The strong Bell's theorem which does not use the assumption of locality

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Abstract
The standard Bell's theorem states that quantum mechanics (QM) cannot be locally realistic. Here we prove the **strong** Bell's theorem which states that QM cannot be realistic. In the proof we use the Mermin's form of the Bell's theorem. Our result solves the old dilemma: non-locality or non-realism. Then we discuss the consequences of the strong Bell's theorem, e.g. that no Bell non-locality of QM exists and that quantum theory is local and non-realistic.

Keywords: Bell's theorem, non-locality, non-realism

1. Introduction.

The standard form of Bell's theorem (see [3]) has the following form:
The assumption of local realism of quantum mechanics (QM) *implies* Bell's inequality which is in the contradiction with experimental facts. As a consequence of this result QM is not local realistic, i.e. either QM is non-local or QM is non-realistic.

We interpret the realism as a counter-factuality: any observable has a value even if it is not measured. (The standard QM rule requires that only the measured observables have a value.)

In this paper we prove the **strong** form of the Bell's theorem: The realism of QM *implies* the Bell's theorem and thus the contradiction with the experiment. The conclusion is that QM must be non-realistic.

The difference between the previous assumption (the local realism of QM) and the new assumption (the realism of QM) is enormous. For many years (and tens of years) QM was imprisoned in this dilemma:

Non-locality or non-realism?

If the non-realism of QM is proved, the problem of non-locality of QM would become irrelevant, i.e. Bell non-locality should not exist.

The proof of non-realism, that we present here, is based on the modification of the Mermin's form of the Bell's theorem (see [1]) and on the simplifications from [2].

The main point is the following: the first part of the proof which usually uses the locality assumption is substituted by the proof based completely on the experimental facts.

The paper is organized as follows. In the second section we define basic concepts. In the third section we present the proof of the strong Bell's theorem. Finally we conclude by revising our arguments and viewing them in a broader context.

2. Simple observables, the EPR entangled state

Let us consider the quantum system S with the (finite dimensional) Hilbert space of states **H(**S) and its observable A. We shall call A a simple observable if its eigenvalues are {1. -1}. Simple observables are in the 1-1 relation with projectors given by formulas P = ½(A+I), A = 2P – I (where I is the identity operator). Thus simple observables can generate general observables using the spectral decomposition.

The assumption that a simple observable A is realistic is equivalent to the so-called counter-factuality: A is assumed to have a value also in the situation where A is not measured.

The EPR system is the composition of two 2-dimensional systems S_1 and S_2 in a special "entangled" state. Let us consider two 2-dimensional systems S_1 and S_2 with a simple observable A_1 in $\mathbf{H}(S_1)$ and a simple observable A_2 in $\mathbf{H}(S_2)$. There exists orthogonal basis $\{u_+^i, u_-^i\}$, i = 1, 2 such that $A_i(u_+^i) = 1$, $A_i(u_-^i) = -1$, i = 1, 2.

Then the entangled state ([4]) is defined as

$$\Psi = 2^{-1/2} (u_+^1 \otimes u_-^2 - u_-^1 \otimes u_+^2)$$

where the state Ψ is an element of the space $\mathbf{H}(S_1) \otimes \mathbf{H}(S_2)$ and $\otimes$ denotes the tensor product.

It is clear that systems S_1 and S_2 are in some sense correlated (in fact, anti-correlated). It is assumed that systems S_1 and S_2 are then sent in opposite directions and then measured by observers Alice and Bob (further developed in part 3).

3. The strong Bell's theorem.

The main goal of this central section is to prove the strong Bell's theorem solely with the assumption of realism. (The usual way is to prove Bell's theorem using the assumption of the local realism.)

The detailed construction of our proof is based on the Mermin's form of the Bell's theorem ([1]).

We shall consider the experiment described in [2], where Alice and Bob are used as independent observers: measurements at Alice in possible three orientations A_1, B_1 and C_1 and the measurements in possible three orientations A_2, B_2 and C_2 at the part of Bob. The orientations are taken at angles 0°, 120° and 240°. The choice of the orientation at Alice' part (among A_1, B_1, C_1) and the choice of the orientation at the Bob's part (among A_2, B_2, C_2) are chosen randomly.

Orientations of the Alice's part and of the Bob's part will be correlated. This means that the orientation of A_1 is the same as the orientation of A_2 and similarly for B_1 and B_2 and for C_1 and C_2.

We shall consider also the combinations of orientations of A_1-B_2, A_1-C_2, C_1-B_2. All observables A_1, B_2, C_1, C_2 are simple observables. For the detail description see [2].

We shall assume that the realism of QM holds, i.e. that observables A_1, B_2, C_1, C_2 has the "pre-existing" (i.e. counter-factual) values.

It is possible to formulate the basic properties which have been tested experimentally and which means that the following facts are experimentally proved:

Experimental fact 1 (EF1). The measurements of A_1-A_2 and correspondingly B_1-B_2 and C_1-C_2 gives the 100% anti-correlation, i.e. that relations $A_1 = -A_2$, $B_1 = -B_2$, $C_1 = -C_2$ are true in all rounds.

Experimental fact 2 (EF2). The correlations between A_1 and B_2, A_1 and C_2 and C_1 and B_2 can be measured. It is obtained that the events $[A_1=B_2]$, $[A_1=C_2]$ and $[C_1=B_2]$ have probabilities approx. 0.75, i.e. at least 0.7 (in consideration of possible experimental errors). (In fact, the probabilities of events $[A_1=-B_2]$, $[A_1=-C_2]$ and $[C_1=-B_2]$ are measured to be approx. 0.25.). The details of the discussion can be found in [2].)

Theorem 3.1. QM is not realistic (i.e. the counter-factual reasoning cannot be used).
Proof.
We shall assume that QM is realistic. This means that each simple observable has its pre-existing value.

<u>The first part of the proof.</u>
Using the standard probabilistic formula

$$\text{prob}\,(X \cup Y) = \text{prob}\,(X) + \text{prob}\,(Y) - \text{prob}\,(X \cap Y)\text{ , i.e.}$$
$$\text{prob}\,(X \cap Y) = \text{prob}\,(X) + \text{prob}\,(Y) - \text{prob}\,(X \cup Y)$$

we obtain that

$$\text{prob}\,(X \cap Y) \geqq \text{prob}\,(X) + \text{prob}\,(Y) - 1.$$

Denote $X = [A_1=B_2]$, $Y = [A_1=C_2]$, $Z = [B_1=C_2] = [C_1=B_2]$ (here we have used **EF1**). From **EF2** we know that probabilities of X, Y, Z are at least 0.7 .

We obtain

$$\text{prob}\,(X \cap Y) \geqq \text{prob}\,(X) + \text{prob}\,(Y) - 1 \geqq 0.7 + 0.7 - 1 = 0.4.$$

Then we obtain

$$\text{prob}\,(X \cap Y \cap Z) \geqq \text{prob}\,(X \cap Y) + \text{prob}\,(Z) - 1 \geqq 0.4 + 0.7 - 1 = 0.1 > 0.$$

If the experiment is repeated the sufficient number of times then the round where $X \cap Y \cap Z$ happens must exist.

The conclusion: there exists a round such that $[A_1=B_2]$, $[A_1=C_2]$, $[C_1=B_2]$ happen.

<u>The second part of the proof</u>
We shall obtain the contradiction. We shall consider the round where

$$A_1=B_2,\ A_1=C_2,\ C_1=B_2$$

which existence was proved in the first part.

If $A_1=1$, then $B_2=1$, $C_2=1$, $C_1=1$ and then $C_1=C_2$ is in the contradiction with **EF1**.
If $A_1=-1$, then $B_2=-1$, $C_2=-1$, $C_1=-1$ and then $C_1=C_2$ is in the contradiction with **EF1.**

This contradiction implies that the assumption of the realism of QM is false. Therefore, we were able to prove that the assumption of realism does not hold. The end of the proof.

We see that the methods, we used for our proof, did not require locality of QM. The first part of the proof was based on the experimental facts (and the elementary probability theory) while the second part was based on the theory of logic. In both parts the realism of QM (i.e. counter-factual reasoning) was the only assumption used.

4. Conclusions

We have proved the strong version of Bell's theorem where only the realism of QM (instead of the realism + locality of QM) is assumed. As a consequence, we obtained the proof of the non-realism of QM.

Some consequences and related considerations of the strong Bell's theorem were discussed, also in [8]. It was shown that no Bell non-locality exists.

We are convinced that the proof of the strong Bell's theorem will change completely the field of Bell's inequalities. Many of the assumptions that were previously used, would need to be fundamentally reconsidered. This reconsideration will be the theme of the future research.

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