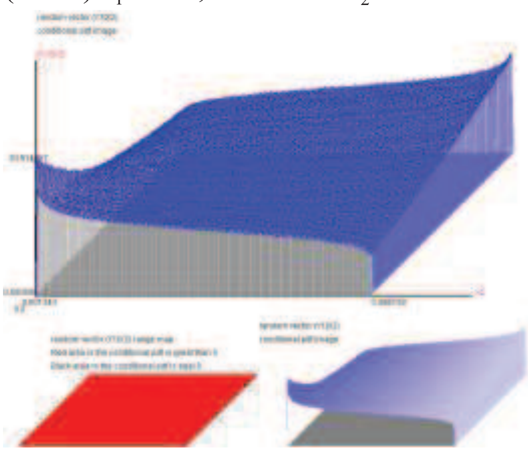
(3-3-3) $\lambda_1 = 0.3$, 0.0001<= λ_2 <=0.6999



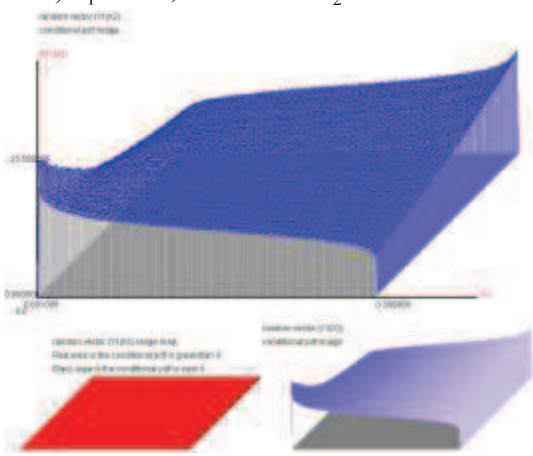
(3-3-4) $\lambda_1 = 0.4$, 0.0001<= λ_2 <=0.5999



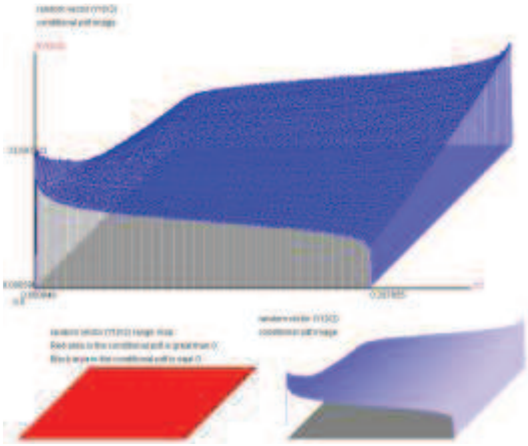
(3-3-5) $\lambda_1 = 0.5$, 0.0001<= λ_2 <=0.4999



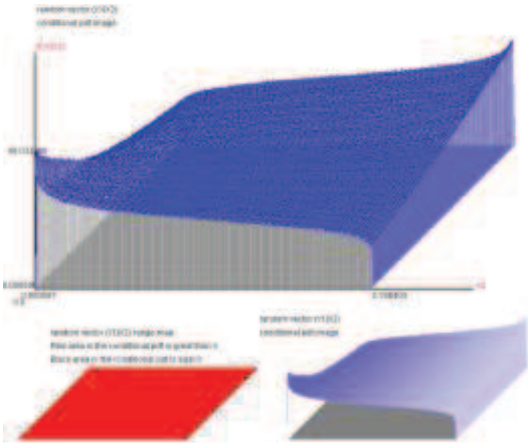
(3-3-6) $\lambda_1 = 0.6$, 0.0001<= λ_2 <=0.3999



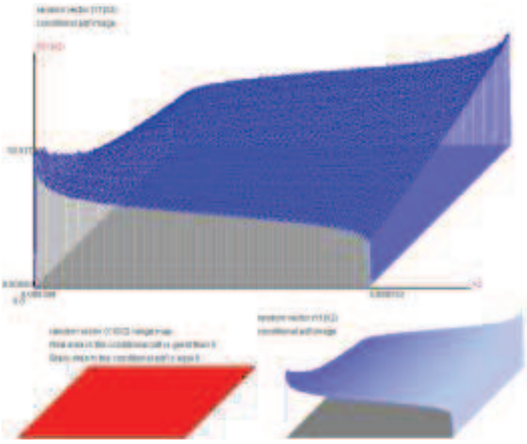
(3-3-7) $\lambda_1 = 0.7$, $0.0001 <= \lambda_2 <= 0.2999$



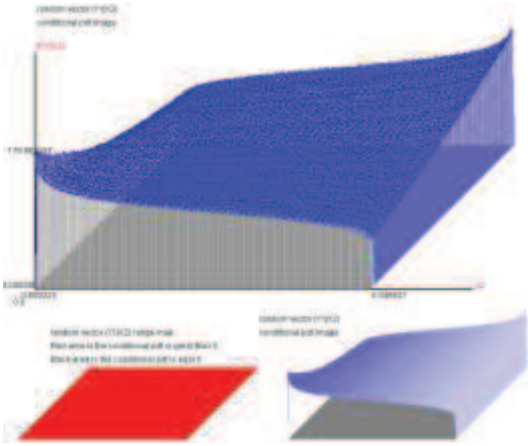
(3-3-8) $\lambda_1 = 0.8$, $0.0001 <= \lambda_2 <= 0.1999$



(3-3-9) $\lambda_1 = 0.9$, $0.0001 <= \lambda_2 <= 0.0999$



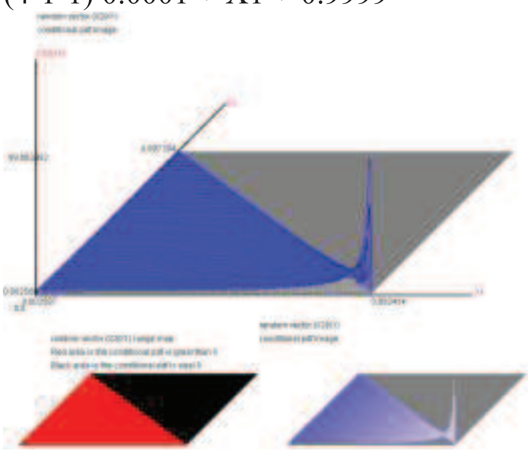
(3-3-10) $\lambda_1 = 0.95$, $0.0001 <= \lambda_2 <= 0.0499$



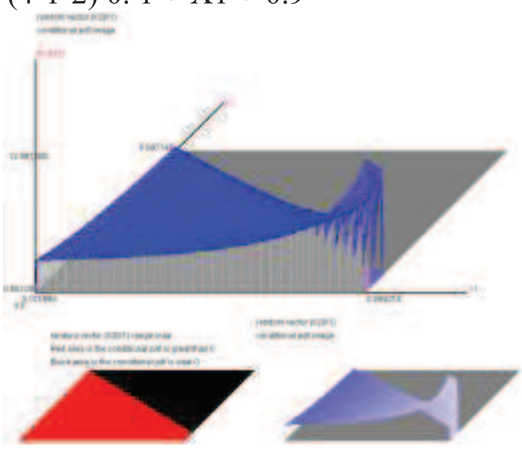
(4) 3D image of $X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1)$ given λ_1 and λ_2 are known,

(4-1)f(X2|X1), $\lambda_1 = 0.1$, λ_2=0.1,

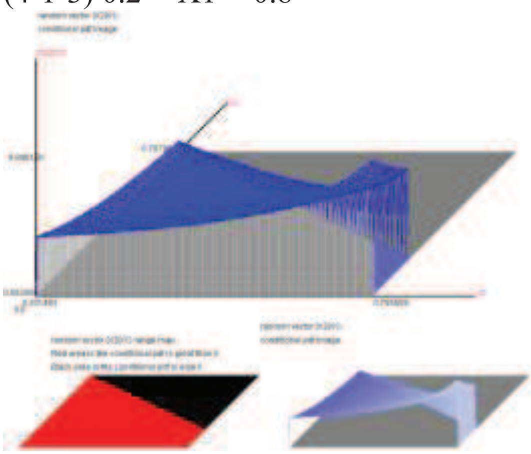
(4-1-1) 0.0001<=X1<=0.9999



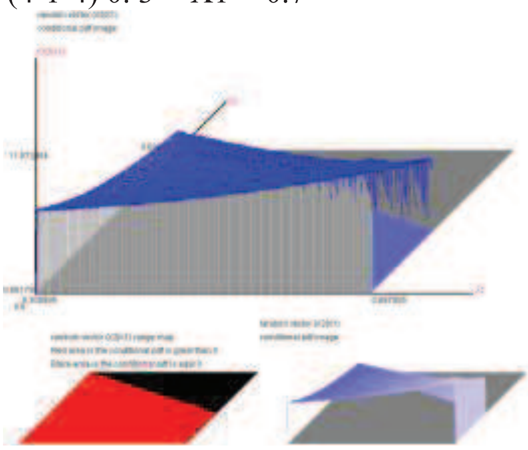
(4-1-2) 0. 1<=X1<=0.9



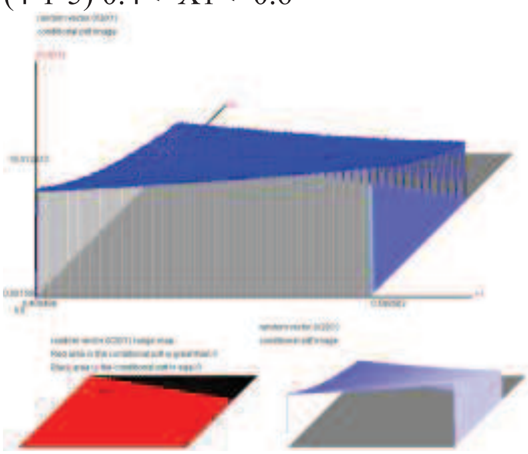
(4-1-3) 0.2<=X1<=0.8



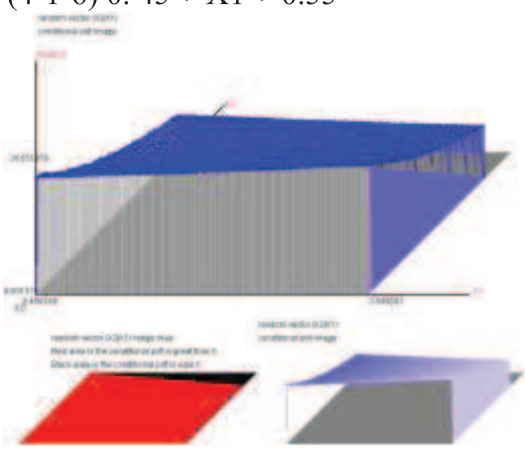
(4-1-4) 0. 3<=X1<=0.7



(4-1-5) 0.4<=X1<=0.6

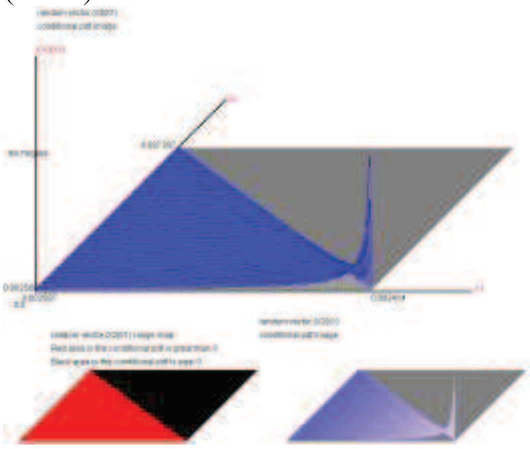


(4-1-6) 0. 45<=X1<=0.55

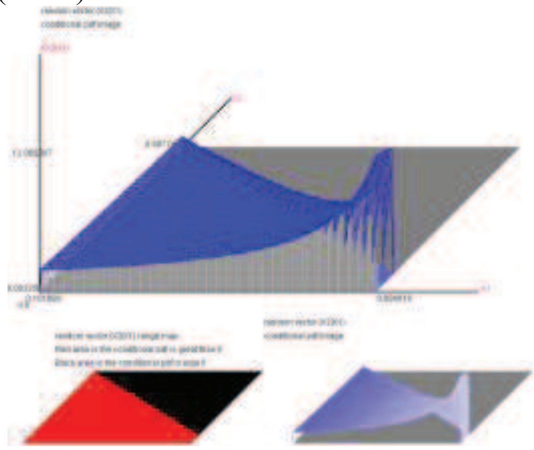


(4-2)f(X2|X1), $\lambda_1 = 0.1$, $\lambda_2 = 0.3$,

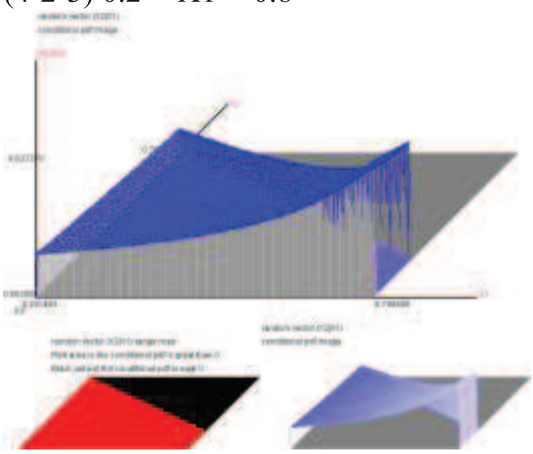
(4-2-1) 0.0001<=X1<=0.9999



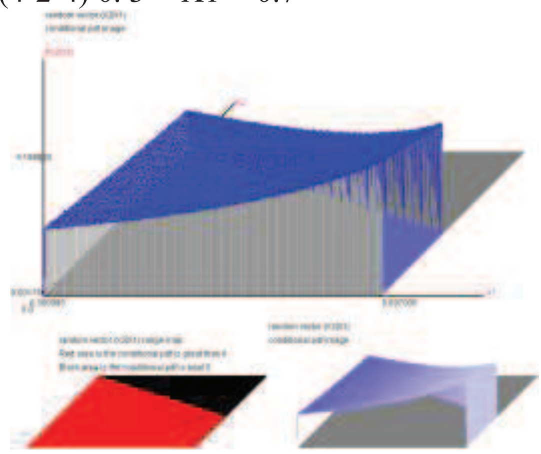
(4-2-2) 0. 1<=X1<=0.9



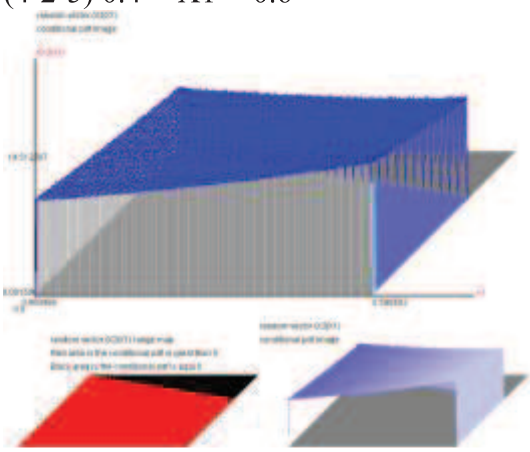
(4-2-3) 0.2<=X1<=0.8



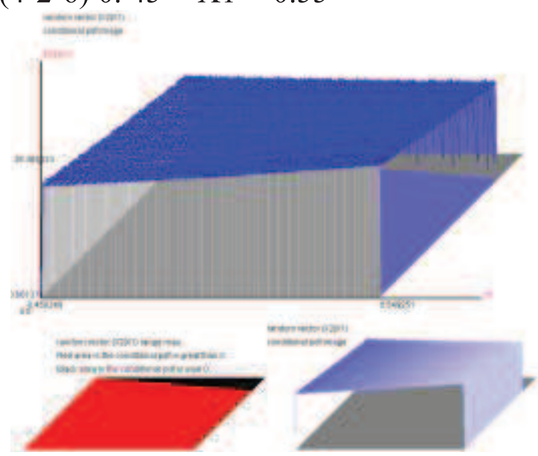
(4-2-4) 0. 3<=X1<=0.7



(4-2-5) 0.4<=X1<=0.6

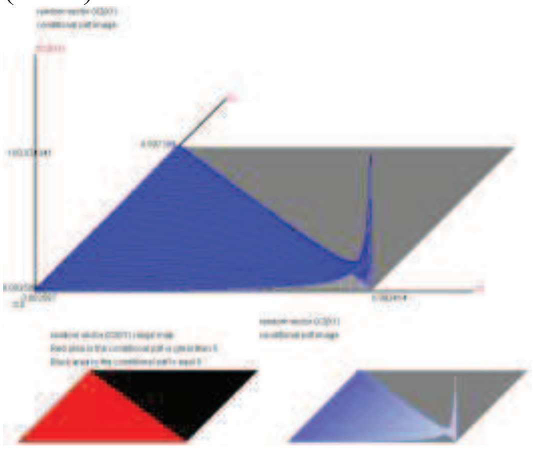


(4-2-6) 0. 45<=X1<=0.55

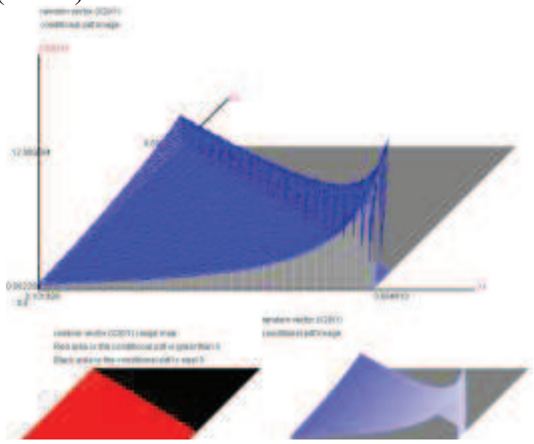


(4-3)f(X2|X1), $\lambda_1 = 0.1$, $\lambda_2 = 0.8$,

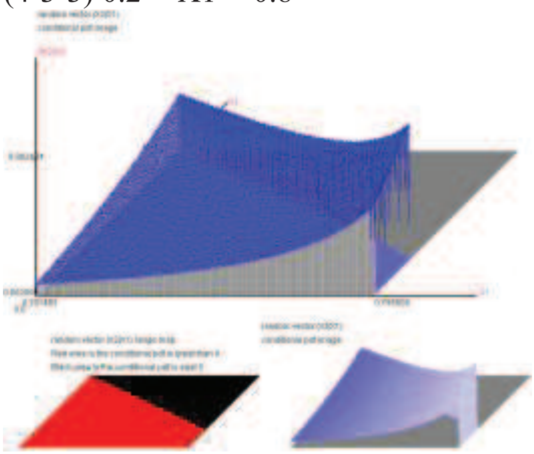
(4-3-1) 0.0001<=X1<=0.9999



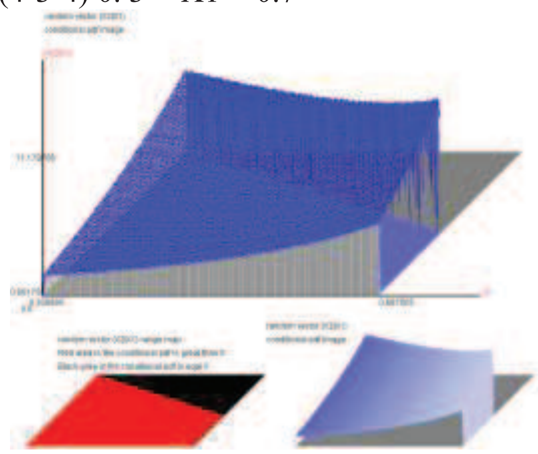
(4-3-2) 0. 1<=X1<=0.9



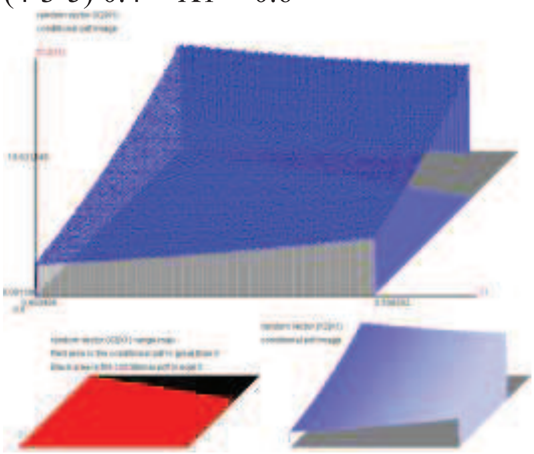
(4-3-3) 0.2<=X1<=0.8



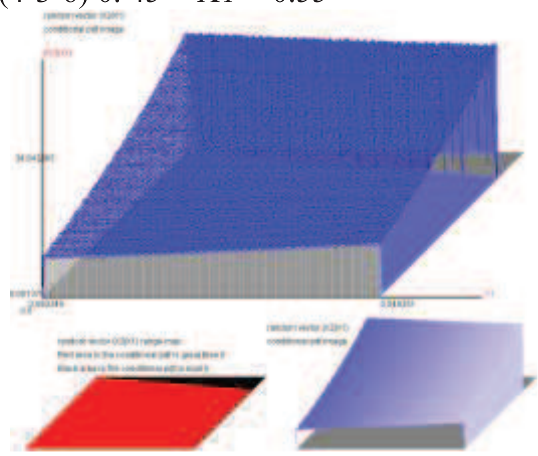
(4-3-4) 0. 3<=X1<=0.7



(4-3-5) 0.4<=X1<=0.6

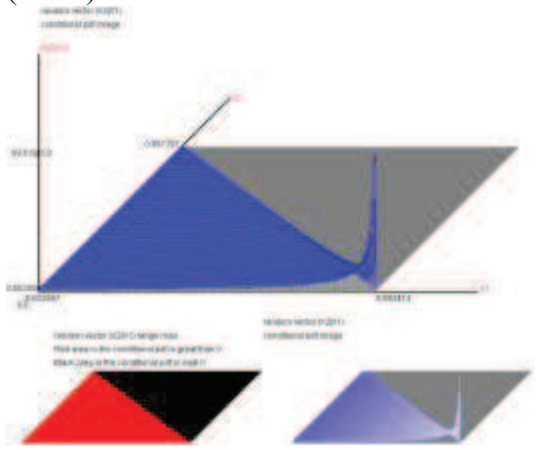


(4-3-6) 0. 45<=X1<=0.55

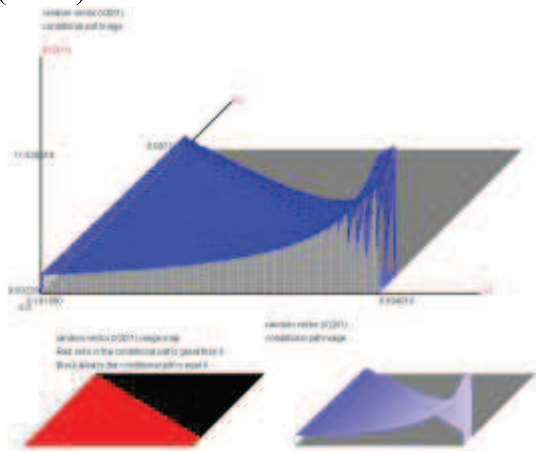


(4-4)f(X2|X1), $\lambda_1 = 0.5$, $\lambda_2 = 0.2$,

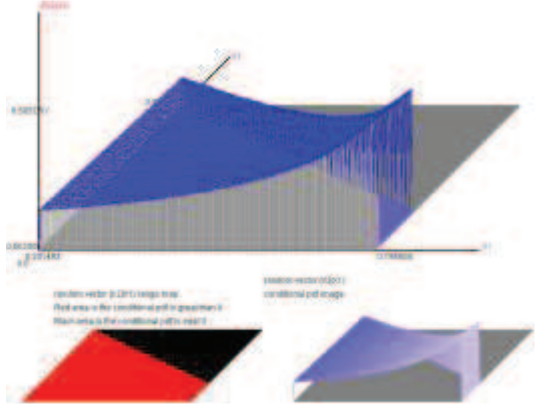
(4-4-1) 0.0001<=X1<=0.9999



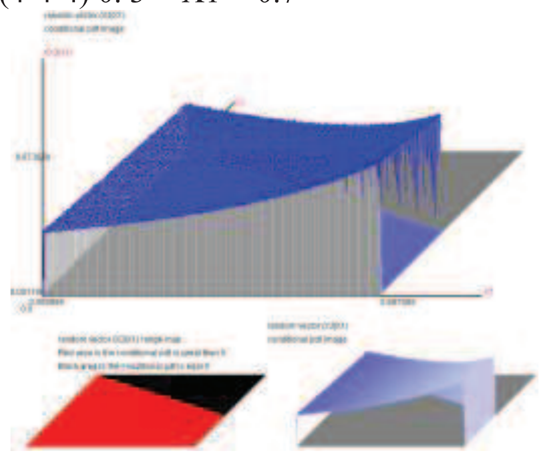
(4-4-2) 0. 1<=X1<=0.9



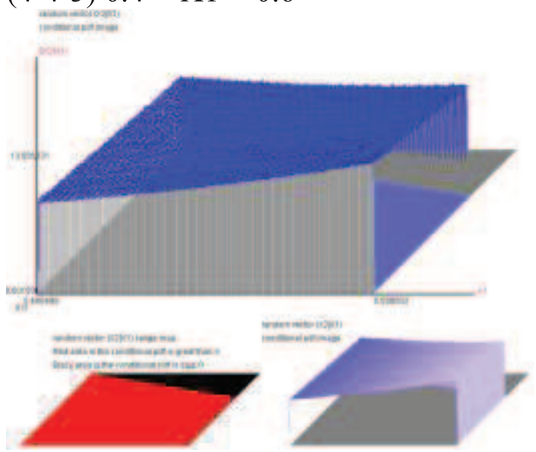
(4-4-3) 0.2<=X1<=0.8



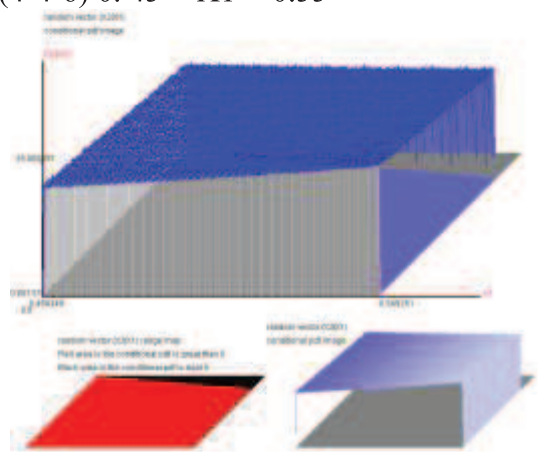
(4-4-4) 0. 3<=X1<=0.7



(4-4-5) 0.4<=X1<=0.6



(4-4-6) 0. 45<=X1<=0.55

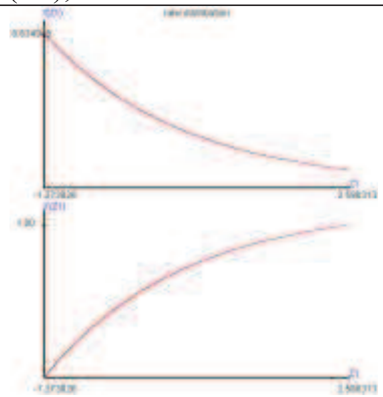


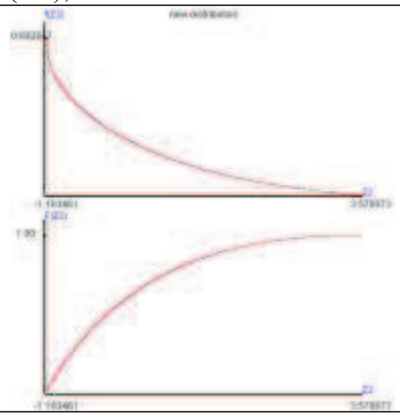
(5)The comparison of X_2 Z score and Y1 Z score

let $X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1 - x_1,$

$$Z_1 = \frac{Y_1 - E(Y_1)}{\sqrt{Var(Y_1)}}, Z_2 = \frac{X_2 - E(X_2|x_1)}{\sqrt{Var(X_2|x_1)}}, Z_2 = \frac{X_2 - E(X_2)}{\sqrt{Var(X_2)}}.$$

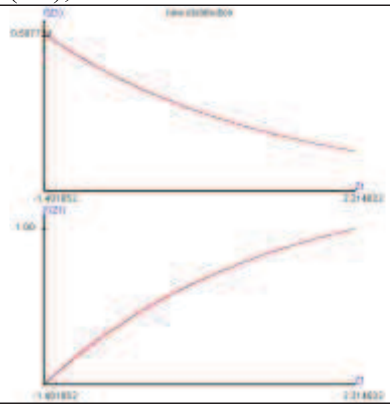
(5-1) λ_1=0.1, λ_2=0.1,

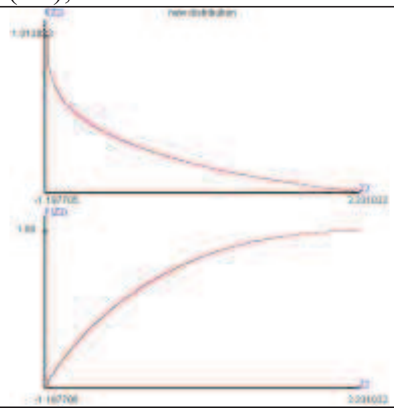
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.83188 Range : 3.87770 Mid_range : 0.65864 Median : -0.24262 Q1 : -0.83664 Q2 : -0.24262 Q3 : 0.65879 IQR : 1.49543 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.98206 Kurtosis Coef. : 3.29203 MAD : 0.81180 Range : 4.77181 Mid_range : 1.19361 Median : -0.26630 Q1 : -0.81302 Q2 : -0.26630 Q3 : 0.58623 IQR : 1.39925 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0068599911
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0003374610,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.985845,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.363515,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.172122,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.034049,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.017063,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.003351,

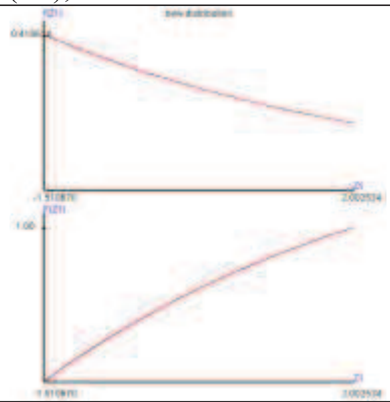
(5-2) λ_1 =0.2, λ_2 =0.2,

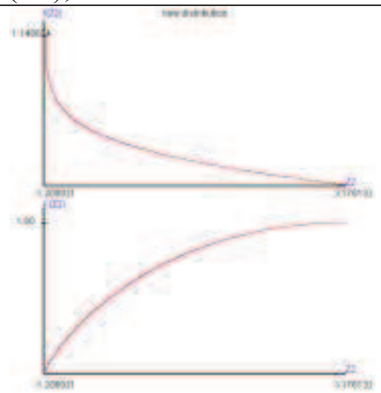
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.47583 Kurtosis Coef. : 2.11562 MAD : 0.85096 Range : 3.62992 Mid_range : 0.40639 Median : -0.17762 Q1 : -0.86483 Q2 : -0.17762 Q3 : 0.75616 IQR : 1.62100 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.89126 Kurtosis Coef. : 3.01149 MAD : 0.82066 Range : 4.54556 Mid_range : 1.06666 Median : -0.25727 Q1 : -0.82845 Q2 : -0.25727 Q3 : 0.61883 IQR : 1.44728 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0199326909
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0012142524,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.878938,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.166971,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.082982,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.016513,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.008250,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.001639,

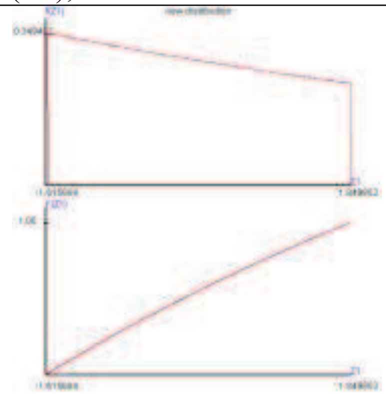
(5-3) λ_1 =0.3, λ_2 =0.3,

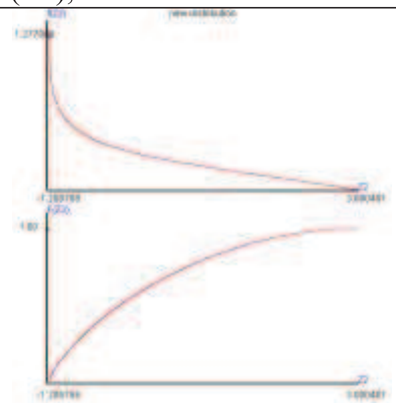
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.29234 Kurtosis Coef. : 1.91849 MAD : 0.86014 Range : 3.52626 Mid_range : 0.24593 Median : -0.11658 Q1 : -0.87560 Q2 : -0.11658 Q3 : 0.81192 IQR : 1.68752 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.83475 Kurtosis Coef. : 2.84011 MAD : 0.82684 Range : 4.39333 Mid_range : 0.98761 Median : -0.25284 Q1 : -0.84001 Q2 : -0.25284 Q3 : 0.64161 IQR : 1.48162 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0348105598
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0021054069,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 0.959591,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.853637,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.120045,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.059713,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.011935,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.005966,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.001198,

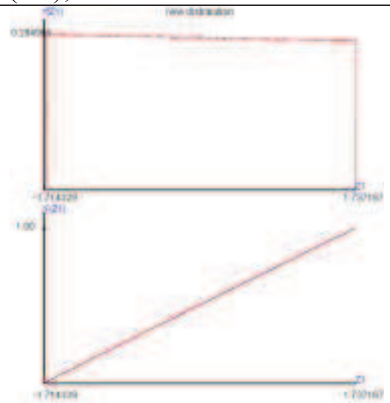
(5-4) λ_1 =0.4, λ_2 =0.4,

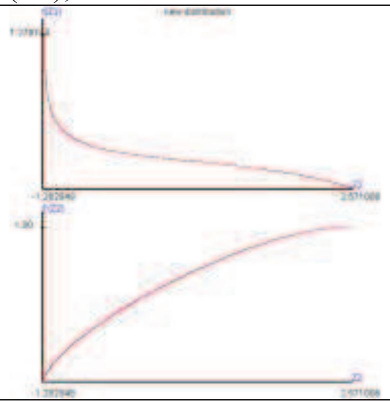
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.14012 Kurtosis Coef. : 1.82717 MAD : 0.86465 Range : 3.47842 Mid_range : 0.11708 Median : -0.05771 Q1 : -0.87560 Q2 : -0.05771 Q3 : 0.84597 IQR : 1.72157 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.76508 Kurtosis Coef. : 2.64778 MAD : 0.83427 Range : 4.22484 Mid_range : 0.89581 Median : -0.24503 Q1 : -0.85424 Q2 : -0.24503 Q3 : 0.67085 IQR : 1.52509 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0479945581
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0028121039,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 0.943466,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.560759,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.099733,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.049703,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.009913,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.004958,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.000997,

(5-5) λ_1 =0.49, λ_2 =0.49,

<table>
<tr><th>f(Z1),F(Z1),</th><th>Coefficinet</th></tr>
<tr><td></td><td>Mathematical Mean: -0.00000
Geometrical Mean : none
Harmonic Mean : none
Variance : 1.00000
S.D. : 1.00000
Skewed Coef. : 0.01362
Kurtosis Coef. : 1.80027
MAD : 0.86601
Range : 3.46433
Mid_range : 0.01142
Median : -0.00559
Q1 : -0.86750
Q2 : -0.00559
Q3 : 0.86453
IQR : 1.73202
C.V. : none</td></tr>
</table>

<table>
<tr><th>f(Z2),F(Z2),</th><th>Coefficinet</th></tr>
<tr><td></td><td>Mathematical Mean: 0.00000
Geometrical Mean : none
Harmonic Mean : none
Variance : 1.00000
S.D. : 1.00000
Skewed Coef. : 0.53567
Kurtosis Coef. : 2.18709
MAD : 0.85308
Range : 3.86828
Mid_range : 0.64403
Median : -0.18998
Q1 : -0.89143
Q2 : -0.18998
Q3 : 0.75605
IQR : 1.64748
C.V. : none</td></tr>
</table>

E(| Z2 distribution - Z1 distribution |^2)=0.0367248808
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0021895112,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 0.965039,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.669863,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.109825,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.054828,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.010936,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.005489,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.001094,

Section 5. The comparison of CB(λ1, λ2, x1) Z score and CB(λ*), λ* = λ2 / (1 - λ1) Z score

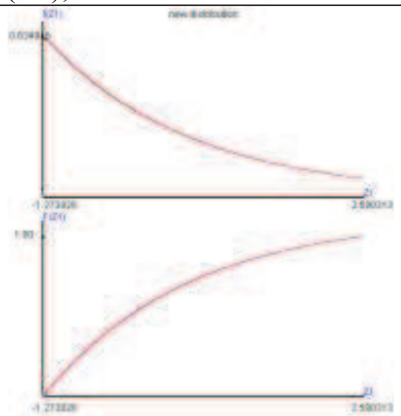
let $X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1)$ and $Y_1 \sim CB(\lambda^*)$ and $\lambda^* = \frac{\lambda_2}{1-\lambda_1}, = \lambda_1$.

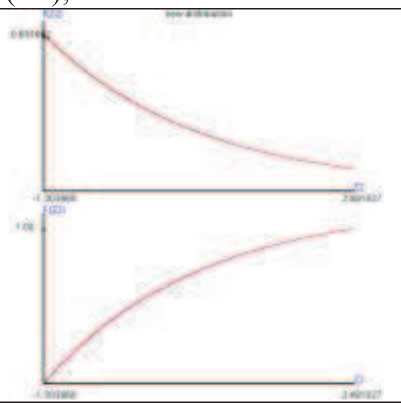
$(X_1, X_2)'$ and Y_1 are independent random variables,

$$Z_1 = \frac{Y_1 - E(Y_1)}{\sqrt{Var(Y_1)}}, Z_2 = \frac{X_2 - E(X_2|x_1)}{\sqrt{Var(X_2|x_1)}}.$$

1. λ_1=0.1, λ_2=0.09, $\lambda^* = \frac{\lambda_2}{1-\lambda_1}$=0.1,

(1) x_1=0.1,

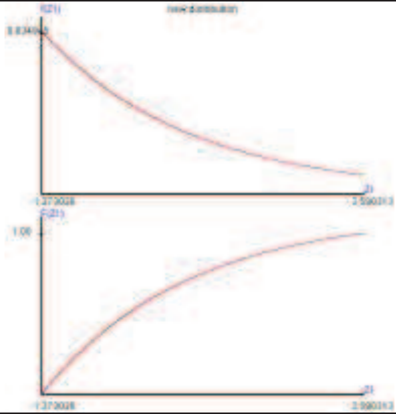
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.83188 Range : 3.87770 Mid_range : 0.65864 Median : -0.24262 Q1 : -0.83664 Q2 : -0.24262 Q3 : 0.65879 IQR : 1.49543 C.V. : none

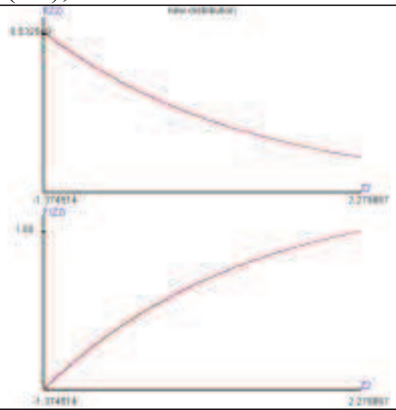
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.67272 Kurtosis Coef. : 2.43638 MAD : 0.83748 Range : 3.79986 Mid_range : 0.58893 Median : -0.22832 Q1 : -0.84494 Q2 : -0.22832 Q3 : 0.68538 IQR : 1.53032 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0005751614
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0000490650,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.865015,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.461568,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.085048,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.042467,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.008358,

Z1 and Z2 are similar probability distribution.

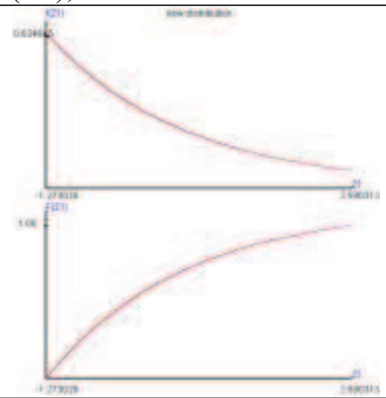
(2) x_1=0.3,

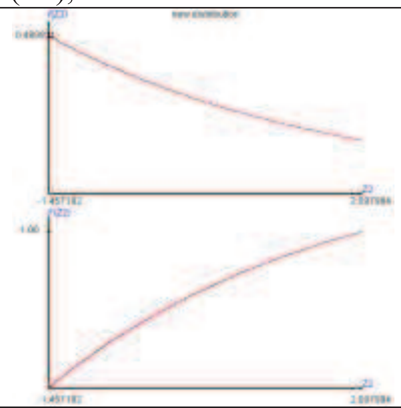
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.83188 Range : 3.87770 Mid_range : 0.65864 Median : -0.24262 Q1 : -0.83664 Q2 : -0.24262 Q3 : 0.65879 IQR : 1.49543 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.52719 Kurtosis Coef. : 2.18800 MAD : 0.84779 Range : 3.66797 Mid_range : 0.45268 Median : -0.19246 Q1 : -0.86023 Q2 : -0.19246 Q3 : 0.73851 IQR : 1.59874 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0055193803
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0004262343,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.985248,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.281122,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.137297,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.027154,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.013403,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.002695,

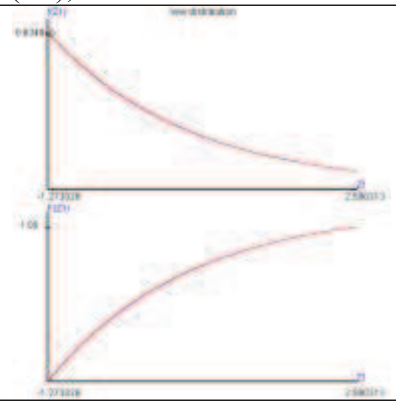
(3) x_1=0.5,

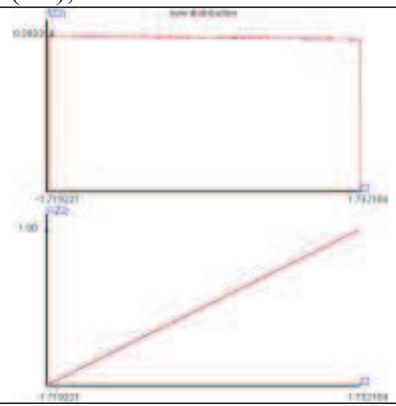
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.83188 Range : 3.87770 Mid_range : 0.65864 Median : -0.24262 Q1 : -0.83664 Q2 : -0.24262 Q3 : 0.65879 IQR : 1.49543 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.37863 Kurtosis Coef. : 1.99903 MAD : 0.85632 Range : 3.56840 Mid_range : 0.32040 Median : -0.14696 Q1 : -0.87169 Q2 : -0.14696 Q3 : 0.78752 IQR : 1.65921 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0160696160
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0011185537,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.891674,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.163144,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.080862,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.016135,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.008054,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.001618,

(4) x_1=0.99,

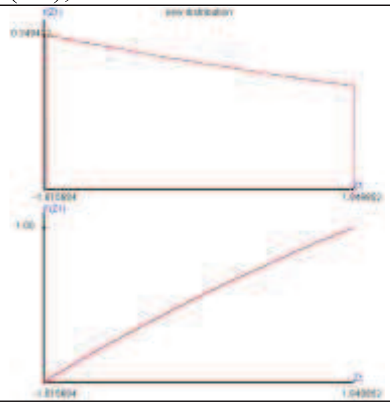
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.74370 Kurtosis Coef. : 2.58133 MAD : 0.83188 Range : 3.87770 Mid_range : 0.65864 Median : -0.24262 Q1 : -0.83664 Q2 : -0.24262 Q3 : 0.65879 IQR : 1.49543 C.V. : none

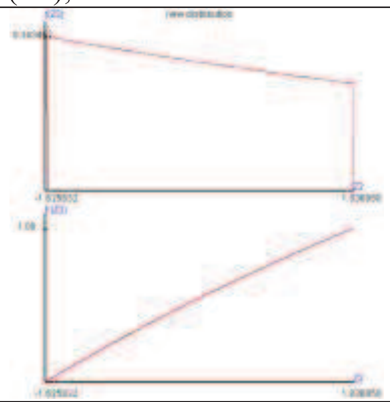
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.00774 Kurtosis Coef. : 1.80005 MAD : 0.86603 Range : 3.46416 Mid_range : 0.00644 Median : -0.00323 Q1 : -0.86682 Q2 : -0.00323 Q3 : 0.86525 IQR : 1.73206 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0666621261
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0037000874,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 0.943939,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.438118,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.083555,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.041747,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.008369,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.004188,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.000837,

2. $\lambda_1=0.4$, $\lambda_2=0.24$, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}=0.4$,

(1) $x_1=0.1$,

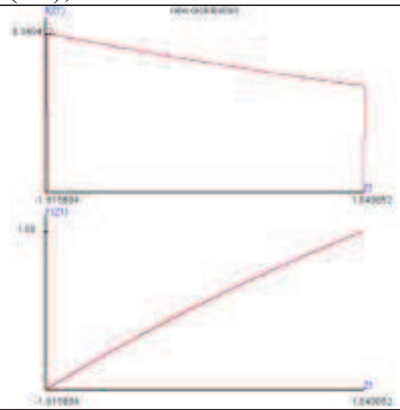
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.14012 Kurtosis Coef. : 1.82717 MAD : 0.86465 Range : 3.47842 Mid_range : 0.11708 Median : -0.05771 Q1 : -0.87560 Q2 : -0.05771 Q3 : 0.84597 IQR : 1.72157 C.V. : none

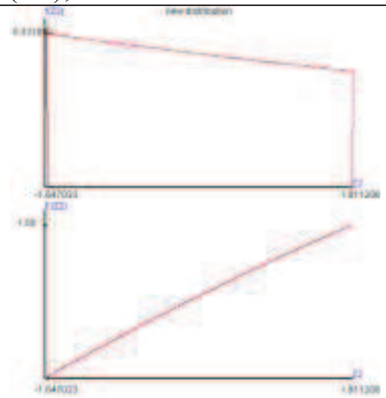
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.12646 Kurtosis Coef. : 1.82205 MAD : 0.86492 Range : 3.47565 Mid_range : 0.10556 Median : -0.05227 Q1 : -0.87502 Q2 : -0.05227 Q3 : 0.84848 IQR : 1.72351 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0000257430
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0000021260,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.371379,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.180894,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.035553,

Z2 is approaching to Z1.

(2) x_1=0.3,

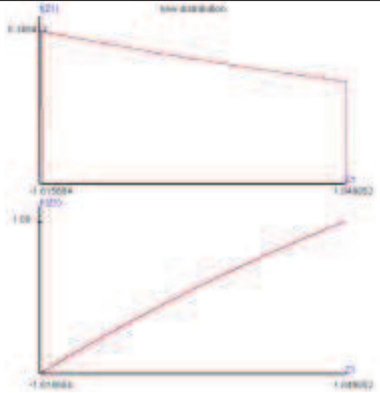
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.14012 Kurtosis Coef. : 1.82717 MAD : 0.86465 Range : 3.47842 Mid_range : 0.11708 Median : -0.05771 Q1 : -0.87560 Q2 : -0.05771 Q3 : 0.84597 IQR : 1.72157 C.V. : none

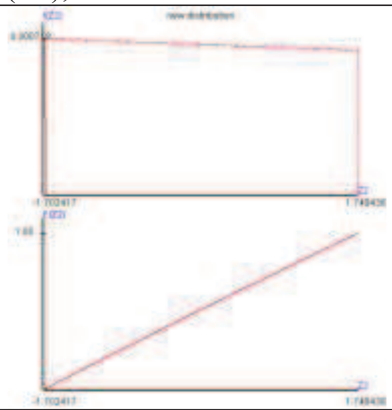
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.09841 Kurtosis Coef. : 1.81333 MAD : 0.86536 Range : 3.47110 Mid_range : 0.08209 Median : -0.04081 Q1 : -0.87373 Q2 : -0.04081 Q3 : 0.85315 IQR : 1.72688 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0002401238
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0000194298,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.993556,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.821560,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.116286,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.057246,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.011789,

Z1 and Z2 are similar probability distribution.

(3) x_1=0.8,

f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.14012 Kurtosis Coef. : 1.82717 MAD : 0.86465 Range : 3.47842 Mid_range : 0.11708 Median : -0.05771 Q1 : -0.87560 Q2 : -0.05771 Q3 : 0.84597 IQR : 1.72157 C.V. : none

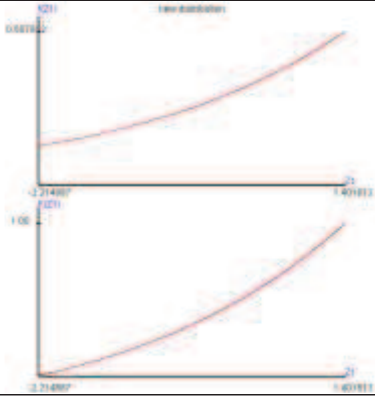
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.02822 Kurtosis Coef. : 1.80106 MAD : 0.86598 Range : 3.46468 Mid_range : 0.02351 Median : -0.01176 Q1 : -0.86875 Q2 : -0.01176 Q3 : 0.86291 IQR : 1.73167 C.V. : none

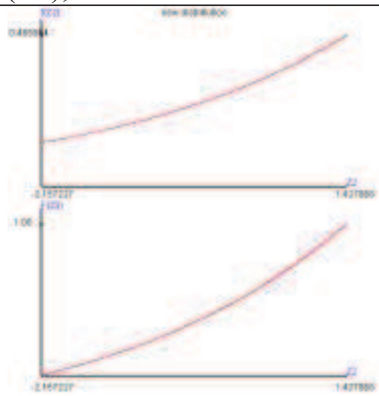
E(| Z2 distribution - Z1 distribution |^2)=0.0017311366
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0001326113,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.482199,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.223912,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.043714,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.021680,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.004322,

Z1 and Z2 are similar probability distribution.

3. λ_1 =0.8, λ_2 =0.16, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.8,

(1) x_1 =0.1,

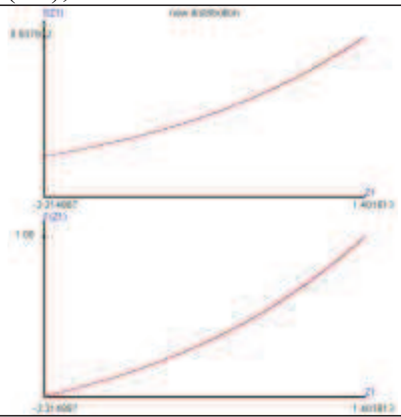
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.47630 Kurtosis Coef. : 2.11604 MAD : 0.85096 Range : 3.62995 Mid_range : -0.40664 Median : 0.17793 Q1 : -0.75619 Q2 : 0.17793 Q3 : 0.86471 IQR : 1.62090 C.V. : none

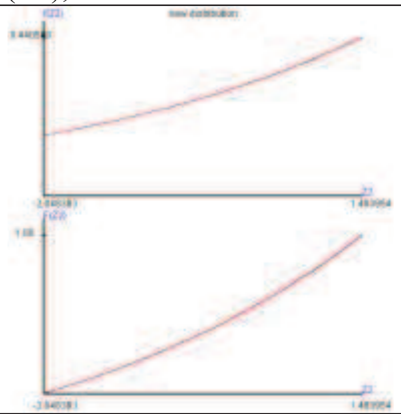
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.42906 Kurtosis Coef. : 2.05615 MAD : 0.85367 Range : 3.59842 Mid_range : -0.36468 Median : 0.16336 Q1 : -0.77168 Q2 : 0.16336 Q3 : 0.86842 IQR : 1.64010 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0002849663
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0000241270,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.948301,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.700642,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.112942,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.056528,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.011538,

Z1 and Z2 are similar probability distribution.

(2) x_1=0.3,

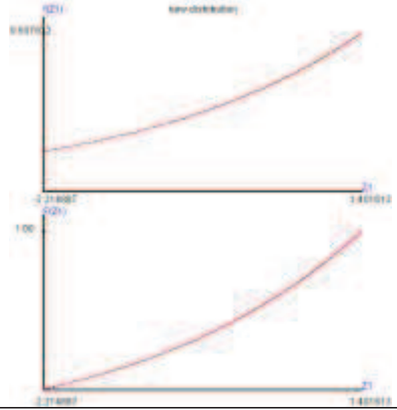
f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.47630 Kurtosis Coef. : 2.11604 MAD : 0.85096 Range : 3.62995 Mid_range : -0.40664 Median : 0.17793 Q1 : -0.75619 Q2 : 0.17793 Q3 : 0.86471 IQR : 1.62090 C.V. : none

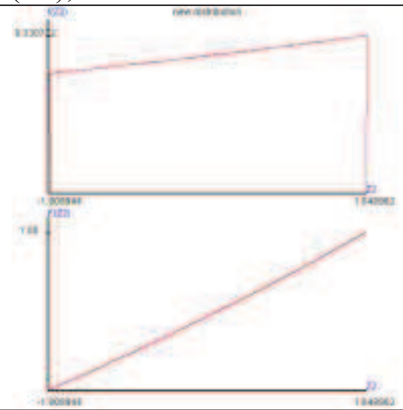
f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.33462 Kurtosis Coef. : 1.95535 MAD : 0.85838 Range : 3.54547 Mid_range : -0.28221 Median : 0.13182 Q1 : -0.80039 Q2 : 0.13182 Q3 : 0.87397 IQR : 1.67436 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0026003622
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0002036696,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.396342,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.189918,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.037650,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.018938,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.003835,

Z1 and Z2 are similar probability distribution.

(3) x_1=0.8,

f(Z1),F(Z1),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.47630 Kurtosis Coef. : 2.11604 MAD : 0.85096 Range : 3.62995 Mid_range : -0.40664 Median : 0.17793 Q1 : -0.75619 Q2 : 0.17793 Q3 : 0.86471 IQR : 1.62090 C.V. : none

f(Z2),F(Z2),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.09588 Kurtosis Coef. : 1.81269 MAD : 0.86539 Range : 3.47076 Mid_range : -0.07999 Median : 0.03975 Q1 : -0.85351 Q2 : 0.03975 Q3 : 0.87363 IQR : 1.72714 C.V. : none

E(| Z2 distribution - Z1 distribution |^2)=0.0190948871
*********** | Z2 distribution function - Z1 distribution function| ***********
The almost surely limiting theory
E(| Z2 distribution function - Z1 distribution function|^2)=0.0012539289,
Pr(| Z2 distribution function - Z1 distribution function|<0.1000000000)= 1.000000,
Pr(| Z2 distribution function - Z1 distribution function|<0.0500000000)= 0.896836,
Pr(| Z2 distribution function - Z1 distribution function|<0.0100000000)= 0.143817,
Pr(| Z2 distribution function - Z1 distribution function|<0.0050000000)= 0.071626,
Pr(| Z2 distribution function - Z1 distribution function|<0.0010000000)= 0.014325,
Pr(| Z2 distribution function - Z1 distribution function|<0.0005000000)= 0.007151,
Pr(| Z2 distribution function - Z1 distribution function|<0.0001000000)= 0.001450,

Z1 and Z2 are similar probability distribution.

Chapter 6 The sampling distribution $\sum_{i=1}^{n} X_{2,i}$ when the population is $X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1)$ of model 3 and test statistic of $\lambda^* = \lambda_2 / (1 - \lambda_1)$

Section 1. The sampling distribution $\sum_{i=1}^{n} X_{2,i}$ when λ_1 and x_1 are known

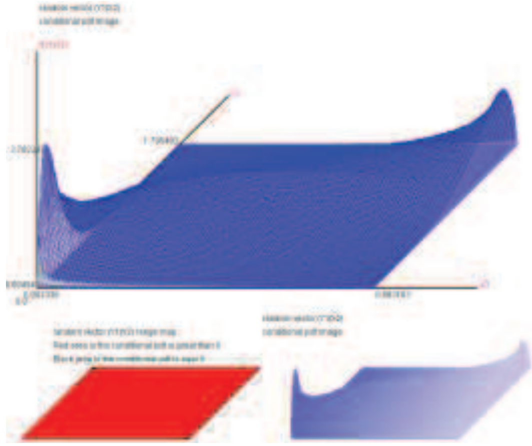
$$X_{2,1}, X_{2,2}, \ldots, X_{2,n} \overset{iid}{\sim} CB(\lambda_1, \lambda_2, x_1),\ \sum_{i=1}^{n} X_{2,i} \xrightarrow[n \to 0]{} Normal\left(E\left(\sum_{i=1}^{n} X_{2,i} \right), Var\left(\sum_{i=1}^{n} X_{2,i} \right) \right).$$

The sampling distribution of $\sum_{i=1}^{n} X_{2,i}$ will be affected by $\lambda_1, \lambda_2, x_1$ and n.

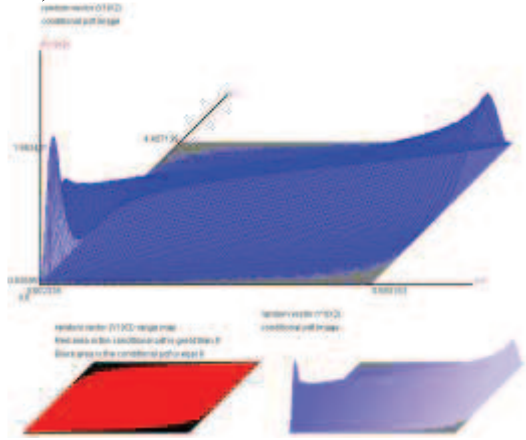
Let $X_2 = \sum_{i=1}^{n} X_{2,i}$,

1.f(X2|Y1= λ_2), $\lambda_1 = 0.1$, 0.0001<= λ_2 <=0.8999, x1=0.1,

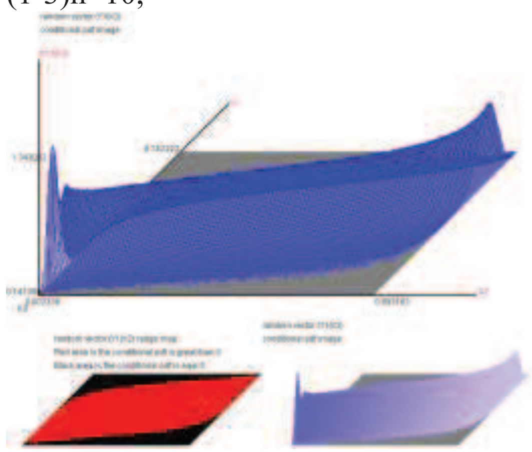
(1-1)n=2,



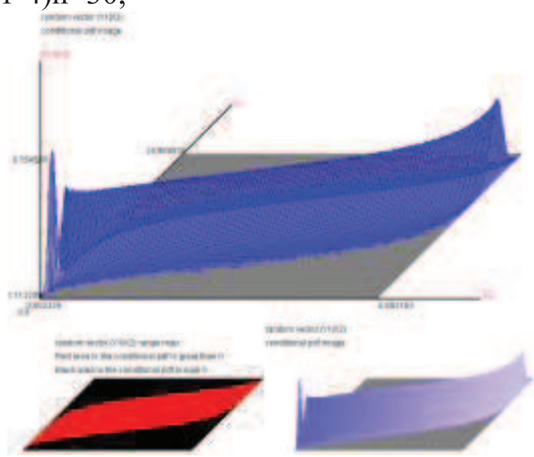
(1-2)n=5,



(1-3)n=10,

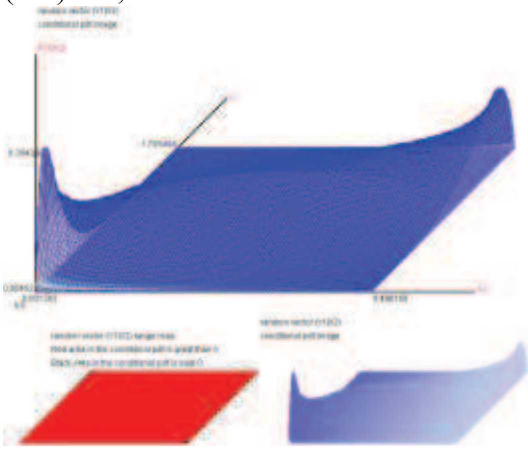


(1-4)n=30,

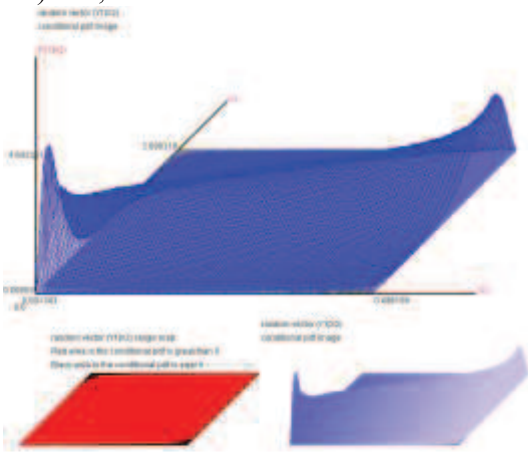


2.f(X2|Y1= λ_2), $\lambda_1 = 0.5$, 0.0001<= λ_2 <=0.4999, x1=0.1,

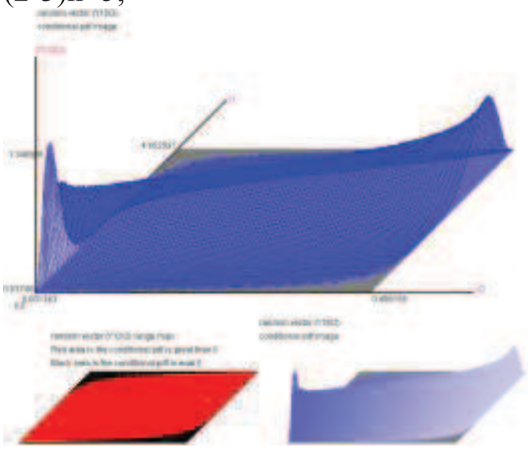
(2-1)n=2,



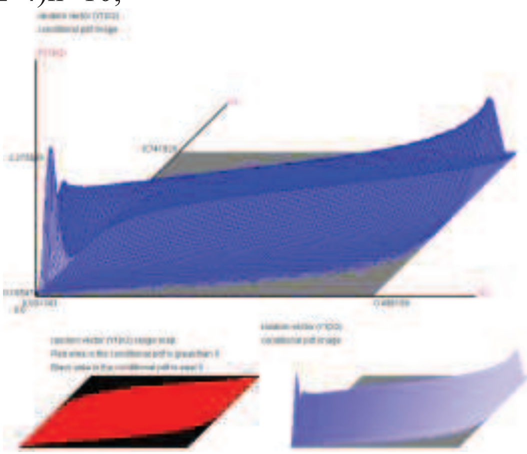
(2-2)n=3,



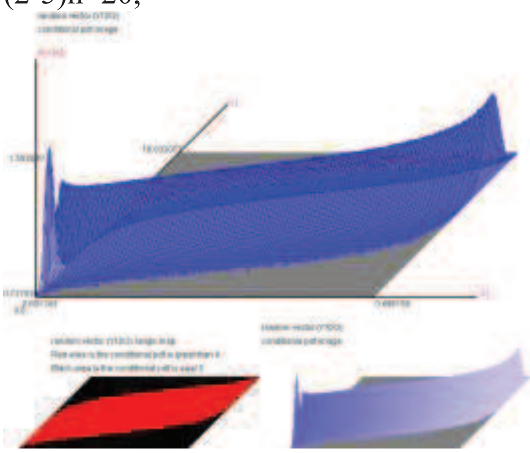
(2-3)n=5,



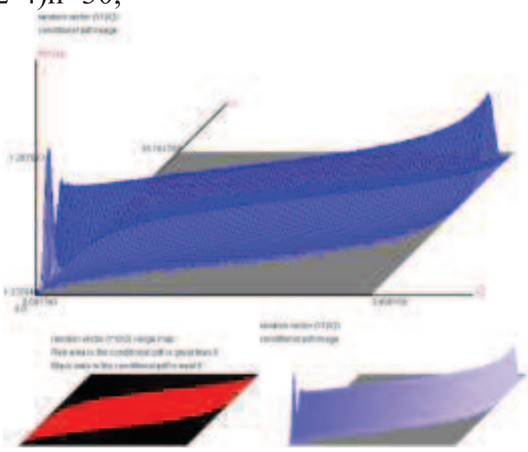
(2-4)n=10,



(2-5)n=20,

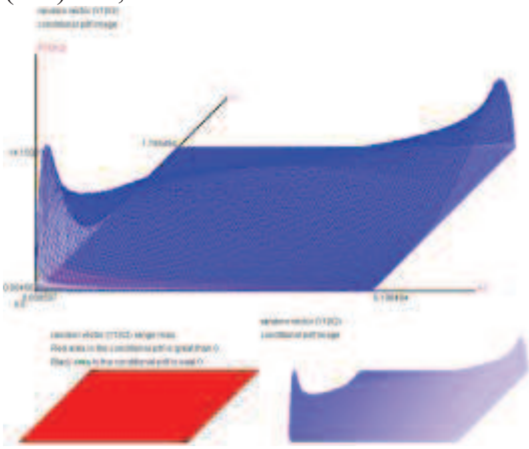


(2-4)n=30,

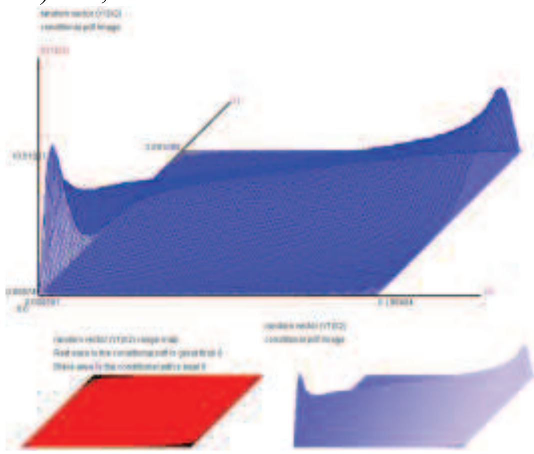


3.f(X2|Y1=λ_2), $\lambda_1 = 0.8$, 0.0001<=λ_2<=0.1999, x1=0.1,

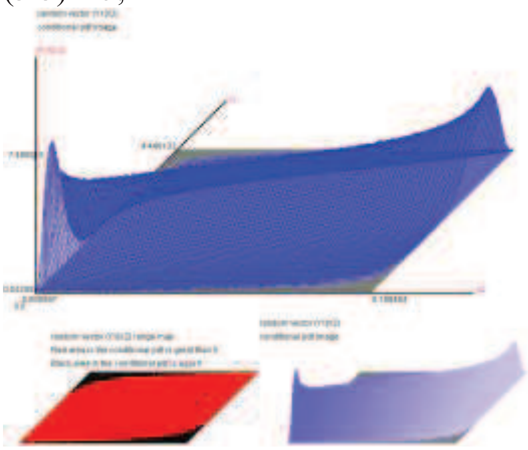
(3-1)n=2,



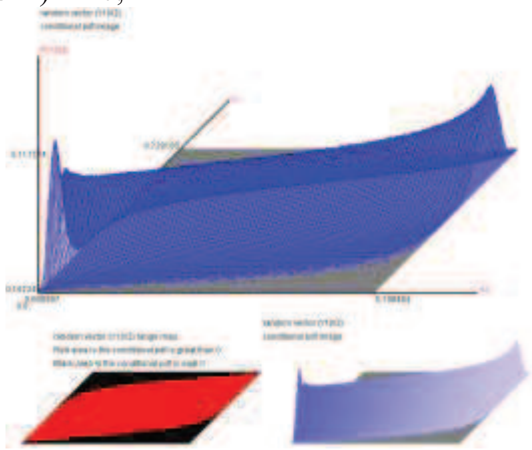
(3-2)n=3,



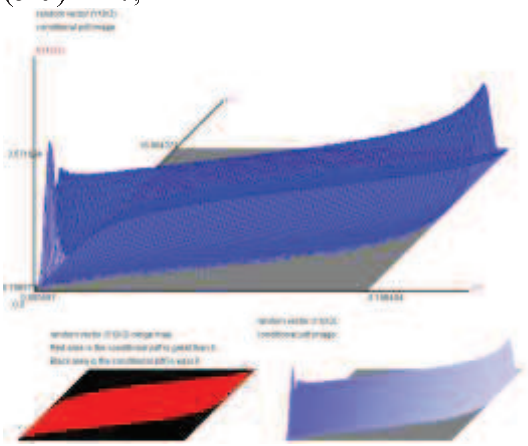
(3-3)n=5,



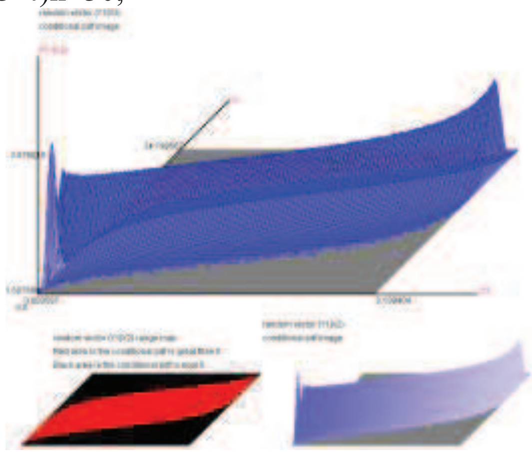
(3-4)n=10,



(3-5)n=20,

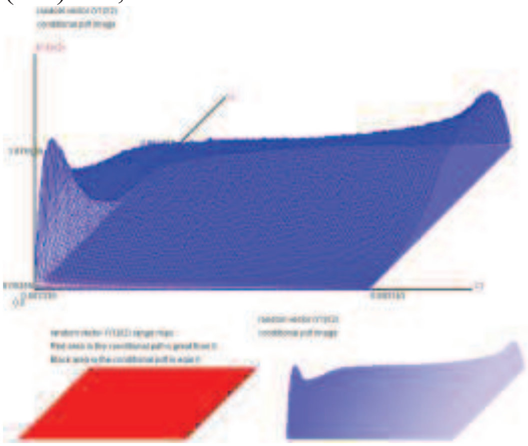


(3-4)n=30,

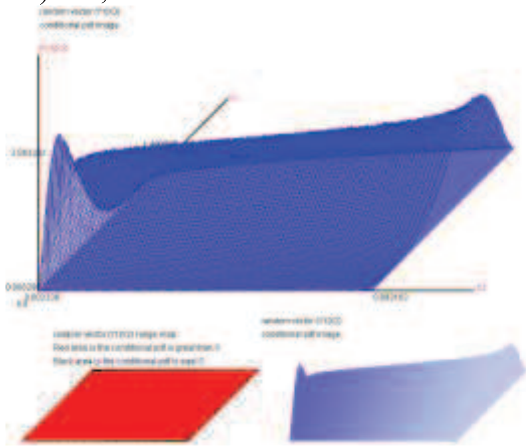


4.f(X2|Y1=λ_2), $\lambda_1 = 0.1$, 0.0001<=λ_2<=0.8999, x1=0.5,

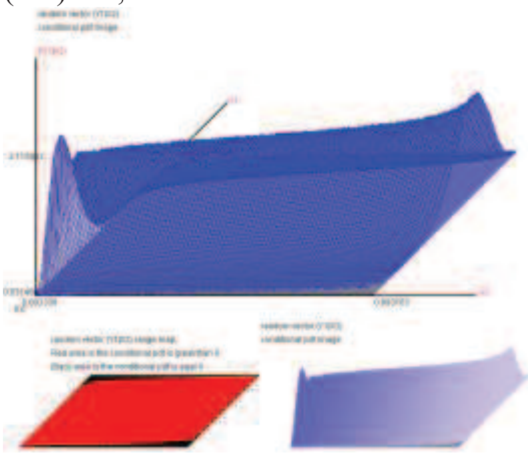
(4-1)n=2,



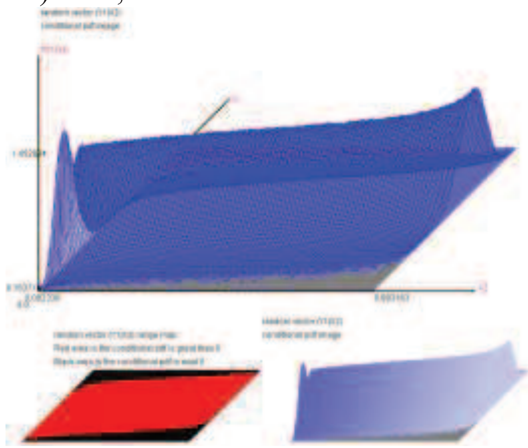
(4-2)n=3,



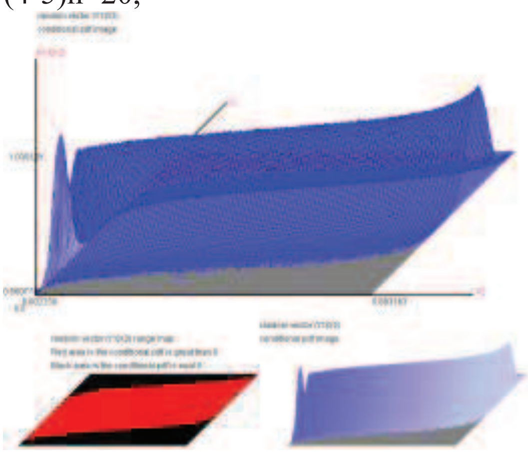
(4-3)n=5,



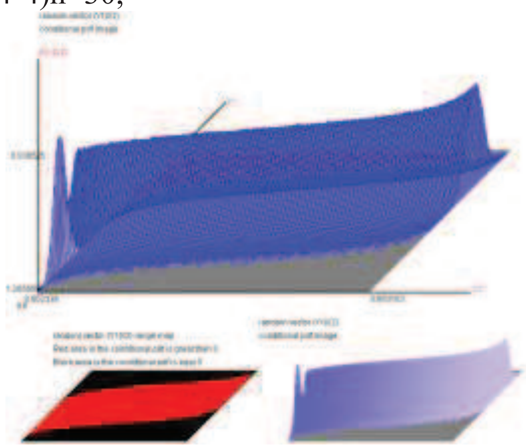
(4-4)n=10,



(4-5)n=20,

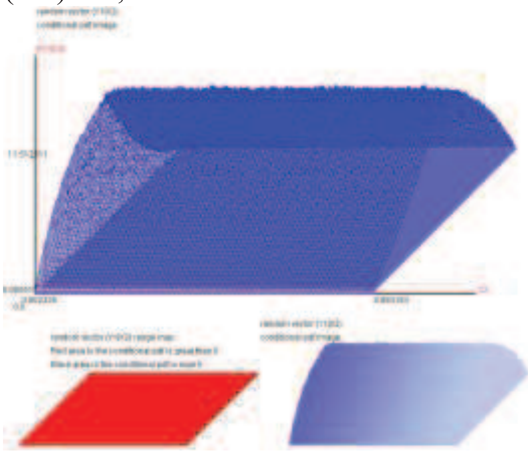


(4-4)n=30,

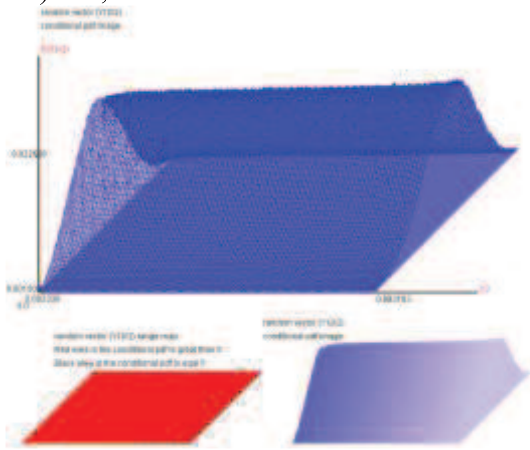


5.f(X2|Y1=λ_2), $\lambda_1 = 0.1$, 0.0001<=λ_2<=0.8999, x1=0.9,

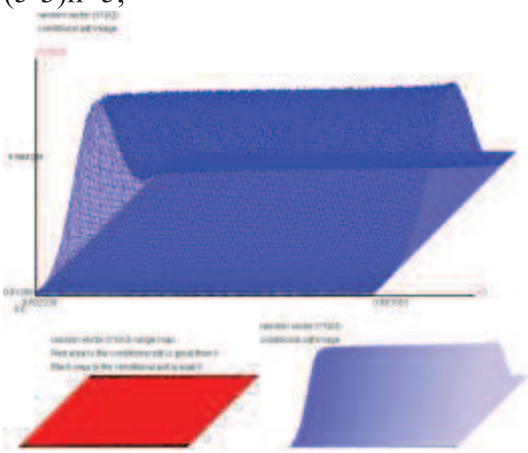
(5-1)n=2,



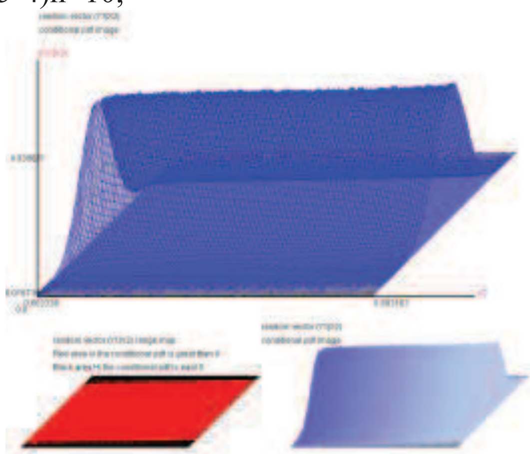
(5-2)n=3,



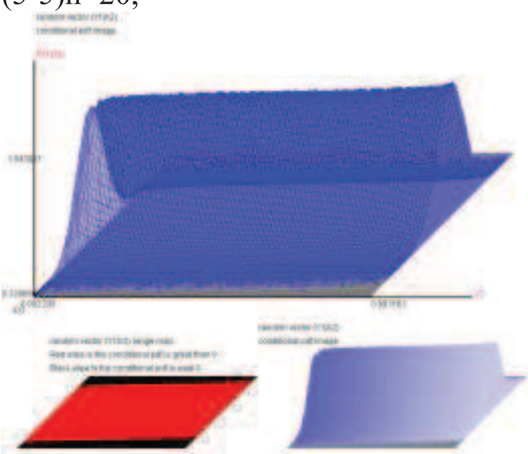
(5-3)n=5,



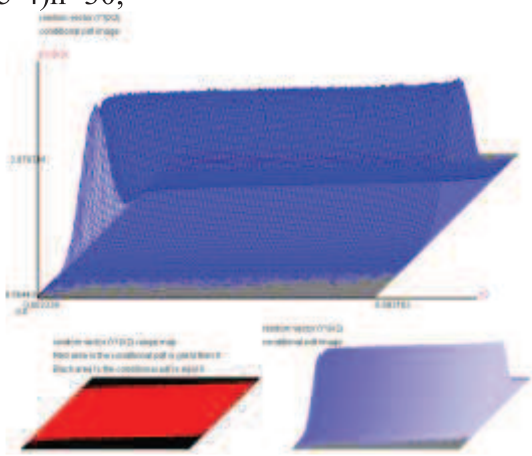
(5-4)n=10,



(5-5)n=20,



(5-4)n=30,



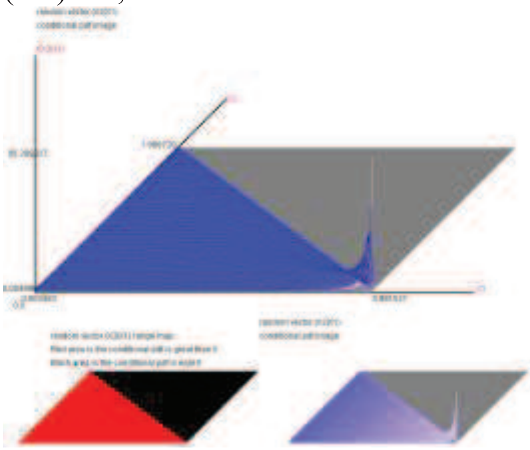
Section 2. The sampling distribution $\sum_{i=1}^{n} X_{2,i}$ when λ1 and λ2 are known

$$X_{2,1}, X_{2,2}, \ldots, X_{2,n} \overset{iid}{\sim} CB\left(\frac{\lambda_2}{1-\lambda_1}\right) = CB(\lambda_1, \lambda_2, x_1),$$

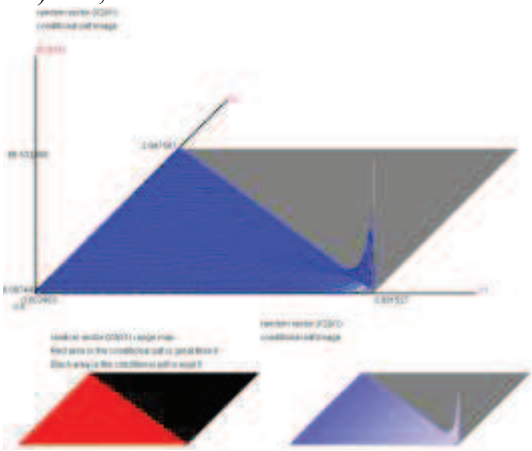
let $X_2 = \sum_{i=1}^{n} X_{2,i}$,

1.f(X2|X1), $\lambda_1 = 0.1$, λ_2=0.2, 0.001<=x1<=0.999

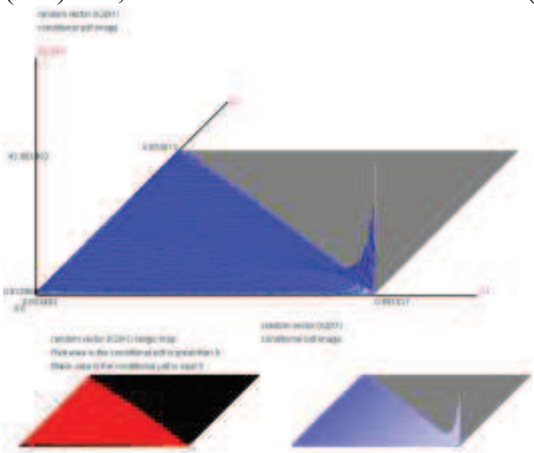
(1-1)n=2,



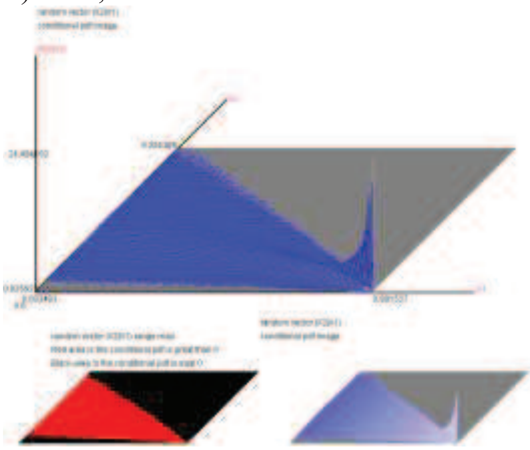
(1-2)n=3,



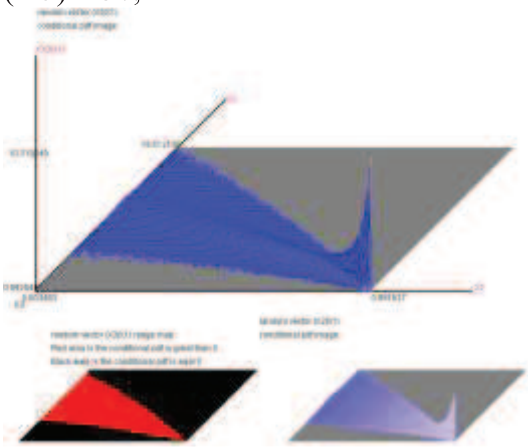
(1-3)n=5,



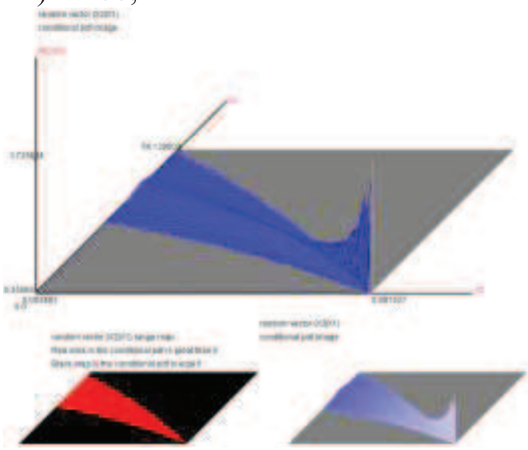
(1-4)n=10,



(1-5)n=30,

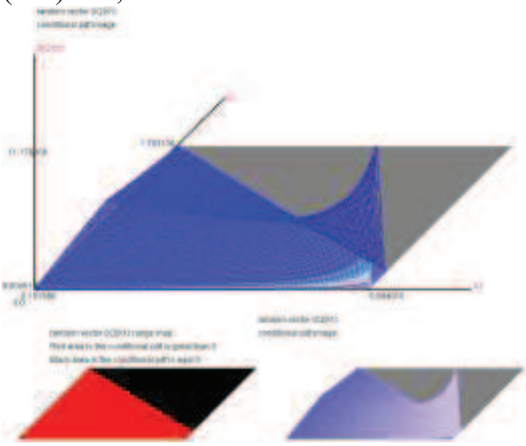


(1-4)n=100,

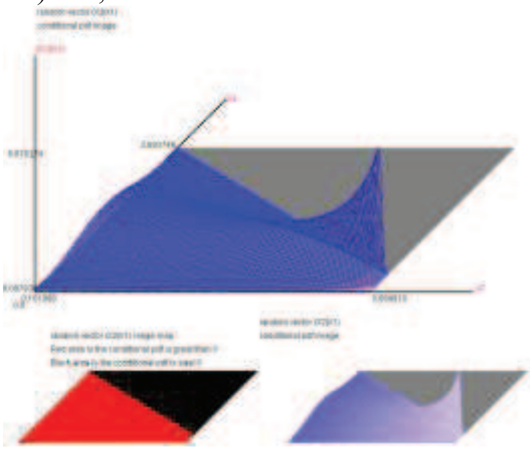


2.f(X2|X1), $\lambda_1 = 0.1$, λ_2=0.4, 0.1<=x1<=0. 9,

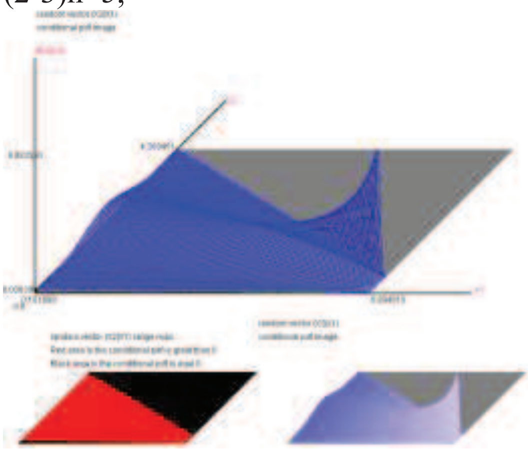
(2-1)n=2,



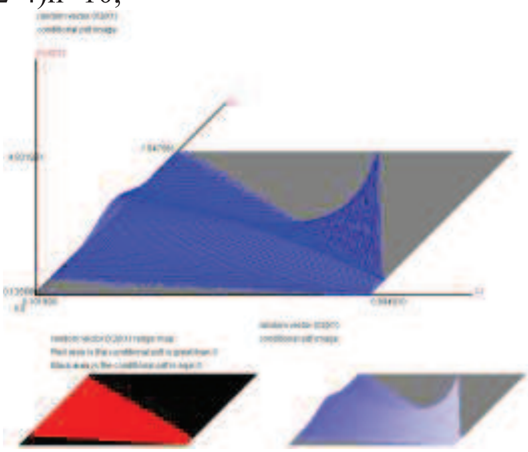
(2-2)n=3,



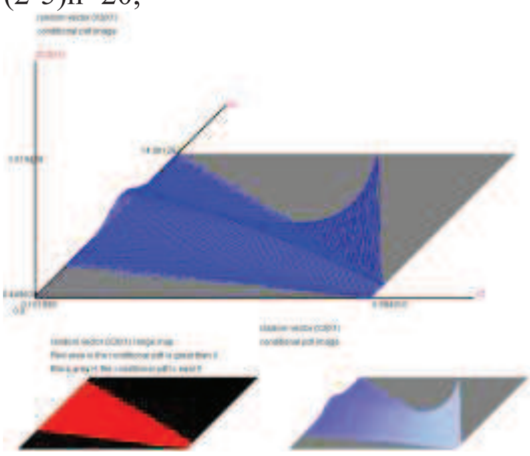
(2-3)n=5,



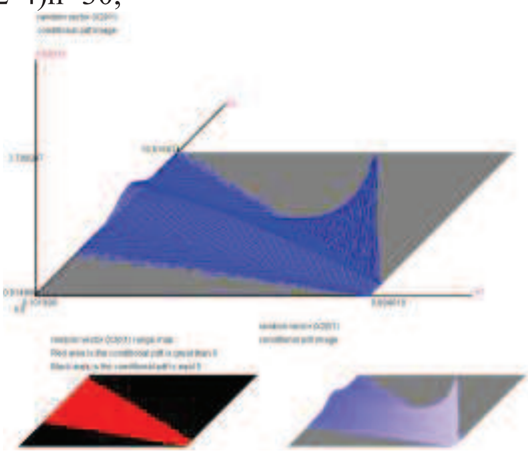
(2-4)n=10,



(2-5)n=20,

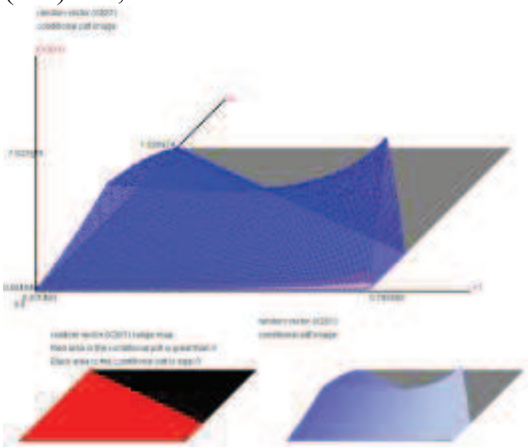


(2-4)n=30,

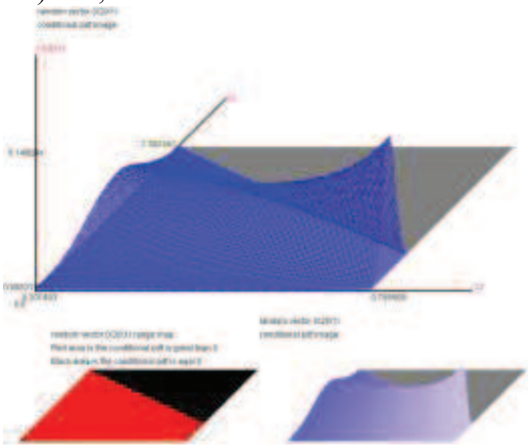


3.f(X2|X1), $\lambda_1 = 0.1$, $\lambda_2 = 0.6$, 0.2<=x1<=0.8,

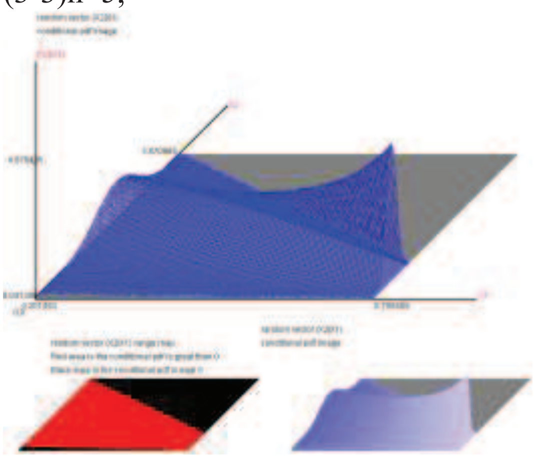
(3-1)n=2,



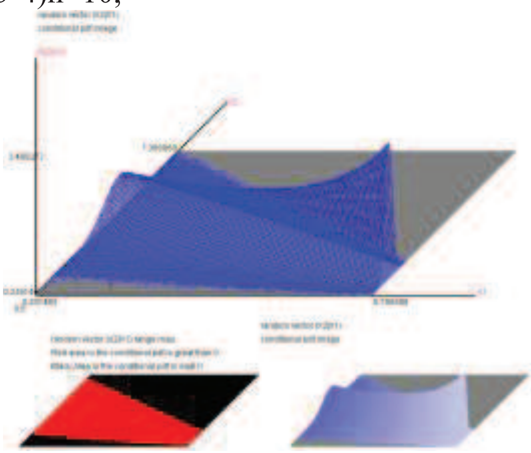
(3-2)n=3,



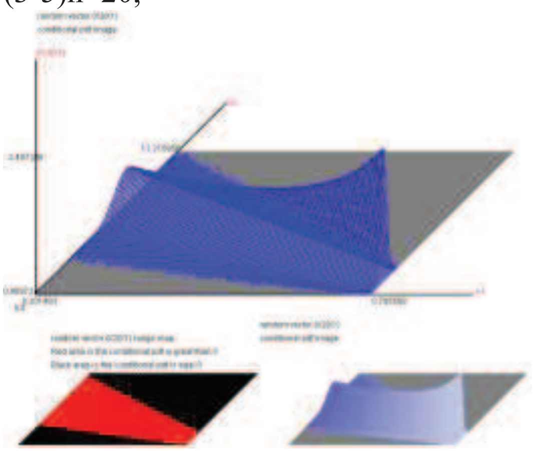
(3-3)n=5,



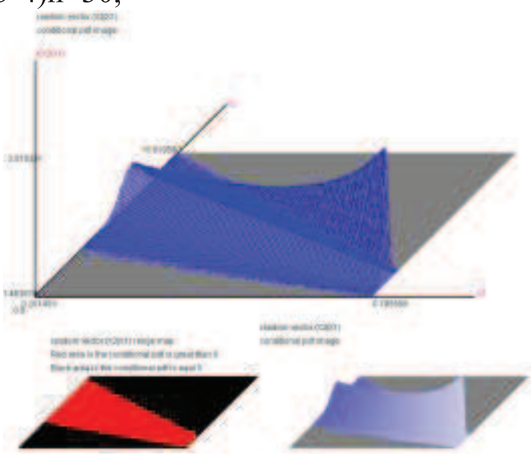
(3-4)n=10,



(3-5)n=20,

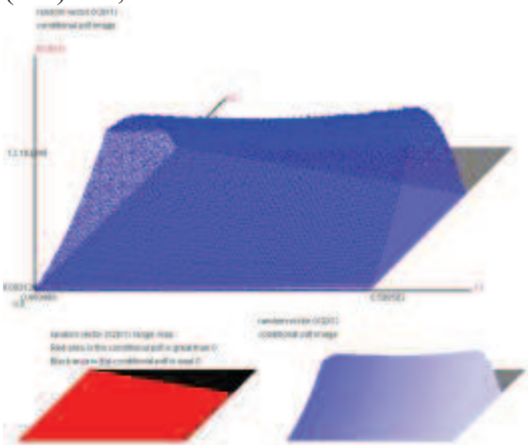


(3-4)n=30,

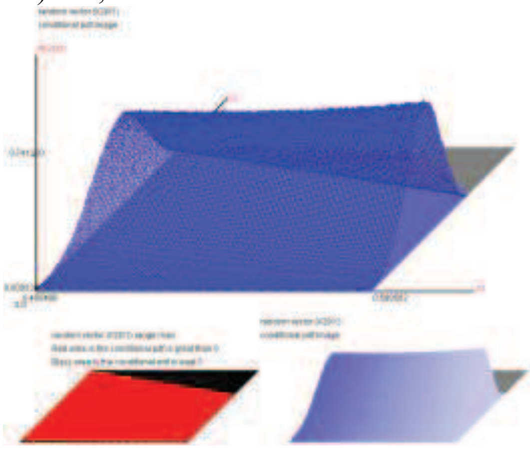


4.f(X2|X1), $\lambda_1 = 0.5$, λ_2=0.4, 0.4 <=x1<=0.6,

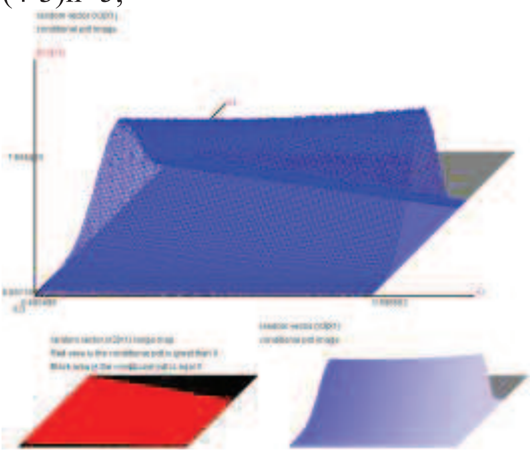
(4-1)n=2,



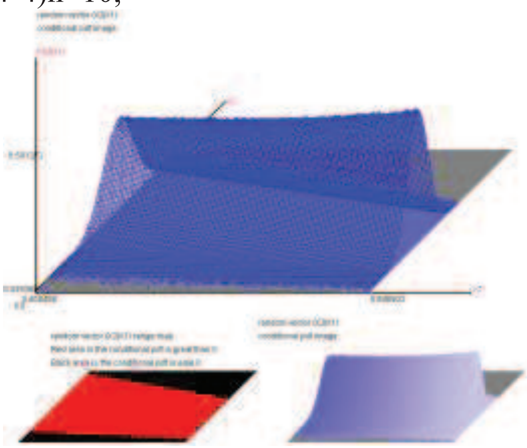
(4-2)n=3,



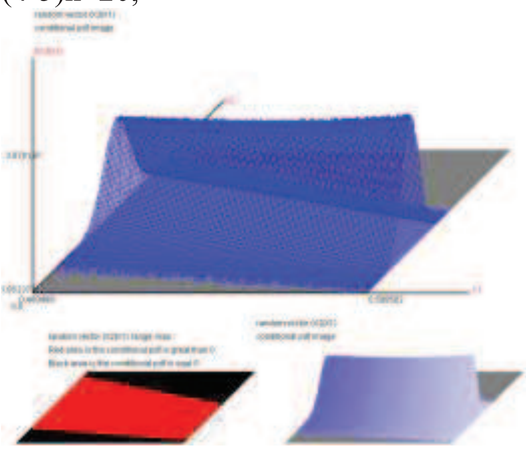
(4-3)n=5,



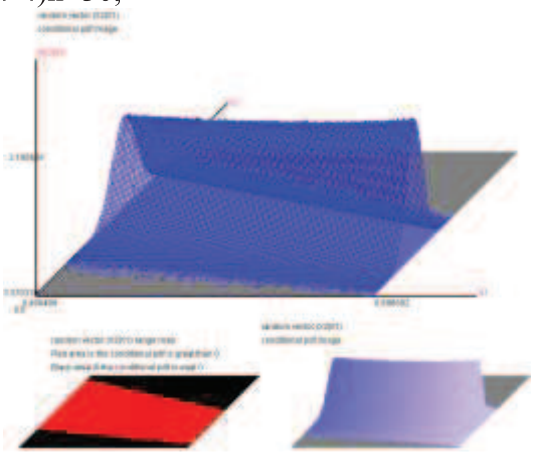
(4-4)n=10,



(4-5)n=20,



(4-4)n=30,



Section 3. The approaching distribution of sample mean

$X_{2,1}, X_{2,2}, ..., X_{2,n} \overset{iid}{\sim} CB(\lambda_1, \lambda_2, x_1)$, n random samples from $CB(\lambda_1, \lambda_2, x_1)$.

Test statistic, $\dfrac{\sqrt{n}\left(\overline{X}_2 - \mu\right)}{\sigma}, \overline{X}_2 = \dfrac{\sum_{j=1}^{2} X_{2,j}}{n}, \mu = E\left(X_{2,j} \middle| x_1\right), \sigma^2 = Var\left(X_{2,j} \middle| x_1\right),$

$\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$, $X_2 | x_1 \sim CB(\lambda_1, \lambda_2, x_1)$,

$(i) \lambda^* \neq 0.5,$

$$\mu = E\left(X_2 \middle| x_1\right) = \frac{(1-x_1)\left(\lambda^*\right)^{1-x_1}}{\left(\lambda^*\right)^{1-x_1} - \left(1-\lambda^*\right)^{1-x_1}} - \frac{1}{ln\left(\lambda^*\right) - ln\left(1-\lambda^*\right)},$$

$$\sigma^2 = Var\left(X_2 \middle| x_1\right) = \frac{1}{\left(ln\left(\lambda^*\right) - ln\left(1-\lambda^*\right)\right)^2} - \frac{(1-x_1)^2 \left(\lambda^*\right)^{1-x_1} \left(1-\lambda^*\right)^{1-x_1}}{\left(\left(\lambda^*\right)^{1-x_1} - \left(1-\lambda^*\right)^{1-x_1}\right)^2},$$

$(ii) \lambda^* = 0.5,$

$$\mu = E\left(X_2 \middle| x_1\right) = \frac{1-x_1}{2}, \sigma^2 = Var\left(X_2 \middle| x_1\right) = \frac{(1-x_1)^2}{12}.$$

$n\left(\overline{X}_2\right) = ?$ when $\dfrac{\sqrt{n}\left(\overline{X}_2 - \mu\right)}{\sigma} \xrightarrow[n \geq n\left(\overline{X}_2\right)]{} Normal(0,1),$

let W15= $\dfrac{\sqrt{n}\left(\overline{X}_2 - \mu\right)}{\sigma}$,

Getting the simulated data of W15 and standard normal distribution using the simulator and the simulated data number=100,000,000.

Calculating the $n\left(\overline{X}\right)$ using the Strong Law of Large Number, the requirement is

$P\left\{\left|F_{W15}(W15) - \Phi(W15)\right| < 0.1\right\} = 1,$ $P\left\{\left|F_{W15}(W15) - \Phi(W15)\right| < 0.05\right\} = 1,$

$P\left\{\left|F_{W15}(W15) - \Phi(W15)\right| < 0.01\right\} = 1,$ $P\left\{\left|F_{W15}(W15) - \Phi(W15)\right| < 0.005\right\} = 1,$

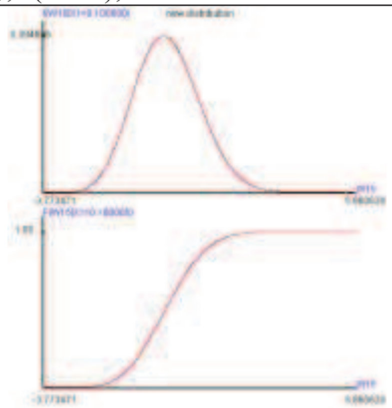
when $\dfrac{\sqrt{n}\left(\overline{X} - \mu(X)\right)}{\sigma(X)} \rightarrow Normal(0,1).$

$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \dfrac{1}{\sqrt{2\pi}} exp\left(-\dfrac{z^2}{2}\right) dz$ is the distribution function of standard normal distribution.

1. λ_1=0.1, λ_2=0.09, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.1, W15=$\dfrac{\sqrt{n}\left(\overline{X}_2-\mu\right)}{\sigma}$,

(1-1) x_1=0.1 , λ_1=0.1, λ_2=0.09, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.1,

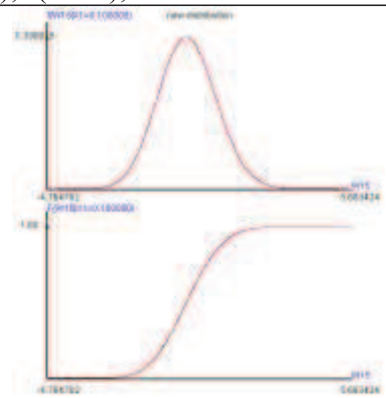
(1-1-1) $n\left(\overline{X}_2\right)=10$,

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.21280 Kurtosis Coef. : 2.94363 MAD : 0.80133 Range : 9.66991 Mid_range : 1.04357 Median : -0.03653 Q1 : -0.70220 Q2 : -0.03653 Q3 : 0.66229 IQR : 1.36449 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0028407863
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000841815,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.629596,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.310324,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.055907,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.027890,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.005631,

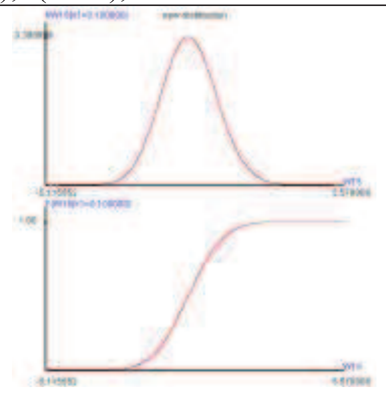
(1-1-2) $n\left(\overline{X}_2\right)=50,\ \ \lambda_1=0.1,\ \ \lambda_2=0.09,\ \ \lambda^*=\dfrac{\lambda_2}{1-\lambda_1}=0.1,$

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.09541 Kurtosis Coef. : 2.98905 MAD : 0.79856 Range : 10.50704 Mid_range : 0.44936 Median : -0.01585 Q1 : -0.68466 Q2 : -0.01585 Q3 : 0.66749 IQR : 1.35215 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0005167765
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000159316,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.687417,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.132994,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.064198,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.011463,

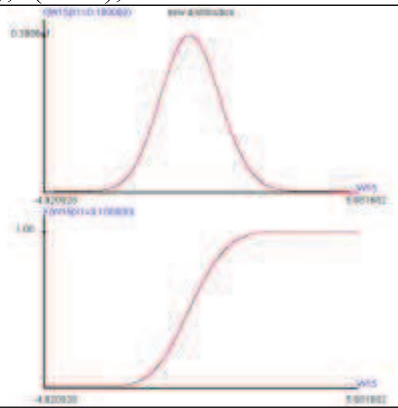
(1-1-3) $n\left(\overline{X}_2\right)=60,\ \ \lambda_1=0.1,\ \ \lambda_2=0.09,\ \ \lambda^*=\dfrac{\lambda_2}{1-\lambda_1}=0.1,$

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.08748 Kurtosis Coef. : 2.99260 MAD : 0.79837 Range : 10.72619 Mid_range : 0.22768 Median : -0.01470 Q1 : -0.68359 Q2 : -0.01470 Q3 : 0.66759 IQR : 1.35118 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0004318626
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000133986,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.733367,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.144672,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.073430,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.013112,

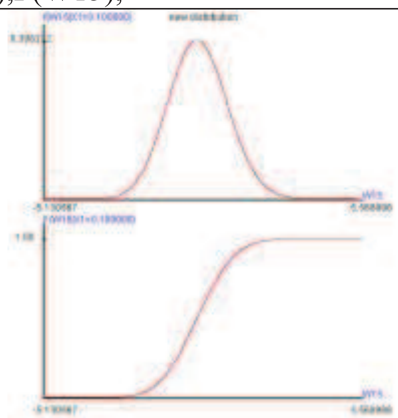
$(1\text{-}1\text{-}4)\ n\left(\overline{X}_2\right)=90,\ \lambda_1=0.1,\ \lambda_2=0.09,\ \lambda^* = \frac{\lambda_2}{1-\lambda_1}=0.1,$

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.07074 Kurtosis Coef. : 2.99268 MAD : 0.79829 Range : 10.64203 Mid_range : 0.38038 Median : -0.01195 Q1 : -0.68181 Q2 : -0.01195 Q3 : 0.66872 IQR : 1.35054 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0002797292
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000088072,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.181123,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.086117,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.018526,

$(1\text{-}1\text{-}5)\ n\left(\overline{X}_2\right)=100,\ \lambda_1=0.1,\ \lambda_2=0.09,\ \lambda^* = \frac{\lambda_2}{1-\lambda_1}=0.1,$

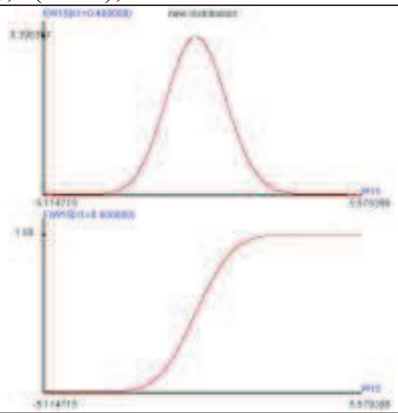
f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.06669 Kurtosis Coef. : 2.99380 MAD : 0.79825 Range : 10.73934 Mid_range : 0.21921 Median : -0.01124 Q1 : -0.68119 Q2 : -0.01124 Q3 : 0.66922 IQR : 1.35042 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0002460346
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000078876,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.193800,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.094333,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.020078,

(1-2) x_1=0.4, λ_1=0.1, λ_2=0.09, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.1,

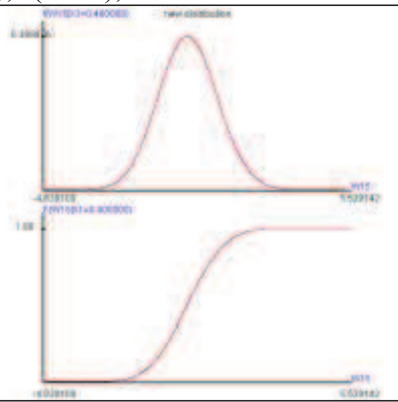
(1-2-1) $n(\overline{X}_2)=80$,

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.05117 Kurtosis Coef. : 2.98875 MAD : 0.79834 Range : 10.73387 Mid_range : 0.23234 Median : -0.00862 Q1 : -0.68010 Q2 : -0.00862 Q3 : 0.67069 IQR : 1.35079 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0001509785
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000048604,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.254363,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.118626,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.022948,

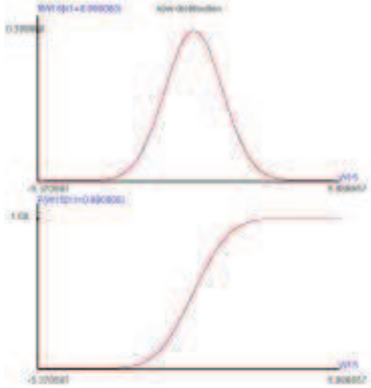
(1-2-2) $n(\overline{X}_2)=90$,

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.04769 Kurtosis Coef. : 2.98988 MAD : 0.79836 Range : 10.39775 Mid_range : 0.34952 Median : -0.00804 Q1 : -0.68006 Q2 : -0.00804 Q3 : 0.67113 IQR : 1.35119 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0001279311
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000040955,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.276106,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.126936,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.026192,

(1-3) x_1=0.99 , λ_1=0.1, λ_2=0.09, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.1,

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.00117 Kurtosis Coef. : 2.98548 MAD : 0.79837 Range : 10.49612 Mid_range : -0.14196 Median : -0.00040 Q1 : -0.67567 Q2 : -0.00040 Q3 : 0.67562 IQR : 1.35129 C.V. : none

Z0~standard normal distribution,

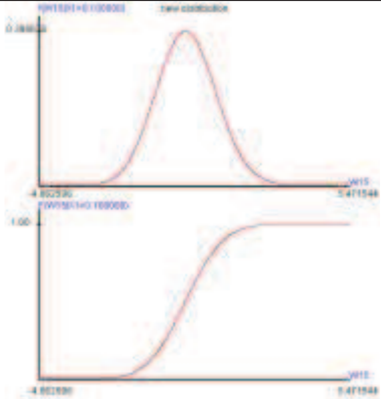
E(| W15 distribution - Z0 distribution |^2)=0.0000030325
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000000754,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.993235,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.241159,

The requirement of sample size is decreasing when x_1 is increasing, if the sample mean can approach normal distribution.

$\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.1 and $n(\overline{X}_2) = 80$ for any x_1, $\dfrac{\sqrt{n}(\overline{X}_2 - \mu)}{\sigma} \xrightarrow[n \ge n(\overline{X}_2)]{} Normal(0,1)$,

2. λ_1=0.2, λ_2=0.16, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.2, W15=$\dfrac{\sqrt{n}\left(\bar{X}_2-\mu\right)}{\sigma}$, x_1=0.1,

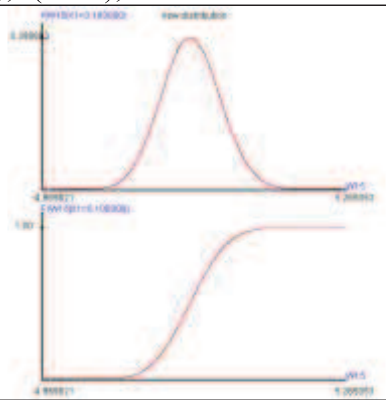
(2-1) $n\left(\bar{X}_2\right)=50$, W15 is approaching Z distribution when $n\left(\bar{X}_2\right)=50$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.06113 Kurtosis Coef. : 2.98103 MAD : 0.79866 Range : 10.37256 Mid_range : 0.30447 Median : -0.01041 Q1 : -0.68172 Q2 : -0.01041 Q3 : 0.67081 IQR : 1.35253 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0002172702
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000068504,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.214796,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.098963,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.017644,

(2-2) $n\left(\bar{X}_2\right)=60$,

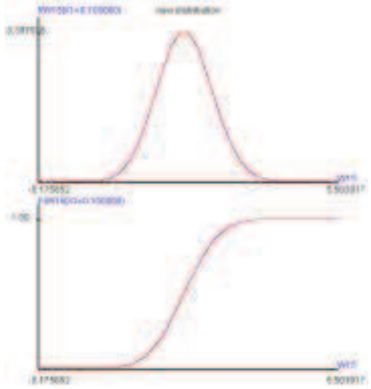
f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.05536 Kurtosis Coef. : 2.98316 MAD : 0.79857 Range : 10.27292 Mid_range : 0.14762 Median : -0.00933 Q1 : -0.68098 Q2 : -0.00933 Q3 : 0.67124 IQR : 1.35223 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0001776132
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000057123,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.252814,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.114670,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.020651,

3. $\lambda_1=0.8$, $\lambda_2=0.06$, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}=0.3$, W15= $\dfrac{\sqrt{n}\left(\overline{X}_2-\mu\right)}{\sigma}$, $x_1=0.1$,

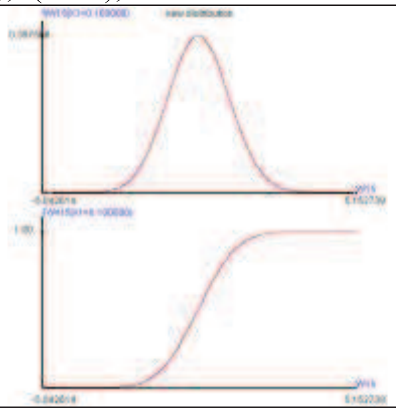
(3-1) $n\left(\overline{X}_2\right)=40$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right)=40$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.04156 Kurtosis Coef. : 2.97206 MAD : 0.79892 Range : 10.71927 Mid_range : 0.16413 Median : -0.00669 Q1 : -0.68055 Q2 : -0.00669 Q3 : 0.67263 IQR : 1.35318 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0001076282
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000032449,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.323084,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.128597,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.023240,

(3-2) $n\left(\overline{X}_2\right)=50$,

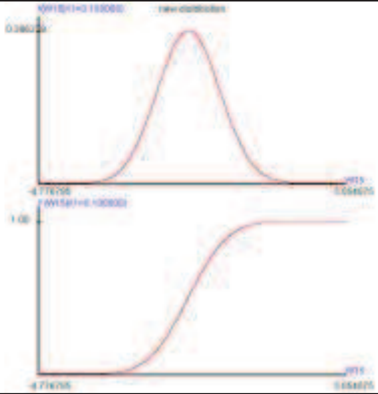
f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.03765 Kurtosis Coef. : 2.97737 MAD : 0.79871 Range : 10.23325 Mid_range : 0.05506 Median : -0.00630 Q1 : -0.67975 Q2 : -0.00630 Q3 : 0.67263 IQR : 1.35239 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0000870745
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000027672,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.369591,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.165485,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.026002,

4. $\lambda_1=0.5$, $\lambda_2=0.2$, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}=0.4$, W15=$\dfrac{\sqrt{n}\left(\overline{X}_2-\mu\right)}{\sigma}$, $x_1=0.1$,

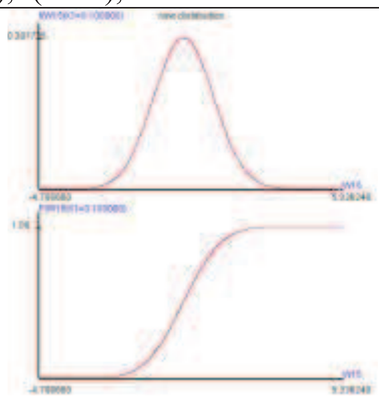
(4-1) $n\left(\overline{X}_2\right)=25$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right)=25$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.02618 Kurtosis Coef. : 2.95285 MAD : 0.79950 Range : 9.86741 Mid_range : 0.13864 Median : -0.00440 Q1 : -0.68036 Q2 : -0.00440 Q3 : 0.67550 IQR : 1.35586 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0000640824
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000018418,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.456493,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.203573,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.044366,

(4-2) $n\left(\overline{X}_2\right)=40$,

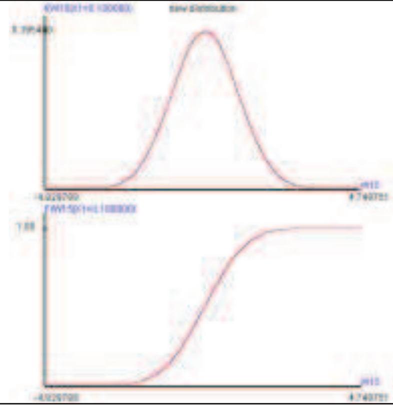
f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : 0.02142 Kurtosis Coef. : 2.97204 MAD : 0.79883 Range : 10.16456 Mid_range : 0.27279 Median : -0.00365 Q1 : -0.67820 Q2 : -0.00365 Q3 : 0.67465 IQR : 1.35285 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0000348261
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000010047,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.616652,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.276034,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.055595,

5. λ_1=0.8, λ_2=0.1, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.5, W15=$\dfrac{\sqrt{n}\left(\overline{X}_2 - \mu\right)}{\sigma}$, x_1=0.1,

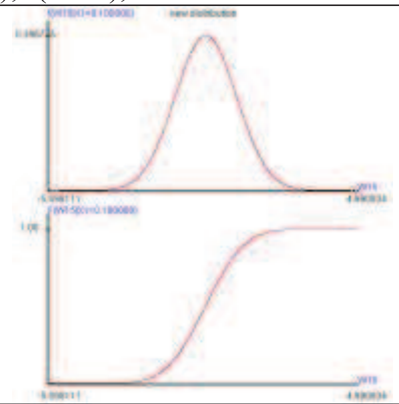
(5-1) $n\left(\overline{X}_2\right) = 15$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right) = 15$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.00001 Kurtosis Coef. : 2.91908 MAD : 0.80062 Range : 9.61515 Mid_range : -0.04002 Median : 0.00009 Q1 : -0.68027 Q2 : 0.00009 Q3 : 0.68027 IQR : 1.36054 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0000752874
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000018144,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.383816,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.208204,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.037580,

(5-2) $n\left(\overline{X}_2\right) = 25$,

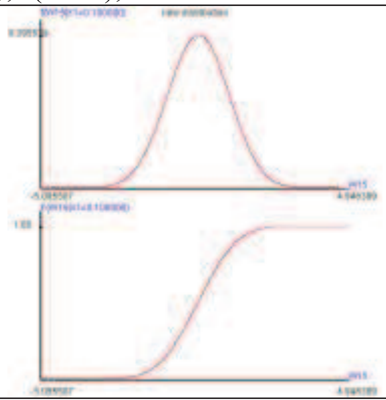
f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.00071 Kurtosis Coef. : 2.95181 MAD : 0.79952 Range : 10.12645 Mid_range : -0.05364 Median : 0.00009 Q1 : -0.67819 Q2 : 0.00009 Q3 : 0.67834 IQR : 1.35653 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0000272932
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000007249,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.625217,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.331431,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.061525,

6. λ_1=0.3, λ_2=0.42, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.6, W15=$\dfrac{\sqrt{n}\left(\overline{X}_2-\mu\right)}{\sigma}$, x_1=0.1,

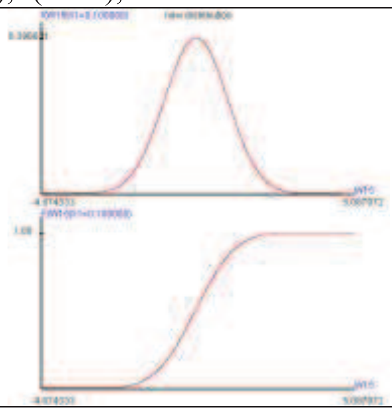
(6-1) $n\left(\overline{X}_2\right)=15$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right)=15$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.03248 Kurtosis Coef. : 2.92064 MAD : 0.80056 Range : 10.08123 Mid_range : -0.07356 Median : 0.00553 Q1 : -0.67716 Q2 : 0.00553 Q3 : 0.68314 IQR : 1.36030 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0001352863
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000034991,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.288426,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.139061,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.031841,

(6-2) $n\left(\overline{X}_2\right)=25$,

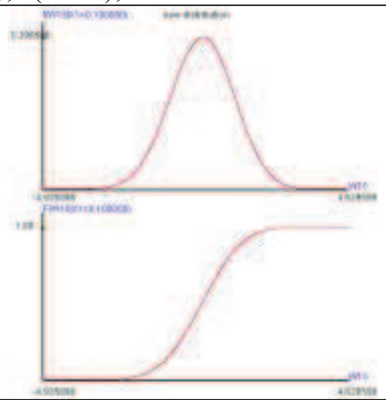
f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.02542 Kurtosis Coef. : 2.95319 MAD : 0.79948 Range : 9.99934 Mid_range : 0.10682 Median : 0.00435 Q1 : -0.67566 Q2 : 0.00435 Q3 : 0.68027 IQR : 1.35593 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0000626316
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000017015,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.468752,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.199303,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.038066,

7. λ_1=0.4, λ_2=0.42, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.7, W15= $\dfrac{\sqrt{n}\left(\overline{X}_2 - \mu\right)}{\sigma}$, x_1=0.1,

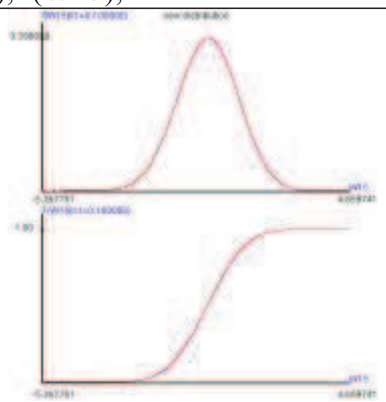
(7-1) $n\left(\overline{X}_2\right) = 18$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right) = 18$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.06103 Kurtosis Coef. : 2.93703 MAD : 0.80014 Range : 9.60105 Mid_range : -0.15315 Median : 0.01043 Q1 : -0.67381 Q2 : 0.01043 Q3 : 0.68512 IQR : 1.35893 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0002653701
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000076660,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.192625,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.092424,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.016698,

(7-2) $n\left(\overline{X}_2\right) = 20$,

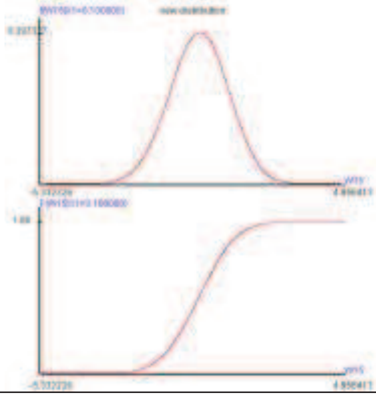
f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.05892 Kurtosis Coef. : 2.94334 MAD : 0.79986 Range : 10.06478 Mid_range : -0.35401 Median : 0.01022 Q1 : -0.67358 Q2 : 0.01022 Q3 : 0.68420 IQR : 1.35778 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0002415124
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000073056,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.198279,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.100185,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.021307,

8. λ_1=0.1, λ_2=0.72, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.8, W15=$\dfrac{\sqrt{n}\left(\overline{X}_2-\mu\right)}{\sigma}$, x_1=0.1,

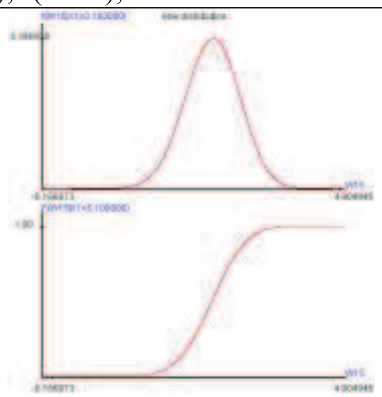
(8-1) $n\left(\overline{X}_2\right)=35$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right)=35$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.07253 Kurtosis Coef. : 2.97236 MAD : 0.79897 Range : 10.32689 Mid_range : -0.18791 Median : 0.01226 Q1 : -0.67023 Q2 : 0.01226 Q3 : 0.68325 IQR : 1.35348 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0003090050
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000092501,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.999267,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.179163,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.086260,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.017262,

(8-2) $n\left(\overline{X}_2\right)=25$,

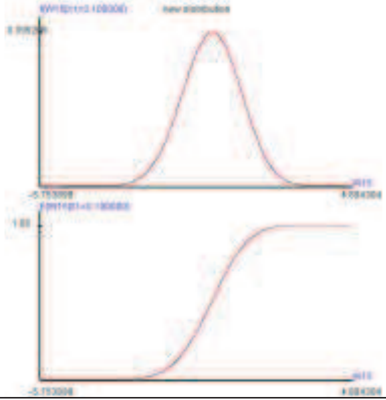
f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.08571 Kurtosis Coef. : 2.96199 MAD : 0.79938 Range : 11.11217 Mid_range : -0.63056 Median : 0.01469 Q1 : -0.66994 Q2 : 0.01469 Q3 : 0.68549 IQR : 1.35543 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0004459321
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000136344,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.746419,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.143397,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.067630,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.013886,

9. $\lambda_1=0.2$, $\lambda_2=0.72$, $\lambda^* = \frac{\lambda_2}{1-\lambda_1}=0.9$, W15=$\frac{\sqrt{n}\left(\overline{X}_2-\mu\right)}{\sigma}$, $x_1=0.1$,

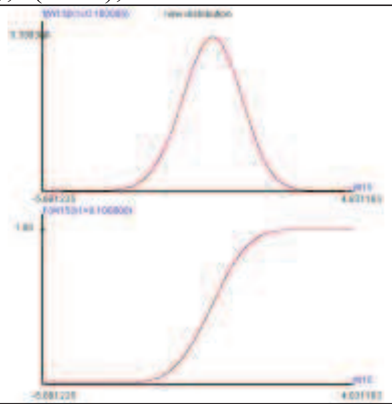
(9-1) $n\left(\overline{X}_2\right)=80$, W15 is approaching Z distribution when $n\left(\overline{X}_2\right)=80$.

f(W15),F(W15),	Coefficinet
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.07528 Kurtosis Coef. : 2.99199 MAD : 0.79829 Range : 10.59745 Mid_range : -0.47480 Median : 0.01270 Q1 : -0.66848 Q2 : 0.01270 Q3 : 0.68219 IQR : 1.35067 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0003199837
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000098908,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.907827,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.165939,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.082244,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.015341,

(9-2) $n\left(\overline{X}_2\right)=50$,

f(W15),F(W15),	Coefficinet
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 1.00000 S.D. : 1.00000 Skewed Coef. : -0.09470 Kurtosis Coef. : 2.98911 MAD : 0.79852 Range : 10.55147 Mid_range : -0.42503 Median : 0.01586 Q1 : -0.66735 Q2 : 0.01586 Q3 : 0.68457 IQR : 1.35192 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0005131452
*********** | W15 distribution function - Z0 distribution function| ***********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000155971,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.694637,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.126577,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.062076,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.012583,

Section 4, λ* = λ2 / (1 – λ1) estimated value

1. $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ estimated method,

$X_{2,1}, X_{2.2}, \ldots, X_{2,n} \overset{iid}{\sim} f_{X_2|x_1}(x_2|x_1), X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1),$

$\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ point estimator= $\hat{\lambda}^*$ and the sample mean= $\overline{X}_2 = \dfrac{\sum_{j=1}^{n} X_{2,j}}{n}$,

The $\hat{\lambda}^*$ computed procees,
lower=0, upper=1, midpoint=0.5,
repeat

{
let $E(X_2|x_1)$ estimated value will be computed by λ^* =midpoint in according to the $E(X_2|x_1)$ function.
if $E(X_2|x_1)$ estimated value< $\overline{X}_2$ then upper=midpoint,
if $E(X_2|x_1)$ estimated value> $\overline{X}_2$ then lower=midpoint,
midpoint=(lower+upper)/2, }

until | $E(X_2|x_1)$ estimated value- $\overline{X}_2$ |<0.000001,

$\hat{\lambda}^*$ =midpoint is the $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ estimated value.

$\hat{\lambda}^*$ is not function of sufficient statistic.

$E(\hat{\lambda}^*) = \lambda^* + b(\lambda^*), b(\lambda^*) \neq 0, b(\lambda^*) \xrightarrow[n\to\Im]{} 0,$

$Var(\hat{\lambda}^*) \xrightarrow[n\to\Im]{} 0, \ \hat{\lambda} \xrightarrow[n\to\Im]{} \lambda^*.$

Note:
$E(X_2|x_1)$ function,

$(i) \lambda^* \neq 0.5,$

$$E(X_2|x_1) = \frac{(1-x_1)(\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}} - \frac{1}{ln(\lambda^*) - ln(1-\lambda^*)},$$

$(ii) \lambda^* = 0.5,$

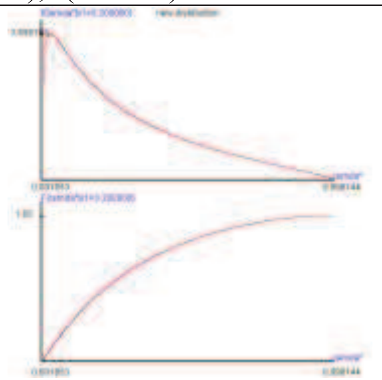
$$E(X_2|x_1) = \frac{1-x_1}{2},.$$

2. The sampling distribution of $\hat{\lambda}^*$,

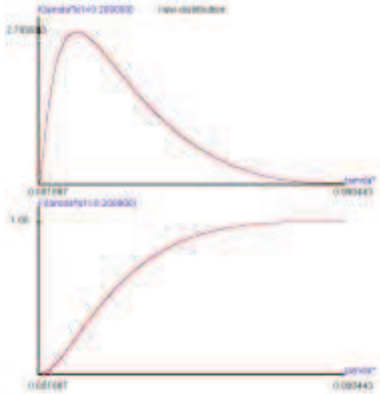
Let lamda*= $\hat{\lambda}^*$,

(1) λ_1 =0.1, λ_2 =0.2, x_1 =0.2, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.222222222,

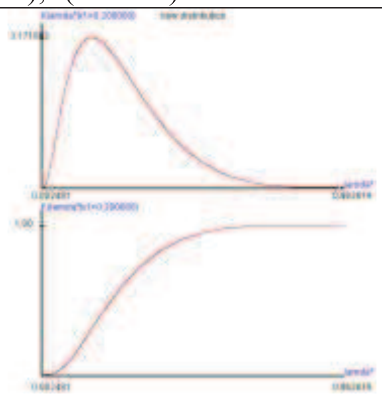
(1-1)n=10,

f(lamda*),F(lamda*)	Coefficient
	Mathematical Mean: 0.27828 Geometrical Mean : 0.17481 Harmonic Mean : 0.03724 Variance : 0.05080 S.D. : 0.22539 Skewed Coef. : 0.89221 Kurtosis Coef. : 2.96569 MAD : 0.18509 Range : 0.99999 Mid_range : 0.50000 Median : 0.21768 Q1 : 0.09323 Q2 : 0.21768 Q3 : 0.41730 IQR : 0.32407 C.V. : 0.80994

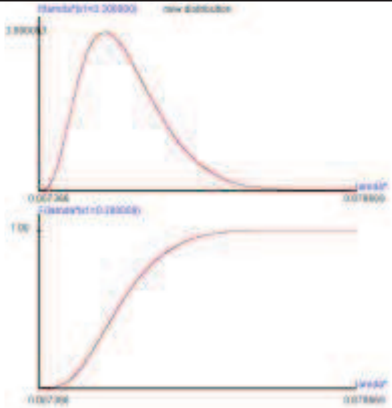
(1-2)n=20,

f(lamda*),F(lamda*)	Coefficient
	Mathematical Mean: 0.25528 Geometrical Mean : 0.19761 Harmonic Mean : 0.13290 Variance : 0.02813 S.D. : 0.16773 Skewed Coef. : 0.89636 Kurtosis Coef. : 3.37802 MAD : 0.13500 Range : 0.99225 Mid_range : 0.49616 Median : 0.21996 Q1 : 0.12394 Q2 : 0.21996 Q3 : 0.35426 IQR : 0.23032 C.V. : 0.65703

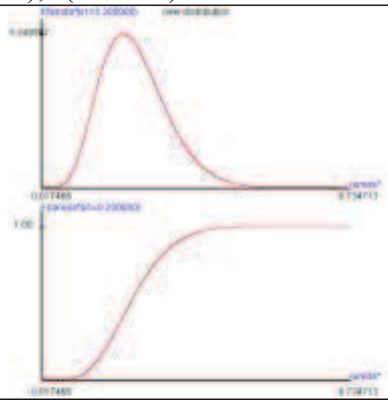
(1-3)n=30,

f(lamda*),F(lamda*)	Coefficient
	Mathematical Mean: 0.24569 Geometrical Mean : 0.20559 Harmonic Mean : 0.16233 Variance : 0.01929 S.D. : 0.13888 Skewed Coef. : 0.84958 Kurtosis Coef. : 3.48413 MAD : 0.11111 Range : 0.96371 Mid_range : 0.48255 Median : 0.22071 Q1 : 0.13936 Q2 : 0.22071 Q3 : 0.32776 IQR : 0.18840 C.V. : 0.56525

(1-4)n=50,

f(lamda*),F(lamda*)	Coefficient	
	Mathematical Mean:	0.23709
	Geometrical Mean :	0.21214
	Harmonic Mean :	0.18598
	Variance :	0.01176
	S.D. :	0.10843
	Skewed Coef. :	0.75290
	Kurtosis Coef. :	3.48222
	MAD :	0.08645
	Range :	0.87474
	Mid_range :	0.44312
	Median :	0.22131
	Q1 :	0.15603
	Q2 :	0.22131
	Q3 :	0.30210
	IQR :	0.14607
	C.V. :	0.45734

(1-5)n=100,

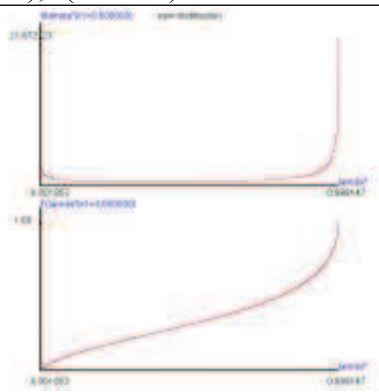
f(lamda*),F(lamda*)	Coefficient	
	Mathematical Mean:	0.23000
	Geometrical Mean :	0.21715
	Harmonic Mean :	0.20399
	Variance :	0.00591
	S.D. :	0.07688
	Skewed Coef. :	0.59592
	Kurtosis Coef. :	3.36119
	MAD :	0.06123
	Range :	0.71991
	Mid_range :	0.37609
	Median :	0.22179
	Q1 :	0.17399
	Q2 :	0.22179
	Q3 :	0.27735
	IQR :	0.10336
	C.V. :	0.33428

(1-6)n=500,

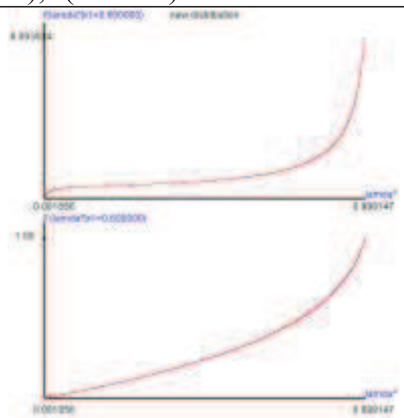
f(lamda*),F(lamda*)	Coefficient	
	Mathematical Mean:	0.22383
	Geometrical Mean :	0.22120
	Harmonic Mean :	0.21856
	Variance :	0.00118
	S.D. :	0.03434
	Skewed Coef. :	0.29225
	Kurtosis Coef. :	3.09686
	MAD :	0.02739
	Range :	0.37142
	Mid_range :	0.27157
	Median :	0.22214
	Q1 :	0.19976
	Q2 :	0.22214
	Q3 :	0.24606
	IQR :	0.04630
	C.V. :	0.15342

(2) λ_1 =0.5, λ_2 =0.4, x_1 =0.6, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.8,

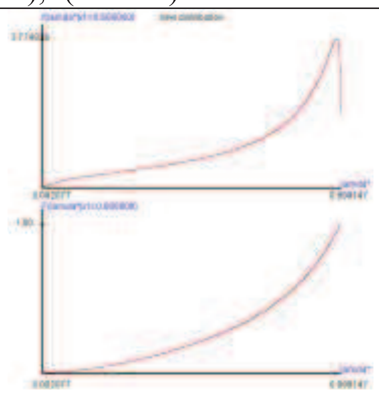
(2-1)n=10,

f(lamda*),F(lamda*)	Coefficient
	Mathematical Mean: 0.66669 Geometrical Mean : 0.47441 Harmonic Mean : 0.02587 Variance : 0.11250 S.D. : 0.33541 Skewed Coef. : -0.70067 Kurtosis Coef. : 2.01478 MAD : 0.29353 Range : 1.00000 Mid_range : 0.50000 Median : 0.80444 Q1 : 0.38632 Q2 : 0.80444 Q3 : 0.96591 IQR : 0.57959 C.V. : 0.50310

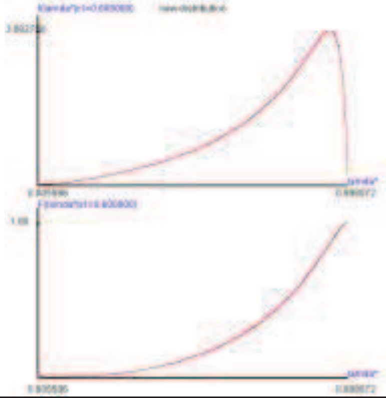
(2-2)n=20,

f(lamda*),F(lamda*)	Coefficient
	Mathematical Mean: 0.70509 Geometrical Mean : 0.60514 Harmonic Mean : 0.34741 Variance : 0.07652 S.D. : 0.27663 Skewed Coef. : -0.87093 Kurtosis Coef. : 2.57873 MAD : 0.23265 Range : 0.99999 Mid_range : 0.50000 Median : 0.80219 Q1 : 0.52063 Q2 : 0.80219 Q3 : 0.93961 IQR : 0.41898 C.V. : 0.39233

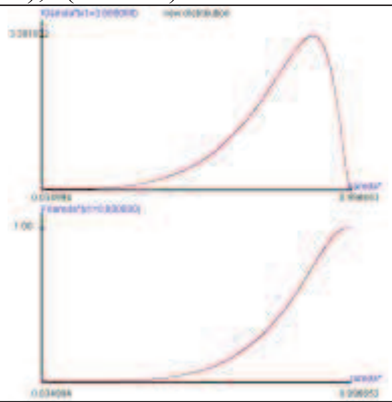
(2-3)n=30,

f(lamda*),F(lamda*)	Coefficient
	Mathematical Mean: 0.72565 Geometrical Mean : 0.66089 Harmonic Mean : 0.52647 Variance : 0.05769 S.D. : 0.24019 Skewed Coef. : -0.95349 Kurtosis Coef. : 2.95870 MAD : 0.19782 Range : 0.99977 Mid_range : 0.50011 Median : 0.80145 Q1 : 0.57977 Q2 : 0.80145 Q3 : 0.92320 IQR : 0.34343 C.V. : 0.33100

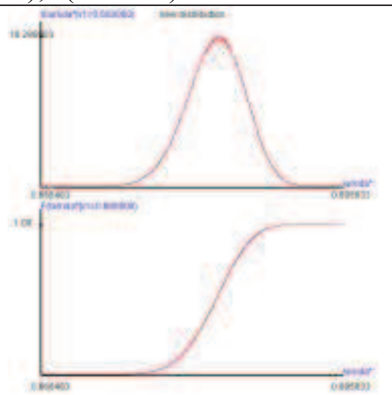
(2-4)n=50,

f(lamda*),F(lamda*)	Coefficient	
	Mathematical Mean:	0.74787
	Geometrical Mean :	0.71172
	Harmonic Mean :	0.65320
	Variance :	0.03819
	S.D. :	0.19542
	Skewed Coef. :	-1.01549
	Kurtosis Coef. :	3.40907
	MAD :	0.15769
	Range :	0.99617
	Mid_range :	0.50183
	Median :	0.80090
	Q1 :	0.63657
	Q2 :	0.80090
	Q3 :	0.90318
	IQR :	0.26662
	C.V. :	0.26130

(2-5)n=100,

f(lamda*),F(lamda*)	Coefficient	
	Mathematical Mean:	0.77004
	Geometrical Mean :	0.75404
	Harmonic Mean :	0.73352
	Variance :	0.02017
	S.D. :	0.14203
	Skewed Coef. :	-0.99551
	Kurtosis Coef. :	3.78339
	MAD :	0.11288
	Range :	0.96523
	Mid_range :	0.51582
	Median :	0.80045
	Q1 :	0.69031
	Q2 :	0.80045
	Q3 :	0.87883
	IQR :	0.18852
	C.V. :	0.18445

(2-6)n=4000,

f(lamda*),F(lamda*)	Coefficient	
	Mathematical Mean:	0.79912
	Geometrical Mean :	0.79882
	Harmonic Mean :	0.79851
	Variance :	0.00049
	S.D. :	0.02212
	Skewed Coef. :	-0.23773
	Kurtosis Coef. :	3.07450
	MAD :	0.01763
	Range :	0.22820
	Mid_range :	0.78216
	Median :	0.79999
	Q1 :	0.78470
	Q2 :	0.79999
	Q3 :	0.81450
	IQR :	0.02980
	C.V. :	0.02768

3. The computed estimated value of $\hat{\lambda}^*$ by simulated data

(1) λ_1=0.2, λ_2=0.4, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.5,

(1-1) x_1=0.2,
(a)n=10, $\hat{\lambda}^*$=0/2599609385, (b)n=50, $\hat{\lambda}^*$=0.56663458654,
(c)n=100, $\hat{\lambda}^*$=0.5440046188, (d)n=500, $\hat{\lambda}^*$=0.4661391509,
(e)n=1,000, $\hat{\lambda}^*$=0.5228015954, (f)n=10,000, $\hat{\lambda}^*$=0.4951802464,
(g)n=100,000, $\hat{\lambda}^*$=0.5011871914, (h)n=1,000,000, $\hat{\lambda}^*$=0.4993828590,

(1-2) x_1=0.9,
(a)n=10, $\hat{\lambda}^*$=0.1793815359, (b)n=50, $\hat{\lambda}^*$=0.1385749710,
(c)n=100, $\hat{\lambda}^*$=0.9617780736, (d)n=500, $\hat{\lambda}^*$=0.5924358750,
(e)n=1,000, $\hat{\lambda}^*$=0.6507483642, (f)n=10,000, $\hat{\lambda}^*$=0.4574610470,
(g)n=100,000, $\hat{\lambda}^*$=0.4760391605, (h)n=1,000,000, $\hat{\lambda}^*$=0.4989973940,

(2) λ_1=0.2, λ_2=0.64, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$=0.8,

(2-1) x_1=0.2,
(a)n=10, $\hat{\lambda}^*$=0.8100391362, (b)n=50, $\hat{\lambda}^*$=0.8542375866,
(c)n=100, $\hat{\lambda}^*$=0.7313282770, (d)n=500, $\hat{\lambda}^*$=0.7952513814,
(e)n=1,000, $\hat{\lambda}^*$=0.7925437660, (f)n=10,000, $\hat{\lambda}^*$=0.7941872043,
(g)n=100,000, $\hat{\lambda}^*$=0.8010429393, (h)n=1,000,000, $\hat{\lambda}^*$=0.7991706425,

(2-2) x_1=0.9,
(a)n=10, $\hat{\lambda}^*$=0.7357834624, (b)n=50, $\hat{\lambda}^*$=0.7748116157,
(c)n=100, $\hat{\lambda}^*$=0.1266213986, (d)n=500, $\hat{\lambda}^*$=0.5257581845,
(e)n=1,000, $\hat{\lambda}^*$=0.8887161310, (f)n=10,000, $\hat{\lambda}^*$=0.7872918001,
(g)n=100,000, $\hat{\lambda}^*$=0.7697467254, (h)n=1,000,000, $\hat{\lambda}^*$=0.7974339414,

Section 5. The test statistic of λ* = λ2 / (1 – λ1)

1. The test statistic,

$X_{2,1}, X_{2,2}, ..., X_{2,n} \overset{iid}{\sim} CB(\lambda_1, \lambda_2, x_1)$, n random samples from $CB(\lambda_1, \lambda_2, x_1)$.

Let . $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$,

The Z test statistic for large sample,

$n \geq 15+15\times|\lambda^*$ -0.5|, if $0.1 \leq \lambda \leq 0.9$,

$$\frac{\sqrt{n}\left(\overline{X}_2 - \mu\right)}{\sigma} \longrightarrow Normal(0,1), \overline{X}_2 = \frac{\sum_{j=1}^{n} X_{2,j}}{n}, E(X_2) = \mu, Var(X_2) = \sigma^2,$$

$X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1)$.

$H_0 : \lambda^* = c \quad H_0 : \lambda^* = c$,

$$Z^* = \frac{\sqrt{n}\left(\overline{X} - \mu_0\right)}{\sigma_0} \rightarrow Z \sim Normal(0,1), \left|Z^*\right| > Z_{\alpha/2} \text{ rejected } H_0 \text{ and } P\left(Z > Z_{\alpha/2}\right) = \frac{\alpha}{2}.$$

$(i) c \neq 0.5$,

$$\mu_0 = E(X_2|x_1) = \frac{(1-x_1)(c)^{1-x_1}}{(c)^{1-x_1} - (1-c)^{1-x_1}} - \frac{1}{ln(c) - ln(1-c)},$$

$$\sigma_0^2 = Var(X_2|x_1) = \frac{1}{\left(ln(c) - ln(1-c)\right)^2} - \frac{(1-x_1)^2 c^{1-x_1}(1-c)^{1-x_1}}{\left(c^{1-x_1} - (1-c)^{1-x_1}\right)^2},$$

$(ii) c = 0.5$,

$$\mu_0 = E(X_2|x_1) = \frac{1-x_1}{2}, \sigma_0^2 = Var(X_2|x_1) = \frac{(1-x_1)^2}{12}.$$

About $E(X_2|x_1)$ and $Var(X_2|x_1)$, please see the appendix 1.

2. Example

The data is stimulated from simulator of $X_2|x_1 \sim CB(\lambda_1,\lambda_2,x_1)$, the sample size is n.

(1) λ_1=0.1, λ_2=0.09, $\lambda^* = \frac{\lambda_2}{1-\lambda_1}$ =0.1

(1-1) x_1=0.1,
(i)n=100,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.2851720435,
Z test=-1.0707570105, p value=0.284252>0.05, failed to reject H0: λ^*=0.1.
(ii)n=200,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.2926309149,
Z test=-1.0689296348, p value=0.284938>0.05, failed to reject H0: λ^*=0.1.
(iii)n=1,000,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.3055531335,
Z test=-0.6649578556, p value=0.506490>0.05, failed to reject H0: λ^*=0.1.
(iv)n=10,000,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.3148167583,
Z test=1.8082722080, p value=0.70962>0.05, failed to reject H0: λ^*=0.1.

(1-2) x_1=0.7, λ_1=0.1, λ_2=0.09, $\lambda^* = \frac{\lambda_2}{1-\lambda_1}$ =0.1,
(i)n=100,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.1330257695,
Z test=-0.0715702100, p value=0.944038>0.05, failed to reject H0: λ^*=0.1.
(ii)n=200,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.1339923282,
Z test=0.0583356969, p value=0.953306>0.05, failed to reject H0: λ^*=0.1.
(iii)n=1,000,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.1351672254,
Z test=0.5641101128, p value=0.572278>0.05, failed to reject H0: λ^*=0.1.
(iv)n=10,000,
H0: λ^*=0.1,
$\overline{X}_2|x_1$ =0.1313600586,
Z test=-2.6599724390, p value=0.0078536<0.01, rejected H0: λ^*=0.1.

(2) λ_1=0.4, λ_2=0.12, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.2,

(2-1) x_1=0.2,
(i)n=100,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.2956513984,
Z test=-1.4230205395, p value=0.154958>0.05, failed to reject H0: λ^*=0.2.
(ii)n=200,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.3393012402,
Z test=0.7425093372, p value=0.457660>0.05, failed to reject H0: λ^*=0.2.
(iii)n=1,000,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.3337244984,
Z test=0.8732576646, p value=0.382534>0.05, failed to reject H0: λ^*=0.2.
(iv)n=10,000,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.3265907042,
Z test=-0.4222661987, p value=0.673264>0.05, failed to reject H0: λ^*=0.2.
(2-2) x_1=0.8, λ_1=0.4, λ_2=0.12, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.2,
(i)n=100,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.0922244833,
Z test=-0.5484569907, p value=0.583312>0.05, failed to reject H0: λ^*=0.2.
(ii)n=200,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.0909258090,
Z test=-1.1188974460, p value=0.263124>0.05, failed to reject H0: λ^*=0.2.
(iii)n=1,000,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.0959329140,
Z test=0.3007208096, p value=0.763090>0.05, failed to reject H0: λ^*=0.2.
(iv)n=10,000,
H0: λ^*=0.2,
$\overline{X}_2|x_1$ =0.0938605119,
Z test=-2.6454402143, p value=0.008264<0.05, rejected H0: λ^*=0.2.

(3) λ_1=0.9, λ_2=0.08, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.8,

(3-1) x_1=0.4, λ_1=0.9, λ_2=0.08, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.8,

(i)n=100,
H0: λ^*=0.8,
$\overline{X}_2|x_1$ =0.3467354308,

Z test=0.3299799122, p value=0.740504>0.05, failed to reject H0: λ^*=0.8.
(ii)n=200,
H0: λ^*=0.8,
$\overline{X}_2|x_1$ =0.3294236629,

Z test=-0.9712489905, p value=0.331488>0.05, failed to reject H0: λ^*=0.8.
(iii)n=1,000,
H0: λ^*=0.8,
$\overline{X}_2|x_1$ =0.3505316801

Z test=1,7485550085, p value=0.080366>0.05, failed to reject H0: λ^*=0.8.
(iv)n=10,000,
H0: λ^*=0.8,
$\overline{X}_2|x_1$ =0.3416652033,

Z test=0.3219474895, p value=0.746660>0.05, failed to reject H0: λ^*=0.8.

(3-2) x_1=0.9, λ_1=0.9, λ_2=0.08, $\lambda^* = \dfrac{\lambda_2}{1-\lambda_1}$ =0.8,

(i)n=100,
H0: λ^*=0.8,
$\overline{X}_2|x_1$ =0.0541042355,

Z test=1.0221791454, p value=0.306736>0.05, failed to reject H0: λ^*=0.8.
(ii)n=200,
H0: λ^*=0. 8,
$\overline{X}_2|x_1$ =0.0505863353,

Z test=-0.2786604815, p value=0.781180>0.05, failed to reject H0: λ^*=0.8.
(iii)n=1,000,
H0: λ^*=0. 8,
$\overline{X}_2|x_1$ =0.0521765631,

Z test=1.1197404029, p value=0.262942>0.05, failed to reject H0: λ^*=0.8.
(iv)n=10,000,
H0: λ^*=0. 8,
$\overline{X}_2|x_1$ =0.0504483476,

Z test=-2.4486601228, p value=0.014718>0.01, failed to reject H0: λ^*=0.8.

Chapter 7 Designed the probability model using model 3

There are 4 categories, X_1, X_2 and X_3 are continuous random variables,

λ_1	λ_2	λ_3	$1-\lambda_1-\lambda_2-\lambda_3$

$$X_i \sim CB(\lambda_i), 0 \le x_i \le 1, i = 1,2,3,$$
$$X_4 \sim CB(1-\lambda_1-\lambda_2-\lambda_3), 0 \le x_4 \le 1,$$

Section 1.The probability models of X1 to X2 and X1 to X3

There two new conditional Continuous Bernoulli distribution,

1. $X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1,$

$$X_2|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_2 \le 1-x_1,$$

$$f_{X_2|x_1}(x_2|x_1) = C_2(\lambda_1, \lambda_2, x_1)\left(\frac{\lambda_2}{1-\lambda_1}\right)^{x_2}\left(1-\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1-x_2},$$

$$0 \le x_1 \le 1, 0 \le x_2 \le 1-x_1,$$

2.

$$X_3|x_1 \sim CB(\lambda_1, \lambda_2, x_1), 0 \le x_3 \le 1-x_1,$$

$$f_{X_3|x_1}(x_3|x_1)C_2(\lambda_1, \lambda_3, x_1)\left(\frac{\lambda_3}{1-\lambda_1}\right)^{x_3}\left(1-\frac{\lambda_3}{1-\lambda_1}\right)^{1-x_1-x_3},$$

$$0 \le x_1 \le 1, 0 \le x_3 \le 1-x_1,$$

The diagram,

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$1-X_1 = X_2 + (1-X_1-X_2)$$

X_2	$1-X_1-X_2$
$\frac{\lambda_2}{1-\lambda_1}$	$1-\frac{\lambda_2}{1-\lambda_1}$

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$1-X_1 = X_3 + (1-X_1-X_3)$$

X_3	$1-X_1-X_3$
$\frac{\lambda_3}{1-\lambda_1}$	$1-\frac{\lambda_3}{1-\lambda_1}$

3.The comparison of $X_2|x_1$ and $X_3|x_1$ at the same condition situation using the estimated line to do statistical analysis.

X1~$CB(\lambda_1)$,

X2|x1~$CB(\lambda_1,\lambda_2,x_1)$,

X3|x1~$CB(\lambda_1,\lambda_3,x_1)$,

X1 is sample data and getting two paired samples (X1,X2|X1), (X1,X3|X1) , the data is simulated data and doing the simple linear model analysis,

(1) λ_2=0.2, λ_3=0.2,

(1-1) λ_1 =0.1, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=100, the simple linear model estimated line as,

X2 estimated=0.4500600570+-0.4887984472*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	1.4571394458	1.4571394458
Error	98	4.4564085230	0.0454735564
Total	99	5.9135479688	

H0:slope=0, test statistic=32.043666 , p value=0.000000

R2=0.246407, R2(adj)=0.238717,MSE=0.045474,

H0:residual population~Rayleigh(lamda=4.261469,c=-0.437442)

chi square test statistic=1.080000, p value=0.897818,

99% C.I. for slope, [-0.7116863489, -0.2583440536]

95% C.I. for slope, [-0.6585572527, -0.3158529419]

90% C.I. for slope, [-0.6314853040, -0.3446247029]

99% C.I. for intercept, [0.3580867559, 0.5374067239]

95% C.I. for intercept, [0.3809345973, 0.5162820756]

90% C.I. for intercept, [0.3923652138, 0.5055576266]

X3 estimated=0.3932578534+-0.3616806821*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	0.7977964619	0.7977964619
Error	98	3.9546759315	0.0403538360
Total	99	4.7524723933	

H0:slope=0, test statistic=19.770028 , p value=0.000016

R2=0.167870, R2(adj)=0.159379,MSE=0.040354,

H0:residual population~Trapezoid(mu=-0.001168,c=0.278428)

chi square test statistic=1.500000, p value=0.826921

99% C.I. for slope, [-0.5750737853, -0.1477797836]

95% C.I. for slope, [-0.5231303020, -0.2003844184]

90% C.I. for slope, [-0.4968014526, -0.2266360011]

99% C.I. for intercept, [0.3087377227, 0.4776501221]

95% C.I. for intercept, [0.3295104412, 0.4569557090]

90% C.I. for intercept, [0.3399349651, 0.4465733327]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2=\lambda_3$ from two CI's about slope and intercept at significant=5% or 1%.

(1-2) $\lambda_1 = 0.2$, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.4529433281+-0.4686956300*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	2.9561293947	2.9561293947
Error	198	7.8611093267	0.0397025724
Total	199	10.8172387214	

H0:slope=0, test statistic=74.456873 , p value=0.000000

R2=0.273279, R2(adj)=0.269609,MSE=0.039703,

H0:residual population~Rayleigh(lamda=5.220063,c=-0.394618)

chi square test statistic=3.520000, p value=0.619667

99% C.I. for slope, [-0.6087130795, -0.3262040597]

95% C.I. for slope, [-0.5752017831, -0.3609226637]

90% C.I. for slope, [-0.5582168275, -0.3784356696]

99% C.I. for intercept, [0.3874919299, 0.5163716146]

95% C.I. for intercept, [0.4035403743, 0.5011430198]

90% C.I. for intercept, [0.4115610692, 0.4933306275]

X3 estimated=0.4128520736+-0.3931965835*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	2.0804682981	2.0804682981
Error	198	7.3559465131	0.0371512450
Total	199	9.4364148112	

H0:slope=0, test statistic=55.999962 , p value=0.000000

R2=0.220472, R2(adj)=0.216535,MSE=0.037151,

H0:residual population~Normal(mu=-0.013843,sigma*sigma=0.031913)

chi square test statistic=1.280000, p value=0.936567,

99% C.I. for slope, [-0.5296571304, -0.2565849164]

95% C.I. for slope, [-0.4966346346, -0.2897239331]

90% C.I. for slope, [-0.4799128948, -0.3063730824]

99% C.I. for intercept, [0.3506593117, 0.4750448231]

95% C.I. for intercept, [0.3656521311, 0.4599891553]

90% C.I. for intercept, [0.3733700878, 0.4522643659]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(1-3) $\lambda_1 = 0.3$, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=400, the simple linear model estimated line as,

X2 estimated=0.4398168494+-0.4282379854*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	5.2322288130	5.2322288130
Error	398	14.3025632565	0.0359360886
Total	399	19.5347920695	

H0:slope=0, test statistic=145.598172 , p value=0.000000

R2=0.267842, R2(adj)=0.266002,MSE=0.035936,

H0:residual population~Double exponential(lamda=6.075102,mu=-0.016875)

chi square test statistic=2.975000, p value=0.812014

99% C.I. for slope,	[-0.5201025537,	-0.3363371751]
95% C.I. for slope,	[-0.4980367502,	-0.3583712835]
90% C.I. for slope,	[-0.4868474151,	-0.3696922516]
99% C.I. for intercept,	[0.3943151964,	0.4851769703]
95% C.I. for intercept,	[0.4052167713,	0.4743447751]
90% C.I. for intercept,	[0.4107948429,	0.4688338055]

X3 estimated=0.4374274284+-0.4147122439*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	4.9069323290	4.9069323290
Error	398	12.4123164395	0.0311867247
Total	399	17.3192487685	

H0:slope=0, test statistic=157.340419 , p value=0.000000

R2=0.283322, R2(adj)=0.281522,MSE=0.031187,

H0:residual population~Hyperbolic secant(mu=-0.003528,sigma=0.195484)

chi square test statistic=2.525000, p value=0.86605,

99% C.I. for slope,	[-0.5000168102,	-0.3293990916]
95% C.I. for slope,	[-0.4794989539,	-0.3498079432]
90% C.I. for slope,	[-0.4690588832,	-0.3602984444]
99% C.I. for intercept,	[0.3952989095,	0.4795184110]
95% C.I. for intercept,	[0.4053309631,	0.4694911442]
90% C.I. for intercept,	[0.4104663772,	0.4643712641]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(1-4) $\lambda_1 = 0.4$, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=600, the simple linear model estimated line as,

X2 estimated=0.4491854617+-0.4366398480*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	9.2367592600	9.2367592600
Error	598	19.2683756739	0.0322213640
Total	599	28.5051349338	

H0:slope=0, test statistic=286.665681 , p value=0.000000

R2=0.324038, R2(adj)=0.322908,MSE=0.032221,

H0:residual population~Double exponential(lamda=6.768197,mu=-0.012384)

chi square test statistic=6.233333, p value=0.513014,

99% C.I. for slope, [-0.5031877405, -0.3698841085]

95% C.I. for slope, [-0.4870872569, -0.3858901854]

90% C.I. for slope, [-0.4790252329, -0.3940294138]

99% C.I. for intercept, [0.4131903612, 0.4849617060]

95% C.I. for intercept, [0.4217917666, 0.4763984502]

90% C.I. for intercept, [0.4262203631, 0.4720513735]

X3 estimated=0.4566418452+-0.4466298823*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	9.6642564026	9.6642564026
Error	598	17.0207424856	0.0284627801
Total	599	26.6849988882	

H0:slope=0, test statistic=339.540142 , p value=0.000000

R2=0.362161, R2(adj)=0.361094,MSE=0.028463,

H0:residual population~Double exponential(lamda=6.935862,mu=-0.004213)

chi square test statistic=4.800000, p value=0.683910,

99% C.I. for slope, [-0.5090200305, -0.3840812166]

95% C.I. for slope, [-0.4940971964, -0.3989777740]

90% C.I. for slope, [-0.4865093762, -0.4066151594]

99% C.I. for intercept, [0.4229012444, 0.4903102891]

95% C.I. for intercept, [0.4309646390, 0.4822696465]

90% C.I. for intercept, [0.4351163981, 0.4781254718]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(1-5) $\lambda_1 = 0.5$, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=1,000, the simple linear model estimated line as,

X2 estimated=0.4630697351+-0.4538824909*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	16.8049934059	16.8049934059
Error	998	27.4608388325	0.0275158706
Total	999	44.2658322384	

H0:slope=0, test statistic=610.738205 , p value=0.000000

R2=0.379638, R2(adj)=0.379016,MSE=0.027516,

H0:residual population~Double exponential(lamda=7.695409,mu=-0.007662)

chi square test statistic=9.200000, p value=0.239140,

99% C.I. for slope, [-0.5011554561, -0.4064404323]

95% C.I. for slope, [-0.4898994567, -0.4178727315]

90% C.I. for slope, [-0.4840951343, -0.4236390197]

99% C.I. for intercept, [0.4353661411, 0.4906974028]

95% C.I. for intercept, [0.4420581130, 0.4840838563]

90% C.I. for intercept, [0.4454288187, 0.4806902407]

X3 estimated=0.4783162121+-0.4751158701*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	18.4141027313	18.4141027313
Error	998	25.3598927231	0.0254107142
Total	999	43.7739954544	

H0:slope=0, test statistic=724.659001 , p value=0.000000

R2=0.420663, R2(adj)=0.420083,MSE=0.025411,

H0:residual population~Double exponential(lamda=7.723562,mu=-0.003510)

chi square test statistic=7.520000, p value=0.376774,

99% C.I. for slope, [-0.5206122948, -0.4296316246]

95% C.I. for slope, [-0.5097508967, -0.4404781011]

90% C.I. for slope, [-0.5041733671, -0.4460973173]

99% C.I. for intercept, [0.4517766585, 0.5049290000]

95% C.I. for intercept, [0.4581328880, 0.4985061270]

90% C.I. for intercept, [0.4613651579, 0.4952664412]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(1-6) $\lambda_1 = 0.6$, λ_2=0.2, lamda3=0.2,
Getting (X1,X2),(X1,X3), the paired sample size=10,000, the simple linear model estimated line as,

X2 estimated=0.4972740446+-0.4963751688*X1---(1),
ANOVA

Source	df	SS	MS
Regression	1	205.2646841590	205.2646841590
Error	9998	249.4085924593	0.0249458484
Total	9999	454.6732766183	

H0:slope=0, test statistic=8228.410625 , p value=0.000000
R2=0.451455, R2(adj)=0.451400,MSE=0.024946,
H0:residual population~Double exponential(lamda=8.059910,mu=-0.000900)
chi square test statistic=153.684800, p value=0.000000,
99% C.I. for slope, [-0.5105275404, -0.4823032986]
95% C.I. for slope, [-0.5071248031, -0.4856406637]
90% C.I. for slope, [-0.5053877760, -0.4873629938]
99% C.I. for intercept, [0.4887571666, 0.5058082031]
95% C.I. for intercept, [0.4907690108, 0.5037733909]
90% C.I. for intercept, [0.4918201040, 0.5027260455]

X3 estimated=0.4984923677+-0.4970769275*X1---(2),
ANOVA

Source	df	SS	MS
Regression	1	205.8454872494	205.8454872494
Error	9998	241.8182997371	0.0241866673
Total	9999	447.6637869865	

H0:slope=0, test statistic=8510.700736 , p value=0.000000
R2=0.459822, R2(adj)=0.459768,MSE=0.024187,
H0:residual population~Double exponential(lamda=8.308334,mu=-0.000965)
chi square test statistic=174.735200, p value=0.000000,
99% C.I. for slope, [-0.5109631119, -0.4831694513]
95% C.I. for slope, [-0.5076272862, -0.4865089631]
90% C.I. for slope, [-0.5059384323, -0.4882173325]
99% C.I. for intercept, [0.4900805287, 0.5069101491]
95% C.I. for intercept, [0.4921015927, 0.5048933574]
90% C.I. for intercept, [0.4931208441, 0.5038680330]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(1-7) $\lambda_1 = 0.7$, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=100,000, the simple linear model estimated line as,

X2 estimated=0.4544066094+-0.4455589390*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	1599.9554899405	1599.9554899405
Error	99998	2180.7799874919	0.0218082360
Total	99999	3780.7354774324	

H0:slope=0, test statistic=73364.736471 , p value=0.000000

R2=0.423186, R2(adj)=0.423181,MSE=0.021808,

H0:residual population~Double exponential(lamda=9.189179,mu=-0.006494)

chi square test statistic=3080.792000, p value=0.000000,

99% C.I. for slope, [-0.4497952344, -0.4413179904]

95% C.I. for slope, [-0.4487783660, -0.4423336246]

90% C.I. for slope, [-0.4482588992, -0.4428564080]

99% C.I. for intercept, [0.4517114431, 0.4570976150]

95% C.I. for intercept, [0.4523512712, 0.4564597594]

90% C.I. for intercept, [0.4526813031, 0.4561304077]

X3 estimated=0.4520659909+-0.4439618493*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	1588.5060832073	1588.5060832073
Error	99998	2149.2945365358	0.0214933752
Total	99999	3737.8006197430	

H0:slope=0, test statistic=73906.776669 , p value=0.000000

R2=0.424984, R2(adj)=0.424978,MSE=0.021493,

H0:residual population~Double exponential(lamda=9.274119,mu=-0.006352)

chi square test statistic=3143.425780, p value=0.000000,

99% C.I. for slope, [-0.4481806887, -0.4397685374]

95% C.I. for slope, [-0.4471687978, -0.4407583139]

90% C.I. for slope, [-0.4466511525, -0.4412736583]

99% C.I. for intercept, [0.4493794171, 0.4547406661]

95% C.I. for intercept, [0.4500239255, 0.4541056786]

90% C.I. for intercept, [0.4503541488, 0.4537796389]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant= 1%.

(2) λ_2=0.2, λ_3=0.3,

(2-1) $\lambda_1 = 0.1$, λ_2=0.2, λ_3=0.3,

(i)Getting (X1,X2),(X1,X3), the paired sample size=100, the simple linear model estimated line as,

X2 estimated=0.4500600570+-0.4887984472*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	1.4571394458	1.4571394458
Error	98	4.4564085230	0.0454735564
Total	99	5.9135479688	

H0:slope=0, test statistic=32.043666 , p value=0.000000
R2=0.246407, R2(adj)=0.238717,MSE=0.045474,
H0:residual population~Rayleigh(lamda=4.261469,c=-0.437442)
chi square test statistic=1.080000, p value=0.897605
99% C.I. for slope, [-0.7117796602, -0.2581742030]
95% C.I. for slope, [-0.6580273028, -0.3156752785]
90% C.I. for slope, [-0.6311015570, -0.3443721105]
99% C.I. for intercept, [0.3579241020, 0.5373913935]
95% C.I. for intercept, [0.3807148704, 0.5162268352]
90% C.I. for intercept, [0.3922227106, 0.5055458723]

X3 estimated=0.4346447103+-0.4153610497*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	1.0521872053	1.0521872053
Error	98	4.2616188999	0.0434859071
Total	99	5.3138061052	

H0:slope=0, test statistic=24.196051 , p value=0.000006
R2=0.198010, R2(adj)=0.189827,MSE=0.043486,
H0:residual population~Trapezoid(mu=-0.016762,c=0.277612)
chi square test statistic=1.920000, p value=0.749848,
99% C.I. for slope, [-0.6372483910, -0.1939934111]
95% C.I. for slope, [-0.5831758073, -0.2478757508]
90% C.I. for slope, [-0.5558250546, -0.2752429781]
99% C.I. for intercept, [0.3468612197, 0.5225016672]
95% C.I. for intercept, [0.3684659125, 0.5008945186]
90% C.I. for intercept, [0.3793055210, 0.4900305551]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(ii)The sample size=2,000,

X2 estimated=0.4092801106+-0.3803705801*X1---(1),
ANOVA

Source	df	SS	MS
Regression	1	18.8541689195	18.8541689195
Error	1998	82.1859008342	0.0411340845
Total	1999	101.0400697537	

H0:slope=0, test statistic=458.358783 , p value=0.000000,
R2=0.186601, R2(adj)=0.186194,MSE=0.041134,
H0:residual population~Rayleigh(lamda=4.670435,c=-0.393868)
chi square test statistic=25.188000, p value=0.001503

99% C.I. for slope,	[-0.4258331321,	-0.3343525206]
95% C.I. for slope,	[-0.4150809223,	-0.3453416849]
90% C.I. for slope,	[-0.4095538842,	-0.3510307124]
99% C.I. for intercept,	[0.3900026261,	0.4283468612]
95% C.I. for intercept,	[0.3946095577,	0.4238069295]
90% C.I. for intercept,	[0.3969567571,	0.4214834426]

X3 estimated=0.4594935989+-0.4446468803*X1---(2),
ANOVA

Source	df	SS	MS
Regression	1	25.7646386150	25.7646386150
Error	1998	81.0093548710	0.0405452227
Total	1999	106.7739934860	

H0:slope=0, test statistic=635.454362 , p value=0.000000
R2=0.241301, R2(adj)=0.240921,MSE=0.040545,
H0:residual population~Normal(mu=0.000805,sigma*sigma=0.045034)
chi square test statistic=18.962000, p value=0.015214,

99% C.I. for slope,	[-0.4902001543,	-0.3992382791]
95% C.I. for slope,	[-0.4792262150,	-0.4100465517]
90% C.I. for slope,	[-0.4736466336,	-0.4156211911]
99% C.I. for intercept,	[0.4404564246,	0.4784882306]
95% C.I. for intercept,	[0.4449852794,	0.4739762627]
90% C.I. for intercept,	[0.4473138580,	0.4716575407]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(iii)The sample size=10,000,

X2 estimated=0.4071366232+-0.3824829770*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	98.4240035353	98.4240035353
Error	9998	405.5401357729	0.0405621260
Total	9999	503.9641393082	

H0:slope=0, test statistic=2426.500118 , p value=0.000000
R2=0.195300, R2(adj)=0.195219,MSE=0.040562,
H0:residual population~Rayleigh(lamda=4.948718,c=-0.396943)
chi square test statistic=137.122800, p value=0.000000,

99% C.I. for slope,	[	-0.4025134828,	-0.3624684937]
95% C.I. for slope,	[	-0.3977309963,	-0.3672793752]
90% C.I. for slope,	[	-0.3952545016,	-0.3697172546]
99% C.I. for intercept,	[	0.3987185486,	0.4155453199]
95% C.I. for intercept,	[	0.4007440413,	0.4135303131]
90% C.I. for intercept,	[	0.4017660409,	0.4125051701]

X3 estimated=0.4470003038+-0.4288725546*X1---(2),

99% C.I. for slope,	[	-0.4489545415,	-0.4087633245]
95% C.I. for slope,	[	-0.4441527809,	-0.4135929874]
90% C.I. for slope,	[	-0.4416912960,	-0.4160485760]
99% C.I. for intercept,	[	0.4385801961,	0.4554371449]
95% C.I. for intercept,	[	0.4405714154,	0.4534161342]
90% C.I. for intercept,	[	0.4416116065,	0.4523905709]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(2-2) $\lambda_1 = 0.2$, $\lambda_2=0.2$, $\lambda_3=0.3$,

Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.4529433281+-0.4686956300*X1---(1),
ANOVA

Source	df	SS	MS
Regression	1	2.9561293947	2.9561293947
Error	198	7.8611093267	0.0397025724
Total	199	10.8172387214	

H0:slope=0, test statistic=74.456873 , p value=0.000000
R2=0.273279, R2(adj)=0.269609,MSE=0.039703,
H0:residual population~Rayleigh(lamda=5.220063,c=-0.394618)
chi square test statistic=3.520000, p value=0.619837
99% C.I. for slope, [-0.6090093255, -0.3261377601]
95% C.I. for slope, [-0.5752669422, -0.3609793378]
90% C.I. for slope, [-0.5581957418, -0.3784859035]
99% C.I. for intercept, [0.3876789559, 0.5163003748]
95% C.I. for intercept, [0.4034327210, 0.5011411285]
90% C.I. for intercept, [0.4114501306, 0.4933438186]

X3 estimated=0.4543067502+-0.4456556395*X2---(2),
ANOVA

Source	df	SS	MS
Regression	1	2.6726398811	2.6726398811
Error	198	7.7136071187	0.0389576117
Total	199	10.3862469998	

H0:slope=0, test statistic=68.603792 , p value=0.000000
R2=0.257325, R2(adj)=0.253574,MSE=0.038958,
H0:residual population~Normal(mu=-0.011025,sigma*sigma=0.039737)
chi square test statistic=1.120000, p value=0.952391,
99% C.I. for slope, [-0.5858048685, -0.3061002757]
95% C.I. for slope, [-0.5518373204, -0.3397737324]
90% C.I. for slope, [-0.5346562462, -0.3567288672]
99% C.I. for intercept, [0.3904057780, 0.5180853485]
95% C.I. for intercept, [0.4059382942, 0.5027084210]
90% C.I. for intercept, [0.4138058192, 0.4948349866]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2=\lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(2-3) $\lambda_1 = 0.3$, λ_2=0.2, λ_3=0.3,

Getting (X1,X2),(X1,X3), the paired sample size=400, the simple linear model estimated line as,

X2 estimated=0.4398168494+-0.4282379854*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	5.2322288130	5.2322288130
Error	398	14.3025632565	0.0359360886
Total	399	19.5347920695	

H0:slope=0, test statistic=145.598172 , p value=0.000000
R2=0.267842, R2(adj)=0.266002,MSE=0.035936,
H0:residual population~Double exponential(lamda=6.075102,mu=-0.016875)
chi square test statistic=2.975000, p value=0.811149,

99% C.I. for slope,	[	-0.5202457790,	-0.3366287684]
95% C.I. for slope,	[	-0.4981122020,	-0.3585451576]
90% C.I. for slope,	[	-0.4868108441,	-0.3697337237]
99% C.I. for intercept,	[	0.3942665880,	0.4854163531]
95% C.I. for intercept,	[	0.4053091283,	0.4744201702]
90% C.I. for intercept,	[	0.4108646946,	0.4688460137]

X3 estimated=0.4836523642+-0.4727123255*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	6.3754406332	6.3754406332
Error	398	12.9560046658	0.0325527755
Total	399	19.3314452990	

H0:slope=0, test statistic=195.849372 , p value=0.000000
R2=0.329796, R2(adj)=0.328112,MSE=0.032553,
H0:residual population~Double exponential(lamda=6.526101,mu=0.002725)
chi square test statistic=1.400000, p value=0.965899,

99% C.I. for slope,	[	-0.5601529871,	-0.3852032277]
95% C.I. for slope,	[	-0.5391175851,	-0.4064034178]
90% C.I. for slope,	[	-0.5283807836,	-0.4171110810]
99% C.I. for intercept,	[	0.4403973726,	0.5270526941]
95% C.I. for intercept,	[	0.4508099297,	0.5165639324]
90% C.I. for intercept,	[	0.4560561108,	0.5112423094]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(2-4) $\lambda_1 = 0.4$, λ_2=0.2, λ_3=0.3,

Getting (X1,X2),(X1,X3), the paired sample size=600, the simple linear model estimated line as,

X2 estimated=0.4491854617+-0.4366398480*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	9.2367592600	9.2367592600
Error	598	19.2683756739	0.0322213640
Total	599	28.5051349338	

H0:slope=0, test statistic=286.665681 , p value=0.000000

R2=0.324038, R2(adj)=0.322908,MSE=0.032221,

H0:residual population~Double exponential(lamda=6.768197,mu=-0.012384)

chi square test statistic=6.233333, p value=0.513262,

99% C.I. for slope, [-0.5033791502, -0.3700214650]

95% C.I. for slope, [-0.4872907143, -0.3859602502]

90% C.I. for slope, [-0.4791375917, -0.3942159993]

99% C.I. for intercept, [0.4133927218, 0.4851031459]

95% C.I. for intercept, [0.4219022863, 0.4765116323]

90% C.I. for intercept, [0.4262895083, 0.4721005284]

X3 estimated=0.5075745576+-0.5090632479*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	12.5549902071	12.5549902071
Error	598	17.5167214888	0.0292921764
Total	599	30.0717116959	

H0:slope=0, test statistic=428.612406 , p value=0.000000

R2=0.417502, R2(adj)=0.416528,MSE=0.029292,

H0:residual population~Double exponential(lamda=6.630370,mu=0.004720)

chi square test statistic=5.133333, p value=0.643633,

99% C.I. for slope, [-0.5724398071, -0.4454198501]

95% C.I. for slope, [-0.5574128538, -0.4608009842]

90% C.I. for slope, [-0.5496151698, -0.4685075740]

99% C.I. for intercept, [0.4732428448, 0.5417933661]

95% C.I. for intercept, [0.4814688696, 0.5336165696]

90% C.I. for intercept, [0.4856627723, 0.5294176441]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(2-5) $\lambda_1 = 0.5$, λ_2=0.2, λ_3=0.3,

Getting (X1,X2),(X1,X3), the paired sample size=1,000, the simple linear model estimated line as,

X2 estimated=0.4630697351+-0.4538824909*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	16.8049934059	16.8049934059
Error	998	27.4608388325	0.0275158706
Total	999	44.2658322384	

H0:slope=0, test statistic=610.738205 , p value=0.000000

R2=0.379638, R2(adj)=0.379016,MSE=0.027516,

H0:residual population~Double exponential(lamda=7.695409,mu=-0.007662)

chi square test statistic=9.200000, p value=0.238097,

99% C.I. for slope, [-0.5013094707, -0.4065059375]

95% C.I. for slope, [-0.4899740890, -0.4178660281]

90% C.I. for slope, [-0.4842268613, -0.4236870533]

99% C.I. for intercept, [0.4354102938, 0.4907341163]

95% C.I. for intercept, [0.4420258760, 0.4841202262]

90% C.I. for intercept, [0.4454210655, 0.4807585743]

X3 estimated=0.5362176107+-0.5450277025*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	24.2319654622	24.2319654622
Error	998	25.5115416112	0.0255626669
Total	999	49.7435070734	

H0:slope=0, test statistic=947.943558 , p value=0.000000

R2=0.487138, R2(adj)=0.486624,MSE=0.025563,

H0:residual population~Double exponential(lamda=7.966259,mu=0.007243)

chi square test statistic=9.400000, p value=0.224676,

99% C.I. for slope, [-0.5909645059, -0.4993107088]

95% C.I. for slope, [-0.5798060655, -0.5103391834]

90% C.I. for slope, [-0.5741645116, -0.5158998724]

99% C.I. for intercept, [0.5096150645, 0.5628980104]

95% C.I. for intercept, [0.5160003616, 0.5564881647]

90% C.I. for intercept, [0.5192440892, 0.5532110390]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(2-6) $\lambda_1 = 0.6$, λ_2=0.2, λ_3=0.3,

Getting (X1,X2),(X1,X3), the paired sample size=10,000, the simple linear model estimated line as,

X2 estimated=0.4972740446+-0.4963751688*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	205.2646841590	205.2646841590
Error	9998	249.4085924593	0.0249458484
Total	9999	454.6732766183	

H0:slope=0, test statistic=8228.410625 , p value=0.000000
R2=0.451455, R2(adj)=0.451400,MSE=0.024946,
H0:residual population~Double exponential(lamda=8.059910,mu=-0.000900)
chi square test statistic=153.684800, p value=0.000000,
99% C.I. for slope, [-0.5104391541, -0.4823078092]
95% C.I. for slope, [-0.5071043324, -0.4856512618]
90% C.I. for slope, [-0.5053767487, -0.4873627544]
99% C.I. for intercept, [0.4887290770, 0.5058270830]
95% C.I. for intercept, [0.4907836193, 0.5037853786]
90% C.I. for intercept, [0.4918187627, 0.5027350406]

X3 estimated=0.5741215969+-0.5868731619*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	286.9344135446	286.9344135446
Error	9998	233.7897345139	0.0233836502
Total	9999	520.7241480585	

H0:slope=0, test statistic=12270.728108 , p value=0.000000
R2=0.551030, R2(adj)=0.550985,MSE=0.023384,
H0:residual population~Double exponential(lamda=8.608603,mu=0.010219)
chi square test statistic=239.398400, p value=0.000000,
99% C.I. for slope, [-0.6005749981, -0.5732216764]
95% C.I. for slope, [-0.5972662426, -0.5764825387]
90% C.I. for slope, [-0.5955815917, -0.5781472918]
99% C.I. for intercept, [0.5658233872, 0.5823890556]
95% C.I. for intercept, [0.5678197849, 0.5804134114]
90% C.I. for intercept, [0.5688396742, 0.5794072895]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(3) λ_2=0.2, λ_3=0.4,

(3-1) $\lambda_1 = 0.1$, λ_2=0.2, λ_3=0.4,

(i)Getting (X1,X2),(X1,X3), the paired sample size=100, the simple linear model estimated line as,

X2 estimated=0.4500600570+-0.4887984472*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	1.4571394458	1.4571394458
Error	98	4.4564085230	0.0454735564
Total	99	5.9135479688	

H0:slope=0, test statistic=32.043666 , p value=0.000000
R2=0.246407, R2(adj)=0.238717,MSE=0.045474,
H0:residual population~Rayleigh(lamda=4.261469,c=-0.437442)
chi square test statistic=1.080000, p value=0.897482,
99% C.I. for slope, [-0.7112864793, -0.2580842344]
95% C.I. for slope, [-0.6582549870, -0.3156318116]
90% C.I. for slope, [-0.6312484617, -0.3442115890]
99% C.I. for intercept, [0.3579836822, 0.5375016183]
95% C.I. for intercept, [0.3806440712, 0.5163456668]
90% C.I. for intercept, [0.3921114570, 0.5055162881]

X3 estimated=0.4698000226+-0.4608558628*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	1.2953040200	1.2953040200
Error	98	4.4715059772	0.0456276120
Total	99	5.7668099972	

H0:slope=0, test statistic=28.388600 , p value=0.000000
R2=0.224614, R2(adj)=0.216702,MSE=0.045628,
H0:residual population~Normal(mu=0.022103,sigma*sigma=0.057588)
chi square test statistic=2.760000, p value=0.599023,
99% C.I. for slope, [-0.6880346733, -0.2333789395]
95% C.I. for slope, [-0.6327473771, -0.2891895383]
90% C.I. for slope, [-0.6046427698, -0.3172111066]
99% C.I. for intercept, [0.3802094912, 0.5595909019]
95% C.I. for intercept, [0.4020697563, 0.5376184285]
90% C.I. for intercept, [0.4131109904, 0.5265629337]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 = \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(ii) The paired sample size=500,

X2 estimated=0.4034269015+-0.3784359630*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	4.3263125190	4.3263125190
Error	498	21.2205817891	0.0426116100
Total	499	25.5468943081	

H0:slope=0, test statistic=101.528962 , p value=0.000000
R2=0.169348, R2(adj)=0.167680,MSE=0.042612,
H0:residual population~Rayleigh(lamda=4.873070,c=-0.383776)
chi square test statistic=3.640000, p value=0.724511,
99% C.I. for slope, [-0.4748576431, -0.2805726640]
95% C.I. for slope, [-0.4519251251, -0.3041832960]
90% C.I. for slope, [0.3636465040, 0.4421678806]
99% C.I. for intercept, [0.3636641958, 0.4423975411]
95% C.I. for intercept, [0.3733152996, 0.4330482374]
90% C.I. for intercept, [0.3781342349, 0.4282808252]

X3 estimated=0.4901590013+-0.4760637962*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	6.8464177830	6.8464177830
Error	498	20.4238133192	0.0410116733
Total	499	27.2702311022	

H0:slope=0, test statistic=166.938270 , p value=0.000000
R2=0.251058, R2(adj)=0.249554,MSE=0.041012,
H0:residual population~Normal(mu=-0.004855,sigma*sigma=0.046933)
chi square test statistic=3.748000, p value=0.710596,
99% C.I. for slope, [-0.5714403732, -0.3806005127]
95% C.I. for slope, [-0.5486777060, -0.4035978364]
90% C.I. for slope, [-0.5368969941, -0.4152420868]
99% C.I. for intercept, [0.4515558020, 0.5288883003]
95% C.I. for intercept, [0.4607824167, 0.5194940073]
90% C.I. for intercept, [0.4655373993, 0.5147475326]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(3-2) $\lambda_1 = 0.2$, $\lambda_2=0.2$, $\lambda_3=0.4$,

(i)Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.4529433281+-0.4686956300*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	2.9561293947	2.9561293947
Error	198	7.8611093267	0.0397025724
Total	199	10.8172387214	

H0:slope=0, test statistic=74.456873 , p value=0.000000
R2=0.273279, R2(adj)=0.269609,MSE=0.039703,
H0:residual population~Rayleigh(lamda=5.220063,c=-0.394618)
chi square test statistic=3.520000, p value=0.620292,

99% C.I. for slope,	[	-0.6091430858,	-0.3263912230]
95% C.I. for slope,	[	-0.5752344923,	-0.3609857203]
90% C.I. for slope,	[	-0.5581187567,	-0.3785649610]
99% C.I. for intercept,	[	0.3877589453,	0.5166330167]
95% C.I. for intercept,	[	0.4035222351,	0.5010519752]
90% C.I. for intercept,	[	0.4114990143,	0.4933080103]

X3 estimated=0.4907498944+-0.4917281553*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	3.2538069536	3.2538069536
Error	198	7.8993487160	0.0398957006
Total	199	11.1531556696	

H0:slope=0, test statistic=81.557835 , p value=0.000000
R2=0.291739, R2(adj)=0.288162,MSE=0.039896,
H0:residual population~Normal(mu=-0.008766,sigma*sigma=0.043072)
chi square test statistic=1.360000, p value=0.928695,

99% C.I. for slope,	[	-0.6334043502,	-0.3500219828]
95% C.I. for slope,	[	-0.5991980759,	-0.3842687447]
90% C.I. for slope,	[	-0.5818681530,	-0.4016529050]
99% C.I. for intercept,	[	0.4262341640,	0.5552685668]
95% C.I. for intercept,	[	0.4417933296,	0.5397213605]
90% C.I. for intercept,	[	0.4497133348,	0.5317688767]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2=\lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(ii)The paired sample size=500,

```
X2 estimated=0.4162456888+-0.3975529019*X1---(1),
ANOVA
   Source          df              SS                  MS
   Regression       1              5.5488279581        5.5488279581
   Error          498              18.9528612654       0.0380579543
   Total          499              24.5016892234
H0:slope=0, test statistic=145.799428 , p value=0.000000
R2=0.226467, R2(adj)=0.224914,MSE=0.038058,
H0:residual population~Normal(mu=-0.014812,sigma*sigma=0.036106)
chi square test statistic=11.812000, p value=0.066126,
99% C.I. for slope,    [     -0.4829273861,      -0.3125882771]
95% C.I. for slope,    [     -0.4622109621,      -0.3328932197]
90% C.I. for slope,    [     -0.4517714889,      -0.3433977489]
99% C.I. for intercept,    [     0.3766153146,      0.4557426753]
95% C.I. for intercept,    [     0.3862131997,      0.4461932382]
90% C.I. for intercept,    [     0.3911087582,      0.4413964327]

X3 estimated=0.5077533659+-0.5025282522*X---(2),
ANOVA
   Source          df              SS                  MS
   Regression       1              8.8660947813        8.8660947813
   Error          498              17.9482170123       0.0360405964
   Total          499              26.8143117936
H0:slope=0, test statistic=246.002998 , p value=0.000000
R2=0.330648, R2(adj)=0.329304,MSE=0.036041,
H0:residual population~Logistic(mu=0.016729,sigma=0.113521)
chi square test statistic=4.648000, p value=0.590465,
99% C.I. for slope,    [     -0.5850556664,      -0.4200209365]
95% C.I. for slope,    [     -0.5653152183,      -0.4396168191]
90% C.I. for slope,    [     -0.5552130572,      -0.4497977282]
99% C.I. for intercept,    [     0.4693800304,      0.5460635755]
95% C.I. for intercept,    [     0.4785714637,      0.5368573880]
90% C.I. for intercept,    [     0.4832577748,      0.5321833855]
```

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(3-3) $\lambda_1 = 0.3$, λ_2=0.2, λ_3=0.4,

Getting (X1,X2),(X1,X3), the paired sample size=400, the simple linear model estimated line as,

X2 estimated=0.4398168494+-0.4282379854*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	5.2322288130	5.2322288130
Error	398	14.3025632565	0.0359360886
Total	399	19.5347920695	

H0:slope=0, test statistic=145.598172 , p value=0.000000

R2=0.267842, R2(adj)=0.266002,MSE=0.035936,

H0:residual population~Double exponential(lamda=6.075102,mu=-0.016875)

chi square test statistic=2.975000, p value=0.812039,

99% C.I. for slope, [-0.5201330733, -0.3365402665]

95% C.I. for slope, [-0.4979490273, -0.3585649066]

90% C.I. for slope, [-0.4867471562, -0.3697993497]

99% C.I. for intercept, [0.3944897135, 0.4852281963]

95% C.I. for intercept, [0.4052880424, 0.4743839438]

90% C.I. for intercept, [0.4108239943, 0.4688036001]

X3 estimated=.5257587288+-0.5254373170*X---(2),

ANOVA

Source	df	SS	MS
Regression	1	7.8769516652	7.8769516652
Error	398	13.1729087827	0.0330977608
Total	399	21.0498604479	

H0:slope=0, test statistic=237.990471 , p value=0.000000

R2=0.374204, R2(adj)=0.372632,MSE=0.033098,

H0:residual population~Double exponential(lamda=6.298914,mu=0.006913)

chi square test statistic=2.165000, p value=0.904184,

99% C.I. for slope, [-0.6135266651, -0.4375055977]

95% C.I. for slope, [-0.5924618735, -0.4586440965]

90% C.I. for slope, [-0.5815801423, -0.4693455642]

99% C.I. for intercept, [0.4820891772, 0.5693595631]

95% C.I. for intercept, [0.4926083658, 0.5589328586]

90% C.I. for intercept, [0.4979981742, 0.5535623804]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(3-4) $\lambda_1 = 0.4$, λ_2=0.2, λ_3=0.4,

Getting (X1,X2),(X1,X3), the paired sample size=600, the simple linear model estimated line as,

X2 estimated=0.4491854617+-0.4366398480*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	9.2367592600	9.2367592600
Error	598	19.2683756739	0.0322213640
Total	599	28.5051349338	

H0:slope=0, test statistic=286.665681 , p value=0.000000

R2=0.324038, R2(adj)=0.322908,MSE=0.032221,

H0:residual population~Double exponential(lamda=6.768197,mu=-0.012384)

chi square test statistic=6.233333, p value=0.511786,

99% C.I. for slope, [-0.5032194659, -0.3699413424]

95% C.I. for slope, [-0.4873864050, -0.3860382699]

90% C.I. for slope, [-0.4791849417, -0.3942013988]

99% C.I. for intercept, [0.4131211387, 0.4850761369]

95% C.I. for intercept, [0.4218690050, 0.4764470851]

90% C.I. for intercept, [0.4262713579, 0.4721048314]

X3 estimated=0.5466550078+-0.5504748116*X,

ANOVA

Source	df	SS	MS
Regression	1	15.1177603457	15.1177603457
Error	598	16.1254624184	0.0269656562
Total	599	31.2432227641	

H0:slope=0, test statistic=560.630167 , p value=0.000000

R2=0.483873, R2(adj)=0.483010,MSE=0.026966,

H0:residual population~Double exponential(lamda=7.839228,mu=0.004343)

chi square test statistic=9.133333, p value=0.243129,

99% C.I. for slope, [-0.6104538383, -0.4906072116]

95% C.I. for slope, [-0.5961604217, -0.5048027776]

90% C.I. for slope, [-0.5888354224, -0.5121144803]

99% C.I. for intercept, [0.5122229833, 0.5810923492]

95% C.I. for intercept, [0.5204589099, 0.5728651492]

90% C.I. for intercept, [0.5247114975, 0.5686316294]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(3-5) $\lambda_1 = 0.5$, λ_2=0.2, λ_3=0.4,

Getting (X1,X2),(X1,X3), the paired sample size=10,000, the simple linear model estimated line as,

X2 estimated=0.4589181377+-0.4486377830*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	167.0015922365	167.0015922365
Error	9998	277.4465455090	0.0277502046
Total	9999	444.4481377455	

H0:slope=0, test statistic=6018.031027 , p value=0.000000

R2=0.375750, R2(adj)=0.375688,MSE=0.027750,

H0:residual population~Double exponential(lamda=7.603999,mu=-0.006377)

chi square test statistic=128.860000, p value=0.000000,

99% C.I. for slope, [-0.4635456530, -0.4337848721]

95% C.I. for slope, [-0.4599767319, -0.4373241257]

90% C.I. for slope, [-0.4581394877, -0.4391357493]

99% C.I. for intercept, [0.4503661695, 0.4675035752]

95% C.I. for intercept, [0.4523906356, 0.4654414546]

90% C.I. for intercept, [0.4534405612, 0.4643943790]

X3 estimated=0.5970149857+-0.6171628311*X---(2),

ANOVA

Source	df	SS	MS
Regression	1	316.0300977594	316.0300977594
Error	9998	264.4405696129	0.0264493468
Total	9999	580.4706673723	

H0:slope=0, test statistic=11948.502917 , p value=0.000000

R2=0.544438, R2(adj)=0.544392,MSE=0.026449,

H0:residual population~Double exponential(lamda=7.964709,mu=0.014488)

chi square test statistic=271.315600, p value=0.000000,

99% C.I. for slope, [-0.6317185283, -0.6026430103]

95% C.I. for slope, [-0.6282224047, -0.6060881360]

90% C.I. for slope, [-0.6264433488, -0.6078782864]

99% C.I. for intercept, [0.5886552131, 0.6054127043]

95% C.I. for intercept, [0.5906517779, 0.6033868532]

90% C.I. for intercept, [0.5916781040, 0.6023674941]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(4) λ_2=0.2, λ_3=0.5,

(4-1) $\lambda_1 = 0.1$, λ_2=0.2, λ_3=0.5,

Getting (X1,X2),(X1,X3), the paired sample size=100, the simple linear model estimated line as,

X2 estimated=0.4102521186+-0.3563533378*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	0.7210065843	0.7210065843
Error	98	4.4823547961	0.0457383142
Total	99	5.2033613804	

H0:slope=0, test statistic=15.763733 , p value=0.000153

R2=0.138566, R2(adj)=0.129775,MSE=0.045738,

H0:residual population~Normal(mu=-0.015320,sigma*sigma=0.056879)

chi square test statistic=0.520000, p value=0.971612,

99% C.I. for slope, [-0.5927151831, -0.1206578480]

95% C.I. for slope, [-0.5345344170, -0.1777416412]

90% C.I. for slope, [-0.5055649800, -0.2068218559]

99% C.I. for intercept, [0.3215401120, 0.4992829523]

95% C.I. for intercept, [0.3431311388, 0.4774256570]

90% C.I. for intercept, [0.3541354356, 0.4663855177]

X3 estimated=0.5564132267+-0.6430398183*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	2.3477578480	2.3477578480
Error	98	4.3764560293	0.0446577146
Total	99	6.7242138773	

H0:slope=0, test statistic=52.572279 , p value=0.000000

R2=0.349150, R2(adj)=0.342508,MSE=0.044658,

H0:residual population~Normal(mu=-0.005046,sigma*sigma=0.048352)

chi square test statistic=2.200000, p value=0.699006,

99% C.I. for slope, [-0.8765475268, -0.4098035110]

95% C.I. for slope, [-0.8193559557, -0.4669252525]

90% C.I. for slope, [-0.7904596294, -0.4955934096]

99% C.I. for intercept, [0.4685477214, 0.6440279786]

95% C.I. for intercept, [0.4901496709, 0.6226710056]

90% C.I. for intercept, [0.5009287422, 0.6118187712]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(4-2) $\lambda_1 = 0.2$, λ_2=0.2, λ_3=0.5,

Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.4480620871+-0.4550130194*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	3.1210162351	3.1210162351
Error	198	8.3468181182	0.0421556471
Total	199	11.4678343533	

H0:slope=0, test statistic=74.035543 , p value=0.000000

R2=0.272154, R2(adj)=0.268478,MSE=0.042156,

H0:residual population~Normal(mu=-0.001638,sigma*sigma=0.041248)

chi square test statistic=1.120000, p value=0.952421

99% C.I. for slope, [-0.5923882529, -0.3174032928]

95% C.I. for slope, [-0.5592743876, -0.3506950352]

90% C.I. for slope, [-0.5423769949, -0.3676992331]

99% C.I. for intercept, [0.3825406390, 0.5136968262]

95% C.I. for intercept, [0.3983488892, 0.4977622747]

90% C.I. for intercept, [0.4064708930, 0.4897192859]

X3 estimated=0.5384528223+-0.5085331368*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	3.8984044652	3.8984044652
Error	198	7.0887154237	0.0358015930
Total	199	10.9871198889	

H0:slope=0, test statistic=108.889134 , p value=0.000000

R2=0.354816, R2(adj)=0.351557,MSE=0.035802,

H0:residual population~Double exponential(lamda=6.278256,mu=-0.003435)

chi square test statistic=2.080000, p value=0.838355,

99% C.I. for slope, [-0.6350131446, -0.3821098871]

95% C.I. for slope, [-0.6046467810, -0.4124930911]

90% C.I. for slope, [-0.5891951584, -0.4279592180]

99% C.I. for intercept, [0.4781851323, 0.5988555569]

95% C.I. for intercept, [0.4927046599, 0.5842022143]

90% C.I. for intercept, [0.5001108445, 0.5768450447]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(4-3) $\lambda_1 = 0.3$, λ_2=0.2, λ_3=0.5,

Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.4962436076+-0.5037276816*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	4.1656356470	4.1656356470
Error	198	6.1033077597	0.0308247867
Total	199	10.2689434067	

0:slope=0, test statistic=135.139156 , p value=0.000000

R2=0.405654, R2(adj)=0.402652,MSE=0.030825,

H0:residual population~Double exponential(lamda=6.978237,mu=0.000006)

chi square test statistic=1.760000, p value=0.881942,

99% C.I. for slope, [-0.6161731064, -0.3911799429]

95% C.I. for slope, [-0.5891839951, -0.4181054916]

90% C.I. for slope, [-0.5753984505, -0.4319928091]

99% C.I. for intercept, [0.4320445489, 0.5605111645]

95% C.I. for intercept, [0.4475547035, 0.5449958171]

90% C.I. for intercept, [0.4554122115, 0.5371673379]

X3 estimated=0.5913909700+-0.6141527199*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	6.1921633571	6.1921633571
Error	198	5.1627213302	0.0260743502
Total	199	11.3548846874	

H0:slope=0, test statistic=237.481023 , p value=0.000000

R2=0.545330, R2(adj)=0.543034,MSE=0.026074,

H0:residual population~Double exponential(lamda=7.176221,mu=0.016168)

chi square test statistic=3.200000, p value=0.669881,

99% C.I. for slope, [-0.7176777581, -0.5112262948]

95% C.I. for slope, [-0.6926396271, -0.5357285978]

90% C.I. for slope, [-0.6800106446, -0.5482990203]

99% C.I. for intercept, [0.5324461474, 0.6503860944]

95% C.I. for intercept, [0.5466495295, 0.6362937594]

90% C.I. for intercept, [0.5539063572, 0.6290071792]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(4-4) $\lambda_1 = 0.4$, $\lambda_2 = 0.2$, $\lambda_3 = 0.5$,

Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.4173234049+-0.4022691858*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	2.9259033391	2.9259033391
Error	198	5.8595987013	0.0295939328
Total	199	8.7855020404	

H0:slope=0, test statistic=98.868351 , p value=0.000000

R2=0.333038, R2(adj)=0.329669,MSE=0.029594,

H0:residual population~Double exponential(lamda=7.052457,mu=-0.010024)

chi square test statistic=3.840000, p value=0.572391,

99% C.I. for slope, [-0.5074486601, -0.2972878746]

95% C.I. for slope, [-0.4819723425, -0.3225717051]

90% C.I. for slope, [-0.4690767275, -0.3355297213]

99% C.I. for intercept, [0.3584115261, 0.4764843639]

95% C.I. for intercept, [0.3726216676, 0.4622718479]

90% C.I. for intercept, [0.3798711455, 0.4549729930]

X3 estimated=0.6905948185+-0.7406017745*X1---(2),

ANOVA

Source	df	SS	MS
Regression	1	9.9173567964	9.9173567964
Error	198	4.1176396173	0.0207961597
Total	199	14.0349964136	

H0:slope=0, test statistic=476.884047 , p value=0.000000

R2=0.706616, R2(adj)=0.705135,MSE=0.020796,

H0:residual population~Logistic(mu=0.005076,sigma=0.080874)

chi square test statistic=7.040000, p value=0.217801,

99% C.I. for slope, [-0.8278040781, -0.6532168947]

95% C.I. for slope, [-0.8069629467, -0.6740951287]

90% C.I. for slope, [-0.7963677857, -0.6847507699]

99% C.I. for intercept, [0.6416852452, 0.7395446915]

95% C.I. for intercept, [0.6533476375, 0.7279121231]

90% C.I. for intercept, [0.6593209670, 0.7219677052]

Checking slope and intercept of X3 estimated whether in the CI of (1) estimated line,
Checking slope and intercept of X1 estimated whether in the CI of (2) estimated line,
Conclusion, $\lambda_2 \neq \lambda_3$ from two CI's about slope and intercept at significant=10% or 5% or 1%.

(5)Heterskedastic analysis

(5-1) $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$,

Getting (X1,X2),(X1,X3), the paired sample size=200, the simple linear model estimated line as,

X2 estimated=0.3160617529+-0.2518647259*X1---(1),

ANOVA

Source	df	SS	MS
Regression	1	0.8172972849	0.8172972849
Error	198	7.2355129259	0.0365429946
Total	199	8.0528102108	

H0:slope=0, test statistic=22.365362 , p value=0.000004

R2=0.101492, R2(adj)=0.096954,MSE=0.036543,

H0:residual population~Gumbel(mu=-0.092967,sigma=0.166022)

chi square test statistic=3.840000, p value=0.571795

|residual|=0.219877+-0.204069*X+resodual*=G(X)+residual*,

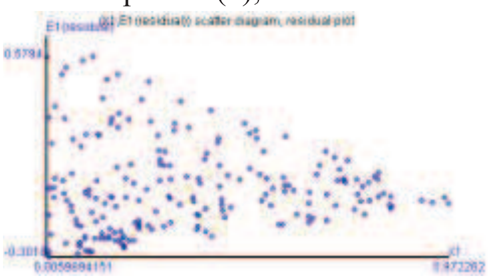
ANOVA

Source	df	SS	MS
Regression	1	0.5365369955	0.5365369955
Error	198	224.5893769122	1.1342897824
Total	199	225.1259139077	

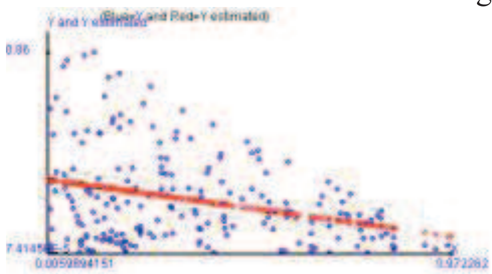
H0:slope=0, SSR/MSE=test statistic=0.473016 , p value=**0.491556**

R2=0.002383, R2(adj)=-0.002655,MSE=1.134290,

Residual plot of (1),



The estimated line and scatter diagram,



X3 estimated=0.3452359296+-0.3016360616*X---(2),

ANOVA

Source	df	SS	MS
Regression	1	1.1722274409	1.1722274409
Error	198	6.6277723596	0.0334735978
Total	199	7.7999998005	

H0:slope=0, test statistic=35.019464 , p value=0.000000

R2=0.150286, R2(adj)=0.145994,MSE=0.033474,

H0:residual population~Rayleigh(lamda=6.922531,c=-0.336291)

chi square test statistic=0.400000, p value=0.995344,

|residual|=0.206310+-0.183159*X+resodual*=G(X)+residual*,

ANOVA

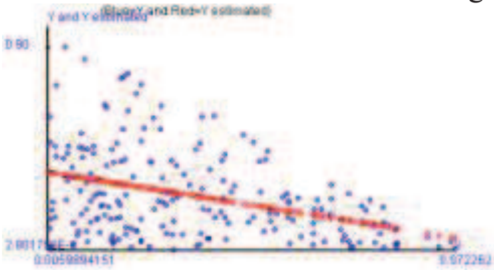
Source	df	SS	MS
Regression	1	0.4322177348	0.4322177348
Error	198	48.2035052421	0.2434520467
Total	199	48.6357229769	

H0:slope=0, SSR/MSE=test statistic=1.775371 , p value=**0.540349**

R2=0.008887, R2(adj)=0.003881,MSE=0.243452,

Residual plot of (2),

The estimated line and scatter diagram,



(5-2) $\lambda_1 = 0.2$, λ_2=0.2, λ_3=0.2,

Getting (X1,X2),(X1,X3), the paired sample size=10,000, the simple linear model estimated line as,

X2 estimated=0.4217986386+-0.4035518362*X1---(1),
ANOVA

Source	df	SS	MS
Regression	1	121.5177399937	121.5177399937
Error	9998	366.2937996725	0.0366367073
Total	9999	487.8115396662	

H0:slope=0, test statistic=3316.830275 , p value=0.000000
R2=0.249108, R2(adj)=0.249033,MSE=0.036637,
H0:residual population~Normal(mu=-0.013781,sigma*sigma=0.037864)
chi square test statistic=324.526800, p value=0.000000,

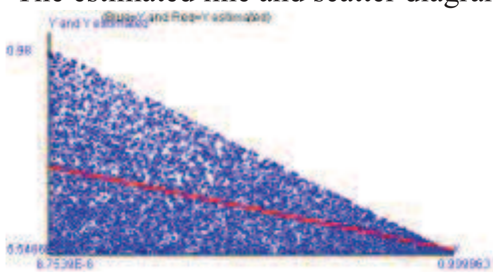
|residual|=0.242311+-0.239039*X+resodual*=G(X)+residual*,
ANOVA

Source	df	SS	MS
Regression	1	42.6361801416	42.6361801416
Error	9998	95.6256012450	0.0095644730
Total	9999	138.2617813866	

H0:slope=0, SSR/MSE=test statistic=4457.765740 , p value=0.000000,
R2=0.308373, R2(adj)=0.308304,MSE=0.009564,
H0:residual* population~Logistic(mu=0.000000,sigma=0.053915),
chi square test statistic=146.945200, p value=0.000000,

Residual plot of (1),

The estimated line and scatter diagram,



X3 estimated=0.4188089952+-0.3992240938*X1---(2),
ANOVA

Source	df	SS	MS
Regression	1	118.9253712559	118.9253712559
Error	9998	364.0191304651	0.0364091949
Total	9999	482.9445017210	

H0:slope=0, test statistic=3266.355426 , p value=0.000000
R2=0.246251, R2(adj)=0.246175,MSE=0.036409,
H0:residual population~Normal(mu=0.003053,sigma*sigma=0.038560)
chi square test statistic=364.396000, p value=0.000000,

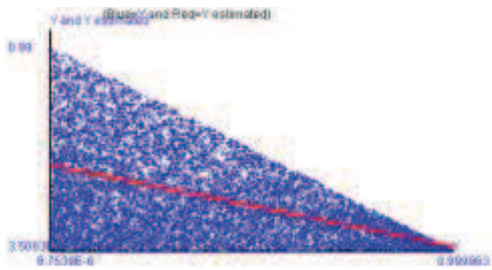
|residual|=0.240881+-0.236303*X+resodual*=G(X)+residual*,
ANOVA

Source	df	SS	MS
Regression	1	41.6659458471	41.6659458471
Error	9998	95.4829124201	0.0095502013
Total	9999	137.1488582672	

H0:slope=0, SSR/MSE=test statistic=4362.834313 , p value=0.000000
R2=0.303801, R2(adj)=0.303731,MSE=0.009550,
H0:residual* population~Normal(mu=-0.002345,sigma*sigma=0.009470)
chi square test statistic=133.421200, p value=0.000000,

Residual plot of (2),

The estimated line and scatter diagram,



Section 2. $X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1, \lambda_2 . x_1), 0 \le x_2 \le 1 - x_1,$

$X_3|x_1, x_2 \sim CB(\lambda_1, \lambda_2, \lambda_3 . x_1 + x_2), 0 \le x_3 \le 1 - x_1 - x_2,$

X_1 and X_2 are two random variables,

X_1	$1 - X_1$
p_1	$1 - p_1$

$X_1 = 0$

X_2	$1 - X_1 - X_2$
$\frac{p_2}{1-p_1}$	$1 - \frac{p_2}{1-p_1}$

$1 - X_1 - X_2 = X_3 + (1 - X_1 - X_2 - X_3)$

X_3	$1 - X_1 - X_2 - X_3$
$\frac{\lambda_3}{1-\lambda_1-\lambda_2}$	$1 - \frac{\lambda_3}{1-\lambda_1-\lambda_2}$

$$f_{X_1}(x_1;\lambda_1) f_{X_2|x_1}(x_2|x_1) f_{X_3|x_1,x_2}(x_3|x_1,x_2)$$

$$= C_1(\lambda_1) C_2(\lambda_1,\lambda_2,x_1)(\lambda_1)^{x_1}(\lambda_2)^{x_2}(1-\lambda_1-\lambda_2)^{1-x_1-x_2} \times$$

$$C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right)^{x_3}\left(1-\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2-x_3}$$

$$= C_1(\lambda_1) C_2(\lambda_1,\lambda_2,x_1) C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2) \times$$

$$(\lambda_1)^{x_1}(\lambda_2)^{x_2}(\lambda_3)^{x_3}(1-\lambda_1-\lambda_2-\lambda_3)^{1-x_1-x_2-x_3}$$

$$0 \le x_1 \le 1, 0 \le x_2 \le 1 - x_1, 0 \le x_3 \le 1 - x_1 - x_2,$$

1.The probability density function,

$$f_{X_3|x_1,x_2}(x_3|x_1,x_2) = C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right)^{x_3}\left(1-\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2-x_3}$$

$(i) 1 - \lambda_1 - \lambda_2 \ne 2\lambda_3,$

$$\int_0^{1-x_1-x_2} f_{X_3|x_1,x_2}(x_3|x_1,x_2)\,dx_3$$

$$= C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\left(\frac{1-\lambda_1-\lambda_2-\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2}\int_0^{1-x_1-x_2}\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2-\lambda_3}\right)^{x_3} dx_3$$

$$= C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\left(\frac{1-\lambda_1-\lambda_2-\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2}\frac{\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2-\lambda_3}\right)^{1-x_1-x_2}-1}{ln\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2-\lambda_3}\right)} = 1,$$

$$C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)=\frac{ln\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2-\lambda_3}\right)}{\left(\frac{1-\lambda_1-\lambda_2-\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2}\left(\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2-\lambda_3}\right)^{1-x_1-x_2}-1\right)},$$

$(ii) 1-\lambda_1-\lambda_2=2\lambda_3,$

$$\int_0^{1-x_1-x_2} f_{X_3|x_1,x_2}(x_3|x_1,x_2)dx_3$$

$$=C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\left(\frac{1-\lambda_1-\lambda_2-\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2}(1-x_1-x_2)=1,$$

$$C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)=\frac{1}{\left(\frac{1-\lambda_1-\lambda_2-\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x_1-x_2}(1-x_1-x_2)},$$

Please see appendix 2.

2. The chosen a random variable each time, the selecting order of random variables having the different joint probability density function.

$$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1,\lambda_2.x_1), 0 \le x_2 \le 1-x_1,$$
$$X_3|x_1,x_2 \sim CB(\lambda_1,\lambda_2,\lambda_3.x_1+x_2), 0 \le x_3 \le 1-x_1-x_2,$$
$$f_{X_1}(x_1;\lambda_1)f_{X_2|x_1}(x_2|x_1)f_{X_3|x_1,x_2}(x_3|x_1,x_2)$$
$$=C_1(\lambda_1)C_2(\lambda_1,\lambda_2,x_1)C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\times$$
$$(\lambda_1)^{x_1}(\lambda_2)^{x_2}(\lambda_3)^{x_3}(1-\lambda_1-\lambda_2-\lambda_3)^{1-x_1-x_2-x_3}$$

and

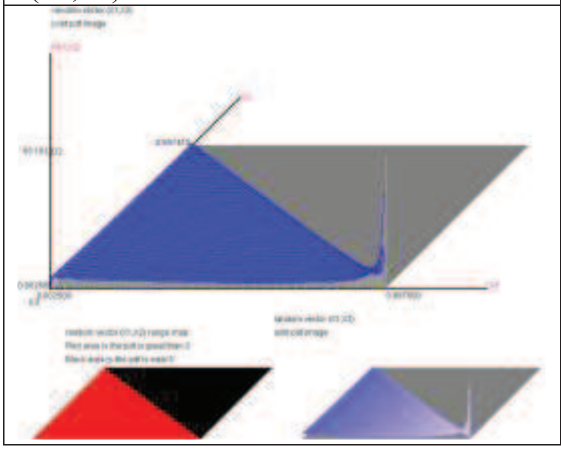
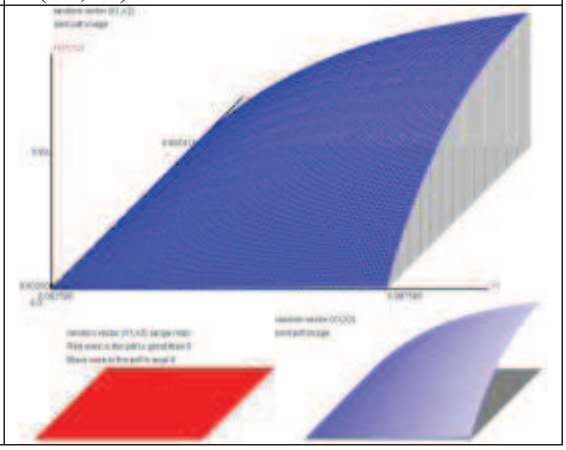
$$X_2 \sim CB(\lambda_2), 0 \le x_2 \le 1, X_1|x_2 \sim CB(\lambda_2,\lambda_1.x_2), 0 \le x_1 \le 1-x_2,$$
$$X_3|x_1,x_2 \sim CB(\lambda_1,\lambda_2,\lambda_3.x_1+x_2), 0 \le x_3 \le 1-x_1-x_2,$$
$$f_{X_2}(x_2;\lambda_2)f_{X_1|x_2}(x_1|x_2)f_{X_3|x_1,x_2}(x_3|x_1,x_2)$$
$$=C_1(\lambda_2)C_2(\lambda_2,\lambda_1,x_2)C_3(\lambda_1,\lambda_2,\lambda_3,x_1+x_2)\times$$
$$(\lambda_1)^{x_1}(\lambda_2)^{x_2}(\lambda_3)^{x_3}(1-\lambda_1-\lambda_2-\lambda_3)^{1-x_1-x_2-x_3}$$

$$C_1(\lambda_2)C_2(\lambda_2,\lambda_1,x_2) \ne C_1(\lambda_1)C_2(\lambda_1,\lambda_2,x_1).$$

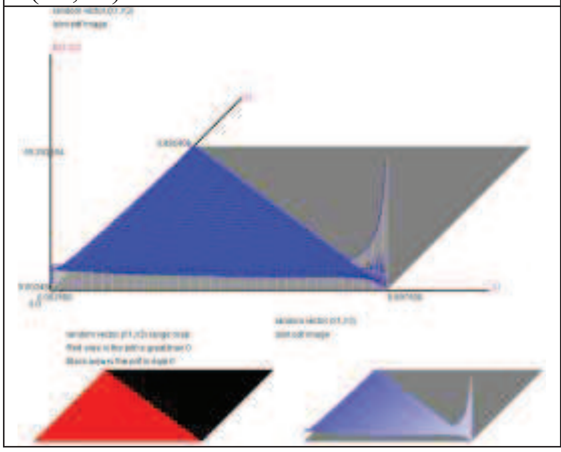
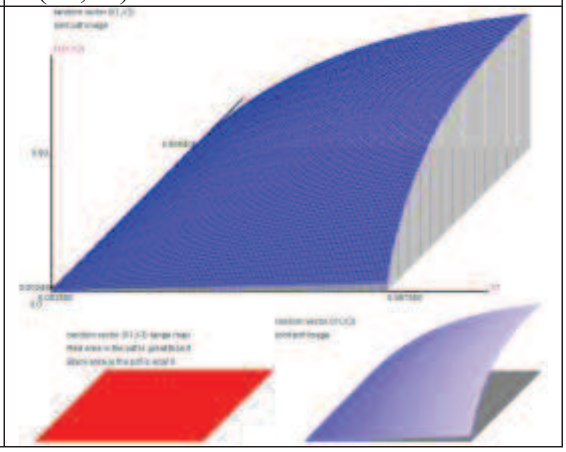
3. The image of $f(x_1,x_2), f(x_1,x_3), f(x_2,x_3)$,

$X_1 \sim CB(\lambda_1), 0 \le x_1 \le 1, X_2|x_1 \sim CB(\lambda_1,\lambda_2.x_1), X_3|x_1,x_2 \sim CB(\lambda_1,\lambda_2,\lambda_3.x_1+x_2)$,

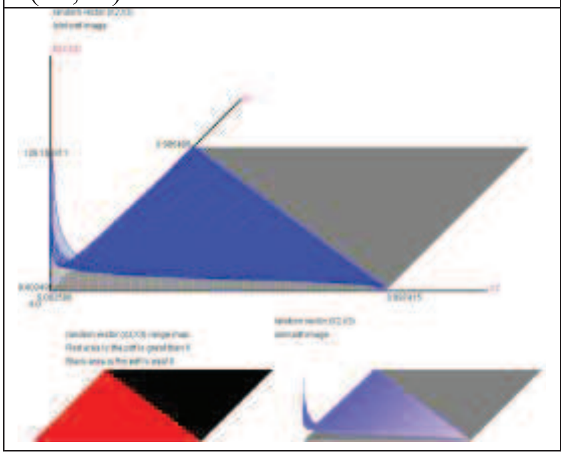
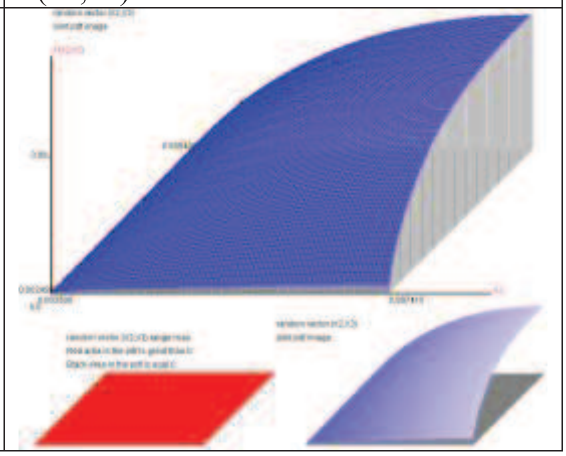
(1) λ_1=0.1, λ_2=0.2, λ_3=0.3,

f(x1,x2)	F(x1,x2)
	

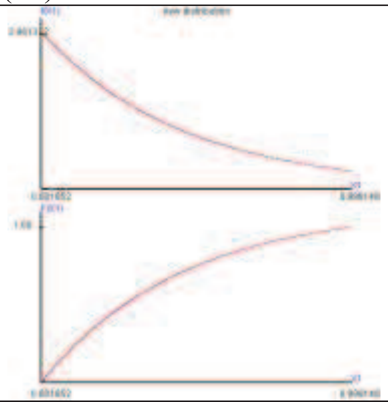
E(X1)= 0.3301, Var(X1)= 0.0665, E(X2)= 0.2821, Var(X2)= 0.0505,
Cov(X1,X2)= -0.0255, X1 and X2 correlation coefficient=-0.4395.

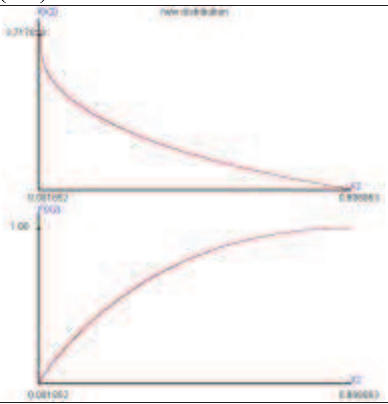
f(x1,x3)	F(x1,x3)
	

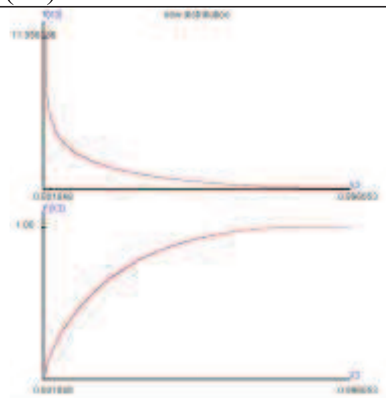
E(X1)= 0.3301, Var(X1)= 0.0665, E(X3)= 0.1887, Var(X3)= 0.0332,
Cov(X1,X3)= -0.0197, X1 and X3 correlation coefficient=-0.4196.

f(x2,x3)	F(x2,x3)
	

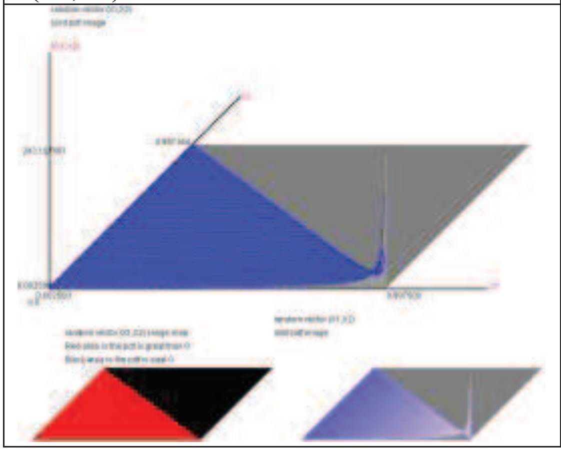
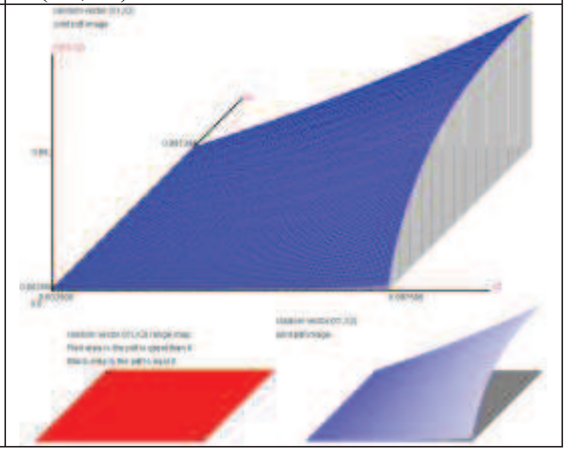
E(X2)= 0.2821, Var(X2)= 0.0505, E(X3)= 0.1887, Var(X3)= 0.0332,
Cov(X2,X3)= -0.0120, X2 and X3 correlation coefficient=-0.2924.

f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.33011
	Geometrical Mean :	0.20661
	Harmonic Mean :	0.02353
	Variance :	0.06650
	S.D. :	0.25788
	Skewed Coef. :	0.74400
	Kurtosis Coef. :	2.58171
	MAD :	0.21453
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.26749
	Q1 :	0.11438
	Q2 :	0.26749
	Q3 :	0.49999
	IQR :	0.38561
	C.V. :	0.78121

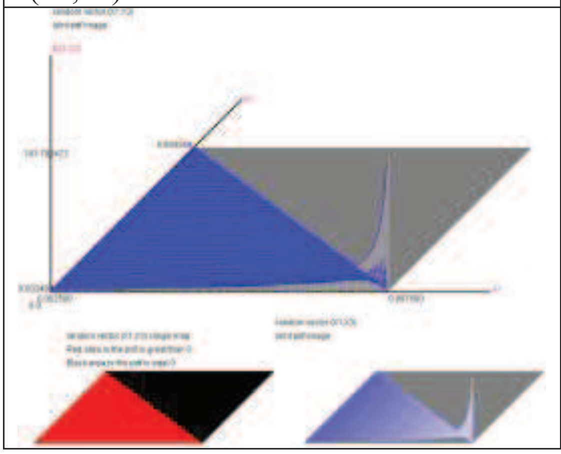
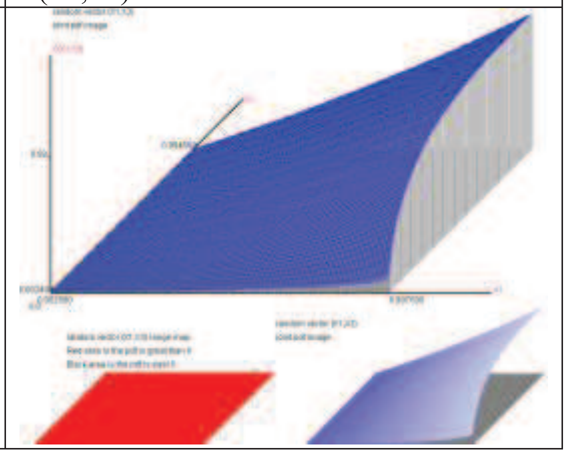
f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.28209
	Geometrical Mean :	0.17158
	Harmonic Mean :	0.00193
	Variance :	0.05051
	S.D. :	0.22474
	Skewed Coef. :	0.80854
	Kurtosis Coef. :	2.82741
	MAD :	0.18557
	Range :	0.99991
	Mid_range :	0.49996
	Median :	0.22865
	Q1 :	0.09459
	Q2 :	0.22865
	Q3 :	0.42647
	IQR :	0.33188
	C.V. :	0.79670

f(x3),F(x3)	Coefficient	
	Mathematical Mean:	0.18871
	Geometrical Mean :	0.09287
	Harmonic Mean :	0.00111
	Variance :	0.03320
	S.D. :	0.18221
	Skewed Coef. :	1.23307
	Kurtosis Coef. :	4.01427
	MAD :	0.14529
	Range :	0.99790
	Mid_range :	0.49895
	Median :	0.13000
	Q1 :	0.04356
	Q2 :	0.13000
	Q3 :	0.28411
	IQR :	0.24055
	C.V. :	0.96557

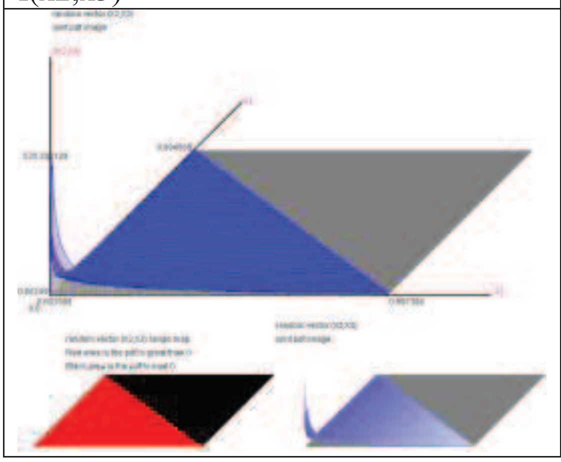
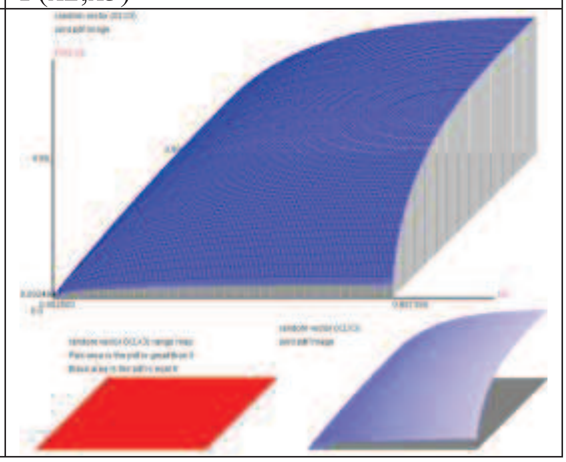
(2) λ_1=0.6, λ_2=0.1, λ_3=0.2,

f(x1,x2)	F(x1,x2)
	

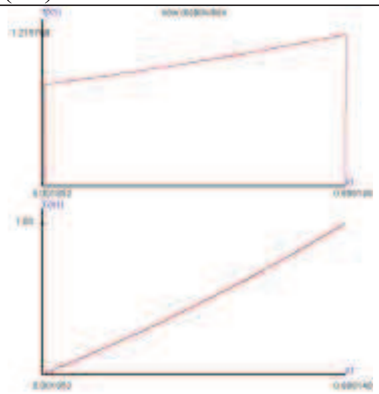
E(X1)= 0.5337, Var(X1)= 0.0826, E(X2)= 0.2060, Var(X2)= 0.0383,
Cov(X1,X2)= -0.0341, X1 and X2 correlation coefficient=-0.6060.

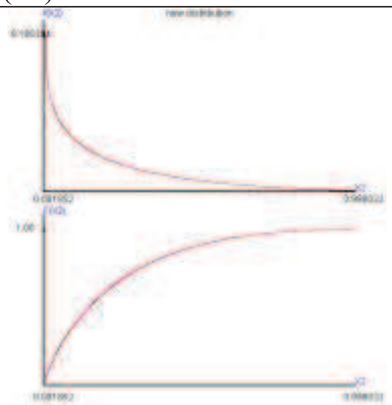
f(x1,x3)	F(x1,x3)
	

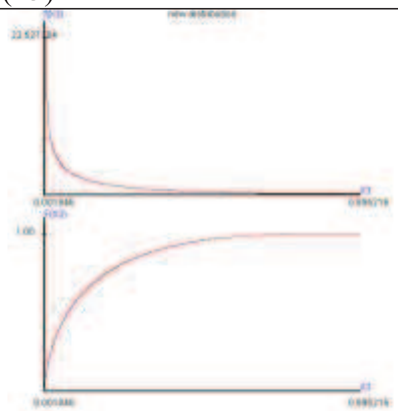
E(X1)= 0.5337, Var(X1)= 0.0826, E(X3)= 0.1371, Var(X3)= 0.0255,
Cov(X1,X3)= -0.0262, X1 and X3 correlation coefficient=-0.5713.

f(x2,x3)	F(x2,x3)
	

E(X2)= 0.2060, Var(X2)= 0.0383, E(X3)= 0.1371, Var(X3)= 0.0255,
Cov(X2,X3)= -0.0024, X2 and X3 correlation coefficient=-0.0765.

f(x1),F(x1)	Coefficient	
	Mathematical Mean:	0.53369
	Geometrical Mean :	0.40612
	Harmonic Mean :	0.06876
	Variance :	0.08265
	S.D. :	0.28748
	Skewed Coef. :	-0.14024
	Kurtosis Coef. :	1.82720
	MAD :	0.24857
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.55031
	Q1 :	0.29049
	Q2 :	0.55031
	Q3 :	0.78540
	IQR :	0.49490
	C.V. :	0.53867

f(x2),F(x2)	Coefficient	
	Mathematical Mean:	0.20599
	Geometrical Mean :	0.10693
	Harmonic Mean :	0.00054
	Variance :	0.03825
	S.D. :	0.19559
	Skewed Coef. :	1.24028
	Kurtosis Coef. :	4.02964
	MAD :	0.15543
	Range :	0.99988
	Mid_range :	0.49994
	Median :	0.14372
	Q1 :	0.05159
	Q2 :	0.14372
	Q3 :	0.30631
	IQR :	0.25473
	C.V. :	0.94951

f(x3),F(x3)	Coefficient	
	Mathematical Mean:	0.13710
	Geometrical Mean :	0.05303
	Harmonic Mean :	0.00032
	Variance :	0.02546
	S.D. :	0.15957
	Skewed Coef. :	1.69083
	Kurtosis Coef. :	5.73387
	MAD :	0.12144
	Range :	0.99706
	Mid_range :	0.49853
	Median :	0.07479
	Q1 :	0.02054
	Q2 :	0.07479
	Q3 :	0.19805
	IQR :	0.17750
	C.V. :	1.16391

Section 3. X~CB(λ1 + λ2), 0 ≤ x ≤ 1

Merge λ_1 and λ_2, let X is new random variable.

λ_1	λ_2	λ_3	$1-\lambda_1-\lambda_2-\lambda_3$

changed to

X	X_3	$1-X-X_3$
$\lambda_1+\lambda_2$	λ_3	$1-\lambda_1-\lambda_2-\lambda_3$

$$X_3|x \sim CB\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right), 0 \le x_3 \le 1-x,$$

X is one random variable.

$$f_{X_3|x}(x_3|x) = C_2(\lambda_1+\lambda_2, \lambda_3, x)\left(\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right)^{x_3}\left(1-\frac{\lambda_3}{1-\lambda_1-\lambda_2}\right)^{1-x-x_3},$$

$$0 \le x_1 \le 1, 0 \le x_3 \le 1-x,$$

X	$1-X$
$\lambda_1+\lambda_2$	$1-\lambda_1-\lambda_2$

$$1-X = X_3 + (1-X-X_3)$$

X_3	$1-X-X_3$
$\frac{\lambda_3}{1-\lambda_1-\lambda_2}$	$1-\frac{\lambda_3}{1-\lambda_1-\lambda_2}$

$$f_X(x;\lambda_1)f_{X_3|x}(x_3|x) \ne f_{X_1}(x_1;\lambda_1)f_{X_2|x_1}(x_2|x_1)f_{X_3|x_1,x_2}(x_3|x_1,x_2)$$

Section 4. 2×2 table

The probability			marginal
	λ_1	λ_3	$\lambda_1+\lambda_3$
	λ_2	λ_4	$\lambda_2+\lambda_4$
marginal	$\lambda_1+\lambda_2$	$\lambda_3+\lambda_4$	1

Random variables			marginal
	X_1	X_3	X_1+X_3
	X_2	X_4	X_2+X_4
marginal	X_1+X_2	X_3+X_4	1

X_1	X_2	X_3	X_4
λ_1	λ_2	λ_3	λ_4

$\lambda_1+\lambda_2+\lambda_3+\lambda_4=1, X_1+X_2+X_3+X_4=1, 0<X_i<1, i=1,2,3,4,$

The comparison of $\lambda_1+\lambda_2$ and $\lambda_1+\lambda_3$, that is about the difference of λ_2 and λ_3,

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$1-X_1=X_2+(1-X_1-X_2)$$

X_2	$1-X_1-X_2$
$\dfrac{\lambda_2}{1-\lambda_1}$	$1-\dfrac{\lambda_2}{1-\lambda_1}$

and

X_1	$1-X_1$
λ_1	$1-\lambda_1$

$$1-X_1=X_3+(1-X_1-X_3)$$

X_3	$1-X_1-X_3$
$\dfrac{\lambda_3}{1-\lambda_1}$	$1-\dfrac{\lambda_3}{1-\lambda_1}$

$X_2|x_1$ and $X_3|x_1$ can be comparison at the same condition situation.

About statistical analysis, please see section 1 of chapter 7.

Appendix 1, $X_2 | x_1 \sim CB(\lambda_1, \lambda_2, x_1)$, $\lambda^* = \lambda_2 / (1 - \lambda_1)$

$$f_{X_2|x_1}(x_2|x_1) = C_2(\lambda_1, \lambda_2, x_1)\left(\frac{\lambda_2}{1-\lambda_1}\right)^{x_2}\left(1-\frac{\lambda_2}{1-\lambda_1}\right)^{1-x_1-x_2}, 0 \le x_2 \le 1-x_1, 0 < \lambda_2 < 1-\lambda_1,$$

$$\lambda^* = \frac{\lambda_2}{1-\lambda_1}, f_{X_2|x_1}(x_2|x_1) = C_2(\lambda^*, x_1)(\lambda^*)^{x_2}(1-\lambda^*)^{1-x_1-x_2}, 0 \le x_2 \le 1-x_1, 0 < \lambda^* < 1$$

$$\int_0^{1-x_1} f_{X_2|x_1}(x_2|x_1)dx_2 = C_2(\lambda^*, x_1)(1-\lambda^*)^{1-x_1}\int_0^{1-x_1}\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_2}dx_2 ---(1.1),$$

$$(i)\lambda^* \ne 0.5, (1.1) = C_2(\lambda^*, x_1)(1-\lambda^*)^{1-x_1}\left.\frac{\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_2}}{ln\left(\frac{\lambda^*}{1-\lambda^*}\right)}\right|_0^{1-x_1}$$

$$= C_2(\lambda^*, x_1)\frac{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}}{ln\left(\frac{\lambda^*}{1-\lambda^*}\right)} = 1, C_2(\lambda^*, x_1) = \frac{ln(\lambda^*) - ln(1-\lambda^*)}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}},$$

$$(ii)\lambda^* = 0.5, (1.1) = C_2(\lambda^*, x_1)\left(\frac{1}{2}\right)^{1-x_1}(1-x_1) = 1, C_2(\lambda^*, x_1) = \frac{2^{1-x_1}}{1-x_1},$$

$$C_2(\lambda^*, x_1) = \begin{cases} \dfrac{ln(\lambda^*) - ln(1-\lambda^*)}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}}, \lambda^* \ne 0.5, \\ \dfrac{2^{1-x_1}}{1-x_1}, \lambda^* \ne 0.5, \end{cases}$$

The cumulative probability distribution function,

$(i)\lambda^* \neq 0.5,$

$$F_{X_2|x_1}(x_2|x_1) = \frac{(1-\lambda^*)^{1-x_1}\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_2} - (1-\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}}, 0 < x_2 < 1-x_1,$$

$(ii)\lambda^* = 0.5,$

$$\int_0^{1-x_1} f_{X_2|x_1}(x_2|x_1)dx_2 = \frac{x_2}{1-x_1}, 0 < x_2 < 1-x_1,$$

The Expected value and variance.

$(i)\lambda^* \neq 0.5,$

$$E(X_2|x_1) = C_2(\lambda^*, x_1)(1-\lambda^*)^{1-x_1}\int_0^{1-x_1} x_2\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_2} dx_2$$

$$= \frac{(1-\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}}\left((1-x_1)\left(\frac{\lambda^*}{1-\lambda^*}\right)^{1-x_1} - \frac{\left(\frac{\lambda^*}{1-\lambda^*}\right)^{1-x_1} - 1}{ln(\lambda^*) - ln(1-\lambda^*)}\right)$$

$$= \frac{(1-x_1)(\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}} - \frac{1}{ln(\lambda^*) - ln(1-\lambda^*)},$$

$$E(X_2^2|x_1) = C_2(\lambda^*, x_1)(1-\lambda^*)^{1-x_1}\int_0^{1-x_1} x_2^2\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_2} dx_2$$

$$= \frac{(1-\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}}\left((1-x_1)^2\left(\frac{\lambda^*}{1-\lambda^*}\right)^{1-x_1} - 2\left(\frac{(1-x_1)\left(\frac{\lambda^*}{1-\lambda^*}\right)^{1-x_1}}{ln(\lambda^*) - ln(1-\lambda^*)} - \frac{\left(\frac{\lambda^*}{1-\lambda^*}\right)^{1-x_1} - 1}{(ln(\lambda^*) - ln(1-\lambda^*))^2}\right)\right)$$

$$= \frac{(1-x_1)^2(\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}} - \frac{2(1-x_1)(\lambda^*)^{1-x_1}}{ln(\lambda^*) - ln(1-\lambda^*)\left((\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}\right)} + \frac{2}{(ln(\lambda^*) - ln(1-\lambda^*))^2},$$

$$Var(X_2|x_1)$$

$$= \frac{(1-x_1)^2(\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}} - \frac{2(1-x_1)(\lambda^*)^{1-x_1}}{ln(\lambda^*) - ln(1-\lambda^*)\left((\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}\right)} + \frac{2}{(ln(\lambda^*) - ln(1-\lambda^*))^2}$$

$$-\left(\frac{(1-x_1)(\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}} - \frac{1}{ln(\lambda^*) - ln(1-\lambda^*)}\right)^2$$

$$= \frac{1}{(ln(\lambda^*) - ln(1-\lambda^*))^2} - \frac{(1-x_1)^2(\lambda^*)^{1-x_1}(1-\lambda^*)^{1-x_1}}{\left((\lambda^*)^{1-x_1} - (1-\lambda^*)^{1-x_1}\right)^2},$$

$$(ii)\lambda^* = 0.5,$$
$$(ii) 1-\lambda_1 = 2\lambda_2,$$
$$E(X_2|x_1) = \frac{1}{1-x_1}\int_0^{1-x_1} x_2 dx_2 = \frac{1-x_1}{2},$$
$$E(X_2^2|x_1) = \frac{1}{1-x_1}\int_0^{1-x_1} x_2^2 dx_2 = \frac{1+x_1+x_1^2}{3},\ Var(X_2|x_1) = \frac{(1-x_1)^2}{12}.$$

$$\lambda^* = \frac{\lambda_2}{1-\lambda_1},$$
$$(i)\lambda^* \neq 0.5,$$
$$E(X_2|x_1) = \frac{(1-x_1)(\lambda^*)^{1-x_1}}{(\lambda^*)^{1-x_1}-(1-\lambda^*)^{1-x_1}} - \frac{1}{ln(\lambda^*)-ln(1-\lambda^*)},$$
$$Var(X_2|x_1) = \frac{1}{(ln(\lambda^*)-ln(1-\lambda^*))^2} - \frac{(1-x_1)^2(\lambda^*)^{1-x_1}(1-\lambda^*)^{1-x_1}}{\left((\lambda^*)^{1-x_1}-(1-\lambda^*)^{1-x_1}\right)^2},$$
$$(ii)\lambda^* = 0.5,$$
$$E(X_2|x_1) = \frac{1-x_1}{2},\ Var(X_2|x_1) = \frac{(1-x_1)^2}{12}.$$

The simulator

The random number= $RND_2 = F_{X_2|x_1}(x_2|x_1) \sim Uniform(0,1),$

x_3 simulated value=

$$\begin{cases} \dfrac{log_e\left((1-\lambda^*)^{1-x_1} + RND_2\left((\lambda^*)^{1-x_1}-(1-\lambda^*)^{1-x_1}\right)\right)-(1-x_1)log_e(1-\lambda^*)}{log_e(\lambda^*)-log_e(1-\lambda^*)}, \lambda^* \neq 0.5 \\ RND_3(1-x_1), \lambda^* = 0.5 \end{cases}$$

Appendix 2, **$X_3| x_1, x_2 \sim CB(\lambda_1, \lambda_2, \lambda_3, x_1, x_2), \lambda^* = \lambda_3 / (1 - \lambda_1 - \lambda_2)$**

The probability density function

$$\lambda^* = \frac{\lambda_3}{1-\lambda_1-\lambda_2},$$

$$f_{X_3|x_1,x_2}(x_3|x_1,x_2) = C_3(\lambda^*,x_1,x_2)(\lambda^*)^{x_3}(1-\lambda^*)^{1-x_1-x_2-x_3}, 0 < x_3 < 1-x_1-x_2,$$

$$(i)\lambda^* \neq 0.5,$$

$$\int_0^{1-x_1-x_2} f_{X_3|x_1,x_2}(x_3|x_1,x_2)dx_3$$

$$= C_3(\lambda^*,x_1,x_2)(1-\lambda^*)^{1-x_1-x_2}\int_0^{1-x_1-x_2}\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_3}dx_3$$

$$= C_3(\lambda^*,x_1,x_2)(1-\lambda^*)^{1-x_1-x_2}\frac{\left(\frac{\lambda^*}{1-\lambda^*}\right)^{1-x_1-x_2}-1}{ln\left(\frac{\lambda^*}{1-\lambda^*}\right)}$$

$$= C_3(\lambda^*,x_1,x_2)\frac{(\lambda^*)^{1-x_1-x_2}-(1-\lambda^*)^{1-x_1-x_2}}{ln\left(\frac{\lambda^*}{1-\lambda^*}\right)} = 1,$$

$$C_3(\lambda^*,x_1,x_2) = \frac{ln\left(\frac{\lambda^*}{1-\lambda^*}\right)}{(\lambda^*)^{1-x_1-x_2}-(1-\lambda^*)^{1-x_1-x_2}},$$

$$(ii)\lambda^* = 0.5,$$

$$\int_0^{1-x_1-x_2} f_{X_3|x_1,x_2}(x_3|x_1,x_2)dx_3$$

$$= C_3(\lambda^*,x_1,x_2)(1-\lambda^*)^{1-x_1-x_2}(1-x_1-x_2) = 1,$$

$$C_3(\lambda^*,x_1,x_2) = \frac{1}{(1-\lambda^*)^{1-x_1-x_2}(1-x_1-x_2)},$$

The cumulative probability distribution function,

$$(i)\lambda^* \neq 0.5,$$

$$F_{X_3|x_1,x_2}(x_3|x_1,x_2) = \frac{(1-\lambda^*)^{1-x_1-x_2}\left(\frac{\lambda^*}{1-\lambda^*}\right)^{x_3}-(1-\lambda^*)^{1-x_1-x_2}}{(\lambda^*)^{1-x_1-x_2}-(1-\lambda^*)^{1-x_1-x_2}}, 0 < x_3 < 1-x_1-x_2,$$

$$(ii)\lambda^* = 0.5,$$

$$F_{X_3|x_1,x_2}(x_3|x_1,x_2) = \frac{x_3}{1-x_1-x_2} 0 < x_3 < 1-x_1-x_2,$$

The simulator

The random number= $RND_3 = F_{X_3|x_1,x_2}\left(x_3|x_1,x_2\right) \sim Uniform(0,1)$,

x_3 simulated value=

$$\begin{cases} \dfrac{log_e\left(\left(1-\lambda^*\right)^{1-x_1-x_2} + RND_3\left(\left(\lambda^*\right)^{1-x_1-x_2} - \left(1-\lambda^*\right)^{1-x_1-x_2}\right)\right) - \left(1-x_1-x_2\right)log_e\left(1-\lambda^*\right)}{log_e\left(\lambda^*\right) - log_e\left(1-\lambda^*\right)} \\ ,\lambda^* \neq 0.5 \\ RND_3\left(1-x_1-x_2\right), \lambda^* = 0.5 \end{cases}$$