Exponents in imaginary numbers different from i

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Abstract:

In this paper I show how it is possible to find the value of an exponent in the square root of a negative number. Using four formulas whose I have develop.

Introduction:

I started to ask myself in we can find a pattern to the negative integer numbers different than -1 inside a square root. I found a group of four formulas whose define the pattern. With this formulas you can develop an application method based in the idea of modulus.

Formulas:

(1.1) $x^{(2k)} \cdot (\sqrt{-x})$

(1.2) $-x^{(2k+1)}$

(1.3) $-x^{(2k+1)} \cdot (\sqrt{-x})$

(1.4) $x^{(2(k-1)+2)}$

We define k as the integer part of the result in the division of the exponent by 4. We define x as the number inside the root in positive.
The formulas 1.1, 1.2, 1.3 and 1.4 says to us which one we should apply following the modulus of the exponent:
- 1.1 if the exponent is congruent to 1 modulus 4.
- 1.2 if the exponent is congruent to 2 modulus 4.
- 1.3 if the exponent is congruent to 3 modulus 4.
- 1.4 if the exponent is congruent to 4 modulus 4.

Examples and applications of the method:

(2.1) $(\sqrt{-3})^0 = 1$

(2.2) $(\sqrt{-3})^1$ $k=1\div 4=0$ $1\equiv 1(mod\,4)$ We should apply the formula 1.1:

$3^{(2\cdot 0)}\cdot(\sqrt{-3})=3^{(0)}\cdot(\sqrt{-3})=1\cdot\sqrt{-3}=\sqrt{-3}$ As we can see the result is the same.

(2.3) $(\sqrt{-3})^2$ $k=2\div 4=0$ $2\equiv 2(mod\,4)$ We should apply the formula 1.2:

$-3^{(2\cdot 0+1)}=-3^1=-3$ Without formula: $(\sqrt{-3})\cdot(\sqrt{-3})=-3$

(2.4) $(\sqrt{-3})^3$ $k=3\div 4=0$ $3\equiv 3(mod\,4)$ We should apply the formula 1.3:

$-3^{(2\cdot 0+1)}\cdot(\sqrt{-3})=(-3)^1\cdot(\sqrt{-3})=-3\cdot\sqrt{-3}$

Without formula: $(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})=-3\cdot\sqrt{-3}$

(2.5) $(\sqrt{-3})^4$ $k=4\div 4=1$ $4\equiv 4(mod\,4)$ We should apply the formula 1.4:

$3^{(2(1-1)+2)}=3^{(2(0)+2)}=3^2=9$

Without formula: $(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})=(-3)\cdot(-3)=9$

(2.6) $(\sqrt{-3})^5$ $k=5\div 4=1$ $5\equiv 1(mod\,4)$ We should apply the formula 1.1:

$3^{(2\cdot 1)}\cdot(\sqrt{-3})=3^2\cdot(\sqrt{-3})=9\cdot\sqrt{-3}$ We check it without formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})=(-3)\cdot(-3)\cdot(\sqrt{-3})=9\cdot\sqrt{-3}$

(2.7) $(\sqrt{-3})^6$ $k=6\div 4=1$ $6\equiv 2(mod\,4)$ We should apply the formula 1.2:

$-3^{(2\cdot 1+1)}=-3^{(2\cdot 1+1)}=-3^3=-27$ We check without formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})=(-3)\cdot(-3)\cdot(-3)=-27$

(2.8) $(\sqrt{-3})^7$ $k=7\div 4=1$ $7\equiv 3(mod\,4)$ We should apply the formula 1.3:

$-3^{(2\cdot 1+1)}\cdot(\sqrt{-3})=-3^3\cdot(\sqrt{-3})=-27\cdot(\sqrt{-3})$ Without formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})=(-3)\cdot(-3)\cdot(-3)\cdot(\sqrt{-3})=-27\cdot\sqrt{-3}$

(2.9) $(\sqrt{-3})^8$ $k=8\div 4=2$ $8\equiv 4(mod\,4)$ We should apply the formula 1.4:

$3^{(2(2-1)+2)}=3^{(2+2)}=3^4=81$ We check it without formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3}) =$

$= (-3)\cdot(-3)\cdot(-3)\cdot(-3)=9\cdot 9=81$

(2.10) $(\sqrt{-3})^9$ $k=9\div 4=2$ $9\equiv 1(mod\,4)$ We should apply the formula 1.1:

$3^{(2\cdot 2)}\cdot(\sqrt{-3})=3^4\cdot(\sqrt{-3})=81\cdot\sqrt{-3}$ Without formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3}) =$

$= (-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(\sqrt{-3})=9\cdot 9\cdot(\sqrt{-3})=81\cdot\sqrt{-3}$

(2.11) $(\sqrt{-3})^{10}$ $k=10\div 4=2$ $10\equiv 2(mod\,4)$ We should apply the formula 1.2:

$-3^{(2\cdot 2+1)}=-3^5=-243$ And again we check without the formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3}) =$

$= (-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(-3)=9\cdot 9\cdot(-3)=-243$

(2.12) $(\sqrt{-3})^{11}$ $k=11\div 4=2$ $11\equiv 3(mod\,4)$ We should apply the formula 1.3:

$-3^{(2\cdot 2+1)}\cdot(\sqrt{-3})=-3^5\cdot(\sqrt{-3})=-243\cdot(\sqrt{-3})$ We check without the formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3}) =$

$= (-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(\sqrt{-3})=9\cdot 9\cdot(-3)\cdot(\sqrt{-3})=-243\cdot(\sqrt{-3})$

(2.13) $(\sqrt{-3})^{12}$ $k=12\div 4=3$ $12\equiv 4(mod\,4)$ We should apply the formula 1.4:

$3^{(2(3-1)+2)}=3^{(2\cdot 2+2)}=3^{(4+2)}=3^6=729$ We check without the formula:

$(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3})\cdot(\sqrt{-3}) =$

$= (-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(-3)=9\cdot 9\cdot 9=729$

Conclusion:

We have seen as example the negative number 3 inside a root from the 0-th exponent to the 12-th exponent, but in my opinion this formulas work with any negative integer number and any exponent.