Klein-Gordon Equation and Wave Function in Robertson-Walker and Schwarzschild space-time

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ABSTRACT

In the general relativity theory, we find Klein-Gordon wave functions in Robertson-Walker and Schwarzschild space-time. Specially, this article is that Klein-Gordon wave equations is treated by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find Klein-Gordon wave equations and functions by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

The gauge fixing equation in general relativity theory

$$A^{\mu}{}_{:\mu} = \frac{\partial A^{\mu}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} A^{\rho}$$

$$\rightarrow \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + \partial^{\rho}\Lambda)$$

$$= \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + g^{\rho\rho}\partial_{\rho}\Lambda) \quad (1)$$

2. Klein-Gordon wave equation in Robertson-Walker space-time

Because the gauge fixing equation is the electro-magnetic wave equation, Klein-Gordon wave equation is in Robertson-Walker space-time.

The Robertson-Walker solution is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \quad (2)$$

In this time, 2-dimensional solution is

$$d\Omega = 0$$

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\frac{dr^2}{1-kr^2} \quad (3)$$

The gauge fixing equation is in 2-dimensional solution[3]

$$\partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + g^{\rho\rho}\partial_{\rho}\Lambda)$$

$$= \partial_{\mu}A^{\mu} + \Gamma^{1}{}_{10}A^{0} + \Gamma^{1}{}_{11}A^{1} + \partial_{\mu}g^{\mu\upsilon}\partial_{\upsilon}\Lambda + g^{\mu\upsilon}\partial_{\mu}\partial_{\upsilon}\Lambda + \Gamma^{1}{}_{10}g^{00}\frac{1}{c}\frac{\partial\Lambda}{\partial t} + \Gamma^{1}{}_{11}g^{11}\frac{\partial\Lambda}{\partial r} \quad (4)$$

Hence, we can find Klein-Gordon wave equation in 2-dimentional Robertson-Walker space-time.

$$\partial_{\mu}g^{\mu\upsilon}\partial_{\upsilon}\phi + g^{\mu\upsilon}\partial_{\mu}\partial_{\upsilon}\phi + \Gamma^{1}{}_{10}g^{00}\frac{1}{c}\frac{\partial}{\partial t}\phi + \Gamma^{1}{}_{11}g^{11}\frac{\partial}{\partial r}\phi$$

$$= \left[\frac{-2kr}{\Omega^2(t)}\frac{\partial}{\partial r} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{1-kr^2}{\Omega^2(t)}\frac{\partial^2}{\partial r^2} - \frac{\dot{\Omega}}{c\Omega}\frac{1}{c}\frac{\partial}{\partial t} + \frac{kr}{\Omega^2(t)}\frac{\partial}{\partial r}\right]\phi = \frac{m^2c^4}{\hbar^2}\phi$$

$$\Gamma^{1}{}_{10} = \frac{\dot{\Omega}}{c\Omega} \quad , \quad \Gamma^{1}{}_{11} = \frac{kr}{1-kr^2} \quad (5)$$

In this time, we can think the shape of Klein-Gordon wave function from 2-dimetional Robertson-Walker space-time. In this case, light is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\frac{dr^2}{1-kr^2} = 0$$

$$\int \frac{dt}{\Omega(t)} = \frac{1}{c}\int \frac{dr}{\sqrt{1-kr^2}} \tag{6}$$

Hence, matter wave function is in 2-dimetional Robertson-Walker space-time.

$\phi = A_0 \exp i\Phi$, A_0 is amplitude

$$\Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 \int \frac{dr}{\sqrt{1-kr^2}},$$ ω_0 is angular frequency, $k_0 = |\vec{k}_0|$ is wave number

i) $k = 1, \Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 \sin^{-1} r$

ii) $k = 0, \Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 r$

iii) $k = -1, \Phi = \omega_0 \int \frac{dt}{\Omega(t)} - k_0 \sinh^{-1} r$ (7)

If the definition of energy and momentum is

$$E = \frac{\hbar\omega_0}{\Omega(t)}, \vec{p} = \frac{\hbar\vec{k}_0}{\Omega^2(t)} \tag{8}$$

Energy-Momentum relation is in Robertson-Walker space-time,

$$m^2c^4 = E^2 - \frac{\Omega^2(t)}{1-kr^2}p^2c^2, E = mc^2\frac{dt}{d\tau}, \vec{p} = m\frac{d\vec{r}}{d\tau} \tag{9}$$

Finally, angular frequency-wave number relation is in Robertson-Walker space-time,

$$\frac{\hbar^2\omega_0{}^2}{\Omega^2(t)} - \frac{\hbar^2 k_0{}^2 c^2}{\Omega^2(t)}\frac{1}{1-kr^2} = m^2c^4 \tag{10}$$

Hence, Klein-Gordon wave equation-Eq(5) is satisfied by matter wave function-Eq(7) in Robertson-Walker space-time.

3.Klein-Gordon wave equation in Schwarzschild space-time

Because the gauge fixing equation is the electro-magnetic wave equation, Klein-Gordon wave equation is in Schwarzschild space-time.

The Schwarzschild solution is

$$d\tau^2 = (1 - \frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\Omega^2] \tag{11}$$

In this time, 2-dimensional solution is

$$d\Omega = 0$$

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} \tag{12}$$

The gauge fixing equation is in 2-dimensional solution[3]

$$\partial_\mu(A^\mu + g^{\mu\upsilon}\partial_\upsilon\Lambda) + \Gamma^\mu{}_{\mu\rho}(A^\rho + g^{\rho\rho}\partial_\rho\Lambda)$$

$$= \partial_\mu A^\mu + \Gamma^0{}_{01}A^1 + \Gamma^1{}_{11}A^1 + \partial_\mu g^{\mu\upsilon}\partial_\upsilon\Lambda + g^{\mu\upsilon}\partial_\mu\partial_\upsilon\Lambda + \Gamma^0{}_{01}g^{11}\frac{\partial\Lambda}{\partial r} + \Gamma^1{}_{11}g^{11}\frac{\partial\Lambda}{\partial r}$$

$$= \partial_\mu A^\mu + \partial_\mu g^{\mu\upsilon}\partial_\upsilon\Lambda + g^{\mu\upsilon}\partial_\mu\partial_\upsilon\Lambda$$

$$\Gamma^0{}_{01} = \frac{GM}{r^2c^2}\frac{1}{1-\frac{2GM}{rc^2}} \quad , \quad \Gamma^1{}_{11} = -\frac{GM}{r^2c^2}\frac{1}{1-\frac{2GM}{rc^2}} \tag{13}$$

Hence, we can find Klein-Gordon wave equation in 2-dimentional Schwarzschild space-time.

$$\partial_\mu g^{\mu\upsilon}\partial_\upsilon\phi + g^{\mu\upsilon}\partial_\mu\partial_\upsilon\phi$$

$$= [\frac{2GM}{r^2c^2}\frac{\partial}{\partial r} - \frac{1}{1-\frac{2GM}{rc^2}}\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + (1-\frac{2GM}{rc^2})\frac{\partial^2}{\partial r^2}]\phi = \frac{m^2c^4}{\hbar^2}\phi \tag{14}$$

In this time, we can think the shape of Klein-Gordon wave function from 2-dimetional Schwarzschild space-time. In this case, light is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} = 0$$

$$t = \frac{1}{c}\int\frac{dr}{1-\frac{2GM}{rc^2}} = \frac{r}{c} + \frac{2GM}{c^3}\ln|\, r - \frac{2GM}{c^2}\,| \tag{15}$$

Hence, Klein-Gordon wave function is in 2-dimetional Schwarzschild space-time-

$\phi = A_0 \exp i\Phi , A_0$ is amplitude

$$\Phi = \omega_0 t - k_0 r - k_0\frac{2GM}{c^2}\ln|\, r - \frac{2GM}{c^2}\,|$$

ω_0 is angular frequency, $k_0 = \left|\vec{k}_0\right|$ is wave number (16)

If the definition of energy and momentum is

$$E=\frac{\hbar\omega_0}{(1-\frac{2GM}{rc^2})},\vec{p}=\hbar\vec{k}_0(1-\frac{2GM}{rc^2}) \quad (17)$$

Energy-Momentum relation is in Schwarzschild space-time,

$$m^2c^4=(1-\frac{2GM}{rc^2})E^2-\frac{p^2c^2}{(1-\frac{2GM}{rc^2})},E=mc^2\frac{dt}{d\tau},\vec{p}=m\frac{d\vec{r}}{d\tau} \quad (18)$$

Finally, angular frequency-wave number relation is in Schwarzschild space-time,

$$\frac{\hbar^2\omega_0{}^2}{(1-\frac{2GM}{rc^2})}-\hbar^2k_0{}^2(1-\frac{2GM}{rc^2})=m^2c^4 \quad (19)$$

Hence, Klein-Gordon wave equation-Eq(14) is satisfied by matter wave function-Eq(16) in Schwarzschild space-time

4. Conclusion

We find Klein-Gordon wave equation and function in Robertson-Walker space-time. We find Klein-Gordon wave equation and function in Schwarzschild space-time..

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