NOTE ON THE COMPARISONS OF GEODETIC AZIMUTHS OF THE TUNISIAN PRIMORDIAL GEODETIC NETWORK

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Abstract

In this paper, we present some comparisons of geodetic azimuths obtained by GPS by classical geodesy and by astronomical observations (Laplace's points). The geodetic azimuths are calculated using 3D coordinates or by plane coordinates (Tunisian Lambert projection). The results obtained confirm that the orientation issued by GPS is identical to astronomical orientation which confirms also the orientation of the Tunisian Primordial Geodetic Network obtained by the definitive adjustment of the French Institut Géographique National in 1984.

Résumé

Dans cette note, nous présentons des comparaisons d'azimuts géodésiques obtenus par GPS avec ceux de la géodésie classique ou par l'astronomie. Les azimuts sont calculés par les coordonnées tridimensionnelles ou issus des coordonnées planes. Les résultats obtenus confirment que l'orientation issue du GPS est identique à celle obtenue par les observations astronomiques. De plus, les résultats GPS confirment l'orientation du réseau géodésique primordial tunisien issu de la compensation définitive de l'IGN de 1984.

1 Introduction

Let $A(\varphi_0, \lambda_0, H)$ with $3D$ coordinates (X_1, Y_1, Z_1) in a given geocentric reference frame, we consider the local geodetic frame in A : From A, we observe the point B of coordinates (X_2, Y_2, Z_2). Let $(x, y, z)^T$ be the components of the vector $\boldsymbol{AB}$ in (A, x, y, z). Then the geodetic azimuth of the direction $\boldsymbol{AB}$ is given by:

$$tg(Az) = \frac{x}{y} \tag{1}$$

Let :

$$DX = X_2 - X_1$$
$$DY = Y_2 - Y_1 \tag{2}$$
$$DZ = Z_2 - Z_1 \tag{3}$$

Between $(DX, DY, DZ)^T$ and $(x, y, z)^T$, we have the relation :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R. \begin{pmatrix} DX \\ DY \\ DZ \end{pmatrix} \tag{4}$$

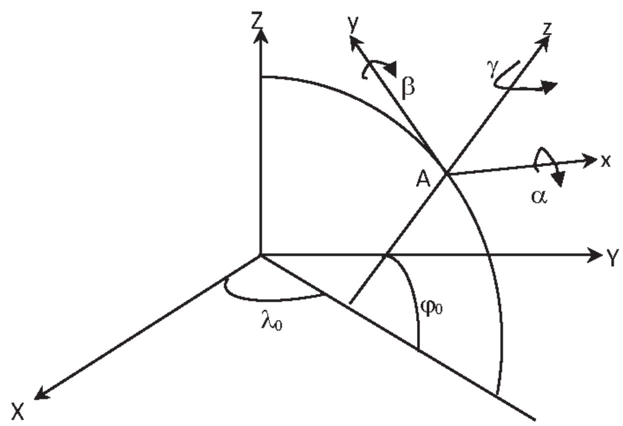

Fig. 1: Le Repère local au point A

Where R is the matrice of the transformation from the geocentric reference to the local geodetic frame in A and is given by:

$$R = \begin{pmatrix} -sin\lambda_0 & cos\lambda_0 & 0 \\ -cos\varphi_0.sin\lambda_0 & -sin\varphi_0.sin\lambda_0 & cos\varphi_0 \\ cos\varphi_0.cos\lambda_0 & cos\varphi_0.sin\lambda_0 & sin\varphi_0 \end{pmatrix} \tag{5}$$

then:

$$\begin{aligned} x = -sin\lambda_0.DX + cos\lambda_0.DY \\ y = -cos\varphi_0.sin\lambda_0.DX - sin\varphi_0.sin\lambda_0.DY + cos\varphi_0.DZ \\ z = cos\varphi_0.cos\lambda_0.DX + cos\varphi_0.sin\lambda_0.DY + sin\varphi_0.DZ \end{aligned} \tag{6}$$

So:

$$tg(Az) = \frac{-sin\lambda_0.DX + cos\lambda_0.DY}{-cos\varphi_0.sin\lambda_0.DX - sin\varphi_0.sin\lambda_0.DY + cos\varphi_0.DZ} \tag{7}$$

2 Applications: Azimuth AIN ABDOUR - H. OUED SEHILI

2.1 Calculations of the azimuths

2.1.1 From $WGS84$ coordinates of the OPAT - Office des Ports Aériens de Tunisie -

The geodetic points AIN ABDOUR and H. OUED SEHILI have been observed by GPS during the connection of the Tunisian airports to $WGS84$ in November 1997 [1]. We have :

AIN ABDOUR : $\lambda_0 = XX°YY'47".38955$, $\varphi_0 = XX°YY'29".14998$. Then we obtain: $DX = 2\,740.786\,m; DY = -16\,245.609\,m; DZ = -534.888\,m$ and :

$$Azg_1 = 267°48'30".7058 \tag{8}$$

2.1.2 From $WGS84$ coordinates of the Tunisian-Polish GPS Campain

The geodetic points AIN ABDOUR et H. OUED SEHILI have been observed during the GPS campaign 19 June - 3 July, 1996 by the OTC and the Space Research Centre of the Polish Academy of Sciences [2], [3] and we have the following results : AIN ABDOUR (φ_0, λ_0) and :

$$\begin{aligned} DX &= 2740.789\,m \\ DY &= -16245.616\,m \\ DZ &= -534.883\,m \end{aligned} \tag{9}$$

Then the geodetic azimuth:

$$Azg_2 = 267°48'30".7474 \tag{10}$$

2.1.3 From the Carthage34 coordinates

We have the following elements:

- at the geodetic point AIN ABDOUR:

$$x_1(Lambert - sud) = 363\,044.79\,m,\;\; y_1(Lambert - sud) = 407\,020.09\,m$$

$$\varphi_0 = XX°YY'23".0712, \quad \lambda_0 = X°YY'45".8138$$

- at the geodetic point H. OUED SEHILI:

$$x_2(Lambert - sud) = 346\,570.13\,m, \quad y_2(Lambert - sud) = 406\,623.60\,m$$

so :

$$\begin{aligned} DX &= 2741.505\,m \\ DY &= -16245.353\,m \\ DZ &= -535.155\,m \end{aligned} \tag{11}$$

Using the formula (7), we obtain:

$$Azg_3 = 267°48'21"709 \tag{12}$$

From the relation between the geodetic azimuth and the bearing angle:

$$G = Az - \gamma + Dv \tag{13}$$

With:
- G the bearing angle of the direction,
- Az the geodetic azimuth of the direction,
- γ the bearing angle of the meridian given by $(\lambda - \lambda'_0).sin\varphi'_0$ with $\varphi'_0 = 33°18' = +40\,gr$ the geodetic latitude of the origin parallel of the Lambert-sud map projection and $\lambda'_0 = 9°54' = 11\,gr$ East of Greenwich the longitude of the origin meridian of the same projection,
- Dv the correction of the chord of the side AIN ABDOUR - H.OUED SEHILI.

Numerically, we obtain:

$$G = 268°37'16".8536$$
$$Dv = -0°00'04".4566$$
$$\gamma = -0°48'59".5704$$

so:

$$Azg_4 = 267°48'21".7399 \tag{14}$$

2.1.4 From the coordinates issued of the IGN84 adjustment

We have the following elements:
- at the geodetic point AIN ABDOUR:

$$x'_1(Lambert - sud) = 363\,039.86\,m, \quad y'_1(Lambert - sud) = 407\,024.97\,m$$

$$\varphi_0 = XX°YY'23".2274, \quad \lambda_0 = X°YY'45".6181$$

- at the geodetic point H.OUED SEHILI:

$$x'_2(Lambert - sud) = 346\,565.09\,m, \quad y'_2(Lamb.sud) = 406\,629.15\,m$$

Then, we obtain:

$$\begin{aligned} DX &= 2741.138\,m \\ DY &= -16245.515\,m \\ DZ &= -534.610\,m \end{aligned} \tag{15}$$

Using the formula (7), we obtain:

$$Azg_5 = 267°48'30".0055 \tag{16}$$

From the equation (13), we obtain:

$$G = 268°37'25".2704$$
$$Dv = -0°00'04".4566$$
$$\gamma = -0°48'59".6778$$

and:

$$Azg_6 = 267°48'30".0491 \tag{17}$$

2.1.5 From the astronomical azimuth of AIN ABDOUR - SELDJA

From the astronomical azimuth of AIN ABDOUR - SELDJA observed during the astronomical campaign in 1982, we calculate the geodetic azimuth from Laplace equation. Having the observed horizontal angle $\alpha = SELDJA - AINABDOUR - H.OUEDSEHILI$, we obtain Azg the geodetic azimuth of the direction AIN ABDOUR - H. OUED SEHILI.

Using the generalized Laplace equation:

$$Azg = Aza + (\lambda - \Lambda).sin\varphi + ((\lambda - \Lambda).cos\varphi.cosAza + (\varphi - \Phi).sinAza).cotgz \tag{18}$$

Where Φ and Λ are respectively the astronomical latitude and longitude, z the zenith angle. We calcule Azg of the direction AIN ABDOUR - SELDJA.

In the point AIN ABDOUR, we have:

$$\Phi = XX°YY'13".8982, \quad \varphi = XX°YY'23".0712$$

$$\Lambda = X°YY'41".8767, \quad \lambda = X°YY'45".8138$$

$Aza = 337°09'08".6956$,

$Zenith\ angle\ z = 87°28'42".8484$.

Then the geodetic azimuth of $AINABDOUR - SELDJA = Azg = 337°09'10".0887$, but $\alpha = 69°20'40".9632$. Then:

$$Azg_7 = Azg - \alpha \Longrightarrow Azg_7 = 267°48'29".9242 \tag{19}$$

2.2 Comparisons of the azimuths

We summarize below the obtained geodetic azimuths:

- for the azimuths by GPS, $Az_{gps} = (Azg_1 + Azg_2)/2 = 267°48'30".727$,

- for the azimuths in Carthage34 frame, $Az_{Cart} = (Azg_3 + Azg_4)/2 = 267°48'21".725$,

- for the azimuths from the IGN84 adjustment, $Az_{Ign} = (Azg_5+Azg_6)/2 = 267°48'30".026$,

- and the azimuth issued from the astronomy, $Azg_a = Azg_7 = 267°48'29".924$.

We obtain these differences:

$$Az_{gps} - Az_{Cart} = 09".0007 \quad (20)$$

$$Az_{gps} - Az_{Ign} = 00".6998 \quad (21)$$

$$Az_{gps} - Azg_a = 00".8035 \quad (22)$$

$$Az_{Ign} - Az_{Cart} = 08".0088 \quad (23)$$

We concluded that:

* the orientation issued by GPS is conformal to those obtained by astronomical observations (equation 22),

* the adjustment of the IGN has put in evidence the disorientation of Carthage34 frame (equation 23),

* the orientation obtained by the adjustment of the IGN84 is similarity to those of the GPS (equation 21),

* the GPS observations put in evidence the disorientation of Carthage34 frame (equation 20).

3 Conclusions

This paper allowed to do some comparisons of the geodetic azimuths obtained by different methods of a direction of the Tunisian Primordial Geodetic Network and to put in evidence the disorientation of the geodetic frame Carthage34 and the contribution of the GPS techniques for the orientation of the geodetic networks.

References

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