Lectures on Physics - Chapter V
Moving charges, electromagnetic waves, radiation, and near and far fields

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Abstract

The special problem we try to get at with these lectures is to maintain the interest of the very enthusiastic and rather smart people trying to understand physics. They have heard a lot about how interesting and exciting physics is—the theory of relativity, quantum mechanics, and other modern ideas—and spend many years studying textbooks or following online courses. Many are discouraged because there are really very few grand, new, modern ideas presented to them. Also, when they ask too many questions in the course, they are usually told to just shut up and calculate. Hence, we were wondering whether or not we can make a course which would save them by maintaining their enthusiasm. This paper is a draft of the fifth chapter of such course. It offers a comprehensive overview of the complementarity of wave- and particle-like perspectives on electromagnetic (EM) waves and radiation. We finish with a few remarks on relativity.

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Lectures on Physics Chapter V : Moving charges, electromagnetic waves, radiation, and near and far fields

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1. The measurement of the position of a charge

A charge – to make matters more specific, we will be talking an electron (e), but it might also be a proton or any other charged particle – moves along a trajectory which we describe in a three-dimensional Cartesian space (an empty *mathematical* space) by measuring its position in terms of its distance from a chosen point of origin to the moving charge at successive points in time *t*. The observer will use an atomic clock or stopwatch to measure and mark the exact time at each position. We assume this measurement does not affect the observer, the charge that is being observed, or the space inbetween them. In other words, we assume there is no exchange or conversion of energy while measuring time.

We have our clock and, therefore, a time unit. Now, we should think about how to measure distance. If you are a DIYer, you will think of a household distance measurement laser measuring the distance to your wall by (1) sending regular bursts of photons (the laser beam) to the wall and (2) receiving some of them back in the receiver. The clock of the device then calculates the distance back and forth by multiplying the time between the sent and receive moment to give you a distance expressed in meter instead of light-seconds: c: $\lambda = c \cdot t$. Consider it done, right? Yes, but such reasoning assumes the photon(s) will travel back and forth in a straight line to the charge and then back to the detector(s).[1] The reasoning also involves the assumption of an elastic or instantaneous collision between the charge and the photon(s)[2], and the charge will, therefore, acquire some extra kinetic energy or momentum which, measured from the zero point ($KE_e = 0$ and $\mathbf{p_e} = \mathbf{0}$) is just the kinetic energy and momentum of the electron after the collision ($KE_{e'} > 0$ and $\mathbf{p_{e'}} \neq 0$). So how does that work, then?

The concept of an elastic collision

We can calculate the extra momentum from the momentum conservation principle while noting we should write the momenta as vectors so as to take the direction of the momentum into account. We can, unfortunately, not *predict* the angle between the incoming and outgoing velocity vector for the photon ($\boldsymbol{v}_\gamma$ and $\boldsymbol{v}_{\gamma'}$, respectively). What we do know, however, is that all of the momentum of the incoming photon ($\mathbf{p}_\gamma$) is being transferred to the outgoing photon ($\mathbf{p}_{\gamma'}$) and the electron ($\mathbf{p}_{e'}$). Because

[1] A straight line of sight assumes the absence of gravitational lensing. Arthur Eddington, Frank Watson Dyson, and their collaborators effectively observed that electromagnetic radiation (light) follows a curved path near a massive object during the total solar eclipse on May 29, 1919 (Dyson, F. W., Eddington, A. S., Davidson C. (1920), *A determination of the deflection of light by the Sun's gravitational field, from observations made at the total eclipse of 29 May 1919*, Philosophical Transactions of the Royal Society 220A (571–581): 291–333.

[2] If there is photon-electron *interaction*, such interaction will take some time which should, therefore, be subtracted from the total travel time to calculate the exact travel time (excluding interaction time).

(linear) momentum is conserved simultaneously in the *x*-, *y*- as well as the *z*-direction, we should write this as a vector equation:

$$\mathbf{p}_{e'} = \mathbf{p}_{\gamma} - \mathbf{p}_{\gamma'}$$

The situation is depicted below (**Figure 1**). As for the precision of the measurement, we agree with the interpretation of the Compton wavelength as the relevant "distance scale within which we can localize the electron in a particle-like sense."[3] All of this, of course, assumes the electron is travelling freely: it is not bound to an atomic or molecular orbital, in other words.

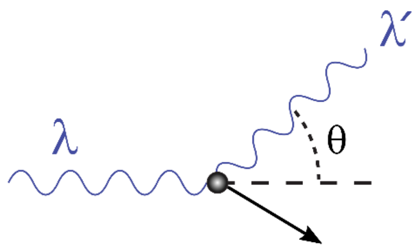

Figure 1: The scattering of a photon from an electron (Compton scattering)

Now, the linear momentum of a photon is equal to $p = m_\gamma c = E_\gamma/c = h \cdot f_\gamma/c = h/c \cdot T_\gamma = h/\lambda_\gamma$. Hence, if the electron momentum changes, then the wavelength, cycle time, and frequency of the incoming and outgoing photon cannot be the same. Hence, if our household laser distance sensor would also be able to measure this wavelength or frequency change, we would be able to tell how the collision has changed the state of motion of the charge – not approximately, but *exactly*. We would, in effect, be able to calculate the *velocity* change (direction as well as magnitude) of the electron by using the $\boldsymbol{v}_{e'} = \mathbf{p}_{e'}/m_{e'}$ formula.[4]

We have come to the conclusion that a fully elastic interaction between an electron and a photon (1) transfers momentum and kinetic energy and (2) causes a wavelength shift between the outgoing and incoming photon. Let us quickly model this before we turn back to our distance measurement problem.

Compton scattering

Electron-photon interaction is modelled by the equation for *Compton* scattering of photons by electrons:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_C(1 - \cos\theta)$$

The $\lambda_C = h/m_e c$ in this equation is referred to as the *Compton* wavelength of the electron and is, a distance: about 2.426 *pico*meter (10^{-12} m). The $1 - \cos\theta$ factor goes from 0 to 2 as θ goes from 0 to π. Hence, the maximum difference between the two wavelengths is about 4.85 pm. This corresponds, unsurprisingly, to *half* the (relativistic) energy of an electron.[5] Hence, a highly energetic photon could lose up to 255 keV while the electron could, potentially, gain as much.[6] That sounds enormous, but

[3] See our analysis of Compton scattering in our paper on [the difference between a theory, a calculation, and an explanation](), April 2020.
[4] We have no need for the Uncertainty Principle here!
[5] The energy is inversely proportional to the wavelength: $E = h \cdot f = hc/\lambda$.
[6] The electron's rest energy is about 511 keV.

Compton scattering is usually done with highly energetic X- or gamma-rays.

When the photon does not interact with the electron, there is no scattering angle (we may enter it as 0 in the equation) and the wavelength shift between the incoming and outgoing photon – which is actually just traveling through – will, therefore, vanish ($1 - \cos 0 = 1 - 1 = 0$). In contrast, when the photon bounces straight back, the scattering angle θ will be equal to $\pm\pi$ (see **Figure 1**) and the wavelength shift between the incoming and outgoing photon will, therefore, attain its maximum value, which is equal to $\lambda_C(1 - \cos\pi) = \lambda_C(1 + 1) = 2\lambda_C$.[7] This is, as mentioned, a rather formidable result. Two more things should be noted here:

1. The wavelength shift $\Delta\lambda = 2\lambda_C$ is independent of the energy of the incoming photon.
2. The outgoing photon will have *longer* wavelength and, therefore, *lower* energy, and the kinetic energy of the electron must change so as to explain the energy difference between the incoming and outgoing photon:

$$E_{e'} + E_{\gamma'} = E_e + E_\gamma \Leftrightarrow \Delta E = E_{e'} - E_e = E_\gamma - E_{\gamma'}$$

Bref (French for 'in short'), if the electron is moving in free space, then the probing of an electron by photons – i.e. the measurement of a position – will result in a *change* of its trajectory.

Of course, one may argue the electron may not be in a free but in a bound state: such bound state may be an atomic or molecular orbital in a crystal lattice.[8] In such case, the photon will be temporarily absorbed as the electron first absorbs and then emits a photon with *exactly* the same wavelength. However, because such absorption and emission of a photon with linear momentum should also respect the conservation of (linear) momentum, the electron should first absorb the incoming momentum of the incoming photon and then return it to the outgoing photon. In such case, we will observe what is referred to as reflection. Such reflection may be specular or diffuse. In the case of specular reflection, the outgoing photon – which we may now refer to as a *reflected* photon – will emerge from the reflecting surface at the same angle to the surface normal as the incident ray, but on the opposing side of the surface normal in the plane formed by the incident and reflected rays, as illustrated below.

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[7] See the *exposé* of Prof. Dr. Patrick R. LeClair on Compton scattering. Prof. LeClair's treatment is precise and offers plenty of other interesting formulas, including the formula for the scattering angle of the electron φ which, as mentioned above, is fully determined from the wavelength shift and the scattering angle θ. We offer a concise discussion of his derivation and arguments in our paper on the difference between a theory, an explanation and a calculation.

[8] The Rutherford-Bohr model of an atom gives us the following formula for the energy level:

$$E_n = -\frac{1}{2}\frac{\alpha^2}{n^2}mc^2 = -\frac{1}{n^2}E_R$$

We, therefore, get the following formula for the energy *difference* between two states with *principal* quantum number n_2 and n_1 respectively:

$$E_{n_2} - E_{n_1} = -\frac{1}{{n_2}^2}E_R + \frac{1}{{n_1}^2}E_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot E_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot\frac{\alpha^2 mc^2}{2}$$

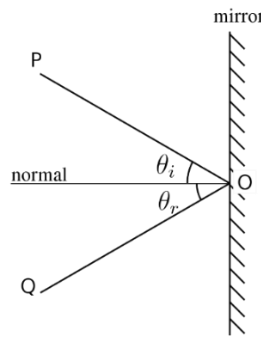

Figure 2: Specular reflection of light photons

Most materials will reflect the light *diffusely*: just like specular reflection, diffuse reflection depends both on the properties of the surface as well as of the properties of the crystal lattice.[9] Before returning to the problem of distance measurement, we should add one final remark on specular reflection: photon interference experiments reveal a phase shift between the incoming and outgoing photons upon *reflection* which is equal to π, and we must assume such *phase* shift is relevant in the context of Compton scattering too.[10] A phase shift effectively suggest the electron will effectively take some time to absorb and re-emit the photon. The time which corresponds to a phase shift equal to 180° (π) can be calculated from the representation of a photon as the vector sum of a sine and a cosine oscillation:

$$a \cdot e^{i\theta} = a \cdot e^{i\omega t} = a \cdot e^{i\frac{E_\gamma}{\hbar}t} = a \cdot \cos\left(\frac{E_\gamma}{\hbar}t\right) + i \cdot a \cdot \sin\left(\frac{E_\gamma}{\hbar}t\right)$$

One may think of these two oscillations as representing the electric and magnetic field respectively when measured using natural time and distance units (c = 1), in which case the (maximum) amplitude of the magnetic field $B = E/c$ will be measured as being identical. The multiplication by i then represents the orthogonality of the ***E*** and ***B*** vectors and the phase difference between the cosine and sine accounts for the phase shift between them, which we know to be equal to 90° (π/2): $\sin(\theta \pm \pi/2) = \cos(\pi/2)$. Any case, if the phase shift is equal to $\Delta\theta = \Delta(\omega\cdot t) = \omega\cdot\Delta t = (E_\gamma/\hbar)\cdot\Delta t = \pi$, then we can calculate Δt as being equal to $\Delta t = \pi\hbar/E_\gamma = (1/2)/f_\gamma = T_\gamma/2$. Unsurprisingly, this is *half* the cycle time of the photon.

Photon and electron spin

So far, we have been discussing spin-zero photons and electrons. Both photons and electrons have spin. To be precise, besides linear momentum, a photon will also have an *angular* momentum, which is either +1 or –1 and which is expressed in units of $\hbar$: hence, this amount of (angular) momentum should be conserved as well throughout the process.[11] In contrast, an electron has spin $\pm\hbar/2$ only. The absorption

[9] We refer the interested reader to the rather instructive Wikipedia article on diffuse reflection (from which we also borrowed the illustration) for more details.

[10] See, for example, K.P. Zetie, S.F. Adams, and R.M. Tocknell, *How does a Mach-Zehnder interferometer work?*, Phys. Educ. 35(1), January 2000.

[11] We find the fact that photons are spin-one particles without a zero-spin state rather striking, especially because it is usually not mentioned very explicitly in (most) physics textbooks. Richard Feynman, for example, hides this fact in a footnote (see: Feynman's Lectures, III-11, footnote 1), and the context of this footnote is rather particular. To explain the interference of a photon with itself in the one-photon Mach-Zehnder interference experiment, we actually do assume a *beam splitter* will actually *split* in two linearly polarized photons which each have spin-1/2 only. Each of these two linearly polarized photons then has the same frequency but carries only half of the total

of the full angular momentum of a photon must, therefore, involve a *spin flip* of the electron, going from + $\hbar/2$ to − $\hbar/2$ and then back again so as to return the full amount of spin to the outgoing photon. In fact, one may speculate the temporary spin flip of the electron explains why the electron – in a configuration where all sub-shells (which are identified using not only the principal quantum number *n* but also the orbital angular momentum number *l*) have been filled by a *pair* of electrons with opposite spin – has to go from one orbital to the next: the line-up of its spin violates the Pauli exclusion principle, according to which two electrons in the same subshell must have opposite spin.[12]

By now, it should be sufficiently clear that the probing of a *free* electron using radiation (photons) is perturbative. We may, therefore, try to find another way to observe the charge's state of motion.

Is non-perturbative measurement of the position of a charge possible at all?

One potentially non-destructive observation might be the use of potential meters: as we will see in a moment, a *moving* charge will *change* the electric and magnetic field all over space and, hence, by measuring how the electric and magnetic field changes at one or various positions.

However, the measurement of a *change* in the electric and/or magnetic field will inevitably involve a charge as well because we must observe be able to observe the force on a charge in order to *measure* the change in potential. A change in electric potential, for example, will result in a simple Coulomb force:

$$\mathrm{F_C} = -\frac{\mathrm{dV}}{\mathrm{d}r} = -\frac{\mathrm{d}\left(-\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r}\right)}{\mathrm{d}r} = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{\mathrm{d}\left(\frac{1}{r}\right)}{\mathrm{d}r} = -\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r^2}$$

We note that the *magnitude* of the force falls off following the inverse square law (F ~ $1/r^2$) while the (electric) potential (V) diminishes linearly (V ~ $1/r$). This makes sense because the energy *flux* is inversely proportional to the *square* of the distance as well. What about the magnetic force? The calculation of the magnetic field and force is more complicated because it depends on the (relative) *motion* of both charges and because it will not only act on the (moving) charge but also on its magnetic moment. We will let the reader review the relevant equations here, but it should be clear that any force acting on a charge will *change* its state of motion.

To be precise, a force acting on a charge will cause it to *accelerate*. The acceleration vector is given by Newton's force law ($\boldsymbol{a} = \mathbf{F}/m_e$) and the total energy expended will be equal to the (line) integral $\Delta E = \int_L \mathbf{F}(\boldsymbol{r})\cdot \mathrm{d}\boldsymbol{r}$.[13] At this point, we should invoke the principle of least action as used in both classical as well as in quantum mechanics.[14] This principle states that, ***in free space***, the charge will *lower* its total energy (kinetic *and* potential) by moving along a path which minimizes the (physical) action $S =$

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energy of the incoming photon: $\frac{E_\gamma}{2} = \frac{\hbar}{2}\cdot\omega_\gamma$. We work out this hypothesis in our [realistic or classical explanation of one-photon Mach-Zehnder interference]().

[12] The electron subshells define the *fine* structure of the atomic or molecular spectrum. The fine spectral line can be further split into two *hyperfine* levels because of the coupling between electron spin and nuclear spin. See our paper on [a classical explanation of the Lamb shift]().

[13] The reader should note that we have a *line* integral here and, therefore, we are integrating a vector dot product ($\mathbf{F}\cdot\mathrm{d}\boldsymbol{r}$) over a curve or a (non-linear) line.

[14] See: Feynman's *Lectures*, *The Least Action Principle* (Vol. II, Chapter 19).

$\int_{t_1}^{t_2}(\mathrm{KE}-\mathrm{PE})\mathrm{d}t$. To be precise, potential energy must be converted to kinetic energy and vice versa: the total energy of the charge (the *sum* of KE and its PE) does, therefore, not change: only its *components* KE and PE, which depend on its velocity and its position respectively, will change. However, in free space, they will add up to a constant. In other words, the situation which we have been considering up to this point, is that of *a charge whose energy state does not change*.

This now changes because we will want to use a *change* in the energy state of the electron to measure a *change* in the electric and magnetic fields or, if one prefers a more elegant representation perhaps, the scalar as well as vector potential.[15]

Quantum-mechanically, this implies the electron we use to measure the change must following *another* trajectory. This trajectory will differ from its *geodesic* – its trajectory in free space, that is – by an amount of physical action equaling one or more units of physical action *h*. Now, this amount of *physical action* – the product of (1) a force, (2) a distance and (3) some time – must be *extracted* from the fields, *somehow*. We refer the reader to Feynman's treatment of the topic for a complete analysis of how a field loses energy to matter or, in the reverse case, how it gains energy, which is just a negative loss.[16] As Feynman puts it, we now need to change the energy conservation law and restate it as follows:

> "Only the total energy in the world – which includes the energy of both matter and fields – is conserved. The field energy will change if there is some work done by matter on the field or, conversely, by the field on matter." (Feynman, Vol. II, page 27-8).

The reader may think this *conversion* from potential into kinetic energy – field energy goes down while the energy of the electron which we use to *measure* the change in potential goes up – should not affect the state of motion of the electron which we are trying to observe. However, this would imply that, sooner or later, all potential energy in the world gets converted to kinetic energy or vice versa: in other words, all of the charges would deplete all potentials and we would be left with kinetic energy and a matter-world only: no fields.

We, therefore, add an additional conservation law—and it will be the final one: in addition to the conservation of total energy as well as (linear and circular) momentum in the world, the total amount of physical action in the world must be conserved. If we, therefore, extract a unit of *h* from the fields, the fields will, somewhere, extract a unit of *h* from matter. If we only have two charges in the world – the one we want to observe and the other one which we want to use to measure any change in potential which the first charge is *causing* – then the fields will have to *extract* one unit of *h* from the charge for every unit of *h* they are transferring to the charge we use to measure the changing potential(s).

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[15] We have reasoned in terms of electric and magnetic fields so far, but we may rewrite Maxwell's equations in terms of the scalar and vector potential. This may or may not simplify the math: the electric and magnetic field *vectors* $\boldsymbol{E}(\boldsymbol{x}, \mathrm{t})$ and $\boldsymbol{B}(\boldsymbol{x}, \mathrm{t})$ effectively have *three* spatial coordinates each (E_x, E_y, E_z, B_x, B_y and B_z respectively), while a description in terms of the scalar and vector potential $\phi(\boldsymbol{x}, t)$ and $\mathbf{A}(\boldsymbol{x}, t)$ involves four numbers only (ϕ, A_x, A_y, and A_z).

[16] See: Feynman's Lectures, [*Field Energy and Momentum*]() (Vol. II, Chapter 27)

Can we prove this? No, but we think Feynman's derivation of the equation of continuity for probabilities comes (very) close to proving this and we, therefore, like to interpret this equation as the conservation law for physical action.[17]

We, therefore, think the extraction of an equivalent energy E = *h·f* from the fields must not only involve the *absorption* of a photon – this photon will, of course, have the same energy E = *h·f* – by the charge we use to measure the changing fields: it must also involve the *emission* of a photon by the charge we are trying to observe. Hence, this will, once again, result in a change of the state of motion of the charge we are trying to observe. We may, therefore, say that the two charges will be *interconnected* or – to use more formidable language – that the two charges will be coupled or *entangled* both classically as well as quantum-mechanically.

Should Einstein worry about 'spooky action at a distance' (*spukhafte Fernwirkung*) here? Not necessarily, because this spooky action should still respect the principle that nothing can travel faster than the speed of light: the *total* effect will push *both* electrons away from each other and may, therefore, be said to involve the exchange of a photon. In order to distinguish such photon from the photons we first wanted to use to observe the charge *directly*, we will refer to such photons as *virtual photons*. We will, however, assume such virtual photons cannot travel any faster than any other electromagnetic wave. In fact, we will assume such virtual photons are – just like real photons – nothing but a pointlike electromagnetic oscillation which propagates at the speed of light—not approximately, but *exactly*.

2. Charges in motion

From the extremely discussion above, the reader should just note the crux of the argument: any measurement of a position will inevitably involve a delay except if the charge and the observer happen to be at the same position, of course). One must, therefore, distinguish between the actual position of a charge and its *retarded position*. This is illustrated in Figure 3. Assuming the observer is positioned at point (1), he can ascertain the position of the charge ***x*** by measuring its distance (denoted as distance *r'* which, just like the position is a retarded rather than an *actual* (right now) distance. This concept is illustrated below (Figure 3).

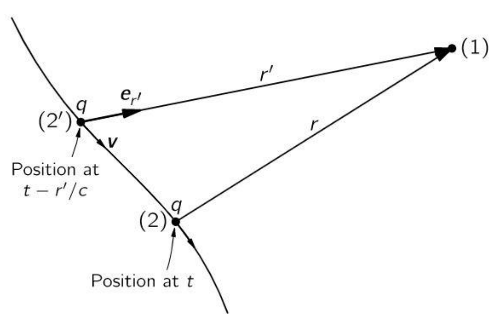

Figure 3: The concepts of retarded time, position, and distance (Feynman, II-21, Fig. 21-1)

[17] See: Feynman's Lectures, [*The equation of continuity for probabilities*]() (Vol. III, Chapter 21, section 2). This equation is closely related to the distinction between kinematic momentum and dynamical momentum ([Feynman's Lectures, II-21-3]()).

The retarded position is written as $\boldsymbol{x}$ and should be written as a function of the retarded time $t - r'/c$:

$$\boldsymbol{x} = \boldsymbol{x}(t - r'/c)$$

The actual position **x**(*t*) *can only be measured in the future* but can be extrapolated by making continuous measurements of the position *right now*. Such measurements then allow to associate a velocity vector $\boldsymbol{v} = \mathrm{d}\boldsymbol{x}/\mathrm{d}t$ with the charge. Just like position, the velocity function will (or *should*) have the retarded time as its argument: $\boldsymbol{v} = \boldsymbol{v}(t - r'/c)$. A first-order approximation of the actual position $\boldsymbol{x}(t)$ is then given by the expression:

$$\boldsymbol{x}(t) \approx \boldsymbol{x}(t - \frac{r'}{c}) + \frac{r'}{c} \cdot \boldsymbol{v}(t - \frac{r'}{c})$$

This first-order approximation may, of course, be complemented by adding second- and higher-order terms by measuring acceleration and using higher-order time derivatives of the position variable. However, let us first stick to the use of the velocity vector. The continuous measurement of the position assumes the measurement of the infinitesimal distance:

$$\Delta\boldsymbol{x} \approx \boldsymbol{x}\left(t - \frac{r'}{c} + \Delta t\right) - \boldsymbol{x}\,(t - \frac{r'}{c})$$

We may, therefore, write the velocity vector as:

$$\boldsymbol{v}(t - \frac{r'}{c}) = \frac{\Delta\boldsymbol{x}}{\Delta t} = \frac{\boldsymbol{x}\left(t - \frac{r'}{c} + \Delta t\right) - \boldsymbol{x}\,(t - \frac{r'}{c})}{\Delta t}$$

Figure 3 shows a unit vector $\boldsymbol{e}_{r'}$ from the retarded position (2') directed towards the observer (1). One might also draw a unit vector from (1) to (2'), which makes it easier to appreciate that the *vector* $\boldsymbol{r'}$ can be written as $\boldsymbol{r'} = r' \cdot \boldsymbol{e}_{r'}$ and, more importantly, that $\Delta\boldsymbol{x} \approx r' \cdot \Delta\boldsymbol{e}_{r'}$. The retarded velocity vector can, therefore, also be approximated by:

$$\boldsymbol{v}\left(t - \frac{r'}{c}\right) = \frac{\Delta\boldsymbol{x}}{\Delta t} = r' \cdot \frac{\Delta\boldsymbol{e}_{\boldsymbol{r}'}}{\Delta t}$$

Moving to differential notation, one can, therefore, write the retarded velocity vector function as:

$$\boldsymbol{v}\left(t - \frac{r'}{c}\right) = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = r' \frac{\mathrm{d}\boldsymbol{e}_{\boldsymbol{r}'}}{\mathrm{d}t}$$

Position, time and, therefore, motion is relative. However, charge is not relative and different observers should, therefore, also agree on a measurement unit for charge, which we may equate to the elementary charge e. This is the charge of a proton or the (negative) charge of the electron. A charge fills empty spacetime (all of it) with a potential which depends on position and evolves in time. This potential is, therefore, also a function of x, y, z, and t.[18] Two equivalent descriptions are possible:

- A description in terms of the electric and magnetic field *vectors* $\boldsymbol{E}(\boldsymbol{x}, \mathrm{t})$ and $\boldsymbol{B}(\boldsymbol{x}, \mathrm{t})$; and
- A description in terms of the scalar and vector potential $\phi(\boldsymbol{x}, t)$ and $\mathbf{A}(\boldsymbol{x}, t)$ respectively.

[18] We will no longer be worried about the relativity of the reference frame and assume the reader will understand what is relative and absolute in our description.

The field vectors ***E*** and ***B*** have three components[19] and we, therefore, have six dependent variables $E_x(\boldsymbol{x}, \mathrm{t})$, $E_y(\boldsymbol{x}, \mathrm{t})$, $E_z(\boldsymbol{x}, \mathrm{t})$, $B_x(\boldsymbol{x}, \mathrm{t})$, $B_y(\boldsymbol{x}, \mathrm{t})$, and $B_z(\boldsymbol{x}, \mathrm{t})$. In contrast, the combined scalar and vector potential give us four dependent variables $\phi(\boldsymbol{x}, t)$, $\mathrm{A}_x(\boldsymbol{x}, t)$, $\mathrm{A}_y(\boldsymbol{x}, t)$, and $\mathrm{A}_z(\boldsymbol{x}, t)$ only, which may appear to be simpler.[20] However, in this paper, we will stick to a description of the fields in terms of the *E* and ***B*** fields.

3. Charges, energy states, potentials, fields, and radiation

We are now ready to analyze Feynman's rather particular formulas for the ***E*** and ***B*** field vectors at point (1):

$$\boldsymbol{E}(1,t) = \frac{\mathrm{q}}{4\pi\epsilon_0}\left[\frac{\boldsymbol{e_{r'}}}{r'^2} + \frac{r'}{c}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{e_{r'}}}{r'^2}\right) + \frac{1}{c^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\boldsymbol{e_{r'}}\right]$$

$$c\boldsymbol{B}(1,t) = \boldsymbol{e_{r'}} \times \boldsymbol{E}(1,t)$$

We refer to Feynman's Lectures[21] for a clear and complete derivation of these functions out from Maxwell's equations for a *single* charge q moving along any arbitrary trajectory, as illustrated in Figure 3. The point to note is that the electric and magnetic field at point (1) *now* will be written as a function of the position and motion of the charge at the *retarded* time $t - r'/c$. The relevant distance is, therefore, also the *retarded* distance *r'*, which is the distance between (1) and (2') – which is *not* the charge's position at time. The latter position is point (2): it is separated from position (2') by a time interval equal to $t - (t - t'/c)$ and a distance interval which depends on the velocity ***v*** of the charge q which will be generally much less than *c*.

The second and third term in the expression for ***E***(1, *t*) are, obviously, equal to zero if the charge is *not* moving, in which case the charge comes with a static (i.e. non-varying in time) Coulomb field only: in this case, the *retarded* field is just the Coulomb field *tout court*. The scalar product which defines the magnetic field is equal to the product:

(1) the *magnitude* of the unit vector $\boldsymbol{e}_{r'}$, whose origin is (2') and which points to (1) and whose magnitude is equal to 1;
(2) the *magnitude* of the electric field vector at point (1) at time *t*;
(3) the cosine of the angle between $\boldsymbol{e}_{r'}$ and ***E***(1, *t*).

The latter factor – the cosine of the angle between $\boldsymbol{e}_{r'}$ and ***E***(1, *t*) – is, obviously, zero *if the second and third term are zero*, which is just a confirmation of the fact that *static* electric fields do *not* come with a magnetic field. However, our charge *is* moving, and the first- and second-order derivative of the $\boldsymbol{e}_{r'}$ will,

[19] **Boldface** symbols denote vector quantities, which have both a magnitude and a direction. Scalar quantities only have a magnitude. However, depending on the reference point for zero potential energy, the *potential* energy of a charge in a potential field may be negative. Potential *energy* is – just like a distance – measured as a *difference*. The plus or minus sign of the potential *energy*, therefore, depends on the *direction* in which we would be moving the charge.

[20] Feynman makes extensive use of the scalar and vector potential for formulas, and they also appear in most quantum-mechanical equations. One, therefore, needs to become intimately familiar with them: the scalar and vector potential are, in many ways, more *real* than the electric and magnetic field vectors.

[21] Richard Feynman, II-26, *Solutions of Maxwell's equations with currents and charges*.

therefore, *not* be equal to zero, which modifies the electric field ***E*** at point (1) at time *t*, and which also gives us a non-zero magnetic field ***B***:

(i) The second term corresponds to what Feynman refers to as a *compensation* for the retardation delay, as it is the product of (*a*) the *rate of change* of the retarded Coulomb field multiplied by (*b*) the retardation delay (the time needed to travel the distance *r′* at the speed of light *c*). In other words, the first two terms correspond to computing the retarded Coulomb field and then extrapolating it (linearly) toward the future by the amount *r′*/*c* – which is right up to time *t*.[22] It should be noted that one might think this second term is (also) inversely proportional to the squared distance r'^2, *but the r′ and* r'^2 *in the numerator and denominator respectively leaves us with an 1/r′ factor only*.

(ii) The third term – the second-order derivative $\mathrm{d}^2(\boldsymbol{e}_{r'})/\mathrm{d}t^2$ – is an acceleration vector which – because of the origin of the unit vector $\boldsymbol{e}_{r'}$ is fixed at point (2′) – can and should be analyzed as the sum of a transverse component and a radial component.[23] Needless to say, this second-order derivative of $\mathrm{d}^2(\boldsymbol{e}_{r'})/\mathrm{d}t^2$ will be zero if the charge moves in a straight line with constant velocity *v*. In other words, *the third term will vanish (be zero) if there is no acceleration*.

In the chapters where Feynman first introduces and uses these equations (Vol. I, Chapters 28 and 29 as well as Vol. II, Chapter 21), the assumption is that the transverse piece of the acceleration vector is far more important than the radial piece, but such statement crucially depends on the assumption that the charge is moving at a more or less right angle to the line of sight, which is not necessarily the case. Feynman corrects for this assumption in Chapter 34 of Vol. I, in which he gives the reader a full treatment of all 'relativistic effects' of the motion of a charge.

Feynman also associates the third term with *radiation* which, as we now know, consists of a stream of photons carrying energy. We must, effectively, assume the charge does not only generate a potential but moves in a potential field itself. Its energy, therefore, must also continually change. To be specific, *in free space*, we must assume the charge will *lower* its total energy (kinetic *and* potential) by moving along a path which minimizes the (physical) action $S = \int_{t_1}^{t_2}(\mathrm{KE} - \mathrm{PE})\mathrm{d}t$. This is just an application of the (classical) *least action principle*.[24]

Of course, in classical physics, potential energy must be converted to kinetic energy and vice versa: the total energy of the charge (the *sum* of KE and PE) does, therefore, not change: only its *components* KE and PE, which depend on its velocity and its position respectively, but they add up to a constant. In other words, the situation which we have been considering up to this point, is that of *a charge whose energy state does not change*.

Such energy state may be the energy state of a free electron or of an electron in a bound state, i.e. an electron in an atomic or molecular orbital. *If and when an electron moves from one energy state to*

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[22] We apologize for quoting quite literally from Feynman's *exposé* here, but we could not find better language.
[23] In the chapters where Feynman uses these equations (Vol. I, Chapters 28 and 29 as well as Vol. II, Chapter 21), he assumes the transverse piece is far more important than the radial piece, but such statement crucially depends on the assumption that the charge is moving at a more or less right angle to the line of sight, which is not necessarily the case. Feynman corrects for this assumption in Chapter 34 of Vol. I, in which he gives the reader a fuller treatment of the 'relativistic effects in radiation'.
[24] See: Feynman's *Lectures*, *The Least Action Principle* (Vol. II, Chapter 19).

another, as it does when hopping from one atomic or molecular orbital to another. Indeed, as the electron moves as a proper *current* in a conductor[25] – whose direction is from high to low potential – it should emit *photons* which will be packing a *discrete* amount of energy which is given by the Planck-Einstein relation:

$$\Delta E = h \cdot f = h/T$$

The frequency f of the photon is, obviously, the inverse of the *cycle time* T, and the Planck-Einstein relation may, therefore, also be written as $h = \Delta E \cdot T$. Because the drop in potential from one atomic or molecular orbital in a crystal structure – i.e. along the conductor – is extremely small, power lines – whether they be high-voltage DC or low-voltage AC lines – emit only extremely low frequency (ELF) *radiation*. Such low-frequency radiation is associated with heat radiation at very low temperature: a photon frequency of 300 Hz, for example, is associated with a wavelength that is equal to $\lambda = c/f \approx (3{\times}10^8 \text{ m/s})/(300 \text{ s}^{-1}) = 1{\times}10^6 \text{ m} = 1000 \text{ km}$.[26]

Hence, yes, we finally dropped the word: *radiation*. Electrons who stay in the same energy state – in a bound atomic or molecular state, for example – do not emit radiation and, hence, do not lose energy. Likewise, the orbital motion (*spin*) of the charge inside a stationary charge does *not* cause any radiation and, therefore, the energy does not leak out.

This, then, combines Maxwell's equations with the Planck-Einstein relation which tells us *energy* comes in quantized packets whose integrity is given by Planck's quantum of action (h). We now have the trio of physical constants in electromagnetic theory (classical as well as quantum physics): c, e, and h.

4. Photons and fields

1. In 1995, W.E. Lamb Jr. wrote the following on the nature of the photon:

> "There is no such thing as a photon. Only a comedy of errors and historical accidents led to its popularity among physicists and optical scientists. I admit that the word is short and convenient. Its use is also habit forming. Similarly, one might find it convenient to speak of the "aether" or "vacuum" to stand for empty space, even if no such thing existed. There are very good substitute words for "photon", (e.g., "radiation" or "light"), and for "photonics" (e.g., "optics" or "quantum optics"). Similar objections are possible to use of the word "phonon", which dates from 1932. Objects like electrons, neutrinos of finite rest mass, or helium atoms can, under suitable conditions, be considered to be particles, since their theories then have viable non-relativistic and non-quantum limits."[27]

The opinion of a Nobel Prize laureate carries some weight, of course, but we think the concept of a photon makes sense. As the electron moves from one (potential) energy state to another – from one

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[25] For a distinction between the concepts of current, electrical *signal*, and (probability) amplitudes, see our paper on electron propagation in a (crystal) lattice (November 2020).

[26] ELF radiation is usually defined as radiation with a (photon) frequency below 300 Hz. Typical field strength near a high-voltage power is typically 2-5 kV/m (1 V/m = 1 J/C·m = 1 N/C) for the electric field strength and up to 40 μT (1 T = 1 (N/C)·(s/m), with the latter factor reflecting the $1/c$ scaling factor and the orthogonality of the ***E*** and ***B*** vectors) for the magnetic field but – as the equations show – diminish rapidly with distance. The typical range for low-voltage lines is 100-400 V/m and 0.5-3 μT, respectively. See, for example: https://ec.europa.eu/health/scientific_committees/opinions_layman/en/electromagnetic-fields07/l-2/7-power-lines-elf.htm

[27] W.E. Lamb Jr., *Anti-photon*, in: Applied Physics B volume 60, pages 77–84 (1995).

atomic or molecular orbital to another – it builds an oscillating electromagnetic field which has an integrity of its own and, therefore, is not only wave-like but also particle-like.

The photon carries no charge but carries energy. We should probably assume its kinetic energy is the same at start and stop of the transition. In other words, at point t_1 and t_2, $(\mathrm{KE})_1$ and $(\mathrm{KE})_2$ are assumed to identical in the (physical) action equation which we introduced above:

$$S = h = \int_{t_1}^{t_2} (\mathrm{KE} - \mathrm{PE})\mathrm{d}t = \int_{t_1}^{t_2} (\mathrm{KE})\mathrm{d}t - \int_{t_1}^{t_2} (\mathrm{PE})\mathrm{d}t$$

This, of course, does not mean that the $\int_{t_1}^{t_2} (\mathrm{KE})\mathrm{d}t$ integral vanishes: it only does so when assuming the velocity in the KE = $m_e v^2/2$ formula[28] is zero everywhere, which cannot be the case because – when everything is said and done – the electron does move from one cell in the crystal lattice to another. However, we will leave it to the reader to draw possible KE, PE and total energy graphs over the electron transition from one crystal cell to another. Such graphs should probably be informed by a profound analysis of the nature of the photon.

We mentioned a photon carries energy, but no charge. While carrying electromagnetic energy, a photon will only exert a force when it meets a charge, in which case its energy will be absorbed as kinetic energy by the charge. In-between the emission and absorption of the photon, we should effectively think of the photon as an oscillating electromagnetic field and, hence, such field can usefully be represented by the electric and a magnetic field vectors ***E*** and ***B***. The magnitudes should not confuse us: field vectors do not take up any space, although we may want to think of them as a force without a charge to act on. Indeed, a non-zero field at some point in space and time – which we describe using the (*x*, *y*, *x*, *t*) coordinates – tell us what the force *would* be *if* we would happen to have a unit charge at the same point in space and in time. This is reflected in the electromagnetic force formula: the Lorentz force equals $\mathbf{F} = q\cdot(\mathbf{E} + \boldsymbol{v}\times\mathbf{B})$. Hence, the electromagnetic force is the sum of two (orthogonal) *component* vectors: $q\cdot\mathbf{E}$ and $q\cdot\boldsymbol{v}\times\mathbf{B}$.

The velocity vector ***v*** in the equation shows *both* of these two component force vectors depend on our frame of reference. Hence, we should think of the separation of the electromagnetic force into an 'electric' (or electrostatic) and a 'magnetic' force *component* as being somewhat artificial: the *electromagnetic* force is (very) *real* – because it determines the *motion* of the charge – but our cutting-up of it in two separate components depends on our frame of reference and is, therefore, (very) *relative*. We should refer to our remarks on the relative *strength* of the electric and magnetic field, however: the reader should not think in terms of the electric or magnetic force being more or less important in the analysis and always analyze *both* as aspects of one and the same *reality*.

Let us get back to our photon: we think the photon is pointlike because the **E** and **B** vectors that describe it will be zero at each and every point in time *and* in space *except if our photon happens to be at the* (*x, y, z*) *location at time t.*

[…] Please read the above again: **our photon is pointlike because the electric and magnetic field**

[28] We use the non-relativistic kinetic energy formula here because the *drift velocity* of the electron is very low. Also, the rather low energy levels involves ensure a particle with rest mass of about 0.51 MeV/c^2 should not reach relativistic velocity levels. The non-relativistic formula simply defines the kinetic energy as the difference between the total energy and the potential energy.

vectors that describe it are zero everywhere except where our photon happens to be.

2. At the same time, we know a photon is defined by its *wavelength*. So how does that work? What is the *physical* meaning of the wavelength? It is, quite simply, the distance over which the electric and magnetic field vectors will go through a full *cycle* of their oscillation. That is all there is to it: nothing more, nothing less.

That distance is, of course, a *linear* distance: to be precise, it is the distance Δs between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) where the ***E*** and ***B*** vectors have the same value. The photon will need some time Δt to travel between these two points, and these intervals in time and space are related through the (constant) velocity of the wave, which is also the velocity of the pointlike photon. That velocity is, of course, the speed of light, and the time interval is the cycle time T = 1/*f*. We, therefore, get the equation that will be familiar to you:

$$c = \frac{\Delta s}{\Delta t} = \frac{\lambda}{\mathrm{T}}$$

We can now relate this to the Planck-Einstein relation. Any (regular) oscillation has a frequency and a cycle time T = 1/*f* = 2π/ω. The Planck-Einstein relation relates *f* and T to the energy (E) through Planck's constant (*h*):

$$\mathrm{E} = h \cdot f = \hbar \cdot \omega \Leftrightarrow \mathrm{E} \cdot \mathrm{T} = h$$

The Planck-Einstein relation does not only apply to matter-particles but also to a photon. In fact, it was *first* applied to a photon.[29] Think of the photon as *packing* not only the energy E but also an amount of *physical action* that is equal to *h*.

3. We have not talked much about the meaning of *h* so far, so let us do that now. *Physical action* is a concept that is not used all that often in physics: physicists will talk about energy or momentum rather than about physical action.[30] However, we find the concept as least as useful. Physical action can *express* itself in two ways: as some energy over some time (E·T) or – alternatively – as some momentum over some distance (p·λ). For example, we know the (pushing) momentum of a photon[31] will be equal to p = E/*c*. We can, therefore, write the Planck-Einstein relation for the photon in two equivalent ways:

$$\mathrm{E} \cdot \mathrm{T} = \frac{E}{c} \cdot c\mathrm{T} = h \Leftrightarrow \mathrm{p} \cdot \lambda = h$$

We could jot down many more relations, but we should not be too long here.[32]

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[29] The application of the Planck-Einstein relation to matter-particles is *implicit* in the *de Broglie* relation. Unfortunately, Louis de Broglie imagined the matter-wave as a linear instead of a circular or orbital oscillation. He also made the mistake of thinking of a particle as a wave *packet*, rather than as a single wave! The latter mistake then led Bohr and Heisenberg to promote uncertainty to a metaphysical principle. See our paper on the meaning of the *de Broglie* wavelength and/or the interpretation of the Uncertainty Principle.

[30] We think the German term for physical action – *Wirkung* – describes the concept much better than English.

[31] For an easily accessible treatment and calculation of the formula, see: *Feynman's Lectures*, Vol. I, Chapter 34, section 9.

[32] We may refer the reader to our manuscript, our paper on the meaning of the fine-structure constant, or various others papers in which we explore the nature of light. We just like to point out one thing that is quite particular for

5. The near- and far-fields

The picture above is quite clear and consistent: a conductor – or a crystal lattice – emits electromagnetic waves as photons, who should be thought of as self-perpetuating through the interplay of the electric and magnetic field vector.[33] The *direction* of propagation equals the line of sight (more or less[34]) and a crystal lattice (conductor) acts as a series of point sources or oscillators. By modulating the voltage (AC or DC), frequency and – and taking into account the spacing and properties of the crystal lattice – one gets photon beams in all directions, whose intensity and energy depends on the above-mentioned factors and – important – can carry a signal through frequency or amplitude modulation (AM or FM). We take, once again, an illustration from Feynman to show how this works (**Figure 4**). It should be noted that the *interference* pattern that emerges does not result from random indeterminism but from an interplay of regular and statistically determined photon emissions from each of the crystals in the conducting lattice. As such, the addition or superposition of photons, electromagnetic waves and probabilities amounts to the same – with the usual *caveat* for the photon picture, of course, which – as particles – do not engage in constructive or destructive interference. The *complementarity* of the different viewpoints, perspectives or *representations* of the same reality is, therefore, clear.

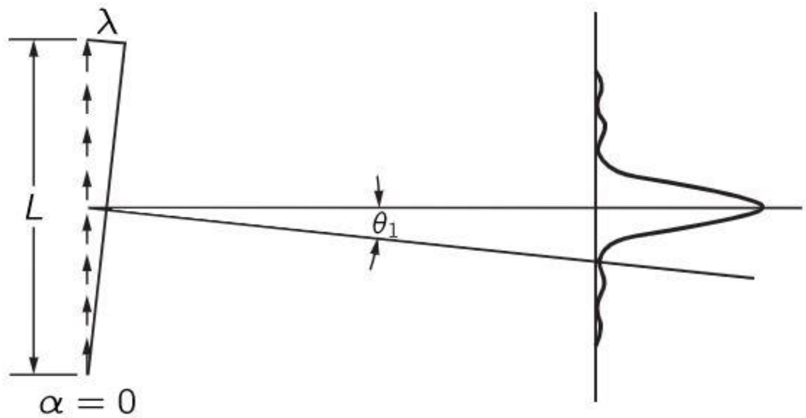

Figure 4: The intensity pattern of a continuous line of oscillators (Feynman, I-30, Fig. 30-5)

However, by way of conclusion, we must probably say something about the oft-used distinction between near- and far-fields. In order to do so, we ask the reader to, once again, carefully look at the relevant equation(s) for the ***E*** and ***B*** field vectors:

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the photon: the reader should note that the E = mc^2 mass-energy equivalence relation and the p = mc = E/c can be very easily related when discussing photons. There is an easy *mathematical* equivalence here. That is not the case for matter-particles: the *de Broglie* wavelength can be interpreted geometrically but the analysis is somewhat more complicated—not impossible (not at all, actually) but just a bit more convoluted because of its circular (as opposed to linear) nature.

[33] We skipped a discussion on photon spin: we think of photon spin as angular momentum, and it is always plus or minus one unit of *h*. Photons do not have a zero-spin state.

[34] Because the lattice consists of several layers, one may think an electron may not always move to the crystal cell right next to it. This is true, it may deviate to left, right, up, or down while moving through the lattice. On the other hand, the conducting electrons will repel each other and will, therefore, tend to travel on the surface of the crystal, which is in agreement with standard theory.

$$\boldsymbol{E}(1,t) = \frac{\mathrm{q}}{4\pi\epsilon_0}\left[\frac{\boldsymbol{e}_{\boldsymbol{r}'}}{r'^2} + \frac{r'}{c}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{e}_{\boldsymbol{r}'}}{r'^2}\right) + \frac{1}{c^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\boldsymbol{e}_{\boldsymbol{r}'}\right]$$

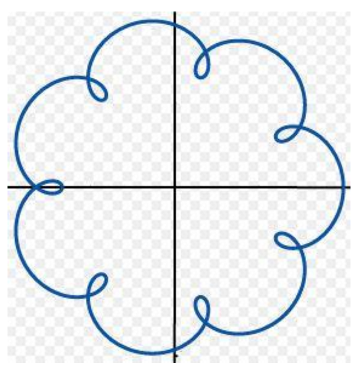
$$c\boldsymbol{B}(1,t) = \boldsymbol{e}_{\boldsymbol{r}'} \times \boldsymbol{E}(1,t)$$

The reader will note the magnitude of the (retarded) Coulomb effect (the first term) diminishes with distance following the inverse square law ($\sim 1/r'^2$) while the second term involves only inverse proportionality ($\sim 1/r^2$).[35] Finally, the third term does not fall off with distance at all! It is this what gives rise to the very different *shape* of the near-field versus the far-field waves, with a transition zone in-between.

In terms of the *shape* of the electromagnetic waves, one should probably think of the first effect (retarded Coulomb effect) as a spherical wavefront, whose energy *density* effectively diminishes as per the inverse square law, while the second effect is a plane wavefront[36]

6. Complicated trajectories

It is important to note that the trajectory of a charge will usually not appear as a *straight line* in the reference frame of the observer: we think of an electron in an atomic or molecular orbital as following a non-linear trajectory. Such non-linear trajectory may be repetitive or cyclical. An example of such cyclical trajectory is the motion of an electron in a Penning or *ion* trap, in which a quadrupole electric field confines the electron while an axial magnetic field causes orbital motion. In addition, when thinking of an electron as a ring current itself, two frequencies of cyclical motion corresponding to two *modes* must be defined, as illustrated below.[37]



In addition, the magnetic moment of his ring current will cause precession in the magnetic field. Such precessional motion will cause the axis of rotation to rotate itself.[38] One should, therefore, appreciate

[35] The coefficient r'/c and the $1/r'^2$ in the argument of the first-order derivative combine to give us a rather straightforward $1/r'$ factor.

[36] The direction of the field vector(s) may be parallel or orthogonal to the direction of propagation, which gives rise to the distinction between longitudinal and transverse waves. The reader may also remember *lenses* can change plane waves into spherical waves and vice versa but such fact is not very relevant for the discussion here.

[37] The illustration was taken from the Wikipedia article on the Penning or ion trap, but we do not expect the reader to review this in depth.

[38] Such precessional motion will be described by a precession frequency and an angle of precession. For more detail, we may refer the reader to course *F47 – Cylotron frequency in a Penning trap*, Heidelberg University, Blaum Group, 28 September 2015.

that a description of motion $\boldsymbol{x} = \boldsymbol{x}(t - r'/c)$ will usually be quite complicated not only involving the velocity, acceleration and jolt or jerk vectors $\boldsymbol{v} = \mathrm{d}\boldsymbol{x}/\mathrm{d}t$, $\boldsymbol{a} = \mathrm{d}\boldsymbol{x}^2/\mathrm{d}t^2$, $\boldsymbol{j} = \mathrm{d}\boldsymbol{x}^3/\mathrm{d}t^3$, but, possibly, even higher-order derivatives. The use of spherical coordinates to describe the position using radial distance from the origin (r) and a polar and azimuthal angle (usually denoted by θ and φ respectively) may or may not make calculations generally easier.[39]

The distance from the (0, 0, 0) origin to the $\boldsymbol{x} = (x, y, z)$ position is the *norm* of $\boldsymbol{x}$ and is given by the Pythagorean Theorem:

$$|\boldsymbol{x}| = +\sqrt{\boldsymbol{x}^2} = +\sqrt{x^2 + y^2 + z^2}$$

The same expression is, obviously, valid in the moving reference frame:

$$|\boldsymbol{x}'| = +\sqrt{\boldsymbol{x}'^2} = +\sqrt{x'^2 + y'^2 + z'^2}$$

Distances may be measured in light-seconds (299,792,458 m) instead of meter by dividing all distances by c. This amounts to measuring the distance in the time that is needed for light to travel from the origin to the position $\boldsymbol{x} = (x, y, z)$ and, therefore, gives a distance measurement in *seconds*.

$$\frac{|\boldsymbol{x}|}{c} = \frac{+\sqrt{\boldsymbol{x}^2}}{c} = +\sqrt{\frac{x^2}{c^2} + \frac{y^2}{c^2} + \frac{z^2}{c^2}}$$

It is tempting to think of the $c^2t^2 = x^2 + y^2 + z^2$ expression as an expression of the Pythagorean Theorem but this can only be done if t is effectively defined as the time that is needed to travel from the origin to $\boldsymbol{x}$, in which case the expression above can effectively be written as:

$$t = \frac{|\boldsymbol{x}|}{c} = +\sqrt{\frac{x^2}{c^2} + \frac{y^2}{c^2} + \frac{z^2}{c^2}} \Leftrightarrow ct = +\sqrt{x^2 + y^2 + z^2} \Leftrightarrow c^2t^2 = x^2 + y^2 + z^2$$

However, we are modeling motion and, hence, the time variable is the time which we associate with an object in motion (usually a charge) and we are, therefore, concerned with the equation of motion only:

$$\boldsymbol{x} = \boldsymbol{x}(t) = (x(t), y(t), z(t))$$

By way of conclusion – and to warn the reader against using relativity theory without much appreciation of what might actually be going on, we make a few remarks on relativity theory.

7. Relativity

Because there is no preferred origin, the coordinate values (x, y, z, t) and (x', y', z', t') have no essential meaning: we are always concerned with *differences* of spatial or temporal coordinate values belonging to two events, which we will label by the subscript 1 and 2. This *difference* is referred to as the *spacetime interval* Δs, whose squared value is given by:

[39] For an overview of how the Lorentz transformation of the position and time variables used to describe motion works for spherical coordinate frames, see: Mukul Chandra Das and Rampada Misra, *Some studies on Lorentz transformation matrix in non-cartesian co-ordinate system*, Journal of Physics and Its Applications, 1(2) 2019, pages 58-61.

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta ct)^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2$$

In the context of this expression, c should be thought of as an invariable mathematical constant which allows us to express the time interval $\Delta t = (t_2 - t_1)$ in equivalent distance units (*meter*). The same spacetime interval in the moving reference frame is measured as:

$$(\Delta s')^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (\Delta ct')^2$$

$$= (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 - c^2(t'_2 - t'_1)^2$$

The spacetime interval is invariant and $(\Delta s)^2$ is, therefore, equal to $(\Delta s')^2$. We can, therefore, combine both expressions above and write:

$$(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta ct)^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (\Delta ct')^2$$

$$\Leftrightarrow [(\Delta x)^2 - (\Delta x')^2] + [(\Delta y)^2 - (\Delta y')^2] + [(\Delta z)^2 - (\Delta z')^2] = c^2[(\Delta ct)^2 - (\Delta ct')^2]$$

This equation shows two observers – in relative motion one to another – can only meaningfully talk about the spacetime interval between two events if they agree on (1) the reality of the events[40], (2) a common understanding of the measurement units for time and distance as well as a conversion factor between the two units so as to establish equivalence.

Because the speed of light is an invariant constant – the *only* measured velocity which does not depend on the reference frame[41] – it will be convenient to measure distance in light-seconds (the distance travelled by light in one second, i.e. 299,792,458 meter *exactly*[42]). This, of course, assumes a common definition of the second which, since last year's revision of the international system of units (SI) only, can be defined with reference to a standard frequency only. This standard frequency was *defined* to be equal to 9,192,631,770 Hz (s^{-1}), *exactly*[43], which is the frequency of the light emitted by a caesium-133 atom when oscillating between the two energy states that are associated with its ground state at a temperature of 0 K.[44]

[40] Both observers need to agree on measuring time along *either* the positive *or* negative direction of the time scale because the *order* of the events (in time) cannot be established in an absolute sense. Even if one reference frame assigns precisely the same time to two events that happen at different points in space, a reference frame that is moving relative to the first will generally assign different times to the two events (the only exception being when motion is exactly perpendicular to the line connecting the locations of both events).

[41] The velocity of light does *not* depend on the motion of the source.

[42] It was only in 1983 – about 120 years after the publication of Maxwell's wave equations, which showed that the velocity of propagation of electromagnetic waves is always measured as c – that the *meter* was redefined in the International System of Units (SI) as the distance travelled by light in vacuum in 1/299,792,458 of a second.

[43] This value was chosen because the caesium 'second' equaled the limit of human measuring ability around 1960, when the caesium atomic clock as built by Louis Essen in 1955 was adopted by various national and international agencies and bodies (e.g. USNO) for measuring time.

[44] These two energy states are associated with the hyperfine splitting resulting from the two possible states of spin in the presence of both nuclear as well as electron spin. Spin is measured in units of h: the nuclear spin of the caesium-atom is 7/6 units of h, while the total electron spin is (because of the pairing of electrons and the presence of one unpaired electron) is equal to $h/2$. Depending on the energy, nuclear and electron spin will either

The rather long introduction on relativity illustrates that two observers need to agree on (1) the use of a (physical) clock to count time and (2) the invariance of the speed of light. The reality of light effectively corresponds to a succession of events – a photon travels the distance $\Delta\boldsymbol{x}$ over a time interval Δt – which are separated by *invariant* spacetime intervals

$$\Delta s = \Delta s' = \sqrt{(\Delta\boldsymbol{x})^2 - c^2(\Delta t)^2} = \sqrt{(\Delta\boldsymbol{x}')^2 - c^2(\Delta t')^2}.$$

It should be noted that the expression under the square root sign cannot be negative because the photon does not travel at superluminal velocity. In fact, for a photon traveling from point A to B the expression above can be multiplied by Δt and $\Delta t'$ respectively so as to yield the following:

$$\frac{\Delta s}{\Delta t} = \frac{\sqrt{(\Delta\boldsymbol{x})^2 - c^2(\Delta t)^2}}{\Delta t} = \sqrt{\frac{(\Delta\boldsymbol{x})^2}{(\Delta t)^2} - \frac{c^2(\Delta t)^2}{(\Delta t)^2}} = \sqrt{c^2 - c^2} = 0$$

$$\frac{\Delta s'}{\Delta t'} = \frac{\sqrt{(\Delta\boldsymbol{x}')^2 - c^2(\Delta t')^2}}{\Delta t'} = \sqrt{\frac{(\Delta\boldsymbol{x}')^2}{(\Delta t')^2} - \frac{c^2(\Delta t')^2}{(\Delta t')^2}} = \sqrt{c^2 - c^2} = 0$$

This establishes the light cone separating time- and spacelike intervals for both observers. We will now no longer be worried about the relativity of the reference frame: we hope the reader got a sufficient understanding of what is relative and absolute in the description of (physical) reality, which consists of matter (charged particles) and light (fields and electromagnetic waves causing *changes* in those fields).[45]

Jean Louis Van Belle, 23 December 2020

have opposite or equal sign. Hence, the two energy states are associated with total spin value (nuclear and electron) *F* = 7/6 – 1/2 = 3 *or* F = 7/6 + 1/2 = 4.

[45] For a short (15 minutes) brief, we refer the reader to our YouTube video on reality, philosophy, and physics.