# About corpuscular-wave dualism and forms of electromagnetic radiation

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It is shown that two forms of electromagnetic radiation follow from the tensor of four-dimensional derivative of canonical electromagnetic potential. One form corresponds to the electromagnetic waves of Maxwell Hertz, and the second form is a spatially localized wave structures of the electromagnetic field, similar to the vortex rings that exist in liquids and gases. These wave structures, moving at the speed of light, carry an impulse and a moment of an impulse, have the properties of a wave-particle and they can be identified with electromagnetic field quanta - photons. It is shown that the cause of corpuscular-wave duality of these structures and the existence of two forms of electromagnetic radiation is pseudo-Euclidian space-time geometry.



## 1 Introduction

The theoretical foundations of classical electrodynamics were outlined by J.K. Maxwell in 1864 in his article "Dynamic theory of electromagnetic field" [1] in the form of a system of mutually related equations that generalized the known by then electromagnetic phenomena. From these equations followed the possibility of existence in nature of electromagnetic waves propagating with the speed of light. Contemporaries met Maxwell's theory indifferently, because at this time already existed W. Weber's electrodynamics, which satisfactorily explained the known electrodynamic phenomena. Only after H. Hertz's experimental discovery of electromagnetic waves, 25 years after the publication of Maxwell's theory, did it gradually gain general acceptance and was eventually elevated to the rank of the greatest scientific achievements. The coincidence of the velocity of light and electromagnetic waves led to the fact that light was identified as electromagnetic waves with very short wavelengths. The nature of light has been a concern for scientists since ancient times. For a long time there were two competing theories - corpuscular and wave - in the struggle.

In the XVIII century it was believed that light is a flow of corpuscles. For the first time the successive wave theory of light was written by R. Hook in 1667. The wave theory of light was studied by such famous physicists and mathematicians as Ch. Huygens, L. Euler, T. Young, A.J. Fresnel, A.L. Cauchy, G. Green, M. Lomonosov and supporters of corpuscular theory were I. Newton, P.S. Laplace, S. Poisson, F. Arago, J.B. Biot [2]. The struggle between the wave and corpuscular theories of light continued for a century and a half and in the early 19th century ended in the victory of the wave theory. The main feature and difference between light and the electromagnetic wave of Maxwell-Hertz, is its propagation from the source in the form of a beam and the absence of attenuation when propagating over vast distances. In late 1900, in the article "On the Law of Energy Distribution in the Normal Spectrum" by M. Planck, based on a study of the thermal radiation of heated bodies, was proposed a hypothesis of the quantum nature of the process of light radiation. This hypothesis was then extended in 1905 by Einstein to the process of light diffusion in space. Discovery in the early twentieth century of new corpuscular properties of light in photovoltaic phenomena, led to new discussions about whether the quantum of light radiation is a particle or a wave. In 1911. H. Lorentz at the First Solvay Conference noted that the quantum hypothesis of light seems completely incompatible with the conclusions of the wave theory, tested experimentally. As within the limits of classical electrodynamics of Maxwell this question remained unsolved, the opinion about dual nature of light, i.e. its corpuscular-wave dualism was established. In the theory of elementary particles, the quantum of light - photon is considered to be a stable full-fledged elementary particle with zero mass, having an internal momentum moment - spin, participating in interactions between other particles and giving rise to electron-positron pairs [3].

Radiation of Maxwell-Hertz waves is made by periodically changing the current in the emitting antenna with the frequency of the wave. In this case, the energy continuously carried by the wave is proportional to the square of the intensity of the excited antenna electromagnetic field. At radiation of electromagnetic wave of Maxwell-Hertz its frequency and energy can be changed smoothly and arbitrarily by technical methods independently of each other. Radiation of the photon energy in the atom occurs at a one-time and jump (quantum) transition of the electron from a high energy level to a lower level, with the emission of electromagnetic energy equal to the difference of energies of these levels [4]. In this case, the photon frequency is proportional to this energy divided by the Planck constant. Thus, the process of photon radiation in the atom is fundamentally different from the process of Maxwell-Hertz electromagnetic wave radiation, because there are no periodic fluctuations of the electron that form the photon radiation frequency. At the same time, the energy for the Maxwell-Hertz wave and for the photon has different expressions. Another important difference between the photon and the Maxwell-Hertz electromagnetic wave is the presence of an angular momentum - a spin, which is absent in the Maxwell-

Hertz plane waves. In order to give them the angular momentum, their circular polarization is necessary. This gave rise to discussions on the issue of angular momentum transmission by a flat electromagnetic wave [5] - [7], but no satisfactory solution to this problem has been found, and it was enrolled into the category of "eternal questions" of classical electrodynamics.

Maxwell-Hertz electromagnetic waves are well studied theoretically and experimentally, as they are described in the classical Dalamber wave equation and are widely used in practice. The corpuscular properties of light required an explanation of the physics of its distribution in space. As a result, the photon was seen as a compact package of flat electromagnetic waves. However, the presence of the photon's spin, the localization of its energy in space, the peculiarity of the process of radiation and transfer of electromagnetic energy indicate that its electromagnetic structure is different from that of the Maxwell-Hertz electromagnetic wave. In his work [8] A. Sommerfield writes, "There is no way to overcome the duality of light wave - light quantum". An attempt to solve this problem was made by J.J. Thomson in his article "The Nature of Light" published in 1925. [9]. In this paper he develops the theory that electromagnetic energy transfer is possible in two ways: waves of Maxwell-Hertz and quanta, which are vortex rings with an internal electromagnetic wave. The size of the vortex rings is related to the wavelength and the Planck constant. In this article Thomson considers the emission and absorption of vortex rings, their interference and diffraction processes on a qualitative level. As will be shown below, Thomson has come surprisingly close to understanding the mechanisms of electromagnetic energy transmission in space and understanding the structure of the photon. To date, the internal structure of the photon has not been strictly described either within the classical theory of EMF or within quantum electrodynamics.

The purpose of this paper is to attempt to describe the electromagnetic structure of the photon without going beyond the scope of classical field theory and to link this structure with the classical Maxwell-Hertz electromagnetic wave and quantum electrodynamics, i.e. the paper attempts to build a bridge between classical and quantum electrodynamics.

The classical theory of EMF operates with three-dimensional power concepts in the form of electric and magnetic field tenseness. Quantum electrodynamics operates with one energy concept in the form of four-dimensional electromagnetic potential. Due to the fact that the photon is a quantum object, and the consideration of electromagnetic field in this paper is carried out in the four-dimensional space Minkovsky (SRT space), the EMF will be described using the canonical four-dimensional electromagnetic potential $A_\nu(\varphi/c, i\cdot\mathbf{A})$, where φ and A - scalar and vector potentials EMF. Four-dimensional current density will be described in a vacuum in the form $J_\nu(\rho\cdot c, i\cdot\mathbf{J})$, where ρ and J, respectively, the density of charges and current. For the accepted representation of space-time geometry with imaginary spatial coordinates [10], the operator of a four-dimensional private derivative is $\partial_\mu(\partial_t / c, i \cdot \nabla)$. In this case, for the accepted description of a four-dimensional electromagnetic potential, current density, and a partial derivative, it is possible not to distinguish between covariant and contravariant indices.

**2 Tensor and equations of motion field electromagnetic potential**

We will find a four-dimensional derivative of electromagnetic potential $\mathbf{A}_\nu(\varphi/c, i \cdot \mathbf{A})$ in the Minkowski space, representing a 4-tensor [11]:

$$A_{\mu\nu} = \partial_\mu \mathbf{A}_\nu = \begin{pmatrix} \frac{1}{c^2}\partial_t\varphi & \frac{1}{c}i\cdot\partial_t A_x & \frac{1}{c}i\cdot\partial_t A_y & \frac{1}{c}i\cdot\partial_t A_z \\ -\frac{1}{c}i\cdot\partial_x\varphi & \partial_x A_x & \partial_x A_y & \partial_x A_z \\ -\frac{1}{c}i\cdot\partial_y\varphi & \partial_y A_x & \partial_y A_y & \partial_y A_z \\ -\frac{1}{c}i\cdot\partial_z\varphi & \partial_z A_x & \partial_z A_y & \partial_z A_z \end{pmatrix} \tag{1}$$

The trace of this tensor is a four-dimensional invariant of electromagnetic potential $I = \delta_{\mu\nu} A_{\mu\nu} = \partial_t \varphi / c^2 + \nabla \cdot \mathbf{A}$. If it is equal to zero, it passes to the Lorentz calibration condition $\partial_t \varphi / c^2 + \nabla \cdot \mathbf{A} = 0$, which can be considered as an equation of electromagnetic potential conservation. The tensor (11) is asymmetric and can be broken down into symmetric and anti-symmetric tensors of derived electromagnetic potentials:

$$A_{\mu\nu} = A_{(\mu\nu)}/2 + A_{[\mu\nu]}/2 = (\partial_\mu \mathbf{A}_\nu + \partial_\nu \mathbf{A}_\mu)/2 + (\partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu)/2 \tag{2}$$

Let's write the symmetric tensor $A_{(\mu\nu)}$ in matrix form:

$$A_{(\mu\nu)} = \partial_\mu \mathbf{A}_\nu + \partial_\nu \mathbf{A}_\mu = \begin{pmatrix} 2\frac{1}{c^2}\partial_t\varphi & \frac{1}{c}i\cdot(\partial_t A_x - \partial_x\varphi) & \frac{1}{c}i\cdot(\partial_t A_y - \partial_y\varphi) & \frac{1}{c}i\cdot(\partial_t A_z - \partial_z\varphi) \\ \frac{1}{c}i\cdot(\partial_t A_x - \partial_x\varphi) & 2\partial_x A_x & (\partial_x A_y + \partial_y A_x) & (\partial_x A_z + \partial_z A_x) \\ \frac{1}{c}i\cdot(\partial_t A_y - \partial_y\varphi) & (\partial_x A_y + \partial_y A_x) & 2\partial_y A_y & (\partial_y A_z + \partial_z A_y) \\ \frac{1}{c}i\cdot(\partial_t A_z - \partial_z\varphi) & (\partial_x A_z + \partial_z A_x) & (\partial_y A_z + \partial_z A_y) & 2\partial_z A_z \end{pmatrix} \tag{3}$$

This symmetric tensor describes the strain stress of an EMF. Its trace $2(\partial_t \varphi / c^2 + \nabla \cdot \mathbf{A})$ describes the stresses of four-dimensional volumetric tensile/compression strain. Let's record the asymmetric tensor $A_{[\mu\nu]}$ in a matrix form:

$$A_{(\mu\nu)} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu = \begin{pmatrix} 0 & \frac{1}{c}i\cdot(\partial_t A_x + \partial_x\varphi) & \frac{1}{c}i\cdot(\partial_t A_y + \partial_y\varphi) & \frac{1}{c}i\cdot(\partial_t A_z + \partial_z\varphi) \\ -\frac{1}{c}i\cdot(\partial_t A_x + \partial_x\varphi) & 0 & (\partial_x A_y - \partial_y A_x) & (\partial_x A_z - \partial_z A_x) \\ -\frac{1}{c}i\cdot(\partial_t A_y + \partial_y\varphi) & (\partial_y A_x - \partial_x A_y) & 0 & (\partial_y A_z - \partial_z A_y) \\ -\frac{1}{c}i\cdot(\partial_t A_z + \partial_z\varphi) & (\partial_z A_x - \partial_x A_z) & (\partial_z A_y - \partial_y A_z) & 0 \end{pmatrix} \tag{4}$$

The antisymmetric tensor $A_{[\mu\nu]}$ is a 4-rotor and describes the four-dimensional rotation of EMFs.

The equations of EMF motion in the absence of sources will be found as a sum divergences of tensor $A_{\mu\nu}$ for each of the indices, equating this sum to zero. The sum of divergences of the symmetric tensor is equal to double the divergence for any of its indices, and the sum of divergences for each of the indices of the antisymmetric tensor is equal to zero, in this case:

$$\partial_\mu A_{\mu\nu} + \partial_\nu A_{\mu\nu} = \partial_\mu (A_{(\mu\nu)}/2 + A_{[\mu\nu]}/2) + \partial_\nu (A_{(\mu\nu)}/2 + A_{[\mu\nu]}/2) = \partial_\mu A_{(\mu\nu)} = \partial_\nu A_{(\mu\nu)} = 0 \quad (5)$$

Let us introduce field sources into the equations of motion by equating the sum of divergences (5) to the 4-vector of current density:

$$\partial_\mu A_{\mu\nu} + \partial_\nu A_{\mu\nu} = \partial_\mu A_{(\mu\nu)} = J_\mu \quad (6)$$

Let us record these equations of motion in unfolded three-dimensional form:

$$(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi) + \partial_t(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}) = \rho/\varepsilon_0 \quad (7)$$

$$(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}) - \nabla(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}) = \mu_0\mathbf{J} \quad (8)$$

After Lorentz calibration is applied to them, they are transformed into Maxwell's canonical equations in potentials [12]:

$$\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi = \rho/\varepsilon_0 \qquad \frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A} = \mu_0\mathbf{J}$$

In Eqs. (7) and (8) the members in the first parentheses have wave character for scalar and vector potentials of EMF, and the members in the second parentheses are included in the equation of electromagnetic potential conservation. Considering that waves do not carry the material medium, Eqs. (7) and (8) can be considered as generalized equations of electromagnetic potential conservation in the Minkowski space.

### 3 Maxwell-Hertz wave equation and photon equation

We take the gradient of both parts of Eq. (7) and the time derivative of both parts of Eq. (8):

$$\nabla(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi) + \nabla\partial_t(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}) = \nabla\rho/\varepsilon_0 \quad (9)$$

$$\partial_t(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}) - \partial_t\nabla(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}) = \mu_0\partial_t\mathbf{J} \quad (10)$$

Then, having found their sum and difference, we will get two equations of EMF motion in form:

$$(\frac{1}{c^2}\partial_{tt}-\Delta)(\nabla\varphi+\partial_t\mathbf{A})=\nabla\rho/\varepsilon_0+\mu_0\mathbf{J} \tag{11}$$

$$(\frac{1}{c^2}\partial_{tt}-\Delta)(\nabla\varphi-\partial_t\mathbf{A})+2\partial_t\nabla(\partial_t\varphi/c^2+\nabla\cdot\mathbf{A})=\nabla\rho/\varepsilon_0-\mu_0\partial_t\mathbf{J} \tag{12}$$

Let us rewrite Eq. (11) as follows:

$$(\Delta-\frac{1}{c^2}\partial_{tt})(-\partial_t\mathbf{A}-\nabla\varphi)=\Delta\mathbf{E}-\frac{1}{c^2}\partial_{tt}\mathbf{E}=\nabla\rho/\varepsilon_0+\mu_0\partial_t\mathbf{J} \tag{13}$$

Here $\mathbf{E}=-\partial_t\mathbf{A}-\nabla\varphi$ - the tenseness of the electric field. Eq. (13) is a canonical wave equation for electric field, describing Maxwell-Hertz electromagnetic waves and their source.

Eq. (12) has a more complex structure than Eq. (11) and is unknown in classical EMF theory, as the author believes. Its right part describes a field source different from the Maxwell-Hertz wave source in Eq. (13). Let us consider Eq. (12) without the source by rewriting it in the form:

$$(\Delta-\frac{1}{c^2}\partial_{tt})(\nabla\varphi-\partial_t\mathbf{A})=2(\partial_{tt}\nabla\varphi/c^2+\nabla\times\nabla\times\partial_t\mathbf{A}+\Delta\partial_t\mathbf{A}) \tag{14}$$

Let's write this equation in terms of electric field:

$$(\Delta-\frac{1}{c^2}\partial_{tt})(\mathbf{E}_P-\mathbf{E}_V)=2(\partial_{tt}\mathbf{E}_P/c^2+\nabla\times\nabla\times\mathbf{E}_V+\Delta\mathbf{E}_V) \tag{15}$$

Here $\mathbf{E}_P=-\nabla\varphi$ and, $\mathbf{E}_V=-\partial_t\mathbf{A}$ accordingly, the potential and vortex electric field. The left part of the waves equation describes the waves of difference of values potential electric field and vortex electric field, propagating at the speed of light. The right part of Eq. (12) is similar to description of vortex ring structure in continuous elastic medium [13], i.e. it can be considered as description of dynamic electromagnetic vortex ring created by electric field. Vortex rings are well known in gas-hydrodynamics, they have high stability and duration of existence in time [14] - [17]. This is explained by the fact that their equation is directly derived from the equation of the law of conservation of energy-impulse of a continuous medium [13]. As mentioned above, Eqs. (7) and (8) and, consequently, Eqs. (11) and (12) can also be considered as generalized equations of conservation of electromagnetic potential. Vortex rings of gas-hydrodynamics are localized in space and have some properties of particles, carry momentum and angular momentum. Eq. (15) reflects the equivalence of the left wave propagating at the speed of light to the right vortex ring. Thus, Eq. (15) can be considered as a description of a local electromagnetic object having a wave and corpuscular vortex part, which can be considered as a manifestation of the wave-particle duality and identify it with the photon.

The electric field is a space-time component of the antisymmetric tensor (4), i.e. a component of a four-dimensional rotor. Therefore, the wave Eq. (13) for the electric field can be considered as a description of the wave four-dimensional rotation of EMF. The electric field corresponds to its energy, therefore, the Maxwell-Hertz waves transfer the energy of the four-dimensional rotation of the EMF.

The left part of Eq. (14) describes a wave of space-time component of symmetric tensor (3), therefore, the left part of this equation can be considered as a description of the stress wave of four-dimensional shear strain EMF. The right side of Eq. (14) is a spatial and temporal derivative of the diagonal components of the symmetric tensor (3). Therefore, it describes the dynamics of the four-dimensional volumetric tensile/compressive deformation of the EMF. It can be concluded that Maxwell-Hertz waves energy transfer of four-dimensional EMF rotation, and photons energy transfer of four-dimensional EMF deformation. This explains the difference in expressions for the energy of the Maxwell-Hertz electromagnetic wave and the electromagnetic radiation quantum.

The trivial solution for the wave part of Eq. (14) is equality $\nabla\varphi - \partial_t \mathbf{A} = 0$, which can be written as $\nabla\varphi - \partial_t \mathbf{A} = -\mathbf{E}_P + \mathbf{E}_V = 0$. With the equality of these electric fields, the wave part is equal to zero and the wave is absent. To these electric fields corresponds the energy proportional $\mathbf{E}_P^2$ and $\mathbf{E}_V^2$ then the wave part of Eq. (12) exists at $\mathbf{E}_V^2 - \mathbf{E}_P^2 \neq 0$. With respect to the atom it can be considered as excitation of a wave in the considered point of space, when the energy of potential electric field of the nucleus in this point is not compensated by the energy of the vortex electric field created by the electron. On the stationary orbit of the electron in the atom this difference of energies is equal to zero and there is no radiation. In case of its transition to a stationary orbit with higher binding energy with the nucleus, photon radiation occurs with a frequency proportional to the energy difference of these stationary orbits. Since the left part of Eq. (14) for the atom changes discretely, the right part describing the vortex ring also changes discretely. The third term of the right part of Eq. (14) can be written as a rotor the vortex electric field $\nabla\times\nabla\times\partial_t \mathbf{A} = -\nabla\times\nabla\times\mathbf{E}_V$. The rotation of this field corresponds to the rotation of energy $\mathbf{E}_V^2$ or mass, indicating that the vortex ring has an internal angular momentum that can be identified as spin. As is known, spin is a purely quantum phenomenon and does not correspond to the angular momentum in mechanics [18]. For example, when rotating the electron spin vector it comes to its initial position through $720^{o}$ instead of $360^{o}$ as in mechanics. This is confirmed by the expression for rotation of the vortex electric field, which suggests that the spin can be identified with toroidal rotation [19]. Thus, equation (12) describes the radiation of an electromagnetic object, which can be identified on a qualitative level as a radiated photon moving in space with the velocity of light, having wave and corpuscular properties, localization in space, the internal moment of the pulse - spin.

In experiments with vortex rings, they are observed wave processes called Kelvin waves [20], which is confirmed by Eq. (14). Since the Kelvin waves are internal oscillations of vortex rings and have the form of spatial standing waves, they can be attributed the properties of monochrome and discrete, associated with the size of the vortex rings and the wavelength of these vibrations. Similar properties can be ascribed to photons. Hence, the frequency of radiation, a photon is determined by the geometry of the vortex ring, i.e. energy, instead of the frequency of electric charge oscillations at radiation, as at waves Maxwell-Hertz. All this is consistent with the views of J.J. Thomson on the structure of light [9].

Eq. (12) does not limit the frequency range of electromagnetic objects it describes. As a result, it is possible to generate them in the low frequency range of electromagnetic radiation with a long wavelength. Such photons can be called macrophotons. Considering that the energy of photons is proportional to the frequency multiplied by the Planck constant, the energy of macrophotons is small. Therefore, macrophotons of the terahertz range of radiation have the greatest perspective for practical use. The methods of generation of macrophotons differ from those of Maxwell-Hertz. Since photons are excited at quantum transitions, i.e., energy surges, jump-shifts of electromagnetic energy are an indispensable condition for generation of macrophotons. Not all existing generators of radiation of the terahertz range fulfill this condition and, despite the pulse mode of their operation, they emit Maxwell-Hertz waves of this range. Generators that use a laser principle, e.g. a quantum cascade laser, meet the condition of macrophoton generation [21]. At present, there are generators of pulsed radiation of the terahertz range, in which femtosecond lasers discharge a precharged electric dipole [22]. These generators correspond in their principle to the requirements for the emission of macrophotons. However, the radiation of this dipole contains a Maxwell-Hertz wave perpendicular to the dipole axis and macrophotons radiated along the dipole axis. The properties of Maxwell-Hertz waves and the properties of macrophotons of the same frequency must be different when interacting with the substance. Therefore, it is necessary to learn to separate these two types of electromagnetic radiation from each other in order to study these properties. Since the terahertz range of electromagnetic radiation is of great interest for practical use, understanding the structure of macrophotons is an important knowledge for its development.

Thus, from the EMF tensor (1) two possible forms of EMF motion and electromagnetic energy transfer through space follow: classical electromagnetic waves of Maxwell-Hertz (13) and wave-corpuscules (15), called photons and being quanta of EMF energy. Figuratively speaking, Eqs. (13) and (15) show that the transmission of electromagnetic energy through space has "two sides of the same coin" - wave and corpuscular-wave.

The conclusion of Eqs. (11) and (12) is made only on the basis of an axiom about four-dimensional pseudo-Euclidean space of Minkowski and four-dimensional electromagnetic potential $\mathbf{A}_\nu(\varphi/c, i\cdot\mathbf{A})$, without any other assumptions. From this we can conclude that the corpuscular-wave duality of photons and two types of electromagnetic radiation are the consequence of pseudo-Euclidean space geometry.

## 4 Conclusion

Consideration of electromagnetic field motion in the Minkowski space using canonical electromagnetic potential $\mathbf{A}_\nu(\varphi/c, i\cdot\mathbf{A})$ and tensor of its derivatives leads to two forms of equations of electromagnetic field motion: in the form of classical Maxwell-Hertz waves and corpuscular-wave form in the form of photons. Thus, the problem of wave-particle duality is solved within the classical electrodynamics. Understanding the internal structure of the photon allows developing the field of photon technique in the low-frequency range of radiation for practical purposes. An important feature of macrophotons distinguishing them from Maxwell-Hertz waves is their natural spatial localization, radial propagation, and principal monochromicity due to their spatial structure.

Corpuscular-wave duality of photons is their fundamental property and is caused by pseudo-Euclidean space-time geometry.

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