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Beal Conjecture Proved by the Scientific Approach

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Abstract

Using the "scientific approach", the author proves directly the original Beal conjecture (and **not** the equivalent conjecture) that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. One will let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$. Then, the equation $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. Seven numerical Beal equations were factored. Based on the consistent pattern of the structure of the relationships between the prime factors on the left sides of the equations and the prime factors on the right sides of the equations in the factorizations, the author conjectured the equalities, $r^x = t^x$ and $s^y = t^y$, which would imply that $r = s = t$, and establish that the Beal conjecture is true. The proof would be complete after showing that $r^x = t^x$ and $s^y = t^y$. The proof in this paper is an expansion of a previous paper (viXra:2001.0694) by the author. The proofs of the above equalities would be complete after showing that the ratios, $\frac{r^x}{t^x} = 1$ and $\frac{s^y}{t^y} = 1$. To accomplish these relationships, one will factor out r^x on the left side of the equation, $D^x r^x + E^y s^y = F^z t^z$, followed by factoring out s^y of the same equation. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of each prime factor on the left side of the equation equals the same power of the prime factor on the right side of the equation. High school students can learn and prove this conjecture as a bonus question on a final class exam.

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Option 1
Preliminaries
Introduction

The following is from the first page of the author's high school practical physics note book: Science is the **systematic observation** of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Beal conjecture, one will apply the scientific approach and be guided by the properties of the factored Beal equations. One will apply a systematic observation of the factorizations of seven numerical Beal equations, and note the consistent pattern of the relationships between the prime factors on the left sides and right sides of the factored equations.

Observation 1: $\boxed{2^3 + 2^3 = 2^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$2^3 + 2^3 = 2^4$$
$$2^3 + 2^3 = 2^3 \bullet 2$$
$$\underbrace{2^3}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side..

Note above that the greatest common power of the prime factors on the left of the equation is the same as a power of the prime factor on the right side of the equation.

Note also, the following

The ratio $\dfrac{K}{M} = \dfrac{2^3}{2^3} = 1$.

If $\dfrac{K}{M} = 1$, then $K = M$

Similarly, $\dfrac{P}{L} = \dfrac{2}{1+1} = 1$.

If $\dfrac{P}{L} = 1$, then $P = L$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 2$$
$$x = 3.y = 3, z = 4$$
$$(D = 1, E = 1, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \quad P = L$$

Observation 2: $\boxed{7^6 + 7^7 = 98^3}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$7^6 + 7^7 = 98^3$$
$$7^6 + 7^6 \bullet 7 = (49 \bullet 2)^3$$
$$7^6 + 7^6 \bullet 7 = 7^6 \bullet 2^3$$
$$7^6(1+7) = 7^6 \bullet 2^3$$
$$\underbrace{7^6}_{K}(\underbrace{1+7}_{L}) = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{7^6}{7^6} = 1$.

Similarly, $\frac{P}{L} = \frac{2^3}{1+7} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 7$$
$$x = 6, y = 7, z = 3$$
$$(D = 1, E = 1, F = 14)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = L$$

Observation 3: $\boxed{3^3 + 6^3 = 3^5}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^3 + 6^3 = 3^5$$
$$3^3 + (3 \bullet 2)^3 = 3^5$$
$$3^3 + 3^3 \bullet 2^3 = 3^5$$
$$3^3(1+2^3) = 3^3 \bullet 3^2$$
$$\underbrace{3^3}_{K}(\underbrace{1+8}_{L}) = \underbrace{3^3}_{M} \bullet \underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{3^3}{3^3} = 1$.

Similarly, $\frac{P}{L} = \frac{3^2}{1+8} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 3$$
$$x = 3, y = 3, z = 5$$
$$(D = 1, E = 2, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = L$$

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Observation 4: $\boxed{2^9 + 8^3 = 4^5}$

Identify the greatest common factor.of all three terms of the equation and factor it out on the left side.

$$2^9 + 8^3 = 4^5$$
$$2^9 + ([2^3])^3 = ([2^2])^5$$
$$2^9 + 2^9 = 2^{10}$$
$$2^9(1+1) = 2^9 \bullet 2$$
$$\underbrace{2^9}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^9}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{2^9}{2^9} = 1$.

Similarly, $\frac{P}{L} = \frac{2}{1+1} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 2$$
$$x = 9, y = 3, z = 5$$
$$(D = 1, E = 4, F = 2)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = L$$

Observation 5: $\boxed{34^5 + 51^4 = 85^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$34^5 + 51^4 = 85^4$$
$$(17 \bullet 2)^5 + (17 \bullet 3)^4 = (17 \bullet 5)^4$$
$$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$$
$$17^4(17 \bullet 2^5 + 3^4) = 17^4 \bullet 5^4$$
$$\underbrace{17^4}_{k}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$$

(Note: $17 \bullet 2^5 + 3^4 = 17 \bullet 32 + 81 = 625$; $5^4 = 625$)

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{17^4}{17^4} = 1$.

Similarly, $\frac{P}{L} = \frac{5^4}{17 \bullet 2^5 + 3^4} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 17$$
$$x = 5, y = 4, z = 4$$
$$(D = 2, E = 3, F = 5)$$
$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$
$$K = M, \ P = L$$

Note above that one factored out s^y .

One will apply the switch from r^x to s^y in the conjecture proof.

Observation 6: $\boxed{3^9 + 54^3 = 3^{11}}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^9 + 54^3 = 3^{11}$$
$$3^9 + (3^3 \bullet 2)^3 = 3^{11}$$
$$3^9 + 3^9 \bullet 2^3 = 3^{11}$$
$$3^9(1+2^3) = 3^9 \bullet 3^2$$
$$\underbrace{3^9}_{k}(\underbrace{1+2^3}_{L}) = \underbrace{3^9}_{M} \bullet \underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side

Note the following

The ratio $\frac{K}{M} = \frac{3^9}{3^9} = 1$.

Similarly, $\frac{P}{L} = \frac{3^2}{1+2^3} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 3$$
$$x = 9, y = 3, z = 11$$
$$(D = 1, E = 18, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M,\ P = L$$

Observation 7: $\boxed{33^5 + 66^5 = 33^6}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$33^5 + 66^5 = 33^6$$
$$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5 = (11 \bullet 3)^6$$
$$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$$
$$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$$
$$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

$\frac{K}{M} = \frac{11^5}{11^5} = 1$. Also $\frac{P}{L} = \frac{11 \bullet 3^6}{3^5 + 2^5 \bullet 3^5} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 11$$
$$x = 5, y = 5, z = 6$$
$$D = 3, E = 6, F = 3$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M,\ P = L$$

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Summary of Observations 1-7 and Conjectures

The most important and useful observation in the above examples is the consistency that the greatest common power of the prime factors on the left side of the equation equals the same power of the prime factor on the right side of the equation. This observation was 100% consistent in all the factored numerical Beal equations, and will be applied in proving Beal conjecture. **P**articularly, one conjectures that $r^x = t^x$ and $s^y = t^y$. After proving these conjectures, one will conclude that $r = s = t$, and establish that Beal conjecture is true.

1.

$2^3 + 2^3 = 2^4$ (a)

$2^3(1+1) = 2^3 \bullet 2$ (b)

$\underbrace{2^3}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$

$x = 3, y = 3, z = 4$

$r = s = t = 2$

$r^x = 2^3;\ \ t^x = 2^3$

$t^{z-x} = 2^{4-3} = 2$

2.

$7^6 + 7^6 \bullet 7 = 7^6 \bullet 2^3$ (a)

$7^6(1+7) = 7^6 \bullet 2^3$ (b)

$\underbrace{7^6}_{K}(\underbrace{1+7}_{L}) = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$

$x = 6, y = 7, z = 3$

$r = s = t = 7$

$r^x = 7^6;\ \ t^x = 7^6$

3.

$3^3 + 3^3 \bullet 2^3 = 3^5$ (a)

$3^3(1+2^3) = 3^3 \bullet 3^2$ (b)

$\underbrace{3^3}_{K}(\underbrace{1+8}_{L}) = \underbrace{3^3}_{M} \bullet \underbrace{3^2}_{P}$

$x = 3, y = 3, z = 5$

$r = s = t = 3$

$r^x = 3^3;\ \ t^x = 3^3$

$t^{z-x} = 3^{5-3} = 3^2$

4.

$2^9 + 2^9 = 2^{10}$ (a)

$2^9(1+1) = 2^9 \bullet 2$ (b)

$\underbrace{2^9}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^9}_{M} \bullet \underbrace{2}_{P}$

$x = 9, y = 3, z = 5$

$r = s = t = 2$

$r^x = 2^9 \ \ t^x = 2^9$

$F = 2$

$t^{z-x} = 2^{5-9} = 2^{-4}$

Note : $t^x \bullet t^{z-x} \bullet F^z$

$= 2^9 \bullet 2^{5-9} \bullet 2^5$

$= 2$

5.

$34^5 + 51^4 = 85^4$

$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$

$17^4(17 \bullet 2^5 + 3^4) = 17^4 \bullet 5^4$

$\underbrace{17^4}_{K}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$

$x = 5, y = 4, z = 4$

$r = s = t = 17$

$s^y = 17^4;\ \ t^y = 17^4$

$t^{z-y} = 17^{4-4} = 17^0 = 1$

Note above that the 17^4 is based on the second term.

6.

$3^9 + 3^9 \bullet 2^3 = 3^{11}$ (a)

(a) $3^9(1+2^3) = 3^9 \bullet 3^2$ (b)

(b) $\underbrace{3^9}_{k}(\underbrace{1+2^3}_{L}) = \underbrace{3^9}_{M} \bullet \underbrace{3^2}_{P}$

$x = 9, y = 3, z = 11$

$r = s = t = 3$

$r^x = 3^9;\ \ t^x = 3^9$

$t^{z-x} = 3^{11-9} = 3^2$

7.

$33^5 + 66^5 = 33^6$

$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$ (a)

$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$ (b)

$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$

$x = 5, y = 5, z = 6$

$r = s = t = 11$

$r^x = 11^5;\ \ t^x = 11^5$

$t^{z-x} = 11^{6-5} = 11$

Corresponding relationship formulas

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^{z}}_{P};\ \ K = M,\ \ P = L$$

or

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^{z}}_{P};K = M,\ \ P = L$$

Properties of the Factored Beal Equation

Let r, s and t be prime factors of A, B and C .respectively, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.** where D, E and F are positive integers; and the equation becomes $(Dr)^x + (Es)^y = (Ft)^z$.

Step 1: Factor out r^x on the left side of the equation and on the right side of the equation, replace t^z by $t^x \bullet t^{z-x}$ (Note $t^x \bullet t^{z-x} = t^z$)

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^y = F^z t^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P};\ K = M,\ \ P = L$$

For the factorization $\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$ with respect to r^x, $r^x = t^x$ $(K = M)$

Step 2: Factor out s^y on the left side of the equation and on the right side of the equation, replace t^z by $t^y \bullet t^{z-y}$ (Note $t^y \bullet t^{z-y} = t^z$)

$$(Es)^y + (Dr)^x + = (Ft)^z$$

$$E^y s^y + D^x r^x = F^z t^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}\ \ ;K = M,\ \ P = L$$

For the factorization $\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$ with respect to s^y, $s^y = t^y$ $(K = M)$

Main Principles Involved in Proving Beal Conjecture

1. From all the above seven numerical factorizations, the ratio $\frac{r^x}{t^x} = 1$, and this implies that $r^x = t^x$

2. Similarly, the ratio, $\frac{s^y}{t^y} = 1$ implies that $s^y = t^y$

3. If $r^x = t^x$ and $s^y = t^y$, then $r = s = t$. (obtained by applying logarithmic properties)

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Option 2
Beal Conjecture Proved by the Scientific Approach

Given: $A^x + B^y = C^z$, A, B, C, x, y, z are positive integers and $x, y, z > 2$.
Required: To prove that A, B and C have a common prime factor.
Plan: Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$. Then, the equation $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after showing that $r = s = t$. The conjectured equalities, $r^x = t^x$ and $s^y = t^y$ from the numerical factorizations will be proved by showing that $\frac{r^x}{t^x} = 1$ and $\frac{s^y}{t^y} = 1$.
The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of each prime factor on the left side of the equation equals the same power of the prime factor on the right side of the equation. Two main steps are involved in the proof. In the first step, one will determine how r and t are related, and in the second step, one will determine how s and t are related.

Proof

Step 1: The conjectured equality, $r^x = t^x$ from the factored numerical factorizations will be true if and only if $\frac{r^x}{t^x} = 1$.

$D^x r^x + E^y s^y = F^z t^z$. (1) (Given)

$\frac{D^x r^x + E^y s^y}{F^z t^z} = 1$ (Dividing both sides by $F^z t^z$) (2)

Approach 1

One will factor out the r^x on the left side of the equation, $D^x r^x + E^y s^y = F^z t^z$, and on the right side, let $t^z = t^x t^{z-x}$, where t^x is of the same power as r^x, to obtain

$r^x(D^x + E^y s^y r^{-x}) = t^x t^{z-x} F^z$ (3)

Replace the t^x on the right side of equation (3) by r^x to obtain

$r^x(D^x + E^y s^y r^{-x}) = r^x t^{z-x} F^z$

or $r^x(D^x + E^y s^y r^{-x}) = r^x t^{-x} F^z t^z$ (4)

Remove the parenthesis on the left side of the equation and on the right side write with positive exponents to obtain

$D^x r^x + E^y s^y = \frac{r^x}{t^x} \bullet F^z t^z$ (5)

$\frac{D^x r^x + E^y s^y}{F^z t^z} = \frac{r^x}{t^x}$ (solving for $\frac{r^x}{t^x}$)

$1 = \frac{r^x}{t^x}$ (From (2), above, $\frac{D^x r^x + E^y s^y}{F^z t^z} = 1$)

Since $\frac{r^x}{t^x} = 1$, $r^x = t^x$.

If $r^x = t^x$, $r = t$. ($\log r^x = \log t^x; x \log r = x \log t; \log r = \log t;\ r = t$)

Approach 2

One will factor out the r^x on the left side of the equation, $D^x r^x + E^y s^y = F^z t^z$, and on the right side, let $t^z = t^x t^{z-x}$, where t^x is of the same power as r^x, to obtain

$r^x(D^x + E^y s^y r^{-x}) = t^x t^{z-x} F^z$ (3)

$$\frac{r^x}{t^x} = \frac{t^{z-x}F^z}{D^x + E^y s^y r^{-x}} \quad \text{(Divide the left side by } t^x \text{ and the right side by } D^x + E^y s^y r^{-x}\text{)}$$

$$\frac{r^x}{t^x} = \frac{t^{-x}F^z t^z}{D^x + E^y s^y r^{-x}}$$

$$\frac{r^x}{t^x} = \frac{F^z t^z}{t^x(D^x + E^y s^y r^{-x})} \quad \text{(The } t^{-x} \text{ in the numerator becomes } t^x \text{ in the denominator)}$$

$$= \frac{F^z t^z}{r^x(D^x + E^y s^y r^{-x})} \quad \text{(Replacing } t^x \text{ by } r^x\text{)}$$

$$\frac{r^x}{t^x} = \frac{F^z t^z}{D^x r^x + E^y s^y} \quad \text{(Removing the parenthesis)}$$

$$\frac{r^x}{t^x} = 1 \quad \text{(From (2) of Step 1, } \frac{F^z t^z}{D^x r^x + E^y s^y} = \frac{D^x r^x + E^y s^y}{F^z t^z} = 1\text{)}$$

Since $\frac{r^x}{t^x} = 1$, $r^x = t^x$

If $r^x = t^x$, $r = t$. $(\log r^x = \log t^x; x\log r = x\log t; \log r = \log t;\ r = t)$

Step 2: The conjectured equality, $s^y = t^y$ from the factored numerical factorizations will be true if and only if $\frac{s^y}{t^y} = 1$.

Approach 1

One will factor out the s^y on the left side of the equation, $D^x r^x + E^y s^y = F^z t^z$ and let $t^z = t^y t^{z-y}$ on the right side, where t^y is of the same power as s^y, to obtain

$s^y(E^y + D^x r^x s^{-y}) = t^y t^{z-y} F^z$. (3)

Replace the t^y on the right side equation (3) by s^y to obtain

$s^y(E^y + D^x r^x s^{-y}) = s^y t^{z-y} F^z$ or $s^y(E^y + D^x r^x s^{-y}) = s^y t^{-y} F^z t^z$

Remove the parenthesis on the left side of the equation and on the right side, write with positive exponents to obtain $E^y s^y + D^x r^x = \frac{s^y}{t^y} \bullet F^z t^z$ **or** $D^x r^x + E^y s^y = \frac{s^y}{t^y} \bullet F^z t^z$

$$\frac{D^x r^x + E^y s^y}{F^z t^z} = \frac{s^y}{t^y} \quad \text{(Solving for } \frac{s^y}{t^y}\text{)}$$

$$1 = \frac{s^y}{t^y} \quad \text{(From (2) in Step 1, } \frac{D^x r^x + E^y s^y}{F^z t^z} = 1\text{)}$$

Since $\frac{s^y}{t^y} = 1$, $s^y = t^y$. If $s^y = t^y$, $s = t$. $(\log s^y = \log t^y;\ y\log s = y\log t;\ = \log s = \log t;\ s = t)$

It has been shown in Step 1 that $r = t$, and in Step 2 that $s = t$; therefore, $r = s = t$.

Since $A = Dr$, $B = Es$, $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, and the proof is complete.

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Approach 2

One will factor out the s^y on the left side of the equation, $D^x r^x + E^y s^y = F^z t^z$, and on the right side, let $t^z = t^y t^{z-y}$; where t^y is of the same power as s^y, to obtain

$s^y(E^y + D^x r^x s^{-y}) = t^y t^{z-y} F^z$. (3) .

$\frac{s^y}{t^y} = \frac{t^{z-y}F^z}{E^y + D^x r^x s^{-y}}$ (Dividing the left side of (3) by t^y and the right side by $E^y + D^x r^x s^{-y}$)

$\frac{s^y}{t^y} = \frac{t^{-y}F^z t^z}{E^y + D^x r^x s^{-y}}$ (Splitting t^{z-y} into t^z and t^{-y})

$\frac{s^y}{t^y} = \frac{F^z t^z}{t^y(E^y + D^x r^x s^{-y})}$ (The t^{-y} in the numerator becomes t^y in the denominator)

$= \frac{F^z t^z}{s^y(E^y + D^x r^x s^{-y})}$ (Replacing t^y by s^y)

$\frac{s^y}{t^y} = \frac{F^z t^z}{E^y s^y + D^x r^x}$ (Removing the parenthesis in the denominator)

$\frac{s^y}{t^y} = 1$ (From (2) of Step 1, $\frac{F^z t^z}{D^x r^x + E^y s^y} = \frac{D^x r^x + E^y s^y}{F^z t^z} = 1$)

Since $\frac{s^y}{t^y} = 1$, $s^y = t^y$**.**

If $s^y = t^y$, $s = t$. ($\log s^y = \log t^y$; $y\log s = y\log t$; $= \log s = \log t$; $s = t$)

It has been shown in Step 1 that $r = t$, and in Step 2 that $s = t$; therefore, $r = s = t$.

Since $A = Dr$ **,** $B = Es$ **,** $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, and the proof is complete..

Discussion

The proof was centered on proving the equalities $r^x = t^x$ and $s^y = t^y$ which imply that $r = s = t$, where r, s and t are prime factors of A, B and C respectively. In the previous paper viXra:2001.0694), the application of these equalities were based only on the properties of factored numerical Beal equations. In the present paper, in addition to the properties from numerical Beal equations, formal proofs of these equalities were covered. Fermat's Last Theorem can be proved by modifying the above proof as follows: For the hypothesis, let $x, y, z = n > 2$, $r \neq s \neq t$ and prove by contradiction (see viXra:2003.0303). Perhaps, the approach used in this paper is the approach to use in solving problems in number theory.

Conclusion

Using the "scientific approach", the original Beal conjecture (and **not** the equivalent conjecture) has been proved in this paper. Since the main concern of this conjecture is a common prime factor, it was appropriate that factorization was the main tool in observing the structure of the factorization of the equations. The factorization of the equations revealed the relationships between the prime factors involved. The proof was based on the two equalities, $r^x = t^x$ and $s^y = t^y$. which were conjectured from the factorizations of the numerical Beal equations.. From these. equalities, it was concluded that $r = s = t$, establishing the truthfulness of the Beal conjecture. High school students can learn and prove this conjecture as a bonus question on a final class exam.

PS: Other proofs of Beal Conjecture by the author are at viXra:2001.0694, viXra:1702.0331; viXra:1609.0383; viXra:1609.0157;

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