ON SOME CHROMATIC PROPERTIES OF SOME CIRCULANT GRAPHS

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Abstract

Here we obtain the list chromatic index, list total chromatic number some of powers of cycles .

1. Introduction

The list chromatic index of a graph is an important chromatic parameter of a graph. It is much more difficult to compute than the normal chromatic index. In fact, the list chromatic index for complete graphs has not yet been determined. A famous conjecture in this field is the List Coloring Conjecture that states that the list chromatic index equals the chromatic index, or, in other words, the line graph of any simple graph is chromatic choosable. The conjecture has been shown to be true for bipartite graphs and complete graphs of odd order[3],[6]. In this work, we see how a useful observation and a classic result on powers of cycles helps us to determine the chromatic indices of powers of cycles.

The list total chromatic number is a parameter that is the list version of the famed total chromatic number. The total chromatic number is the chromatic number of the total graph of the graph, which consists of vertices equal to the total number of vertices and edges in the original graph; the vertices are adjacent if either two vertices correspond to adjacent vertices, or incident edges, or a vertex and its incident edge in the original graph. The total graph can also be seen as the square (in the distance sense) of the subdivision graph of the original graph. The list total coloring conjecture ([4]) states the total graphs are also chromatic choosable. This is also widely open and not yet proved for complete graphs also. There is also another famous problem concerning the total chromatic number, which is the

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Total coloring conjecture[1] stating that any graph with maximum degree Δ is total colorable with $\Delta+2$ colors, or $\chi''(G) \leq \Delta+2$, where $\chi''(G)$ denotes the total chromatic number. By a simple greedy approach, it can be seen that the list total chromatic number $\chi''_l(G)$ can bounded by $\chi'(G)+2$, where $\chi'(G)$ denotes the list chromatic index of G.

Even speaking specifically to powers of cycles, still all the above three coloring parameters are undetermined. The closest correct result we have till date for total colorings is the results by Campos-de Mello ([2]).

2. The main theorems

Theorem 2.1. *The powers of cycles are chromatic edge choosable.*

Proof. It can be observed that the line graph of a power of a cycle is another power of a cycle. In particular, if we have $G = C_n^k$, then the line graph of G, $L(G) = C_{2n}^{2k-1}$. This can be seen as follows: If $(a, a+i) \quad, 1 \leq i \leq k$ is any edge of G, then its neighbors are exactly the union of the edges $(a, a+j) \quad, -k \leq j \neq i \leq k \pmod{n}$ and $(a+m, a+i) \quad, -k \leq m \neq i \leq k \pmod{n}$, as the adjacencies of a vertex in the line graph correspond exactly those edges in the original graph which are incident with either of the end vertices of corresponding the edge. If distance between the two vertices in line graph be defined as the distance between the unequal coordinates in (v_1, v_2). As vertices with similar distances are adjacent in the line graph, we can say that the line graph is circulant, in particular, it is a $(2k-1)th$ power of a $2n$ cycle. Now, from the result in [5], it is clear that all powers of cycles are chromatic choosable. Thus, the line graphs of powers of cycles are chromatic choosable, or, in other words, the power of cycle graph is list edge colorable. $\square$

Theorem 2.2. *The list total chromatic number of powers of cycles $G = C_n^k$ is bounded above by $\Delta+3$, where Δ is the maximum degree. In particular, if n is even, then the list total chromatic number is bounded by $\Delta+2$*

Proof. Since an edge is exactly incident with two vertices, and the any graph can be list colored with $\geq \Delta+1$ colors, when we give a list of cardinality $\Delta+3$ to each graph element, we can have at most two blockages to color the edge incident to the the same two vertices. Since the previous theorem giveus us a bound of $\Delta+1$ on the list edge chromatic number of

G (by Vizing's theorem), we see that the bound on the list total chromatic number is $\Delta + 3$.

Now, if the graph were of even order, then, by [7], we have that the chromatic index of the graph is equal to Δ. This, by the previous theorem and the discussion above, implies that the bound on the list total chromatic number of G in the even order case is $\Delta + 2$. This gives yet another quick way to prove the total coloring conjecture for powers of cycles on even order. $\square$

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