The c^2 Gravitational Potential Limit

and its implications for G as well as general relativity

Martin Mayer
ma.mayer.physics@outlook.com
Augsburg, Germany
Rev 1.02, Jan 2024

Abstract

The relationship between gravitational potentials, black holes and the squared light speed c^2 is examined in this paper as well as the implications of the presented findings for the universal gravitational constant G and general relativity theory. It is common knowledge that the velocity limit in our universe is defined by light speed c and as shown in this work c^2 plays a similar role for the gravitational potential since c^2/G is linked to the mass density of black holes and our local Hubble sphere. Furthermore, it is demonstrated that the rift between cosmology and quantum physics can possibly be reconciled by acknowledging the physical meaning of the Planck units which proposedly define the characteristics of a quantized space-time, including its gravitational impedance. This notion is also supported by the presented logarithmic relationships between the cosmological scale and the quantum scale. Moreover, the presented findings uncover a previously unknown physical meaning for the constituents of Dirac's mysterious large number hypothesis.

Keywords: light speed; gravity; gravitational constant; gravitational potential; gravitational force; general relativity; black holes; gravitational waves; Hubble sphere; space-time; quantization; Planck units; Euler's number; Sommerfeld constant; CMB temperature

1 Introduction

The term c^2 is mostly known for its appearance in the famous energy mass equivalence $E = mc^2$, which was proposed by Albert Einstein in 1905 after other physicists, like Oliver Heaviside, Fritz Hasenöhrl and Henri Poincaré, had already proposed similar results. Another contemporary of Einstein, Erwin Schrödinger, noticed in 1925 that c^2 can also be expressed in the physical units of a gravitational potential, i.e. J/kg or equivalently $\mathrm{N\,m}/\mathrm{kg}$, besides the "raw" units $\mathrm{m}^2/\mathrm{s}^2$, as reported by Alexander Unzicker in (2). Such a coincidence of units may point towards an important physical relationship, like it was the case with the units of Planck's constant h which can be interpreted as $\mathrm{J/Hz}$ or as the units of angular momentum, i.e. $\mathrm{kg\,m^2/s}$. Niels Bohr scrutinized this congruence in 1913 which then led him to the discovery of quantized electron shells in atoms. Similar to Bohr's endeavour the purpose of this paper is to examine the gravitational potential interpretation of c^2 and its implications for the gravitational constant G as well as general relativity theory. To achieve this objective first some standard equations are repeated in the next section for later reference.

2 Prerequisites

The force of Newtonian gravity for two spherical masses m_1 and m_2 is usually expressed as

$$F_{gm} = G\frac{m_1 m_2}{d^2} \tag{2.1}$$

whereby d denotes the distance between their center of mass. Using $E_1 = m_1\,c^2$ and $E_2 = m_2\,c^2$ it is also possible to express Newtonian gravity as follows with respect to energy:

$$F_{ge} = \frac{G}{c^4}\frac{E_1 E_2}{d^2} \tag{2.2}$$

The associated gravitational potential energy can subsequently be expressed in the following two ways:

$$U_{gm} = -G\frac{m_1 m_2}{d} \qquad \text{with} \qquad F_{gm} = -\frac{\mathsf{d}\,U_{gm}}{\mathsf{d}\,d} \tag{2.3}$$

$$U_{ge} = -\frac{G}{c^4}\frac{E_1 E_2}{d} \qquad \text{with} \qquad F_{ge} = -\frac{\mathsf{d}\,U_{ge}}{\mathsf{d}\,d} \tag{2.4}$$

The gravitational potential is subsequently given by the following two equations, whereby equation 2.5 describes gravitational potential energy per unit mass, i.e. $\mathrm{J}/(1\,\mathrm{kg})$,

$$V_{gm} = \frac{U_{gm}}{m_2} = -G\frac{m_1}{d} \tag{2.5}$$

and equation 2.6 describes gravitational potential energy per unit energy, i.e. $\mathrm{J}/(1\,\mathrm{J})$.

$$V_{ge} = \frac{U_{ge}}{E_2} = -\frac{G}{c^4}\frac{E_1}{d} \tag{2.6}$$

Please note that the G/c^4 term in equation 2.2, 2.4 and 2.6 is also present in the Einstein constant κ, which is used in the equations of general relativity theory.

$$\kappa = 8\pi\frac{G}{c^4} \tag{2.7}$$

This correlation already demonstrates the relatedness of Newtonian gravity with general relativity theory and it's unfortunate that this obvious connection is not pointed out in physics literature. The implicit presence of 4π in the Einstein constant indicates a relationship with spherical geometry as shown hereafter in section 4. Moreover, it's noteworthy that $c^4/\,G$ has the physical units of force, i.e. N or $\mathrm{J}/\,\mathrm{m}$. The physical units of the gravitational constant G itself are $\mathrm{N\,m^2\,kg^{-2}}$, whereby its strikingly difficult to make sense of G's "raw" units $\mathrm{m^3\,s^{-2}\,kg^{-1}}$ until regarding equation 3.2 rearranged for G.

This paper also uses some key results of general relativity theory, in particular the mass for a static Schwarzschild black hole with radius r_s

$$m_s = \frac{r_s}{2}\frac{c^2}{G} \tag{2.8}$$

and the mass of an extreme Kerr black hole with radius r_k which rotates with light speed c at its equatorial ring.

$$m_k = r_k\frac{c^2}{G} \tag{2.9}$$

It's noteworthy that these two types of black hole mass are simply related by a factor of two for an identical radius. Kerr black holes with equatorial velocities other than c are not relevant for this paper. Technically, a Kerr black hole cannot rotate that fast, but in the context of this paper we are mainly interested in limiting cases.

3 The c² Limit

The value of c^2 expressed as gravitational potential energy per unit mass is given by:

$$c^2 = 8.988 \times 10^{16}\,\mathrm{J}\,/\,\mathrm{kg} \tag{3.1}$$

Since light speed c is considered to be the maximum possible velocity in our universe, because c^2 also has an extraordinarily high value and because of the matching units it seems sensible to postulate that c^2 denotes the magnitude of maximum gravitational potential energy per unit mass in our universe. Utilizing this assumption by setting equation 2.5 equal to $-c^2$ then gives the following expression which will be useful later on:

$$G\frac{m}{d} = c^2 \tag{3.2}$$

Finding the maximum value for the gravitational potential energy per unit energy is less straightforward, but comparing equation 2.6 with 3.2 reveals that the appropriate value is $-c^2/c^2 = -1$. This expression evaluates to a pure number but here the appropriate physical units can be added without violating any rules.

$$\frac{G}{c^4}\frac{mc^2}{d} = 1\frac{\mathrm{J}}{\mathrm{J}} \tag{3.3}$$

Surprisingly, the last two equations result in a well known relationship when rearranging either of them for mass m:

$$m = \frac{dc^2}{G} \tag{3.4}$$

This relation equals the extreme Kerr black hole mass equation 2.9 when interpreting distance d as radius r_k. Due to its spherical symmetry, and since the mass in a black hole is either compressed into its center or evenly distributed, depending on which argumentation one tends to follow, it is valid to set d equal to r_k. Consequently, these findings suggest that black holes form when the gravitational potential for a spherical region of space is between $-c^2/\,2$, like for a Schwarzschild black hole, and $-c^2$. The primary characteristic of regions with such a strong gravitational potential is that they cannot be destroyed by spinning them up since this would require a tangential velocity greater than light speed c, as explained in more detail in section 5. Moreover, the escape velocity $v_{esc} = \sqrt{2Gm/d}$ already equals the speed of light c for Schwarzschild black holes, as suggested to me by Bernhard Foltz, and therefore objects as well as fundamental particles cannot leave a black hole even according to physics that came before general relativity theory.

It's worth emphasizing here that the presented findings imply that the existence of black holes can be inferred from Newtonian gravity when also acknowledging a limiting velocity c, but surprisingly there is no real necessity for assuming any space curvature. Since an extreme Kerr black hole poses a limit to the amount of energy for any spherical volume of space, and absorption of mass and angular momentum makes it grow, the gravitational potential of $-c^2$ must represent a general upper limit, i.e.

$$-V_{gm} \leqq c^2 \quad \text{and} \quad -V_{ge} \leqq 1\mathrm{J/J} \tag{3.5}$$

is always given. Presumably, electrically charged black holes cannot violate these constraints either.

4 Our Hubble Sphere

The expansion of our universe creates an invisible and intangible spherical boundary centred around our location in space at which objects are moving away from us with light speed c and objects outside of this boundary are receding even faster. The current rate of this expansion is denoted by the Hubble constant H_0 which can also be used to calculate the distance to this boundary, whereby the enclosed volume is called the Hubble sphere whose radius is in turn denoted as Hubble radius r_H.

$$H_0 \cong 74.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{4.1}$$

$$r_H = \frac{c}{H_0} \cong 1.25 \times 10^{26}\,\mathrm{m} \tag{4.2}$$

The correct value of H_0 is still being disputed, but the value used here seems to fit well with the following calculations.

Calculating the ordinary mass density for a Schwarzschild black hole that has the extent of our Hubble sphere gives the following mass density

$$\rho_H = \frac{m_H}{V_H} = \frac{m_s(r_s = r_H)}{4\pi r_H^3/3} = \frac{3H_0^2}{8\pi G} = 1.0 \times 10^{-26}\,\mathrm{kg/m^3} \tag{4.3}$$

whereby m_H denotes the Hubble sphere mass and V_H denotes the Hubble sphere volume. Surprisingly, the result $3H_0^2/(8\pi G)$ matches with the so called "critical density" which characterizes a flat universe according to general relativity theory. This, in turn, suggests that a Schwarzschild black hole possesses a flat space geometry with a homogeneous mass distribution and consequently doesn't have a central singularity. Moreover, NASA reports that the mass density of our universe is around $1.0 \times 10^{-26}\,\mathrm{kg/m^3}$ (see footnote 1) and thus our local Hubble sphere is either technically a Schwarzschild black hole or very close to being one. This makes sense since black holes set an upper limit for the amount of energy and information that can be contained in a particular volume of space. Moreover, nothing can leave our Hubble sphere, due to the expansion of its space, which also matches with the accepted presumption that no object can leave a black hole. Thus it might make sense to define the Hubble constant H_0 in terms of Schwarzschild black hole properties. This can be achieved by rearranging the last equation and leads to the following expression that contains a mass density term:

$$H_0 = \sqrt{2G\,\frac{m_s(r_s = r_H)}{r_H^3}} = \frac{c}{r_H} \tag{4.4}$$

This interesting formulation for H_0 implies that our Hubble sphere's expansion is linked to its mass density m_H/r_H^3. Later in this section it is shown that the gravitational constant G itself also depends on a similar mass density, which is why at any point in time H_0 should be c/r_H even if the mass density of our Hubble sphere changes with time, but there is also a possibility that the last equation might be incomplete or superfluous.

The black hole analogy for the Hubble sphere can be taken one step further by also applying equation

1 http://map.gsfc.nasa.gov/universe/uni_matter.html

2.8 to it. Moreover, that equation can also be multiplied by c^2 and rearranged so that the result coincides with the gravitational potential equation 2.6 adapted for the case of a Schwarzschild black hole:

$$\frac{G}{c^4}\left(\frac{m_H c^2}{r_H}\right) = \frac{1\mathrm{J}}{2\mathrm{J}} \tag{4.5}$$

Interestingly, Erwin Schrödinger was already proposing a similar relationship for analysis in 1925, although there was much less knowledge about the observable universe back then (6). The interesting part of the last equation is the term in brackets, which has the physical units of $\mathrm{J/m}$, and can be dissected to analyse our Hubble sphere's internal energy distribution:

$$\frac{1\mathrm{J}}{2\mathrm{J}}\frac{c^4}{G} = \frac{m_H c^2}{r_H} = \frac{\rho_H V_H c^2}{r_H} = \frac{4\pi}{3}\rho_H r_H^2 c^2 = \frac{2}{3} \times 2\pi\rho_H r_H^2 c^2 \tag{4.6}$$

The calculations in this document usually treat masses as point particles, but here it is necessary to consider the effect of a changing gravitational force inside the Hubble sphere. Therefore the factor $2/3$ appears at the right side of equation 4.6 which is characteristic for the gravitational potential of a sphere with uniform energy density. The remaining part fits with the ideas of Mach, Dicke, Sciama and Schrödinger who speculated that all masses in the observable universe and their distance to us should be causal for the gravitational constant G (2), which is a sensible conjecture according to the next equation which follows from rearranging 4.6:

$$\frac{3}{2}\frac{1\mathrm{J}}{2\mathrm{J}}\frac{c^4}{G} = 2\pi r_H^2 \rho_H c^2 = \int_0^{r_H} \frac{4\pi r^2 \rho_H c^2 \,\mathrm{d}r}{r} = \lim_{\delta r \to 0} \sum_{i=0}^{\infty} \frac{4\pi r_i^2 \rho_H c^2 \,\delta r}{r_i} = \sum_{i=0}^{\infty} \frac{m_i c^2}{r_i} \tag{4.7}$$

It is assumed here that the mass density of our Hubble sphere can be treated as being approximately homogeneous on large scales. This is why the last equation divides our Hubble sphere into a series of spherical shells, which ideally should be infinitely thin, whereby $m_i = 4\pi r_i^2 \rho_H \,\delta r$ denotes the mass of one such spherical shell whose distance to us is given by r_i and whose thickness is δr. The resulting sum term on the right side of equation 4.7 is what typically appears in a gravitational potential equation when several masses are involved and equation 4.5 can thus also be expressed as follows:

$$\frac{G}{c^4}\left(\frac{2}{3}\sum_{i=0}^{\infty}\frac{m_i c^2}{r_i}\right) = \frac{1\mathrm{J}}{2\mathrm{J}} \tag{4.8}$$

The infinity symbol above the summation operator indicates that this sum ideally takes infinitely many spherical shells into consideration. In this limit, and in case the mass distribution is homogeneous, the masses m_i do not have to denote the mass of spherical shells, they could refer to small chunks of matter instead which ideally are infinitely many again. Using spherical shells was just helpful to be able to use a simple integral in equation 4.7.

The findings which were presented above suggest that gravitational potential, G, c^2 and the physical parameters of our local Hubble sphere are all interconnected. This leads to the notion that the gravitational constant G can be regarded as a result of those relationships, in particular of our local Hubble sphere's energy density and its associated gravitational potential of $-c^2/2$. Consequently, the gravitational constant G can be defined as an emergent constant in the following ways

$$G = \frac{c^2}{2}\bigg/\left(\frac{m_H}{r_H}\right) = \frac{c^2}{2}\bigg/\left(\frac{4\pi r_H^2}{3}\rho_H\right) = \frac{c^2}{2}\bigg/\left(\frac{2}{3}\sum_{i=0}^{\infty}\frac{m_i}{r_i}\right) \tag{4.9}$$

in case our Hubble sphere really qualifies as a Schwarzschild black hole. Otherwise the proportionality constant $(1/2)/(2/3) = 3/4$ would have to be changed. On the same grounds the Einstein constant can also be rewritten with respect to the parameters of our local Hubble sphere as follows:

$$1/\kappa = \frac{1}{8\pi}\frac{c^4}{G} = \frac{1}{4\pi}\frac{m_H c^2}{r_H} = \frac{1}{4\pi}\frac{2}{3}\sum_{i=0}^{\infty}\frac{m_i c^2}{r_i} = \frac{1}{3}\rho_H r_H^2 c^2 \tag{4.10}$$

The implications of these expressions will be treated later on in the discussion section.

The emergent constant notion for the gravitational constant G implies that equation 2.1 to 2.6 are connected to cosmological quantities. Equation 2.5 and 2.6, for example, can subsequently be rewritten in a form without G as follows, whereby $E_1 = m_1 c^2$, $E_i = m_i c^2$ and $E_H = m_H c^2$:

$$V_{gm} = -G\frac{m_1}{d} = -\frac{3}{4}\frac{m_1 c^2}{d}\sum_{i=0}^{\infty}\frac{r_i}{m_i} = -\frac{c^2}{2}\frac{m_1/m_H}{d/r_H} \tag{4.11}$$

$$V_{ge} = -\frac{G}{c^4}\frac{E_1}{d} = -\frac{3}{4}\frac{E_1}{d}\sum_{i=0}^{\infty}\frac{r_i}{E_i} = -\frac{1}{2}\frac{E_1/E_H}{d/r_H} = -\frac{1}{2}\frac{m_1/m_H}{d/r_H} \tag{4.12}$$

These equations reveal that the presence of G conceals a normalization relationship for mass m_1 and distance d with respect to the Hubble mass m_H and the Hubble radius r_H, respectively. Put another way, the mass gradients $m_1/\,d$ and m_H/r_H are both contributing to the gravitational potential functions V_{ge} and V_{gm}. The calculation results obtained from these functions would consequently change inversely proportional with an alteration in the mass density of our local Hubble sphere, in case such a density change would not also affect the quantities of length, time, light speed or mass - which might be the case though (see section 5 for more details).

The gravitational potential of a static Schwarzschild black hole is $-c^2/\,2$ according to equation 4.5. Since our local Hubble sphere may qualify as Schwarzschild black hole this in turn gives rise to the speculation that the overall universe, which contains our Hubble sphere, might be a rotating extreme Kerr black hole which has a gravitational potential of $-c^2$. This notion receives some support from the possibility to express Hubble's constant as an angular frequency:

$$H_0 \cong 2.41 \times 10^{-18}\,\mathrm{rad/s} \tag{4.13}$$

Since a frequency is always linked to some kind of energy it must be possible to calculate the Hubble sphere's mass and energy by explicitly using the Hubble constant H_0. Using equation 2.8 and 4.2 it turns out that the Hubble mass m_H can indeed be calculated from H_0 as follows

$$m_H = m_s(r_s = r_H) = \frac{r_H}{2}\frac{c^2}{G} = \frac{\hbar}{2}\frac{1}{H_0\,l_l^2} \tag{4.14}$$

whereat the so called Planck length $l_l = \sqrt{\hbar G/c^3} = 1.62 \times 10^{-35}\,\mathrm{m}$ appears naturally in the resulting equation as well as Planck's constant h. The corresponding Hubble energy E_H can be expressed as follows

$$E_H = m_H c^2 = \frac{\hbar}{2}\frac{1}{H_0\,t_l^2} = \frac{E_l}{4\pi}\frac{t_H}{t_l} = \frac{E_l}{2}\frac{r_H}{l_l} \tag{4.15}$$

whereby $t_l = l_l/c = 5.4 \times 10^{-44}\,\mathrm{s}$ is the Planck time, $t_H = 2\pi/H_0 = 2.61 \times 10^{18}\,\mathrm{s}$ is a rotation period of roughly 82 billion years associated with H_0, when interpreted as angular frequency, and $E_l = c\hbar/l_l$ denotes the so called Planck energy.
Please note that the natural appearance of the super short Planck length and Planck time in equation 4.14 and 4.15 suggests that the cosmological scale is linked to the quantum scale via the Planck units - a notion which is also explored further in section 5.

Interestingly, equation 4.15 suggests that the energy content of our Hubble sphere is related to the Planck energy E_l, which is related to the Planck temperature $T_l = 1.4168 \times 10^{32}\,\mathrm{K}$ through the expression $E_l = k_b\,T_l$, whereby k_b denotes the Boltzmann constant. The Planck temperature, on the other hand, together with the temperature of the cosmic microwave background $T_{cmb} = 2.725\,\mathrm{K}$, seems to be related to our Hubble sphere's mass density ρ_H through the following equation

$$\frac{m_l}{l_l^3}\frac{1}{\alpha^2}\left(\frac{T_{cmb}}{T_l}\right)^4 = 1.3 \times 10^{-26}\,\mathrm{kg/m^3} \cong \rho_H \tag{4.16}$$

whereby $\alpha \cong 1/137.036$ denotes the Sommerfeld constant and T_{cmb} practically is the temperature of empty space due to residual photons from the early universe. The calculated density is not matching ρ_H exactly, but considering the huge exponents that are involved here, e.g. 10^{32^4}, it is intriguingly close. What the last equation implies is that our universe initially had an energy density of approximately $m_l/l_l^3 \cong 10^{113}\mathrm{J/m^3}$, which according to quantum field theory is still the (unobservable) intrinsic energy density of a seemingly empty vacuum in our present time (see also equation 7.1), and a temperature of T_l, instead of an infinite temperature as suggested by modern physics. Our universe then cooled down as it expanded until its average temperature became the cosmic microwave background temperature T_{cmb}, when disregarding other energy sources like suns. Together with the decreasing temperature also the average energy density of our observable universe decreased until it reached ρ_H. Since an energy E is generally power P summed over time and power P is related to temperature T under ideal conditions via $P \propto T^4$ the ratio T_{cmb}/T_l requires an exponent of four in equation 4.16. Thus in total the energy density of our observable universe seems to have decreased by a factor of around $(T_{cmb}/T_l)^4/\alpha^2 = 2.6 \times 10^{-123}$ since it began. The presence of the Sommerfeld constant α might be unjustified here, but it could be related to the electric field energy in our universe since this constant is related to the electric field constant ϵ_0 and the fundamental charge e. Finally, it should be pointed out that equation 4.16 can also be reworked to get an approximate expression for the cosmic microwave background temperate itself, which is given by:

$$T_{cmb} \cong \frac{c\hbar}{k_b}\left(\frac{1}{4\pi}\frac{3}{2}\frac{\alpha^2}{r_H^2\,l_l^2}\right)^{1/4} = 2.563\,\mathrm{K} \tag{4.17}$$

5 Quantized Space

Quantum physics uncovered that physical quantities are often discrete, instead of being continuous, and therefore it also makes sense to assume that space is not infinitely divisible. A sensible candidate for the smallest possible length in our universe is the so called Planck length l_l which already appeared in equation 4.14.

$$l_l = \sqrt{\frac{G}{c^2}\frac{\hbar}{c}} = 1.62 \times 10^{-35}\,\mathrm{m} \tag{5.1}$$

Interestingly, a hypothetical extreme Kerr black hole which has a radius of one Planck length l_l and rotates with light speed c at its equatorial ring would have a mass of one Planck mass m_l:

$$m_l = m_k(r_k = l_l) = \sqrt{\frac{c^2}{G}\frac{\hbar}{c}} = 2.18 \times 10^{-8}\,\mathrm{kg} \tag{5.2}$$

Despite its comparatively high mass it is assumed here that the Planck mass defines the mass of a single quanta of space. In fact, the resultant mass density matches with the so called zero-point energy as proposed by quantum physics. Moreover, the Planck mass can also be used to define the Planck energy $E_l = m_l c^2$ which appeared first in equation 4.15.

It was already suggested in (1) that the Planck units define the properties of the proposed quanta of space. This proposal makes sense since equation 5.1 and 5.2 only contain the most important physical quantities, i.e. light speed c, Planck's constant h, the gravitational constant G and the gravitational potential limit c^2, whereby both equations contain a G/c^2 term which in turn suggests a relationship to the Hubble sphere according to equation 4.9. This leads to the speculation that every single quanta of space exhibits, or mirrors, the characteristic properties of our universe in its own "micro-verse". If this notion is appropriate then equation 2.9 is not only valid on cosmological scales but also for the quantized structure of space itself, which is reflected by the following equalities:

$$\frac{c^2}{G} = \frac{m_k}{r_k} = \frac{m_l}{l_l} \tag{5.3}$$

$$\frac{c^4}{G} = \frac{m_k c^2}{r_k} = \frac{E_l}{l_l} \tag{5.4}$$

Consequently, equation 2.5 and 4.11, which describe gravitational potential energy per unit mass, as well as equation 2.6 and 4.12, which describe gravitational potential energy per unit energy, can also be expressed without the gravitational constant G as follows:

$$V_{gm} = -G\frac{m_1}{d} = -c^2\frac{m_1/m_l}{d/l_l} \tag{5.5}$$

$$V_{ge} = -\frac{G}{c^4}\frac{E_1}{d} = -\frac{E_1/E_l}{d/l_l} = -\frac{m_1/m_l}{d/l_l} \tag{5.6}$$

The last two equations demonstrate that a gravitational potential can also be defined using only properties of the local quantized space, which elegantly solves the problem how all the masses in our observable universe can influence local gravitational interactions as Mach, Dicke, Sciama and Schrödinger once suggested. Colloquially speaking: as above so below.

The proposed "mirroring" becomes even more apparent when expressing the Planck length and Planck mass in terms of the Hubble sphere properties (*please note that the following two equations are only exact in case our Hubble sphere really qualifies as a Schwarzschild black hole*):

$$l_l = \sqrt{\frac{1}{2}\frac{\hbar}{c}\frac{r_H}{m_H}} \tag{5.7}$$

$$m_l = \sqrt{2\frac{\hbar}{c}\frac{m_H}{r_H}} \tag{5.8}$$

These two relations suggest that a change in our Hubble sphere's density can result in a different Planck length and Planck mass, but a definitive conclusion on that matter is difficult since light speed c and Planck's constant h may also be affected by such a density change.

The term c^4/G, which has the physical unit of force, has even more physical meanings than previously discussed. This becomes obvious when dissecting that term into an acceleration part and a mass part:

$$\frac{c^4}{G} = a \times m = \frac{c^2}{r_k}m_k = \frac{c^2}{l_l}m_l \tag{5.9}$$

The last equation has physical meaning for the macro as well as the micro scale of space.

- On the macro scale: objects can be disintegrated by spinning them up to the point where they can overcome their gravitational self attraction when flying apart (molecular bindings are disregarded here for simplicity). For everyday objects this is not a surprising feature but this process is theoretically also possible with very large objects. For example, if a planet could be spun up enough it could disperse itself in space permanently when finally breaking apart. The same process could theoretically be tried with a Schwarzschild black hole which would gradually turn into an extreme Kerr black hole until its equatorial ring velocity nearly reaches light speed c and its centripetal acceleration gets close to $c^2/\,r_k$, but since light speed c cannot be exceeded in our universe it is not possible to destroy an extreme Kerr black hole by spinning it up. Thus for any spherical object with an arbitrary radius of r_k the maximum possible centripetal acceleration is given by $c^2/\,r_k$ and the upper limit for that object's mass is m_k.
- On the micro scale: The acceleration term $c^2/\,l_l = 5.56{\times}10^{51}\ \mathrm{m/s^2}$, which is also known as the Planck acceleration a_l, presumably denotes the maximum possible rotational & translational acceleration in our universe. The case is clear for rotation: there is a limit for centripetal acceleration which results from the circular motion equation $v^2/\,r$ and the meaning of c as well as l_l as limit for their respective physical domain. For comprehending maximum translational acceleration it is necessary to realize that the minimal amount of time needed to travel the fundamental distance l_l is $t_l = l_l/\,c$. Consequently, the maximum possible translational acceleration is also given by:

$$a_l = \frac{\delta v}{\delta t} = \frac{c - 0}{l_l/c} = \frac{c^2}{l_l} \tag{5.10}$$

Thus every time G/c^4 appears in an equation its usage either conceals a normalization with respect to some macro limits of space-time, i.e. $c^2/\,r_k$ and m_k, or alternatively it conceals a normalization with respect to the acceleration limit a_l and the presumed mass of a quanta of space, i.e. the Planck mass m_l.

6 Logarithmic Relations

Hartmut Müller brought up the idea that the mass ratios of fundamental particles are described by exponential relationships that feature Euler's number e_n *(note: the subscript n is used here to distinguish Euler's number from the fundamental charge e)*, whereby the the exponent of each relation is always close to an integer number or an integer number plus one half. For example, proton mass m_p and electron mass m_e are related to each other by the exponent 7.5,

$$\ln\left(\frac{m_p}{m_e}\right) = 7.5 \qquad \text{or} \qquad m_e \times e_n^{7+0.5} \cong m_p \tag{6.1}$$

whereby all numerical results in this section are rounded to one decimal place. Müller thinks that exponential Euler number relationships, which follow the stated exponent scheme, avoid destructive resonance from standing gravitational waves because e_n is an irrational as well as transcendental number and this adherence then in turn stabilizes fundamental particles, and even planetary orbits, at certain sizes. Moreover, Müller showed that such exponential relationships also exist with respect to the Planck mass m_l:

$$\ln\left(\frac{m_l}{m_p}\right) = 44.0 \qquad \text{or} \qquad m_p \times e_n^{44} \cong m_l \tag{6.2}$$

$$\ln\left(\frac{m_l}{m_e}\right) = 51.5 \qquad \text{or} \qquad m_e \times e_n^{51+0.5} \cong m_l \tag{6.3}$$

Due to the properties of logarithms the last three equations are related numerically, i.e. $51.5 - 44.0 = 7.5$.

The relationships shown so far in this section represent only a small selection of Müller's findings and more of his findings can be found in (7). The remainder of this section is dedicated to novel logarithmic relations between the quanta of space, the Hubble sphere, the proton and the electron which also seem to adhere to Müller's concept.

$$\ln\left(\frac{m_H}{m_l}\right) = 139.5 \qquad \text{or} \qquad m_l \times e_n^{139+0.5} \approxeq m_H \tag{6.4}$$

$$\ln\left(\frac{m_H}{m_p}\right) = 183.5 \qquad \text{or} \qquad m_p \times e_n^{183+0.5} \approxeq m_H \tag{6.5}$$

$$\ln\left(\frac{m_H}{m_e}\right) = 191.0 \qquad \text{or} \qquad m_e \times e_n^{191} \approxeq m_H \tag{6.6}$$

Again, due to the properties of logarithms there are numerical relations between the various results, i.e. $191.0 - 183.5 = 7.5$, $183.5 - 139.5 = 44.0$ and $191.0 - 139.5 = 51.5$.

Surprisingly, similar exponential relationships also exist for the proton and the electron with respect to the Sommerfeld constant $\alpha \cong 1/137.036$, whereby these relationships again exhibit integer, or integer plus one half, exponents:

$$\log_\alpha \left(\frac{m_e}{m_p}\right) = \log_\alpha \left(\frac{r_p}{r_e}\right) = 1.5 \qquad \text{or} \qquad m_p \times \alpha^{1+0.5} \cong m_e \tag{6.7}$$

It's worth emphasizing that an α based relationship between electron mass and proton mass, as described by equation 6.7, has been unrecognized in physics so far. Here r_p and r_e denote the reduced Compton wavelength for the proton and the electron which are related to their respective mass according to $r_p = \hbar/(m_p\, c)$ and $r_e = \hbar/(m_e\, c)$. Interestingly, these wavelengths also exhibit exponential relationships for α with hydrogen's radius a_0 , aka the Bohr radius, as well as the Hubble radius r_H:

$$\log_\alpha \left(\frac{r_e}{a_0}\right) = 1.0 \qquad \text{or} \qquad a_0 \times \alpha = r_e \tag{6.8}$$

$$\log_\alpha \left(\frac{r_p}{a_0}\right) = 2.5 \qquad \text{or} \qquad a_0 \times \alpha^{2+0.5} \cong r_p \tag{6.9}$$

$$\log_\alpha \left(\frac{r_e}{r_H}\right) = 18.0 \qquad \text{or} \qquad r_H \times \alpha^{18} \approxeq r_e \tag{6.10}$$

$$\log_\alpha \left(\frac{r_p}{r_H}\right) = 19.5 \qquad \text{or} \qquad r_H \times \alpha^{19+0.5} \approxeq r_p \tag{6.11}$$

Further noteworthy $\log_\alpha$ relationships exist with respect to the Planck mass m_l and the Planck length l_l:

$$\log_\alpha \left(\frac{m_e}{m_l}\right) = \log_\alpha \left(\frac{l_l}{r_e}\right) = 10.5 \qquad \text{or} \qquad m_l \times \alpha^{10+0.5} \cong m_e \tag{6.12}$$

$$\log_\alpha \left(\frac{m_p}{m_l}\right) = \log_\alpha \left(\frac{l_l}{r_p}\right) = 8.9 \qquad \text{or} \qquad m_l \times \alpha^{9} \cong m_p \tag{6.13}$$

These results are also related numerically, i.e. $2.5 - 1.0 = 19.5 - 18.0 = 10.5 - 9 = 1.5$. Another logarithmic relationship which should not be missing in this line-up is the exact $\sqrt{\alpha}$ relationship of the fundamental charge e with the Planck charge $q_l = \sqrt{2\epsilon_0 hc} = 1.88 \times 10^{-18}\,\mathrm{C}$:

$$\log_\alpha \left(\frac{e}{q_l}\right) = 0.5 \qquad \text{or} \qquad q_l \times \alpha^{0.5} = e \tag{6.14}$$

The proton has even more noteworthy exponential relationships, which is indicative of its extraordinary role in our universe. Besides its exponential relations with e_n and α it also exhibits exponential relationships with $\sqrt{2}$ which again follow the previously stated exponent scheme.

$$\log_{\sqrt{2}} \left(\frac{m_l}{m_p}\right) = \log_{\sqrt{2}} \left(\frac{r_p}{l_l}\right) = 127.0 \qquad \text{or} \qquad m_p \times \sqrt{2}^{127} \cong m_l \tag{6.15}$$

$$\log_{\sqrt{2}} \left(\frac{m_H}{m_p}\right) = 529.5 \qquad \text{or} \qquad m_p \times \sqrt{2}^{529+0.5} \approxeq m_H \tag{6.16}$$

$$\log_{\sqrt{2}} \left(\frac{r_H}{r_p}\right) = 277.5 \qquad \text{or} \qquad r_p \times \sqrt{2}^{277+0.5} \approxeq r_H \tag{6.17}$$

The fact that the exponents of all the presented logarithms are close to an integer number, or an integer number plus one half, cannot be the result of chance and is certainly indicative of an underlying physical mechanism that presumably is related to an anti-resonance effect of standing gravitational waves.

7 Gravitational Waves

In his intriguing paper "A Single Field Model of the Universe" John A. Macken proposed that the amplitude of gravitational waves is always one Planck length (9), which matches well with what was proposed in this paper. The gravitational wave intensity equation that he used, which was introduced back in 1968 by A.D. Sakharov, did not reflect that proposition, though. Moreover, that equation did not match exactly with the form of an equation for acoustic wave intensity. These shortcomings will be rectified in this section by starting from the energy density of space for deriving an equation for gravitational wave intensity. The (unobservable) energy density of space itself is given by

$$\rho_s = \frac{1}{2}\frac{m_l}{l_l^3} = 2.577\,59 \times 10^{96}\,\mathrm{kg/m^3} \tag{7.1}$$

as demonstrated in (1). Therefore the impedance of space itself, in which gravitational waves are assumed to propagate with light speed c, evaluates to:

$$Z_s = \rho_s\, c = \frac{c}{2}\frac{m_l}{l_l^3} = \frac{c^3}{2G\, l_l^2} = 7.727\,42 \times 10^{104}\,\mathrm{s\,Pa/m} \tag{7.2}$$

This novel property of space must not be confused with the electric impedance of space $Z_0 = 2\alpha h/e^2$, which is usually just called the impedance of space, and therefore Z_s should always be referred to as the gravitational impedance of space. These two quantities can be related, though, by $Z_0\, e^2/\alpha = Z_s\, 8\pi l_l^4$ and there might be physical meaning hidden in this relation. The value that Macken used for Z_s was c^3/G, but that expression does not result in the appropriate physical units and is off by a factor of $1/(2l_l^2)$.
The revised Z_s makes it possible to propose an equation for the intensity of gravitational waves which has a structure that matches with the expected form of an equation for the sound intensity of an acoustic wave:

$$I = \frac{1}{8\pi} l_l^2 \omega^2 Z_s \left(\frac{\Delta L}{L}\right)^2 \tag{7.3}$$

Here $1/8\pi$ is a proportionality constant, l_l is the wave's amplitude, ω is the angular frequency of the wave and $\Delta L/L$ is an additional dimensionless factor that accounts for the strain in space due to a gravitational wave, whereby L denotes a reference length transverse to the wave's propagation direction and ΔL refers to the change of that length due to the wave. In case $\Delta L = 0$ there is also zero intensity, which makes perfect sense for gravitational waves. The variant of equation 7.3 that John A. Macken used produced the same results, though, because the deviations in the expressions that he uses for Z_s and I cancel each other out. His findings in the aforementioned paper should also not be invalidated by these issues.

Finally, it should be noted that expressing Z_s without the Planck units leads to the following expression,

$$Z_s = \frac{1}{2\hbar}\frac{c^6}{G^2} \tag{7.4}$$

which is a weird formulation since it involves a squared gravitational constant G and c to the power of six. This weirdness suggests that using the Planck units for expressing Z_s is more natural and this notion in turn supports the idea of a quantized space with the proposed properties. Moreover, the weirdness of equation 7.4 is probably also the reason why it was not discovered earlier.

8 Large Number Coincidences

The previous sections proposed that the very large and the very small are interconnected. Weyl and Dirac were among the first physicists who noted numerical coincidences in certain fundamental ratios that curiously also evaluated to extremely large numbers, which led them to the presumption that these coincidences cannot be the result of chance. Dirac, for example, provided the following coincidence

$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} = 2.269 \times 10^{39} \tag{8.1}$$

$$\frac{r_H}{4r_p} = 1.48 \times 10^{41} \tag{8.2}$$

whereby both results are in the vicinity of 10^{40}. Here m_p denotes the proton's mass, r_p denotes the proton's reduced Compton wavelength, m_e denotes the electron's mass, e denotes the fundamental electric charge and ϵ_0 denotes the electric field constant. The last two equations can be brought into much closer alignment when replacing e with the Planck charge q_l and doubling the equation which involves r_H.

$$\frac{q_l^2}{4\pi\epsilon_0 G m_p m_e} = \frac{e^2}{4\pi\epsilon_0 \alpha G m_p m_e} = 3.1 \times 10^{41} \tag{8.3}$$

$$\frac{1}{2}\frac{r_H}{r_p} = 3.0 \times 10^{41} \tag{8.4}$$

These two results are remarkably close, although the correct value for the Hubble radius r_H is still somewhat in dispute. Using the reduced Compton wavelength of the proton and electron it's then possible to reformulate equation 8.3 into something surprisingly simple:

$$\frac{q_l^2}{4\pi\epsilon_0 G m_p m_e} = \frac{r_p r_e}{l_l^2} \tag{8.5}$$

Relating equation 8.4 with 8.5 then leads to a geometric relationship which demonstrates that the properties of the most fundamental objects in our universe are related to each other in an orderly fashion:

$$\frac{r_p}{l_l} \cong \sqrt{\frac{1}{2}\frac{r_H}{r_e}} \tag{8.6}$$

It's worth emphasizing that this remarkable connection can only be obtained when utilizing the Planck units. Moreover, it is also possible to express the last equation in terms of frequencies using equation 4.2 and the angular frequency relationship $\omega = c/r$:

$$\frac{\omega_l}{\omega_p} \cong \sqrt{\frac{1}{2}\frac{\omega_e}{H_0}} \tag{8.7}$$

This relation features the Hubble constant H_0 again, which can also be expressed as an angular frequency.

9 Discussion

The previous sections uncovered several noteworthy physical relationships which seemingly contradict general relativity theory. These conflicts mainly revolve around the following three issues:

1. What is the meaning of the Einstein constant?
2. Is space curvature physically real and do gravitational singularities exist?
3. How can dark energy and dark matter be incorporated into the presented ideas?

Regarding issue 1: it can be argued that general relativity theory should have got rid of the gravitational constant G since its usage leads to the situation that general relativity theory predicts black holes but unknowingly already uses the physical reality of black holes in disguise of a G/c^4 term, which is contained in the Einstein constant κ (equation 2.7). This circumstance makes general relativity theory somewhat circular conceptually. Einstein also would have liked general relativity theory to be more in line with Ernst Mach's thinking, i.e. that local gravity is related to the relative relationships with all the masses in our universe. Replacing the Einstein constant κ with Hubble sphere parameters (see equation 4.10) would have been a step in that direction, but back then there was much less knowledge about cosmology. Einstein also was not able to unify general relativity theory with quantum physics. It was proposed in this paper that the Planck units are the key to this unification, as the Planck units presumably define the properties of the quanta of space which essentially also are micro black holes that implicitly "encode" the gravitational constant. This notion opens the door towards a thermodynamic understanding of quantum gravity, whereby thermodynamic gravity is not dealing with the states of atoms or molecules but instead it operates on basis of the proposed quanta of space (1).

Regarding issue 2: space curvature is a consequence of the modelling approach chosen for general relativity theory and its physical reality is not proven unequivocally, which also implies that gravitational singularities may not be physically real. Few people know that Albert Einstein was initially conceiving a gravitational theory with a variable speed of light which doesn't require curved space (2). This approach doesn't necessarily violate special relativity theory either, as Alexander Unzicker notes in his book "Einstein's lost key". Light speed c would still be a general upper limit in the absence of a gravitational field and non-accelerating observers in an approximately homogeneous gravitational field would all measure the same speed of light. These observers would not even be aware of the reduced speed of light because time is also slowed down correspondingly in their gravitational field. However, in a varying gravitational field the subsequent change in the speed of light results in bent motion trajectories. Back in the day Einstein was even able to derive an optical refraction index for light whose trajectory is bent by a central mass m which was only off from the correct result by a factor of two. Therefore it's rather incomprehensible why Einstein did not pursue this idea again later. Dicke, however, was able to derive the correct refraction index n_l in 1957 with a different calculation approach (5), seemingly without knowing Einstein's prior work on that topic (see Unzicker's book for more details).

$$n_l = 1 + \frac{G}{c^2}\frac{2m}{d} = 1 + \frac{G}{c^4}\frac{2mc^2}{d} \qquad \text{result in range } [1,2] \text{ for static spheres} \tag{9.1}$$

Since the optical refraction index is defined as $n = c/v$ and we have $n_l \geqq 1$ the speed of light decreases to $v_l = c/n_l$ in the vicinity of a uniform spherical mass, whereby the predicted slowdown is still less than 0.4% close to our sun. Moreover, it's obvious that the last equation is related to the gravitational potential analysis that was carried out in this paper since the occurring mathematical terms are very similar (see equation 2.6, for example). In case equation 9.1 doesn't apply to rotating black holes the theoretical maximum value of n_l evaluates to 2 in close vicinity to a static Schwarzschild black hole using equation 2.8. In case equation 9.1 is also valid for rotating black holes then the theoretical maximum value of n_l would be 3.

Dicke was remarkably far sighted as he even speculated that the $1+$ part of equation 9.1 might be related to the gravitational potential of our universe itself since the second term was identified by him as a gravitational potential whose value is also much smaller than 1 for any realistic use-case. Like it was the case in section three the number 1 may actually represent the gravitational potential limit $1\mathrm{J}/\mathrm{J}$, which, in accordance with Dicke's presumption, gives equation 9.1 the meaning of a sum of gravitational potential magnitudes.

In addition to these findings a scientific paper from 1960 demonstrates that the mathematical framework of general relativity is not necessary to describe undeniable gravitational effects like gravitational lensing (4). The notion of time in general and special relativity theory is also problematic since no satisfying justification is given why time should be treated like a spatial dimension. Considering all these arguments it seems reasonable that the physical relationships which are presented in this work do not require the mathematics associated with space curvature.

Regarding issue 3: it has been shown that the gravitational potential energy per unit mass of our local Hubble sphere is $-c^2/2$ whereas the overall gravitational potential limit is $-c^2$. The latter potential value may be related to a rotating universe in which our non-rotating but expanding local Hubble sphere is embedded in. In case that notion is appropriate this gravitational potential difference could be due to our overall universe's rotational energy, which in turn might account for the major portion of the so called dark energy. This also makes sense conceptually since a centrifugal force due to rotation tends to break things apart, which could be reflected in the expansion of space. The remaining dark energy portion could be related to our Hubble sphere's translational kinetic energy in case it moves relative to the enclosing overall universe. Dark matter, on the other hand, might have a quite unspectacular explanation as Randell Mills suggests that dark matter is just made from interstellar clouds of a rather unreactive form of hydrogen which he calls hydrino (8).

Critics may argue that this paper represents the same equations in different ways, but this is unavoidable in case there is an interconnected and repeating underlying physical structure that can be viewed from different perspectives and at different scales.

10 Conclusions

The main findings of this paper are:

- The units of c^2 match with the units of a gravitational potential, i.e. $\mathrm{m}^2/\mathrm{s}^2 = \mathrm{J/kg}$.
- The general limit for gravitational potential energy per unit mass in our universe is $-c^2$. This limit can also be expressed as gravitational potential energy per unit energy with a value of $-1\,\mathrm{J/J}$.
- Newtonian gravity already predicts black holes when considering a velocity limit.
- A gravitational potential of exactly $-c^2/2$ corresponds to a static black hole, aka Schwarzschild hole. An even deeper gravitational potential corresponds to a rotating black hole, aka Kerr hole.
- The Newtonian equation for gravity is not limited to calculations involving mass since it can be adapted to energy using $E = mc^2$. Then it exhibits a G/c^4 term which is also present in the Einstein constant κ.
- Our local Hubble sphere might be a Schwarzschild black hole, or it is at least close to being one. Its gravitational potential per unit mass is subsequently equal to, or close to, $-c^2/2$.
- Space-time is quantized and mirrors cosmological properties at the quantum scale. The Planck units presumably define the properties of the quanta of space.
- Gravitational potentials should best be expressed in the form of equation 5.5 or 5.6 as this presumably is their most natural form that also does not directly involve all the masses present in our visible universe.
- The gravitational constant G is related to all the masses in our observable universe according to equation 4.9, as suspected by Dicke, Mach, Sciama and Schrödinger. Still this relationship might not be directly responsible for its value, which presumably results from the properties of quantized space.
- The gravitational constant G can be regarded as an emergent constant that follows from from other natural constants. It can be substituted by $c^2 l_l / m_l$.
- Space curvature does not exist physically, as indicated by the findings in this paper. Consequently, gravitational singularities shouldn't exist either.
- The sizes and the masses of the fundamental objects in our universe are not coincidental. Underlying patterns exist, in particular various exponential relationships.
- The cosmic microwave background temperature can be calculated approximately by using the Planck density and Planck temperature.
- The intensity of a gravitational wave is given by equation 7.3. The associated gravitational impedance of space is given by $m_l\, c/(2l_l^3)$.
- Physical units are an important tool in physics and they should not be disregarded or omitted in calculations.

Acknowledgments

The development of this paper has been greatly aided by Andrey Ivashov's "SMath Studio".
http://smath.com

Symbols

Speed of light c
Gravitational constant G
Planck length l_l
Planck mass m_l
Planck time t_l
Planck energy E_l
Planck temperature T_l
Hubble constant H_0
Sommerfeld constant α
Boltzmann constant k_b
Euler's number e_n
Fundamental charge e
Electric field constant ϵ_0

References

[1] Mayer, Martin. (2020). Compton Particles and Quantum Forces in a holo-fractal universe.
http://vixra.org/abs/1906.0490

[2] Unzicker and Preuss. (2015). A Machian Version of Einstein's Variable Speed of Light Theory.
http://arxiv.org/abs/1503.06763

[3] Unzicker, Alexander. (2008). A Look at the Abandoned Contributions to Cosmology of Dirac, Sciama and Dicke.
https://arxiv.org/abs/0708.3518

[4] Dehnen and Hönl and Westpfahl. (1960). Ein heuristischer Zugang zur allgemeinen Relativitätstheorie.
Annalen der Physik, Volume 461, Issue 7-8, Pages 370 - 406
http://doi.org/10.1002/andp.19604610705

[5] Dicke, Robert Henry. (1957). Gravitation without a Principle of Equivalence.
Reviews of Modern Physics, Volume 29, Issue 363
https://doi.org/10.1103/RevModPhys.29.363

[6] Schrödinger, Erwin. (1925). Die Erfüllbarkeit der Relativitätsforderung in der klassischen Mechanik.
Annalen der Physik, Volume 382, Issue 11, Pages 325 - 336
https://doi.org/10.1002/andp.19253821109

[7] Müller, Hartmut. (2019). On the Cosmological Significance of Euler's number.
Progress in Physics, Volume 15, Pages 17 - 21
http://www.ptep-online.com/2019/PP-56-03.PDF

[8] Mills, Randell. (2016). The grand unified theory of classical physics.
ISBN 978-0-9635171-5-9
https://brilliantlightpower.com/book-download-and-streaming/

[9] Macken, John (2023). A Single Field Model of the Universe.
http://dx.doi.org/10.13140/RG.2.2.30242.61125