Determinism, Quantum Mechanics and Asymmetric Visible Matter

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Abstract

The focus of this note is in the formation of matter-antimatter asymmetric universe without antimatter in the first place. To avoid problems of best known published preon models we utilize ´t Hooft theory's deterministic Hilbert space methods to preons. Inflation is started in a ultra dense graviton phase predominating the very early universe and producing supersymmetric preons, axion like particles and torsion in spacetime. All standard model and dark sector fermions are created as spectators during early inflation from the preons. The dark sector particles are spectators all the way beyond reheating while the visible sector particles couple to the inflaton. Before reheating is reached supersymmetry is broken to the minimal supersymmetric standard model by gravitational mediation from the preon sector. Consequently, asymmetric visible matter, symmetric dark matter and dark energy are produced, and much later nucleons and light nuclei are formed. The deterministic preon level structure is necessary for the mechanism which creates from C symmetric preons the asymmetric standard model visible matter directly, without notable amount of antimatter and without the Sakharov conditions.

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1 Introduction

The Standard Model of particles (SM) is a quantum field theory and therefore matter-antimatter symmetric. But no such symmetry has been found in the universe. Secondly, visible matter consists only 5% of all matter/energy in the universe. The remaining, unknown components are called dark matter and dark energy, with fractions of 27% and 68%, respectively.

In this note we take the humble attitude that Nature is so efficient as to produce the SM matter predominantly directly without antimatter. Our scenario is based on the proposal that preons are produced respecting matter-antimatter symmetry but once formed preons can be combined into SM particles in a matter-antimatter *asymmetric* way. A second general feature of the preon scenario is that it can be defined within the *deterministic* quantum mechanics of ´t Hooft, shedding new light to the old question of the nature of quantum mechanics.

The main time period considered below is the era of inflation, and we only briefly mention supersymmetry breaking, reheating and later phases ending to thermalization of matter, and quarks forming nucleons and nucleons making the nuclei of the three lightest elements. We conclude that global supersymmetry is supported by observations given that the standard model superpartners are found some day. Supergravity scalar potential has been found for inflation by other people. A number of bosons that can be associated with string theory are needed in this scenario.

The article is organized as follows. In subsections 2.1, 2.2, 2.5 and 2.4 we summarize briefly the concepts used to derive our scenario: non-relativistic phase space, Born's reciprocity symmetry, Clifford algebra and color, deterministic quantum preons, emergent supersymmetry and very minimum of bosonic strings. In later sections we connect these theoretical concepts with observations. The structure of visible matter in terms of preons is recapped and sharpened in section 3. In section 4 candidates for dark matter are discussed. The visible standard model matter is produced in reheating by coupling to the inflaton in a no-scale supergravity model, with hints from string theory, as described in section 5. The central point of this note, the scenario for the creation of matter-antimatter asymmetric universe by charge symmetric preons is proposed in section 6. The idea behind the asymmetry is that the *same* C symmetric preons may form matter at one time and antimatter at another time, see (6.1). A prefatory mechanism is described why matter was chosen for our universe. The dark sector contains possibilities for large scale celestial annihilation processes. Gravitationally mediated supersymmetry breaking for SM particles is proposed in section 7. Conclusions are given in section 8.

The original contributions of this author are the supersymmetric preon (superon from now on) scenario for the visible (see footnote 11 to [14]) and dark sector particles, and the mechanism for directly producing the asymmetric universe. The novelty of this note is composing the right concepts together, in particular the methods of treating preons as (i) t Hooft's Hilbert space deterministically behaving states and (ii) the Wess-Zumino supersymmetric objects. The mathematics needed for (i) and (ii) is available in the literature. To make the presentation self-contained phenomenological results obtained by other people are added, making this note also a mini-review.

2 Theory Concepts

In this section we present a brief description of the theoretical tools and results needed in later sections. The full mathematical treatment of these concepts is available in the literature and textbooks, see e.g. [1, 2, 3, 4, 5, 6].

2.1 Clifford Algebra, Hypercharge and Color

To provide solid mathematical basis for preons we start from non-relativistic phase space considerations and end up to spin and a formula for charge.[1] Born [7] studied the symmetrization of the roles of momenta and positions by the transformation $\mathbf{x} \to \mathbf{p}, \mathbf{p} \to -\mathbf{x}$. The symmetry holds in the zero mass limit. There are eight different orderings for the canonical positions and momenta. To us the interesting cases are the four even permutations shown in Table 1.

[1] We discuss spin and internal quantum numbers, which are valid concepts also non-relativistically.

Position	Momentum
(x_1, x_2, x_3)	(p_1, p_2, p_3)
(x_1, p_2, p_3)	(p_1, x_2, x_3)
(p_1, x_2, p_3)	(x_1, p_2, x_3)
(p_1, p_2, x_3)	(x_1, x_2, p_3)

Table 1: Position-momentum even permutations.

Sixty years later Żenczykowski [8] proposed the nimble conjecture that the four possibilities shown in Table 1 correspond to the first generation leptons and three superons (see (2.6)). Let us unveil why this leap could be so. The spin of a particle was first discovered by the Dirac trick. Here one may try the linearization of the 3D invariant $\mathbf{p}^2 = (\mathbf{p}\cdot\boldsymbol{\sigma})(\mathbf{p}\cdot\boldsymbol{\sigma})$. Linearization of the $x \leftrightarrow p$ symmetric expression $\mathbf{A}\cdot\mathbf{p}+\mathbf{B}\cdot\mathbf{x}$, where $\mathbf{A}$ and $\mathbf{B}$ are anticommuting objects, yields the result

$$\mathbf{A}\cdot\mathbf{p}+\mathbf{B}\cdot\mathbf{x} = \mathbf{p}^2+\mathbf{x}^2+R, \tag{2.1}$$

where the term R appears because x and p do not commute. A and B are eight-dimensional matrices

$$\begin{aligned} A_k &= \sigma_k\otimes\sigma_0\otimes\sigma_1 \\ B_j &= \sigma_0\otimes\sigma_j\otimes\sigma_2 \end{aligned} \tag{2.2}$$

R is the commutator of these matrices $R = -\frac{i}{2}\Sigma_k[A_k, B_k] = \Sigma_k\sigma_k\otimes\sigma_k\otimes\sigma_3$.

The seventh anticommuting element of the Clifford algebra in question is denoted as $B = iA_1A_2A_3B_1B_2B_3 = \sigma_0\otimes\sigma_0\otimes\sigma_3$. We define now

$$I_3 = \frac{1}{2}B, \quad Y = \frac{1}{3}RB \tag{2.3}$$

I_3 and Y commute with the operators describing ordinary 3D rotations and 3D reflections in phase space. The eigenvalues of I_3 and Y are

$$I_3 = \pm\frac{1}{2}, \quad Y = -1, +\frac{1}{3}, +\frac{1}{3}, +\frac{1}{3} \tag{2.4}$$

I_3 and Y are candidates for two new quantum numbers. A reasonable conjecture is that (2.4) could be identified with the Gell-Mann–Nishijima formula for charge Q

$$Q \equiv \frac{1}{6}\Big[(\mathbf{p}^2+\mathbf{x}^2)_{vac}+R\Big]B = I_3+\frac{Y}{2} \tag{2.5}$$

where the first term denotes its the lowest eigenvalue of $\mathbf{p}^2+\mathbf{x}^2$, which is three. I_3 is the weak isospin and Y hypercharge. The eigenvalues of Q are therefore (0, +2/3, +2/3, +2/3, -1, -1/3, -1/3, -1/3). They are the charges of a single generation Standard Model particles.

The correspondence between the phase-space approach and the superon model is obtained from (2.4)

$$\begin{aligned} Y &= -1 \leftrightarrow m^0 m^0 m^0 \\ Y_R &= 1/3 \leftrightarrow m^+ m^+ m^0 \\ Y_G &= 1/3 \leftrightarrow m^+ m^0 m^+ \\ Y_B &= 1/3 \leftrightarrow m^0 m^+ m^+ \end{aligned} \tag{2.6}$$

where the lines 2-4 are labeled by the position of the m^0 superon, see also Table 2. Having no antisymmetrization indices the m's in (2.6) look like classical, i.e. determnistic particles. There has been from time to time a hope to discover a deterministic theory behind the present quantum theory. We discuss this question next in subsection 2.2. Deterministic superon behavior would allow to give a label to them and release us from the requirements of uncertainty relations (requiring high superon mass) and wave function antisymmetrization.

Finkelstein has given arguments, consistent with the ones in this subsection, for the possible existence of preons based on the quantum group $SLq(2)$ [9].

2.2 Deterministic Superons

Having introduced the superons in (2.6) we must ask what kind of equations of motion do they obey. And what mechanism keeps them together? We follow here ideas proposed by ´t Hooft [**?**, 10]. Somewhat surprisingly, but excellently fulfilling our ambition, we end up to the roots of quantum mechanics. It is here considered fundamentally a deterministic theory when defined in terms of classical particles, superons. Quantum behavior enters when some information from the system is lost, of either position, momentum or due to a constraint. Referring to (2.4), we are interested here in three state systems. We follow in this subsection closely the treatment presented by Blasone, Jizba, and Kleinert in [11].

Discrete-time version. A relevant simple case is the three-state system with a cyclic deterministic evolution of states $|1\rangle \to |2\rangle \to |3\rangle \to |1\rangle$, as indicated in Figure 1. This system is interpreted as the basis for superon confinement. It is not limited to discontinuous time. The Hilbert space is associated with this

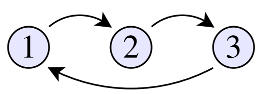

Figure 1: Cyclic three-state system.

system is

$$|\psi\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle \tag{2.7}$$

The time evolution $t_i \to t_{i+1}$ may be represented by the following unitary operator

$$\psi_{t+1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \psi_t \tag{2.8}$$

In a basis in which U is diagonal, it has for a single time step the form

$$U(t+1, t) = \exp(-iH\Delta t) \tag{2.9}$$

where

$$H = \begin{pmatrix} 1 & o & o \\ 0 & -2\pi/3 & 0 \\ 0 & 0 & 2\pi/3 \end{pmatrix} \tag{2.10}$$

A quantum theory in the Heisenberg picture is deterministic if a complete set of operators $O_i(t)(i = 1, .., N)$ exist, such that

$$[O_i(t), O_j(t')] = 0 \;\; \forall t, t'; i, j = 1, .., N \tag{2.11}$$

These operators are called beables. The above three-state system is obviously deterministic in this sense.

Continuous-time version. Classical systems of the form

$$H = p_a f^a(q) \tag{2.12}$$

evolve deterministically even after quantization [10, 11]. This happens since in the Hamiltonian equations of motion

$$\begin{aligned} \dot{q}^a &= \{q^a, H\} = f^a(q) \\ \dot{p}_a &= \{p_a, H\} = -p_a \partial f^a(q)/\partial q^a \end{aligned} \tag{2.13}$$

the equation for the q^a does not contain p^a, making the q^a beables.

Now we have to stop because the Hamiltonian is not bounded from below. This defect can be revised by a constraint [10, 11]. Consider a function $\rho(q_a) > 0$ with $[\rho, H] = 0$ and divide the Hamiltonian in two parts

$$\begin{aligned} H &= H_+ - H_- \\ H_+ &= \frac{1}{4\rho}(\rho + H)^2 \\ H_- &= \frac{1}{4\rho}(\rho - H)^2 \end{aligned} \tag{2.14}$$

where H_+ and H_- are positive definite operators satisfying

$$[H_+, H_-] = [\rho, H] = 0 \tag{2.15}$$

We may now enforce the following constraint to the Hamiltonian to get rid of the spectrum problem

$$H_-|\psi\rangle = 0 \tag{2.16}$$

Then the eigenvalues of H in $H|\psi\rangle = H_+|\psi\rangle = \rho|\psi\rangle$ are positive and the equation of motion

$$\frac{d}{dt}|\psi\rangle = -iH|\psi\rangle \tag{2.17}$$

has only positive frequencies. If there are stable orbits with period $T(\rho)$, then $|\psi\rangle$ satisfies

$$e^{iHT}|\psi\rangle = |\psi\rangle, \quad \rho T(\rho) = 2\pi n, \ n \in \mathbb{Z} \tag{2.18}$$

so that the associated eigenvalues are discrete. 't Hooft motivated the constraint (2.16) by information loss. More details of information loss and periodicity, energy spectra, equivalence classes, limit cycles etc. are in [10].[2]

Path Integral Quantization. A powerful technique for quantization is proposed by Faddeev and Jackiw in [12]. The authors start by observing that a Lagrangian for 't Hooft's equations of motion (2.13) can be simply taken as follows

$$L(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{p}, \dot{\mathbf{p}}) = p \cdot \dot{\mathbf{q}} - H(\mathbf{p}, \mathbf{q}) \tag{2.19}$$

with $\mathbf{q}$ and $\mathbf{p}$ being Lagrangian variables. Note that L does not depend on $\dot{p}$. It is easily verified that the Euler-Lagrange equations for the Lagrangian (2.19) indeed coincide with the Hamiltonian equations (2.13). Thus given 't Hooft's Hamiltonian (2.12) one can always construct a first-order Lagrangian (2.19) whose configuration space coincides with the Hamiltonian phase space. By defining 2N configuration-space coordinates as

$$\xi^a = p_a, \ a = 1, ..., N; \quad \xi^a = q_a, \ a = N+1, ..., 2N \tag{2.20}$$

the Lagrangian (2.19) can be cast into the more expedient form, namely

$$L(\xi, \dot{\xi}) = \frac{1}{2}\xi^a \omega_{ab} \dot{\xi}^b - H(\xi) \tag{2.21}$$

where ω is the $2N \times 2N$ matrix

$$\omega_{ab} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}_{ab} \tag{2.22}$$

which has an inverse $\omega^{-1}_{ab} \equiv \omega^{ab}$. The equations of motion read

$$\dot{\xi}^a = \omega^{ab} \frac{\partial H(\xi)}{\partial \xi^b} \tag{2.23}$$

indicating that there are no constraints on ξ. Thus the procedure of [12] makes the system unconstrained, so that the path integral quantization may proceed in a standard way. The time evolution amplitude is simply [6]

$$\langle \xi_2, t_2 | \xi_1, t_1 \rangle = \mathcal{N} \int_{\xi_1}^{\xi_2} \mathcal{D}\xi \exp\left(\frac{i}{\hbar} \int_{t_1}^{t_2} dt L(\xi, \dot{\xi}) \right) \tag{2.24}$$

[2]One may contemplate that non-linear phenomena, including solitons, may explain the three generations of SM particles.

where $\mathcal{N}$ is a normalization factor. Since the Lagrangian 2.19 is linear in p, we may integrate these variables out and obtain

$$\langle q_2, t_2 | q_1, t_1 \rangle = \mathcal{N} \int_{q_1}^{q_2} \mathcal{D}q \prod_a \delta[\dot{q}^a - f^a(q)] \tag{2.25}$$

where $\delta[f] \equiv \Pi_t \delta(f(t))$ is the functional version of Dirac's δ-function. Hence the system described by the Hamiltonian (2.12) retains its deterministic character even after quantization. The paths are squeezed onto the classical trajectories determined by the differential equations $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q})$. The time evolution amplitude (2.30) contains a sum over only the classical trajectories. There are no quantum fluctuations driving the system away from the classical paths.

The equation (2.25) can be brought into more intuitive form by utilizing the identity

$$\delta[f(q) - \dot{q}] = \delta[q - q_{\text{cl}}](\det(M)^{-1} \tag{2.26}$$

where where M is a functional matrix formed by the second functional derivatives of the action $\mathcal{A}[\xi] \equiv \int dt L(\xi, \dot{\xi})$

$$M_{a,b}(t, t') = \frac{\delta^2}{\delta\xi^a(t)\delta\xi^b(t')} \,|_{\mathbf{q}=\mathbf{q}_{\text{cl}}} \tag{2.27}$$

Morse index theorem [13] ensures that for sufficiently short time intervals $t_2 - t_1$ (before the system reaches its first focal point), the classical solution with the initial condition $q(t_1) = q_1$ is unique. In such a case (2.25) can be brought in the form

$$\langle q_2, t_2 | q_1, t_1 \rangle = \frac{\mathcal{N}}{\det M} \int_{q_1}^{q_2} \mathcal{D}\delta(q - q_{\text{cl}}) \tag{2.28}$$

indicating transparently the classical behavior.

2.3 Emergent Supersymmetry

We now turn to an interesting implication of the result (2.28) [11]. If we had started in (2.25) with an external current

$$\tilde{L}(\xi, \dot{\xi}) = L(\xi, \dot{\xi}) + i\hbar \mathbf{J} \cdot \mathbf{q} \tag{2.29}$$

integrated again over $\mathbf{p}$, and took the trace over $\mathbf{q}$, we would end up with a generating functional

$$\mathcal{Z}[\mathbf{J}] = \frac{\mathcal{N}}{\det M} \int \mathcal{D}\delta(q - q_{\text{cl}}) \exp\left(\int_{t_1}^{t_2} dt \; \mathbf{J} \cdot \mathbf{q} \right) \tag{2.30}$$

The path integral (2.30) has an interesting mathematical structure. We may rewrite it as

$$\mathcal{Z}[\mathbf{J}] = \frac{\mathcal{N}}{\det M} \int \mathcal{D}\mathbf{q} \delta\left[\frac{\delta\mathcal{A}}{\delta\mathbf{q}}\right] \left| \frac{\delta^2 \mathcal{A}}{\delta q_a(t) \delta q_a(t')} \right| \times \exp\left[\int_{t_1}^{t_2} dt \mathbf{J} \cdot \mathbf{q} \right] \tag{2.31}$$

Introduce two real time dependent Grassman ghost variables $c_a(t)$ and $\bar{c}_a(t)$, fermion field λ_a, and two anticommuting coordinates θ and $\bar{\theta}$. The latter pair of variables extends the configuration space of $\mathbf{q}$ variables into superspace. The superfield is defined

$$\Phi_a(t,\theta,\bar{\theta}) = q_a(t) + i\theta c_a(t) - i\bar{\theta}\bar{c}_a(t) + i\bar{\theta}\theta\lambda_a(t) \tag{2.32}$$

Together with the identity $\mathcal{D}\Phi = \mathcal{D}q\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}\lambda$ we may therefore express the classical partition functions (2.30) and (2.31) as a supersymmetric path integral with fully fluctuating paths in superspace

$$\mathcal{Z}_{CM}[\mathbf{J}] = \int \mathcal{D}\Phi \exp\left\{-\int d\theta d\bar{\theta}\mathcal{A}[\Phi](\theta,\bar{\theta})\right\} \times \exp\left\{\int dtd\theta d\bar{\theta}\,\Gamma(t,\theta,\bar{\theta})\Phi(t,\theta,\bar{\theta})\right\} \tag{2.33}$$

where the supercurrent is $\Gamma(t,\theta,\bar{\theta}) = \bar{\theta}\theta\mathbf{J}(t)$. A specific case of supersymmetry, namely Wess-ZUmino supergravity, is discussed in the next section 2.4. There we write the kinetic Lagrangians for our scenario.

2.4 Supergravity

We briefly recap the superon scenario of [14, 15], which turned out to have close resemblance to the simplest N=1 globally supersymmetric 4D model, namely the free, massless Wess-Zumino model [16, 17] with the kinetic Lagrangian including three neutral fields m, s, and p with $J^P = \frac{1}{2}^+, 0^+$, and 0^-, respectively

$$\mathcal{L}_{\text{WZ}} = -\frac{1}{2}\bar{m}\partial\!\!\!/ m - \frac{1}{2}(\partial s)^2 - \frac{1}{2}(\partial p)^2 \tag{2.34}$$

where m is a Majorana spinor, s and p are real fields (metric is mostly plus).

We assume that the pseudoscalar p is the axion [18], and denote it below as a. It has a fermionic superparther, the axino n, and a bosonic superpartner, the saxion s^0.

In order to have visible matter we assume the following charged chiral field Lagrangian

$$\mathcal{L}_{-} = -\frac{1}{2}m^{-}\partial\!\!\!/ m^{-} - \frac{1}{2}(\partial s_i^{-})^2, \quad i = 1,2 \tag{2.35}$$

The first generation standard model particles are formed combinatorially (mod 3) of three superons, the charged $m^{\pm}$, with charge $\pm\frac{1}{3}$, and the neutral m^0, as composite states below an energy scale Λ_{cr} [15], see lower part of Table 1.

The deconfnement temperature Λ_{cr} is in principle calculable but at present it has to be accepted as a free parameter. Numerically $\Lambda_{cr} \sim 10^{10-16}$ GeV, somewhat above the reheating temperature (at reheating there must be SM particles, i.e. visible matter). The R-parity of superons is simply $P_R = (-1)^{2\times spin}$.

Introducing local supersymmetry for superons is an open question in our scenario at the moment. It is a task for the future. In section 5 we discuss a boson sector interaction potential for inflation within a mini supergravity model, and in section 6 we propose a tentative superon-superon gauge interaction.

2.5 Bosonic String

A point particle has one dimensional world line with a tangent vector $dx^\mu(\tau)/d\tau$, where τ is the world line parameter. The tangent vector and the Maxwell field can be multiplied to form a Lorentz scalar. The interaction of a point particle of charge e with the Maxwell gauge field is written as $e\int \frac{dx^\mu(\tau)}{d\tau} A_\mu(x(\tau))d\tau$.

The endpoints of open strings may carry electric charge. But having two Lorentz indexes we hope to discover a new kind of charge that could be contracted with the string indexes. Such a field is the Kalb-Ramond antisymmetric tensor $B_{\mu\nu} = -B_{\nu\mu}$. It is a massless closed string. The obvious way to write a Lorentz scalar with two string tangent vectors of the form $\partial X^\lambda/d\rho$ is

$$-\int \frac{\partial X^\mu}{d\tau}\frac{\partial X^\nu}{d\sigma} B_{\mu\nu}\big(X(\tau,\sigma)\big)d\tau d\sigma \tag{2.36}$$

This describes how a string carrying electric Kalb-Ramond charge couples to the antisymmetric Kalb-Ramond field. The new field strength associated to $B_{\mu\nu}$ is $H_{\mu\nu\rho}$ is defined by

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \tag{2.37}$$

The $H_{\mu\nu\rho}$ plays the same role as torsion in general relativity providing an antisymmetric component to the affine connection.

The total action, analogous to the corresponding Maxwell action, is

$$S = S_{str} - \frac{1}{2}\int \frac{\partial X^{[\mu}}{d\tau}\frac{\partial X^{\nu]}}{d\sigma} B_{\mu\nu}\big(X(\tau,\sigma)\big)d\tau d\sigma + \int d^D x\Big(-\frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\Big) \tag{2.38}$$

where $x^{[\mu}y^{\nu]} \equiv x^\mu y^\nu - x^\nu y^\mu$. S_{str} includes general relativity. In summary, the bosonic string oscillation include these (26D) quantum fields: the symmetric metric tensor $G_{\mu\nu}(X)$, the antisymmetric $B_{\mu\nu}(X)$, and the scalar $\phi(X)$.

In 4D the equations of motion imply that the dual of H field strength, $\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho}$ can be represented as $\partial^\sigma b(x)$, where $b(x)$ is a pseudoscalar, the Kalb-Ramond axion. It is a generalization of Peccei-Quinn axion. We will discuss axions and torsion in later sections.

3 Visible Matter

Visible, or the SM matter, has been discussed in [14, 15]. Here we only change to clarify the notation for superons towards what is used in [8] as seen in Table 2. There it is seen that for u quarks the m^0 is permutated from position three to two. Similarly for the d quark the m^- is rotated between the same positions. Leptons consist of three like superons and can be rotated only as classical particles. In fact, both quarks and leptons are to be considered in this scenario as consisting of classical superons obeying determinitic equation of motion.

SM Matter	Superon state
ν_e	$m^0m^0m^0$
u_R	$m^+m^+m^0$
u_G	$m^+m^0m^+$
u_B	$m^0m^+m^+$
e^-	$m^-m^-m^-$
d_R	$m^0m^0m^-$
d_G	$m^0m^-m^0$
d_B	$m^-m^0m^0$
Dark Matter	Particle
boson(system)	axion(s), s^0
o-fermion(system)	n

Table 2: Visible and Dark Matter particles.

4 Dark Matter

For a general introduction to particle dark matter, see e.g. [21]. Literature on dark matter, dark energy, and axions is extensive, see e.g. [22, 23, 24, 25]. In this section we patch our shortage in [15] to consider the pseudoscalar of (2.34). So we start from the Lagrangian (2.34).

As stated in the previous section 2.4, the superpartners of the axion a are the fermionic axino n, and the scalar saxion s^0, also indicated in Table 1.[3] Particle dark matter consists of all these three particles. The axino n may appear physically as single particle dust or three n composite o dust, gas, or a large astronomical object. The fermionic DM behaves naturally very differently from bosonic DM, which may form in addition Bose-Einstein condensates.

Other candidate forms of DM include primordial black holes (PBH). They can be produced by gravitational instabilities induced from scalar fields such as axion-like particles or multi-field inflation. It is shown in [26] that PBH DM can be produced only in two limited ranges of 10^{-15} or 10^{-12} Solar masses (2×10^{30} kg). Dark photons open a rich phenomenology described [27]. We also mention another supergravity (the graviton-gravitino supermultiplet) based model [28], which may help to relieve the observed Hubble tension [29].

The axion was originally introduced to solve the strong CP problem in quantum chromodynamics (QCD) [18], see also [30, 31]. The PQ axion has a mass in the range 10^{-5} eV to 10^{-3} eV. Axions, or axion-like particles (ALP), occur also in string theory in large numbers (in the hundreds), they form the axiverse.

The axion-like particle masses extend over many orders of magnitude making them distinct candidate components of dark matter. Ultra-light axions (ULA), with masses 10^{-33} eV $< M_a < 10^{-20}$ eV, roll slowly during inflation and behave like dark energy before beginning to oscillate (as we see below). The lightest

[3] In this note we mostly talk about all spin zero particles freely as scalars.

ULAs with $M_a \lessapprox 10^{-32}$ eV are indistinguishable from dark energy. Higher mass ALPs, $M_a \gtrapprox 10^{-25}$ eV behave like cold dark matter [25]. Quantum mechanically, an axion of mass of, say 10^{-22} eV, has a Compton wavelength of 10^{16} m.

Ultra-light bosons with masses $\ll$ eV can form macroscopic systems like Bose-Einstein condensates, such as axion stars [19, 20]. Due to the small mass the occupation numbers of these objects are large, and consequently, they can be described classically.

The fermionic axino n is supposed to appear, like the m superons, as free particle if $T > \Lambda_{cr}$ and when $T \lessapprox \Lambda_{cr}$ in composite states. If the mass of the axino composite state o is closer to the electron mass rather than the neutrino mass it may form 'lifeless' dark stars in a wide mass range. In general, dark matter forms haloes with galaxies residing within.

To obtain a feeling of the possible roles of axions let us go briefly to the early universe. Axions are treated as spectator fields during inflation [22, 23, 24].[4] In fact, all superons are spectators until reheating, which in turn heats the visible matter only. The axion is massless as long as non-perturbative effects are absent. When these effects are turned on the PQ symmetry is broken and the axion acquires a mass. A minimally coupled scalar field ϕ in General Relativity has an action

$$S = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\Big] \tag{4.1}$$

In the Friedmann-Lemaitre-Robertson-Walker metric with potential $V = \frac{1}{2}M_a^2\phi^2$ [5] the axion equation of motion is

$$\ddot{\phi}_0 + 2\mathcal{H}\dot{\phi}_0 + M_a^2 a^2\phi_0 = 0 \tag{4.2}$$

where ϕ_0 is the homogeneous value of the scalar field as a function of the conformal time τ, a is here the cosmological scale factor, and dots denote derivatives with respect to conformal time.

At an early time $t_i \gtrapprox 10^{-36}$s, $M_a \ll H$ and the axion rolls slowly. If the initial velocity is zero it has equation of state $w_a \equiv P_a/\rho_a \simeq -1$. Consequently, the axion is a component of dark energy. With $t > t_i$ the temperature and H decrease and the axion field begins to oscillate coherently at the bottom of the potential. This happens when

$$M_a = 3H(a_{osc}) \tag{4.3}$$

which defines the scale factor a_{osc}. Now the number of axions is roughly constant and the axion energy density redshifts like matter with $\rho_a \propto a^{-3}$. The relic density parameter Ω_a is

$$\Omega_a = \left[\frac{1}{2a^2}\dot{\phi}_0^2 + \frac{M^2 a}{2}\phi_0^2\right]_{M_a^2=3H} a_{osc}^3/\rho_{crit} \tag{4.4}$$

[4] On the other hand, the axion can be modeled as causing the inflation [32].

[5] This is an adequate approximation over most of the parameter space observationally allowed provided $f_a < M_{\rm Pl}$. The potential is anyway unknown away from the minimum without a model for nonperturbative effects.

where ρ_{crit} is the cosmological critical density today. Explicit estimates for the relic density are given in [25]. This applies to all axion-like particles, if there are many like in string theory.

When radiation and matter match in ΛCDM model the Hubble rate is $H(a_{eq}) \sim 10^{-28}$ eV. Axions with mass larger than 10^{-28} eV begin to oscillate in the radiation era and may provide for even all of dark matter. The upper limit of the ultralight axion mass fraction Ω_a/Ω_{DM}, where Ω_a is the axion relic density and Ω_{DM} is the total DM energy density parameter, varies from 0.6 in the low mas end 10^{-33} eV to 1.0 in the high mass limit 10^{-24} eV. In the middle region Ω_a/Ω_{DM} is constrained to be below about 0.05 [25].

The dark fermions may be at this stage be approximated as scalars or as fermion-antifermion pairs. Their behavior follows that of scalar particles until reheating at which time the composite states o may form (without heating up).

5 Inflation and Supergravity

This section is a brief review of work done by other authors. It is included because CMB measurements offer data of inflation in the relevant energy region for testing supergravity.

The era of the universe before inflation (t $< 10^{-36}$ s) is largely unknown. A possible assumption is that it is a phase of strings of gravitation, with quantum fluctuating energy. Within this scenario it would be a condensed state of gravitons. From section 2.5, we assume that some scalar ϕ will initiate inflation, which is discussed below in terms of supergravity, the low energy limit of string theory.[6]

At the beginning of inflation, $t = t_i \sim 10^{-36}$ s, the universe is modeled by gravity and a scalar inflaton ϕ with some potential $V(\phi)$. The Einstein-Hilbert action is

$$S = \int d^4x\sqrt{-g}\Big(\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big) \tag{5.1}$$

Inflation ends at $t_R \approx 10^{-32}$ s when the inflaton, which is actually coherently oscillating homogeneous field, a Bose condensate, reaches the minimum of its potential. There it oscillates and decays by coupling to SM particles produced from m superons at the end of inflation. This causes the reheating phase, or the Bang, giving visible matter particles more kinetic energy than dark matter particles have.

The CMB measurements of inflation can be well described by a few simple slow-roll single scalar potentials in (5.1). One of the best fits to Planck data [34] is obtained by one of the very oldest models, the Starobinsky model [35]. The action is

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\Big(R + \frac{R^2}{6M^2}\Big) \tag{5.2}$$

[6] The issues of the trans-Planckian zone for inflationary models are reviewed in [33] but they are beyond the scope of this note.

where $M \ll M_{\rm Pl}$ is a mass scale. Current CMB measurements indicate scale invariant spectrum with a small tilt in scalar density $n_s = 0.965 \pm 0.004$ and an upper limit for tensor-to-scalar ratio $r < 0.06$. These values are fully consistent with the Starobinsky model (5.2) which predicts $r \simeq 0.003$.

The model (5.2) has the virtue of being based on gravity only physics. Furthermore, the Starobinsky model has been shown to correspond to no-scale supergravity coupled to two chiral supermultiplets. Some obstacles have to be sorted out before reaching supergravity. In this section we follow the review by Ellis, García, Nagata, Nanopoulos, Olive and Verner [36].

The first problem with generic supergravity models with matter fields is that their effective potentials do not provide slow-roll inflation as needed. Secondly, they may have anti-deSitter vacua instead of deSitter ones. Thirdly, looking into the future, any new model of particles and inflation should preferably be consistent with some string model properties. These problems can be overcome by no-scale supergravity models. No-scale property comes from their effective potentials having flat directions without specific dynamical scale at the tree level. This has been derived from string models, whose low energy effective theory supergravity is.

Other authors have studied other implications of superstring theory to inflationary model building focusing on scalar fields in curved spacetime [32] and the swampland criteria [37, 38, 39]. These studies point out the inadequacy of slow roll single field inflation. We find it important to establish first a connection between the Starobinsky model and (two field) supergravity.

The bosonic supergravity Lagrangian includes a Hermitian function of complex chiral scalar fields ϕ_i which is called the Kähler potential $K(\phi^i, \phi^*_j)$. It describes the geometry of the model. In minimal supergravity (mSUGRA) $K = \phi^i \phi^*_i$. Secondly the Lagrangian includes a holomorphic function called the superpotential $W(\phi^i)$. This gives the interactions among the fields ϕ^i and their fermionic partners. K and W can be combined into a function $G \equiv K + \ln |W|^2$. The bosonic Lagrangian is of the form

$$\mathcal{L} = -\frac{1}{2}R + K^j_i \partial_\mu \phi^i \partial^\mu \phi^*_j - V - \frac{1}{4}\mathrm{Re}(\mathrm{f}_{\alpha\beta})\mathrm{F}^\alpha_{\mu\nu}\mathrm{F}^{\beta\mu\nu} - \frac{1}{4}\mathrm{Im}(\mathrm{f}_{\alpha\beta})\mathrm{F}^\alpha_{\mu\nu}\tilde{\mathrm{F}}^{\beta\mu\nu} \quad (5.3)$$

where $K^j_i \equiv \partial^2 K / \partial\phi^i \partial\phi^*_j$ and $\mathrm{Im}(\mathrm{f}_{\alpha\beta})$ is the gauge kinetic function of the chiral fields ϕ^i. In mSUGRA the effective potential is

$$V(\phi^i, \phi^*_j) = e^K \left[|W_i + \phi^*_i W|^2 - 3|W|^2 \right] \quad (5.4)$$

where $W_i \equiv \partial W / \partial \phi^i$. It is seen in (5.4) that the last term with negative sign may generate AdS holes with depth $-\mathcal{O}(m^2_{3/2} M^2_{\rm Pl})$ and cosmological instability. Solution to this and the slow-roll problem is provided by no-scale supergravity models. The simplest such model is the single field case with

$$K = -3 \ln(T + T^*) \quad (5.5)$$

where T is a volume modulus in a string compactification.

The single field (5.5) model can be generalized to include matter fields ϕ^i with the followng Kähler potential

$$K = -3\ln(T + T^* - \frac{1}{3}|\phi_i|^2) \tag{5.6}$$

The no-scale Starobinsky model is now obtained with some extra work from the potential (5.4) and assuming $\langle T \rangle = \frac{1}{2}$. For the superpotential the Wess-Zumino form is introduced [40]

$$W = \frac{1}{2}M\phi^2 - \frac{1}{3}\lambda\phi^3 \tag{5.7}$$

which is a function of ϕ only. Then $W_T = 0$ and from $V' = |W_\phi|^2$ the potential becomes as

$$V(\phi) = M^2\frac{|\phi|^2|1 - \lambda\phi/M|^2}{(1 - |\phi|^2/3)^2} \tag{5.8}$$

The kinetic terms in the scalar field Lagrangian can be written now

$$\mathcal{L} = (\partial_\mu\phi^*, \partial_\mu T^*)\Big(\frac{3}{(T + T^* - |\phi|^2/3)^2}\Big)\begin{pmatrix} (T+T^*)/3 & -\phi/3 \\ -\phi^*/3 & 1 \end{pmatrix}\begin{pmatrix} \partial^\mu\phi \\ \partial^\mu T \end{pmatrix} \tag{5.9}$$

Fixing T to some alue one can define the canonically normalized field χ

$$\chi \equiv \sqrt{3}\tanh^{-1}\left(\frac{\phi}{\sqrt{3}}\right) \tag{5.10}$$

By analyzing the real and imaginary parts of χ one finds that the potential (5.8) reaches its minimum for Im$\chi = 0$. Reχ is of the same form as the Starobinsky potential in conformally transformed Einstein-Hilbert action [41] with a potential of the form $V = \frac{3}{4}M^2(1 - e^{-\sqrt{2/3}\phi})^2$. when

$$\lambda = \frac{M}{\sqrt{3}} \tag{5.11}$$

Most interestingly, λ/M has to be very accurately $1/\sqrt{3}$, better than one part in 10^{-4}, for the potential to agree with measurements.

This is briefly the basic mechanism behind inflation in the Wess-Zumino mSUGRA model, which foreruns reheating for visible matter. Up to now, model dependence in our scenario has been rather mild. Essential during inflation is that none of the fields have interactions, apart from gravity. All particles in (2.34) and (2.35) fulfill this condition. At $T \sim \Lambda_{cr}$ the m and n superons form composite states. But only the particles containing m superons, i.e. the visible matter gets reheated. The dark sector is going through reheating unaffected and is distributed smoothly all over space. The quantum fluctuations of the dark fields are enhanced by gravitation and provide a clumpy underlay for visible matter to form objects of various sizes, from stars to large scale structures.

6 Matter-Antimatter Asymmetry

The crucial fact enabling the asymmetric creation of matter in the early universe is that the *same* twelve superons, namely four m^+, four m^- and four m^0, may form both hydrogen and anti-hydrogen atom by organizing the superons differently in sets of three using table 1:

$$\begin{aligned} p + e^- &:= u^{2/3} + u^{2/3} + d^{-1/3} + e^- \\ &:= \sum_{l=1}^{4} \left[m_l^+ + m_l^- + m_l^0 \right] =: \bar{p} + e^+ \end{aligned} \tag{6.1}$$

where the superscript is the charge of the particle and $\pm$ indicates charge $\pm\frac{1}{3}$ (the $=:$ on the second line must be read from right to left). In this scenario neither baryon number B nor lepton number L is fundamental but the difference of baryon and lepton number is, which can be read from (6.1)

$$\mathrm{B} - \mathrm{L} \equiv 0 \tag{6.2}$$

If (6.2) is elevated as a rule of nature the proton decay $p \to e^+\pi^0$ is forbidden. Here the present scenario differs from the MSSM, which deserves a study of its own.

One may consider B$-$L as a continuous gauge symmetry $U(1)_{B-L}$ [1, 42] above the energy scale Λ_{cr}. We call it $U(1)_{superon}$ because superons are available above Λ_{cr}, not baryons or leptons. The corresponding gauge boson couples only to superons and is not therefore detectable with current detectors.

The superon content of the early universe evolves as follows. The wave function of the universe at $t = t_i$ is an initial state $\Psi = c_0(t_i)\Psi_{superon}$ described by the gravitational superon sector of subsection 2.5.

Towards the end of inflation the phase transition takes place and Ψ develops into standard model universe

$$\Psi = c_1(t)\Psi_{matter} + c_2(t)\Psi_{antimatter} + c_3(t)\Psi_{radiation} \tag{6.3}$$

Nature has chosen the first line of (6.1), or $c_1 \approx 1$ and $c_2 \approx 0 \neq c_3$, but how?[7]

When a large number of superon-antisuperon pairs are created from vacuum the question is which way they will organize themselves: will they be mostly hydrogen, or anti-hydrogen, or mostly radiation? Observations favor the first alternative (the second means only charge redefinition). We try to develop a precursory mechanism for this case.

In this scenario fermionic superons m and n are created as spectator quantum fields when inflation starts and the metric still has significant quantum fluctuations. By the combinatorial (mod 3) rule, there is non-zero quantum

[7] The superon scenario offers a rudimentary candidate solution by assuming first that $c_1 \gtrapprox c_2 > 0$. Later the antimatter section would annihilate its part of the matter section and the rest of matter remains.

probability for three m^- superons to gravitationally, or even spontaneously, form an electron at time $t \gtrapprox t_i$. This probability is increased if there is a transient C asymmetry in spacetime like one caused by torsion which leads to a difference in fermion masses. The superon density is high enough in the early phase of inflation for torsion to be effective. The torsional correction to a fermion mass is $M_t = M + a/M_{\rm Pl}^2$ where $a \propto 1$ [43]. For an antifermion the correction term is negative. In the environment at $t \gtrapprox t_i$ this mass difference needs not be small. The newly formed e^- is expected to create subtle order by causing movement of the lighter superons in spacetime towards it. It generates a small correlation length λ_{cor}, and a corresponding 3D volume, within which different superon charge states are differentiated. Therefore the electron causes the formation of a correlated region, or bubble, contains antifermions m^+ and m^0, which in turn form u and d quarks and much later hydrogen atoms with the electron.

Inflation is advanced by the potential (5.7). After the first electron-quark pair correlation has formed the correlation length scale λ_{cor} and the corresponding bubble volume expand exponentially due to inflation.[8] Particles move away, in their co-moving frames, from each other due to inflationary expansion of space. Inside the first bubble, every new bubble, which contains twelve, or in fact a myriad more, superons at high density in the formation point, the torsion induced correlation occurs again between the three heavier m^- and the lighter two m^+ and an m^0 (or an m^+ and two m^0). Consequently, predominantly standard model matter production occurs during inflation.

The inflaton decay takes place after the inflaton has reached the minimum of its potential and it couples to the quarks and leptons while vibrating in its ground state causing reheating. The SM particles have now only a few antiparticles to annihilate with. Without further interactions we have $r_B \approx 0$. The expansion, reheating and all the later processes ultimately produce what we see as the observed universe.

All dark matter is smoothly distributed, apart from quantum fluctuations of the corresponding fields, in the universe after inflation because they were unaffected by the reheating. Gravity strengthens, however, clumps in dark matter. Visible matter fields in turn loose their original quantum fluctuations and are remodulated by reheating towards uniform distribution in space. Quantum fluctuations in the dark fields during inflation may lead to formation of primordial black holes in the universe. These density variations of DM provide attractive gravitational potential regions for visible matter to accumulate in the various formations we observe [21].

Fermionic dark matter has in this scenario no mechanism to become 'baryon' asymmetric like visible matter. Therefore we expect that part of dark matter has annihilated into bosonic dark matter. Secondly, there should exist both dark matter and anti-dark matter clumps in the universe. Collisions of anti-dark matter and dark matter celestial bodies would give us a new source for

[8] This idea of λ_{cor} growing exponentially during inflation was suggested to us by R. Brandenberger.

wide spectrum gravitational wave production (the lunar mass alone is $\sim 10^{49}$ GeV). High dark matter density is found only in the solitonic halo centers. Such collisions are obviously rare.

7 Supersymmetry Breaking

There are several ways supersymmetry may get broken, and they are described extensively in a number of articles, reviews and textbooks [1, 2, 3, 4]. The first and to us the relevant method is the gravitationally mediated scenario. Supersymmetry is unbroken in the superon sector and is mediated by gravitational interaction to the visible minimal supersymmetric standard model sector by soft term contributions, which means that the Lagrangian has two terms: symmetric and symmetry breaking

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{soft} \tag{7.1}$$

where $\mathcal{L}_{soft}$ violates supersymmetry but only by mass terms and coupling constants having positive mass dimension. It can be done consistently with the section 5.[9]

The brief description is that if supersymmetry is broken in the superon sector by a vev $\langle F \rangle$ then the soft terms in the visible sector are expected to be approximately $M_{soft} \sim \langle F \rangle / M_{\rm Pl}$. For $M_{soft} \sim 200$ GeV one would estimate that the scale associated with supersymmetry breaking in the superon sector is about $\sqrt{\langle F \rangle} \sim 10^{10}$ or 10^{11} GeV, which must be below Λ_{cr} for consistency. This way the MSSM soft terms arise indirectly or radiatively, instead of tree-level renormalizable couplings to the supersymmetry breaking parameters. The gravitino mass is of the order of the masses of the MSSM sparticles. The gravitino in turn mediates the symmetry breaking with gravitational coupling to the MSSM. A gravitino mass of the order of TeV gives a lifetime 10^5 s, long enough not to disturb nucleosynthesis by decay products.

8 Conclusions

By defining the fundamental fields as superons in (2.34) and (2.35) in section 2 it has been possible to develop a scenario for asymmetric visible matter as well as for the symmetric dark sector. The latter includes both fermionic and bosonic fields, which may and many will conglomerate. The bosonic sector of (2.34) contains axion-like particles, a string theory concept. They are obvious candidates for bosonic dark matter are axions when $M_a \gtrapprox 10^{-25}$ eV and dark energy when $M_a \lessapprox 10^{-32}$ eV. The deterministic nature of superons provides interesting insight to the origin and nature of quantum mechanics.

The matter-antimatter asymmetry is, according to our proposal, created from C symmetric, baryon and lepton neutral superons without the Sakharov

[9] If needed, the MSSM superpartners can be thought of in terms of superons by adding an m^0 to the three m composites.

symmetry breaking conditions. Below the transition energy Λ_{cr} fractional charge three superon composites form quarks while charge zero and one states are leptons. These composite states are to a good approximation point-like, radius between 10^{-18} cm and the electron Cartan radius. Baryons and electrons are produced towards the end of inflation in equal amounts (B−L=0) by the matter production process described in section 6. Dark matter is insensitive to reheating and therefore occurs in the universe as a background gravitational potential for visible matter to form the astronomical objects we observe. Dark matter celestial body annihilation phenomena would provide a new source for observing gravitational waves.

In nutshell, starting from the Wess-Zumino supergravity Lagrangians with three fermions (m^+, m^0, n), the mSUGRA potential (5.7) and some stringy hints for bosons we have constructed a unified picture of quarks, leptons and the dark sector. The main point, the creation of the matter-antimatter asymmetric universe has been made plausible. The dark sector, instead, is predicted to be C symmetric.

In this analysis the role of superstring theory remains tenuous. This is not surprising since we have discussed a non-GUT 4D low energy model. Should one start from 10D or 11D remains to be seen. Torsion is an ultra high energy density spacetime property in general relativity and string theory. For decisive experimental tests one may have to wait for the next generation neutrino and gravitational wave detection experiments that are able to measure the energy range above EeV.

To build this scenario to an acceptable level, or to prove or disprove the scenario, extensive simulations have to be done, more detailed Lagrangians be written and calculated. Phenomenological work is to be carried out with current data for many details like supersymmetry breaking and particle masses while waiting for future precision experiments to be carried out in the years to come. Machine learning and quantum computing may provide new methods for quantitative studies.

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