A revision to the Brocard's problem.

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0- Abstract:

Henri Brocard posed two articles in 1876 and 1885 exposing the diophantine equation $n!+1=m^2$. It was also propose by Ramanujan. The unsolved problem says that it has not possible other solutions than n=4,5,7. In this paper I want to show a revision of the problem with the Stirling's approximation to factorials.

1- Introduction.

We are going to use the classic definition of the Brocard's problem:

$$(1) \quad n!+1=m^2$$

And the approximation formula of Stirling:

$$(2) \quad n!\sim\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

If we substitute (2) in (1):

$$(3) \quad \sqrt{2\pi n}\left(\frac{n}{e}\right)^n+1\sim m^2$$

We can do next an algebraic clearance in (3) to obtain the base of the power by itself:

$$(4) \quad \sqrt{\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n+1\right)}\sim m$$

We can go further and take the result of the approximation in the ceiling function to get the true result of the equation:

$$(5) \quad \sqrt{\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n+1\right)}=\lceil m\rceil$$

2- Numerical results:

In this part we are going to analyze the classical results of Brocard’s problem using (5):

$$(6) \quad \sqrt{(\sqrt{2\pi 4}(\frac{4}{e})^{4}+1)}=\lceil 4{,}9503\rceil=5$$

$$(7) \quad \sqrt{(\sqrt{2\pi 5}(\frac{5}{e})^{5}+1)}=\lceil 10{,}9095\rceil=11$$

$$(8) \quad \sqrt{(\sqrt{2\pi 7}(\frac{7}{e})^{7}+1)}=\lceil 70{,}5790\rceil=71$$

3- Conclusions.

As we can see in (6), (7) and (8) the result of the equation proposed in (5) is the base of the quadratic equation which is result of the Brocard’s problem solutions known nowadays. Anyway is an open problem and maybe there are more solutions.

References:

https://en.wikipedia.org/wiki/Brocard%27s_problem
https://en.wikipedia.org/wiki/Floor_and_ceiling_functions
https://en.wikipedia.org/wiki/Stirling%27s_approximation