Klein-Gordon Equation and Wave Function in Cosmological Special Theory of Relativity

Sangwha-Yi

Department of Math , Taejon University 300-716, South Korea

ABSTRACT

In the Cosmological Special Theory of Relativity, we study energy-momentum relations, Klein-Gordon equation and wave function.

PACS Number:**03.30, 41.20**

Key words: Cosmological special relativity theory;

Klein-Gordon equation;

Energy-momentum relation

e-mail address:sangwha1**@nate.com**

Tel:010-2496-3953

1. Introduction

Our article's aim is that we make Klein-Gordon equation and wave function in cosmological special theory of relativity.

At first, space-time relations are in cosmological special theory of relativity (CSTR).[7]

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega^2(t_0)x'),\ x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{array}{l}\Omega(t_0)y = \Omega(t_0)y', \\ \Omega(t_0)z = \Omega(t_0)z'\end{array}\ ,\ \gamma = 1/\sqrt{1-\frac{v_0^{\,2}}{c^2}\Omega^2(t_0)},\ t_0 \text{ is cosmological time} \qquad (1)$$

Therefore, proper time is[7]

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2],\ t_0 \text{ is cosmological time} \qquad (2)$$

Hence, energy-momentum relations are by the fact that energy-momentum are 4-vector in CSTR,

$$E = \gamma(E' + v_0\Omega^2(t_0)p_x'),\ p_x\Omega(t_0) = \gamma(\Omega(t_0)p_x' + \frac{v_0}{c^2}\Omega(t_0)E')$$

$$\begin{array}{l}\Omega(t_0)p_y = \Omega(t_0)p_y', \\ \Omega(t_0)p_z = \Omega(t_0)p_z'\end{array},\ \gamma = 1/\sqrt{1-\frac{v_0^{\,2}}{c^2}\Omega^2(t_0)},\ E = m_0c^2\frac{dt}{d\tau},\ \vec{p} = m_0\frac{d\vec{x}}{d\tau} \qquad (3)$$

Therefore, energy-momentum-mass relation is in CSTR,

$$m_0^{\,2}c^4 = E^2 - \Omega^2(t_0)p^2c^2 \qquad (4)$$

2. Klein-Gordon Equation and Wave Fuction in CSTR

According to [7], matter wave function is in CSTR,

$$\phi = \phi_0 \exp i\Phi = \phi_0 \exp i[\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)}]$$

$$= \phi' = \phi_0 \exp i\Phi' = \phi_0 \exp i[\frac{\omega' t'}{\sqrt{\Omega(t_0)}} - \vec{k}'\cdot\vec{x}'\sqrt{\Omega(t_0)}]$$

ϕ_0 is amplitude, ω is angular frequency, $k = |\vec{k}|$ is wave number. (5)

If we use Eq(1) in Eq(5), we obtain angular frequency-wave number relation.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1),\ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)} \qquad (6)$$

In this time, if we define energy-momentum by angular frequency-wave number,

$$E = \hbar\omega, \vec{p} = \frac{\hbar\vec{k}}{\Omega(t_0)} \qquad (7)$$

Hence, we obtain the angular frequency-wave number relation about the energy-momentum-mass relation in CSTR,

$$m_0^2c^4 = E^2 - \Omega^2(t_0)p^2c^2 = \hbar^2\omega^2 - \hbar^2k^2c^2 \qquad (8)$$

We obtain next result by the transformation of the angular frequency-wave number relation, Eq(6) in CSTR.

$$m_0^2c^4 = \hbar^2\omega^2 - \hbar^2k^2c^2 = \hbar^2\omega'^2 - \hbar^2k'^2c^2 \qquad (9)$$

If we define the differential operator about energy-momentum in CSTR,

$$E = i\hbar\sqrt{\Omega(t_0)}\frac{\partial}{\partial t}, \vec{p} = -i\hbar\frac{1}{\Omega(t_0)\sqrt{\Omega(t_0)}}\vec{\nabla} \qquad (10)$$

If we apply Eq(10) to Eq(4),

$$m_0^2c^4 = E^2 - \Omega^2(t_0)p^2c^2 = \hbar^2[-\Omega(t_0)(\frac{\partial}{\partial t})^2 + \frac{1}{\Omega(t_0)}c^2\nabla^2]$$

We finally obtain Klein-Gordon equation in CSTR.

$$\frac{m_0^2c^2}{\hbar^2}\phi = [-\Omega(t_0)\frac{1}{c^2}(\frac{\partial}{\partial t})^2 + \frac{1}{\Omega(t_0)}\nabla^2]\phi \qquad (11)$$

Wave function, Eq(5) satisfy Klein-Gordon equation, Eq(11) in CSTR.

3. Conclusion

We are able to describe free particle by Klein-Gordon equation and wave function in CSTR.

References

[1]A. Einstein, " Zur Elektrodynamik bewegter K¨orper", Annalen der Physik. 17:891(1905)

[2]Friedman-Lemaitre-Robertson-Walker metric-Wikipedia

[3]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[4]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)

[5]D.J. Griffith," Introduction To Electrodynamics",(2nd ed.,Prentice Hall,Inc.1981)

[6]Lambda-CDM model –Wikipedia

[7]S.Yi, ”Cosmological Special Theory of Relativity”, International Journal of Advanced research in Physical Science**,7,**11(2020),pp4-9