

A New Theoretical and Technological System of Imprecise-Information Processing

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Abstract Imprecise-information processing will play an indispensable role in intelligent systems, especially in the anthropomorphic intelligent systems (as human-machine dialogue and intelligent robots). Traditionally, the fuzzy set theory is used to deal with imprecise information, but which has some important theoretical and technical problems not solved very well. Recently, a new theoretical and technological system of imprecise-information processing has been founded (see literature [1]) which is different from fuzzy technology. The system results from the formation principle of imprecise information and has solid mathematical and logical bases, so which has many advantages beyond fuzzy technology. The system provides a technological platform for relevant applications and lays a theoretical foundation for further research.

Keywords Imprecise-Information Processing; Artificial Intelligence; Anthropomorphic Intelligent Systems; Intelligent Robots; Natural Language Processing; Machine Learning

1. Genesis

As with uncertain-information processing, imprecise-information processing is also an important component of artificial intelligence (including computational

intelligence and machine learning) technology. With the development of information technology and artificial intelligence technology as well as the rise in social requirements, imprecise-information processing is becoming more and more important and urgent, and it will play an indispensable role in intelligent systems, especially in the anthropomorphic intelligent systems such as machine perceptions, machine behaviors, human-machine dialogue, intelligent robots, etc.

In recent years, the uncertain-information processing technology has made great progress and achieved a lot of practical results, greatly promoting the rapid development of artificial intelligence. However, imprecise-information processing technology is still relatively weak. Although people presented many theories and methods for imprecise-information processing (for instance, Fuzzy Technology), a theoretical and technological system had not yet been formed that is widely approved and has solid foundation of mathematics and logic like that for uncertain-information processing. In fact, since the concept of fuzzy sets was proposed by Zadeh in 1965, the fuzzy technology based on fuzzy set theory has developed rapidly and made some achievements. However, so far, some important theoretical and technical problems in fuzzy information processing have not been well solved, such as the shape of the membership function of a fuzzy set, the objective basis and the logical basis of fuzzy logic operators, the logical basis of fuzzy reasoning, and “the dilemma between interpretability and precision” neural-fuzzy systems encountered^[2]. For this reason, not a few scholars work to improve and develop fuzzy set theory, and presented many new ideas, theories and methods, which all have their respective angles of views and characteristics. But on the whole, people haven't yet reached a common view, and the existed problems are neither solved really.

Recently, a new theoretical and technological system of imprecise-information processing founded in *Principles of Imprecise-Information Processing: A New Theoretical and Technological System*^[1] just emerges as the time requires. In the following, we will present and comment on the system in architecture, innovations, advantages, applications, and so on.

2. Architecture

The architecture of the theoretical and technological system of imprecise-information processing is shown in Figure 2-1. It can be seen from the figure that the system consists of seven modules and the logical relations among them are indicated by arrows. Of them, the module located in the bottom, i.e., *the formation of flexible linguistic values and their mathematical models*, is the basis of the whole system, and it is the origin of other modules. And the upper second model, i.e., *Imprecise-Problem Solving, Imprecise-Knowledge Discovery, and Anthropomorphic Intelligent Systems*, is the goal and application interface of the system, and it is supported by other modules. And the three modules located in the middle, i.e., *Approximate Reasoning with Flexible Linguistic Rules and Approximate Evaluation of Flexible Linguistic Functions, Truth-Degreed Logic and Flexible-Linguistic-Truth-Valued Logic*, and *Fundamental Theories of Flexible Sets and Flexible Linguistic Values*, are the main body and the core of the whole system. In addition, the first module located at the top is the extension of scope of imprecise-information processing and the vertical module located at the right is the overlap of and correlation between imprecise-information processing and uncertain-information processing. So, the hierarchy of the whole system is clear and the structure is rigorous.

Viewed from another angle, the system also has a feature, that is, in which there are many of symmetrical, antithetical or corresponding concepts and terminologies such as “flexible linguistic value” and “flexible set”, “membership function” and “consistency function”, “geometric model” and “algebraic model”, “combined linguistic value” and “synthetic linguistic value”, “form of possession” and “form of membership”, “logical composition” and “algebraical composition”, “conjunction-type rule” and “disjunction-type rule”, “complementary flexible partition” and “exclusive flexible partition”, “flexible linguistic value” and “rigid linguistic value”, “medium value” and “neutral value”, “L-N function” and “N-L function”, “certain rule” and “uncertain rule”, “natural logical semantics” and “extended logical semantics”, “reasoning with truth-degrees” and “reasoning with

believability-degrees”, “degree-true inference” and “near-true inference”, “numerical $\times\times$ ” and “linguistic $\times\times$ ”, “conceptual $\times\times$ ” and “practical $\times\times$ ”, “ $\times\times$ of single conclusion” and “ $\times\times$ of multiple conclusions”, “ $\times\times$ on the same space” and “ $\times\times$ from distinct spaces”, “one-dimensional $\times\times$ ” and “multi-dimensional $\times\times$ ”, “typical $\times\times$ ” and “non-typical $\times\times$ ”, and so forth, thus forming many pairs of parallel or complementary theories and methods — they are arranged in a crisscross pattern, and together constitute a multidimensional system of theories and technologies.

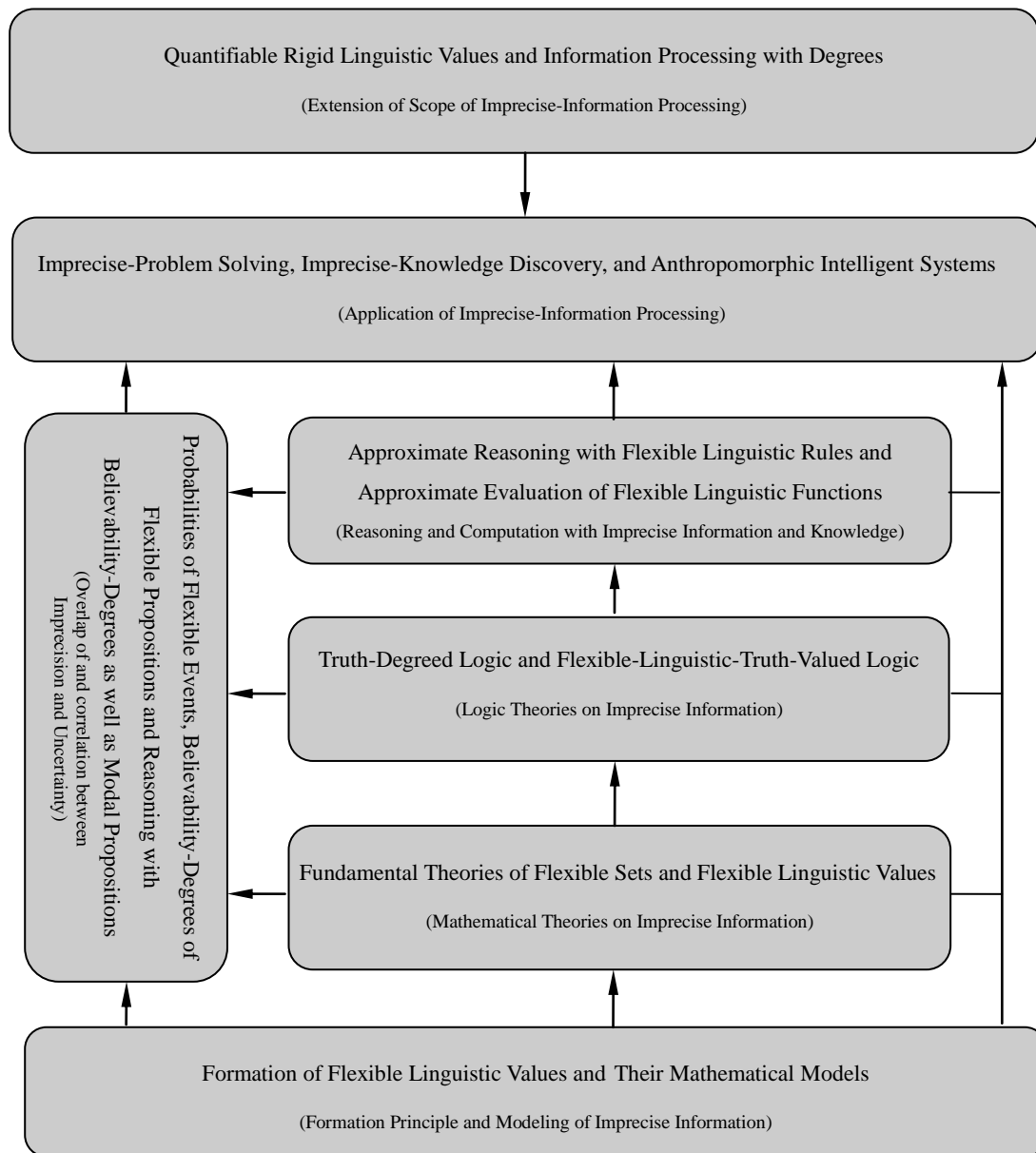


Figure 2-1 Architecture of the theoretical and technological system of imprecise-information processing

3. Innovations

As shown in Figure 2-1, the theoretical and technological system is founded on the basis of formation principle of imprecise information, and from literature [1], imprecise information originates from the continuity of magnitudes of features of things, or the uniform chain similarity of things (see Figure 3-1). These are all revealed and proposed in literature [1] firstly, so the whole system is new naturally. Specifically, the system has mainly the following innovations^[3]:

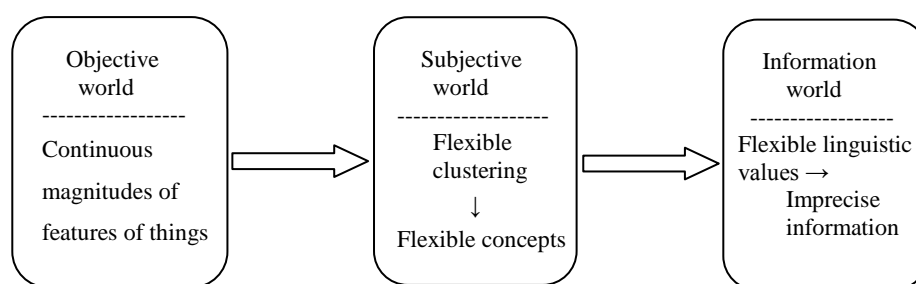


Figure 3-1 The diagram of the origin of imprecise information

(1) Treats explicitly imprecise-information processing as an independent research field outside of uncertain-information processing, and then, discriminates between vagueness (fuzziness) and flexibility of concepts, proposes the terminologies of “flexible concepts” and “flexible linguistic values”, and rectifies the so-called “vague (fuzzy) concepts” as “flexible concepts”.

(2) Presents general mathematical models and modeling methods for flexible concepts and flexible linguistic values according to the revealed formation principles of them.

(3) Proposes the concepts of flexible sets and flexible relations, and finds corresponding theories and methods.

(4) Finds the corresponding theories and methods of flexible linguistic values.

(5) Proposes the concepts of flexible linguistic function and correlation and the concepts of flexible number and flexible function, and finds corresponding theories and methods.

(6) Introduces truth-degrees, founds the basic theory of truth-degreed logic, and finds and presents the principles and methods of corresponding inference.

(7) Introduces flexible linguistic truth values, founds the basic theory of flexible-linguistic-truth-valued logic, and presents the principles and methods of the corresponding inference.

(8) Proposes the terminology and principle of logical semantics of propositions, and based on which establishes the computation models of truth values of basic compound propositions in two-valued logic and truth-degreed logic. In addition, proposes the concepts of negative-type logic and opposite-type logic.

(9) Introduces flexible linguistic rules and founds corresponding theories and methods.

(10) Introduces the adjoint functions of a flexible rule, and gives some acquiring methods and reference models.

(11) Clarifies logical and mathematical principles of the reasoning and computation with flexible linguistic rules, and gives a series of reasoning and computation approaches.

(12) Presents some approaches and ideas of the approximate evaluation of flexible linguistic functions according to the basic principles revealed.

(13) Presents the techniques and methods of problem solving according to the practical problems involving imprecise-information processing.

(14) Presents some methods and ideas of imprecise-knowledge discovery.

(15) Introduces several measures to sets and flexible sets, and founds corresponding theories.

(16) Introduces quantifiable rigid linguistic values and founds corresponding theories.

(17) Introduces the methodology of imprecise-information processing, and cites several application topics.

(18) Introduces random flexible events, and founds corresponding probability theory.

(19) Founds the believability-degree theory of flexible propositions and presents

the corresponding principle and method of reasoning with believability-degrees.

(20) Clears up the correlation between imprecise information (processing) and uncertain information (processing).

The above (1) and (2) are the basic principles of imprecise information; (3), (4) and (5) are the mathematical basic theory on imprecise information; (6), (7) and (8) are the logic basic theory on imprecise information; (9), (10), (11) and (12) are principles and techniques of inference and computation with imprecise information; (13) is the application techniques of imprecise-information processing; (14) is the acquiring techniques of imprecise knowledge; (15), (16) and (17) extend the extent of imprecise-information processing and give some basic methods and techniques of imprecise-information processing; (18), (19) and (20) are the overlapping of imprecise-information processing and uncertain-information processing, and clear up the connections and relations between the two.

It can be seen from above stated that the system is a new theoretical and technological system of imprecise-information processing which is different from fuzzy technology indeed. Of course, as viewed from the relationship between flexible sets and fuzzy sets, this system can also be regarded as an “amendment” to fuzzy technology. However, it is not obtained by following traditional thinking to make modifications and supplementations in the existing framework of fuzzy set theory, but rather, by tracing to the source and opening a new path to research and explore imprecise-information processing with a new perspective and idea.

Examining from the perspective of quantity, how much new knowledge does this system contain? It is not hard to see that the amount of new knowledge is consistent to the meaning of information content called in information theory. With the aid of this term in information theory, the “information content” of the system is as follows:

- Over 100 important concepts.
- Over 40 theorems.
- More than 100 formulas, functions and rules.
- Over 70 specific methods and algorithms.

4. Advantages

From the above stated, it can be seen that the new theoretical and technological system results from the formation principle of imprecise information and has solid mathematical and logical bases, and it avoids those problems occurring in the fuzzy technology. And from 3 above and theoretical analysis and example tests, it is shown that the system has many advantages beyond fuzzy technology. In the following we only make a simple comparison of approximate reasoning and computation in this new system and fuzzy reasoning and computation.

(1) Rationales

It's well known that the basic principle of traditional fuzzy reasoning is CRI (Compositional Rule of Inference) proposed by Zadeh, that is, using the called composition of fuzzy relations to realize approximate reasoning. Specifically, it is

$$B' = A' \circ R \quad (4.1)$$

Where R is a fuzzy relation (i.e., flexible relation called in the system) representing a fuzzy rule $A \rightarrow B$, A' is approximate to A which is viewed as a fuzzy relation, symbol ' \circ ' represents composition operator, and B' also a fuzzy set which is the result deduced by rule $A \rightarrow B$ and fact A' .

The membership functional representation form of equation (4.1) is

$$\mu_{B'}(y) = f(\mu_R(x, y), \mu_{A'}(x)) \quad (4.1')$$

where $\mu_R(x, y)$ is the so-called "implicational operator".

However, author's research shows that rule $A \rightarrow B$ can merely be represented as a binary rigid relation, but not a binary flexible relation, or a binary fuzzy relation (For details, see Sects. 13.5 and 13.6^[6] in [1]). In fact, in theory, rule $A \rightarrow B$ is equivalent to a binary rigid relation, but the binary rigid relation can not be definitely written out in general, so can only be represented by universal relation $\text{core}(A)^+ \times \text{core}(B)^+$ or $\text{supp}(A) \times \text{supp}(B)$.

In addition, we see that to obtain approximate conclusion B' , the rule $A \rightarrow B$ and fact A' would be used of course, but the conduct of directly making the two intersect

is improper perhaps. Because $A \rightarrow B$ is only a rule with linguistic values but not a linguistic function, and the fact A' does not match the antecedent A of $A \rightarrow B$, then, in this situation, doing directly composition of A' and $A \rightarrow B$ is unreasonable no matter from the logic or mathematics point of view. No wonder CRI is not compatible with traditional *modus ponens*, despite CRI is called generalized modus ponens in fuzzy set theory.

Actually, the reasoning with one rule $A \rightarrow B$ is an inference on properties, while the reasoning with two rules $A \rightarrow B$ and $B \rightarrow C$ just is an inference on relations. In CRI, the minor premise, namely fact A' , is extended as a binary relation, but the binary relation is only a pseudo binary relation, which is still a monadic relation really. In addition, the composition of relations is conditional, not any two relations can be composed; as for composing two rules of binary relation, they must satisfy transitivity.

Also, viewed from the proposed various implicational operators and formulas of relation composition, it can not be helped the objectivity of fuzzy reasoning arousing suspicion. In fact, from the implicational operator (presented by Zadeh)

$$\mu_R(x, y) = \max\{1 - \mu_A(x), \min\{\mu_A(x), \mu_B(y)\}\} \quad (4.3)$$

and the formula of relation composition

$$\mu_{B'}(y) = \min\{\mu_{A'}(x), \max\{1 - \mu_A(x), \min\{\mu_A(x), \mu_B(y)\}\}\} \quad (4.4)$$

it is not hard to see that $\min\{\mu_{A'}(x), \max\{1 - \mu_A(x), \min\{\mu_A(x), \mu_B(y)\}\}\}$ is actually the truth-degree of premise $(A \rightarrow B) \wedge A'$ of reasoning, while also treating it as the truth-degree of conclusion B' is then entirely artificially set.

And also, fuzzy reasoning does not consider the orientation of linguistic value A' relatively to antecedent linguistic value A of rule $A \rightarrow B$, but which is closely related to the accuracy of conclusion linguistic value B' and the exactness of corresponding number y' . Because viewed from the perspective of flexible linguistic functions, the approximate reasoning is also the approximate evaluation of a flexible linguistic function with a single pair of corresponding values.

Correspondingly, the approximate reasoning and computation approaches in the new system have logical and mathematical rationales.

In fact, the inferring part of the natural inference^[7] in the system is just usual *modus ponens*; and the reasoning with degrees^[7] in the system is following inference rule “truth-degree-level (degree-level)-UMP^[7]”, and in which the numerical computation models and methods in which are founded naturally on the bases of the mathematical essence of flexible rules and approximate reasoning, and under the constraints of the logical semantics of rules and the inference rules of “near-true”^[4] and “rough-true”^[5]. So it is logical. Just because of this, it is completely compatible with *modus ponens* in traditional logic, which is really the thinning and generalization of latter.

And AT method in the new system is then presented on the basis of the numerical models and the approximate evaluation principle of linguistic functions in viewpoint of mathematics completely, so its rationality is obvious (for the specific demonstration see Sect.16.2.3^[8] in [1]). As to other reasoning and computation methods (as interpolation method and approximate global function method) based on linguistic functions, since they are presented at the level of linguistic functions, so they have solid mathematical basis, and these methods are all closely related to mathematical backgrounds of practical problems, and the approximate computation with the adjoint measured functions of rules in the system is then directly applying the approximate background functions of corresponding rules to be realized. Therefore, the rationality of these methods is also beyond doubt.

Actually, from the perspective of logic, natural inference, reasoning with degrees and AT method are all the reasoning in the sense of near-true (truth-degree>0.5), that is, near-true inference said in the system (see Sect. 11.6^[4] in [1]); while fuzzy reasoning can roughly be counted as the reasoning in the sense of degree-true (truth-degree>0), that is, degree-true inference said in the system (see Sect. 11.5^[4] in [1]), and, the fuzzy reasoning with multiple rules (such as Mamdani-style inference and Sugeno-style inference) can roughly be included in the parallel reasoning with degrees said in the system (see Sect.15.6^[7] in [1]). From Chaps. 11^[4], 12^[5], 13^[6], and 15^[7] in [1] we see that near-true inference is consistent with the natural logical semantics of propositions, so it tallies with inference mechanism of human brain;

while degree-true inference is not consistent with the natural logical semantics of propositions (it corresponds to extended logical semantics of propositions), hence it does not tally with inference mechanism of human brain, particularly, the parallel reasoning with degrees based on the degree-true inference is more different from the inference mechanism of human brain; in addition, after parallel reasoning with degrees, for the synthesizing of multiple conclusions, there is no a unified method or model having theoretical basis.

(2) Efficiencies and effects

Comparing near-true inference and degree-true inference, we see that using near-true inference, one and the same problem can be solved by reasoning once, but if using degree-true inference, then it needs to be done by (parallel) reasoning generally twice or multiply. That is to say, the efficiency of natural inference, reasoning with degrees and AT method in the sense of near-true, generally speaking, will be higher than that of fuzzy reasoning in the sense of degree-true.

Also, let us observe ranges of truth-degrees, the range of truth-degrees of near-true inference is $(0.5, \beta]$ ($\beta \geq 1$), and the range of truth-degrees of degree-true inference is $(0, \beta]$; while the range of truth-degrees of fuzzy reasoning is $[0, 1]$. Note that the 1 in $[0, 1]$ here is equivalent to $[1, \beta]$ in $(0.5, \beta]$. From the three ranges of truth-degrees, it can roughly be observed that the effect (i.e., accuracy) of reasoning methods in the system, generally speaking, will be better than that of fuzzy reasoning.

In the following we will further analyze the efficiencies and effects of the two kinds of reasoning.

First, in fuzzy reasoning rule $A \rightarrow B$ is treated as a couple or implication relation (namely, “if A then B , else $B \vee \neg B$ ”). As a couple relation, the rule $A \rightarrow B$ is represented as the Cartesian product of corresponding fuzzy sets, $A \times B$, the geometric space this fuzzy set occupies in real is the region $\text{supp}(A) \times \text{supp}(B)$ in corresponding universe of discourse (as shown in Figure 4-1(a)). While in the inference methods in the system, the rule $A \rightarrow B$ is then further represented as smaller region $\text{core}(A)^+ \times \text{core}(B)^+$ (as shown in Figure 4-1(b)).

As an implication relation, rule $A \rightarrow B$ is also the following two correspondences:

$$\begin{cases} A \mapsto B \\ \neg A \mapsto B \vee \neg B \end{cases}$$

Which is represented as region $(A^c \times V) \cup (U \times B)$ (as shown in Figure 4-1(c)) in fuzzy reasoning. Since the evidence fact A' of the reasoning is an approximate value of A , so the above second correspondence “ $\neg A \mapsto B \vee \neg B$ ” is really superfluous. That is to say, when rule $A \rightarrow B$ is treated as an implication relation there is redundancy in fuzzy reasoning.

Yet the reasoning methods in the system only use correspondence $A \mapsto B$ in implication relation like the usual logical inference. Therefore, the reasoning can be implemented but also there is no any redundancy.

Secondly, seeing from processes, fuzzy reasoning is based on region $\text{supp}(A) \times \text{supp}(B)$ or $(A^c \times V) \cup (U \times B)$ to realize approximate reasoning and computation. From the analysis above it can be known that region $(A^c \times V) \cup (U \times B) - \text{core}(A)^+ \times \text{core}(B)^+$ is practically useless in reasoning. Then, the effect is conceivable finding the approximate value B' or corresponding number y' based on such a space much bigger than the actual requirement.

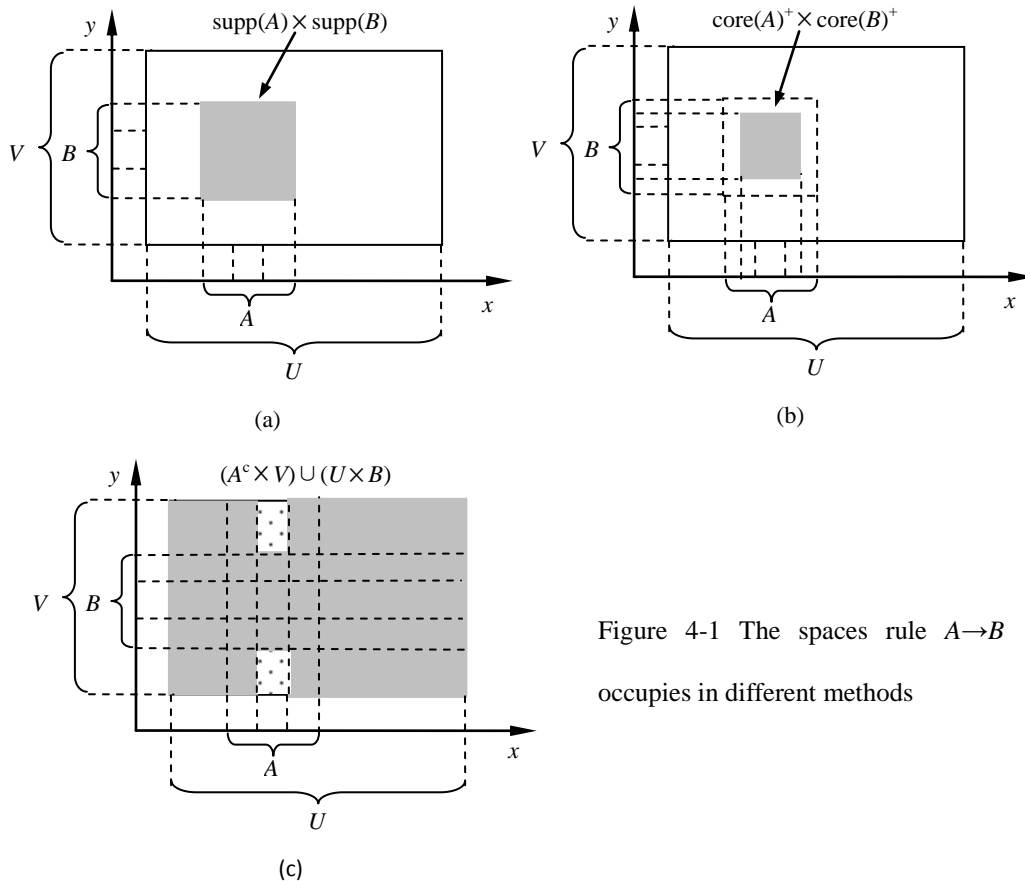


Figure 4-1 The spaces rule $A \rightarrow B$ occupies in different methods

Yet natural inference and reasoning with degrees realize approximate reasoning and computation in the sub-region $\text{core}(A)^+ \times \text{core}(B)^+$ of $(A^c \times V) \cup (U \times B)$; and AT method realizes approximate reasoning and computation round the peak-value point (ξ_A, ξ_B) in region $\text{core}(A)^+ \times \text{core}(B)^+$. From Sect. 13.5^[6] in [1] it's known that region $\text{core}(A)^+ \times \text{core}(B)^+$ is the smallest space that includes the background function or background correlation of rule $A \rightarrow B$. Thus, viewed only from problem-solving space, the error caused by the approximate reasoning methods in the system is certainly not more than that by fuzzy reasoning, on the whole. As to other approximate reasoning and computation methods in the system utilizing linguistic functions, because of their mathematical rationality, therefore their efficiency and effect are also assured.

When fuzzy reasoning is applied to approximate computation of numerical values (as auto-control), the properties of the input and output data generally are needed to be changed through fuzzification and defuzzification. But AT method can be directly applied to approximate evaluation of a function, which is tantamount to omitting two procedures of fuzzification and defuzzification. As for the approximate evaluation methods in the system that utilize natural inference, reasoning with degrees and flexible linguistic function, although they have also the conversion procedure from a linguistic value to a numerical value, that is, L-N conversion, the conversion can always guarantee the numerical result (y_0') obtained falls in the extended core of corresponding flexible linguistic value (B) (as shown in Figure 4-2(a)), so it is always effective and satisfactory. However, the methods of fuzzy reasoning and follow-up defuzzification can not guarantee that the obtained numerical result (y_0') falls within the extended core of corresponding flexible linguistic value (B) (as shown in Figure 4-2(b)), as a result, there would occur the phenomenon of sometimes valid but some other times invalid as well as valid for some problems but invalid for some other problems. We think, it just is an important cause of the fuzzy control being not reliable and stable enough.

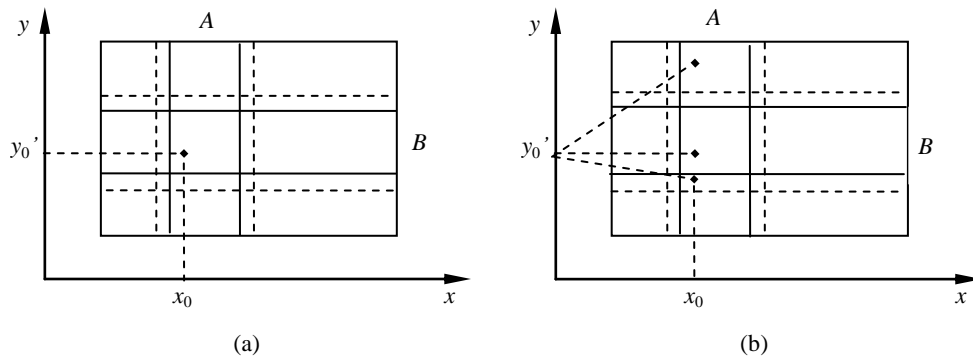


Figure 4-2 Examples of the positions of numerical result (y_0') obtained by two different approximate computing models

The region between two vertical dotted lines is support set of flexible linguistic value A , and between two solid lines is the extended core of A ;

The region between two horizontal dotted lines is support set of flexible linguistic value B , and between two solid lines is the extended core of B .

Actually, fuzzy controller is a kind of interpolator essentially (this is just the cause why fuzzy reasoning can be used to approximate computation to solve the practical problems such as automatic control). But viewed from approximate computation, we can't say much for the effect of this kind of interpolators. Correspondingly, the approximate computation methods in the new system can also be used for interpolation, and the effect is quite well. The graphs as shown in Figures 4-3, 4-4 and 4-5 below are three effect drawings of the interpolator achieved based on AT method. The accuracy is obvious.

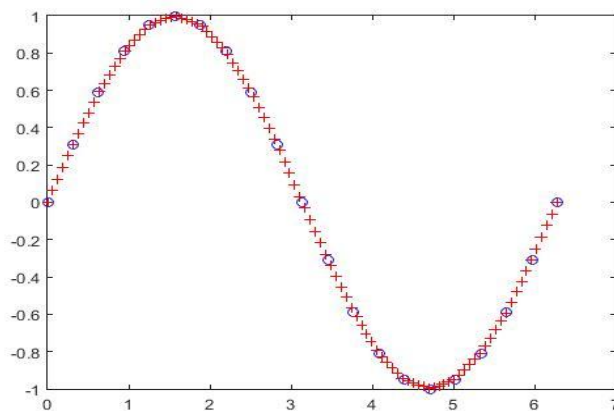


Figure 4-3 An effect drawing of the one-dimensional interpolator with AT method

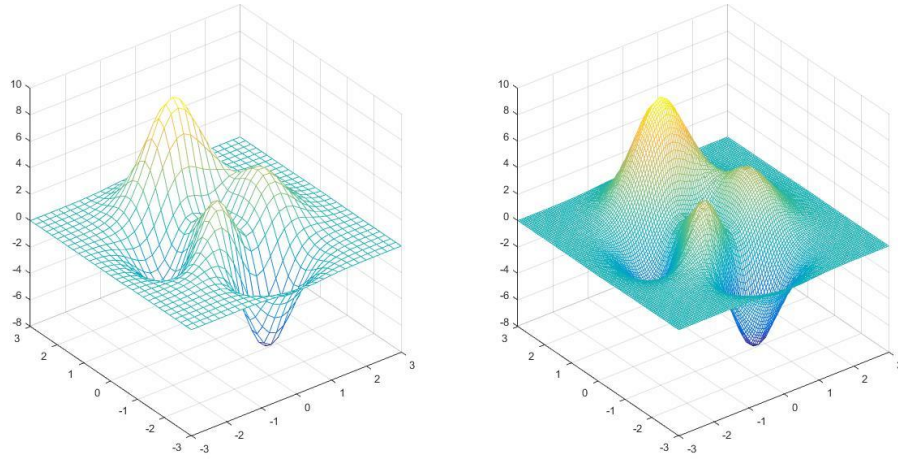


Figure 4-4 An effect drawing of the two-dimensional interpolator with AT method

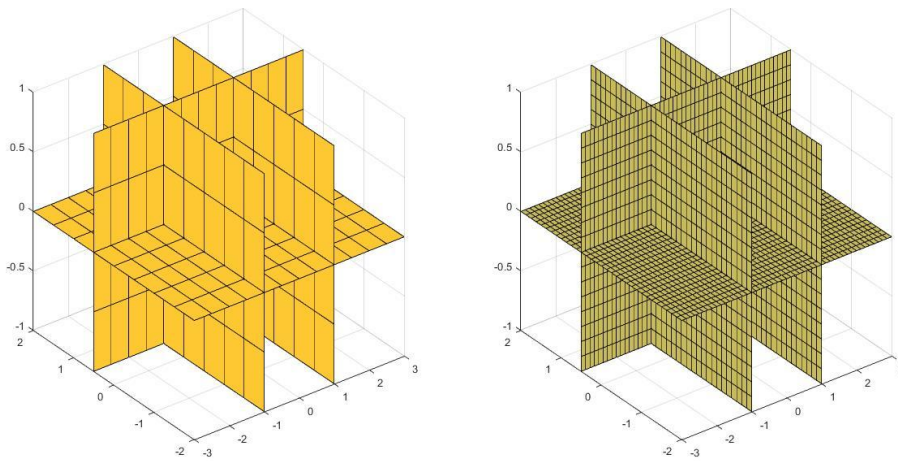


Figure 4-5 An effect drawing of the three-dimensional interpolator with AT method

The approximate computing model of a fuzzy controller is considered to be able to further develop into a fuzzy logic system which can be used to realize a “universal approximator”, but this kind of approximator has some severe problems (For details, see Sect. 16.4.3^[8] in [1]), for example, there occurs “the dilemma between precision and interpretability”^[2]. But the approximate computing models given in Chaps. 15^[7], 16^[8] and 17^[9] in [1], only need to add respectively a function to reduce dynamically granule sizes of flexible linguistic values, then, in principle, they can all be used to realize the “universal approximator” and there won’t occur “the dilemma between precision and interpretability”, and the functions approximated are even more

widespread, for instance, which can also be a multiple valued function and vector function.

(3) Research methods

In research of fuzzy reasoning, the mathematical backgrounds of practical problems are considered rarely, the orientation of an approximate linguistic value is still less considered, but it is tried in the same way that at the level of logic and language to research a fuzzy logic method that can treat the practical problems in different poses and with different expressions.

However, since $(A \rightarrow B) \wedge A' \Rightarrow B'$ is not a valid argument form, so it is impossible to deduce B' directly from $A \rightarrow B$ and A' . On the other hand, logical relation can not reflect, or in other words, covers, the orientation relation when one linguistic value approaches to another linguistic value, but the orientation relation is of great importance to the B' to be obtained.

We know that a linguistic rule actually summarizes an infinite number of practical binary relations —— correlations or functions. And relative to the practical binary relation summarized by rule $A \rightarrow B$, the binary relation $\text{core}(A)^+ \times \text{core}(B)^+$ is a universal relation, while binary relation $(A^c \times V) \cup (U \times B)$ is a bigger universal relation. Therefore, the approximate reasoning at the level of linguistic values should have a guide of the lower-level numerical values (i.e., the mathematical background). Otherwise, with only logic or mathematical methods, it's difficult to obtain a desired effect. On the other hand, the approximation of flexible linguistic value A' to A also involves the orientation problem; different orientations may bring different inference results. However, in the methods of pure logical inference, the information about orientation can not be reflected, so A' approximates to A no matter from which direction, the result B' is all the same. Obviously, the efficiency of the result of such approximate reasoning can only go by luck.

But the approximate reasoning methods in the new system are just guided by the relevant mathematical background information of practical problems, where linguistic values only have the effect of macroscopic positioning, while the specific

computations are also different with different problems.

(4) Scopes and abilities

The rules with multiple conditions involved in fuzzy reasoning are only two types of conjunction and disjunction, but the flexible rules in the new system have a type of synthesis-type rule besides the two types of rules. In addition, there are flexible linguistic functions, especially, flexible functions, in the system. That is to say, compared with fuzzy reasoning, the methods in the new system have a more wide scope of application and more powerful processing ability.

5. Applications and Envisagement

Application is the goal of the system. Actually, the system provides a technological platform for relevant applications. In fact, using the theories, techniques and methods in the system we can directly develop related applications such as intelligent robots, expert (knowledge) systems, machine learning, natural language processing as well as various anthropomorphic computer application systems used for classifying, recognition, judging, decision-making, controlling, diagnosis, forecasting, translating, etc. Especially, the conversion techniques between flexible linguistic values and numerical values will display skills to the full in machine perceptions, machine behaviors, human-machine dialogue, etc.

And then, the system also lays a theoretical foundation for further research. In fact, on the basis of the system we can also carry out further researches. Specifically, there are at least six research directions (For details, see [10]):

(1) Development of anthropomorphic computer application systems and intelligent systems with imprecise-information processing ability.

(2) Imprecise-knowledge discovery, and machine learning with imprecise information.

Actually, preliminary study shows that based on this theoretical and technological system, in principle, we can also achieve a function approximator as a

kind of alternative “Deep Learning” and obtain flexible linguistic rules or functions with some sample data.

(3) Natural language understanding and generation with flexible concepts.

(4) Flexible logic circuits and flexible computer languages.

(5) Exploring on the brain model of flexible concepts and the qualitative thinking mechanism of human brain.

(6) Related mathematical and logic theories.

In conclusion, this new system has broad application and development prospects.

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