Double Relativity: An Inconsistent reflection of light

(Adapted from a draft chapter of the book *Discrete Relativity*, in preparation)

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Abstract.-This paper discusses the role of Lorentz transformation in two inconsistent changes in the velocity of a photon moving through a standard fluid each time the photon is reflected by a mirror inside the fluid, being the fluid at rest in its container and the container observed at rest and in uniform relative motion.

1.-Conventions

All reference frames (frames hereafter) will be assumed to be inertial. RF_o will denote the frame of any object or observer at rest in that frame. RF_v will denote another frame in relative motion with respect to RF_o, whose axes coincide with the corresponding axes of RF_o at a certain instant. The axes in the plane XY of RF_o and RF_v will be denoted respectively by X_o, Y_o and X_v, Y_v. From the perspective of RF_v, RF_o will be assumed to move at a uniform velocity v parallel to X_v and such that $v = kc, 0 < k < 1$, where c is the speed of light in a vacuum. Lengths, times and refractive indices measured in RF_o and RF_v will be respectively sub-indexed by o and v. Lorentz transformation will be denoted by LT. And, unless otherwise indicated, the term "velocity" will be used to refer to the module of the vector velocity, i.e. as a synonym of speed.

2.-Double relativity

At the beginning of this century, Amelino Camelia proposed a solution [3] to the problem of the incompatibility of LT with the character of universal constants of Planck length and Planck time, a solution now known as Doubly Special Relativity, so called because it includes a minimum length and a maximum energy as universal constants (apart from a maximum velocity). Though the theory was not enthusiastically received [[5], [2], [1], [4]], it is a well known relativistic refinement associated with its original name (it is also associated with the names Deformed Special Relativity and Extra Special Relativity). It is for this reason that I would like to stress the discussion that follows has nothing to do with Doubly Special Relativity, but it is a discussion on an aspect of the special relativity that should be termed Double Relativity. Indeed, the discussion that follows deals with objects that move inside (through) other objects that in turn are in relative motion with respect certain frames, but focusing the attention on the relative motion of the first objects with respect to second ones. For instance the motion of a photon through a transparent media at rest in its container while the container moves relative to a given frame RF_v. The calculation of the velocity of the first object with respect to the frame RF_v is a classical relativistic problem, but it is not the problem we are here interested in. Here, we are interested in the velocity of the first object with respect to second one calculated by means of the rulers and the clocks of RF_v, or by making an appropriate use of LT.

A key concept in the discussion that follows will be the concept of velocity: the ratio of the distance an object traverses to the time taken [6, p. 514]. Though in our case the distance can be a moving distance, for example the distance an object O_1 traverses inside a second object O_2 in relative motion. But, moving as it may be, it will always be a fixed distance; for example, the length of the traversed object O_2. A distance that can be calculated according to all relativistic requirements within a frame, for instance RF_v, respect to which both objects move. Obviously, the velocity of O_1 through O_2 is different from the velocity of O_1 with respect to RF_v, which is the relativistic sum of the velocity of O_1 with respect to O_2, plus the velocity of O_2 with respect to RF_v. The distance O_1 traverses through O_2 (for example the length of O_2) is also different from the distance O_1 traverses with respect to RF_v. Indeed, the distance O_1 traverses with respect to RF_v is the sum of the distance O_1 traverses through O_2 plus the distance O_2 moves with respect to RF_v while O_1 completes its trip though O_2 (whenever O_1 and O_2 move along parallel trajectories). Obvious as it may seem, it will be proved the time elapsed while traversing both distances is the same and equal to the time given by LT for the second of them. On the other hand, if the definition of *speed through an object* only holds for objects observed at rest, this *ad hoc* restriction should be explicitly declared in both the physical definition of speed and the First Principle of relativity: the laws of physics are the same in all frames, unless the involved speeds are speeds through objects in relative motion. Evidently, according to this restriction of the First Principle of relativity, certain physical phenomena as the reflection or the refraction of light moving through two transparent media, air and water for instance, could only be examined and interpreted in physical terms in the rest frame of the corresponding transparent media.

3.-The Scenario of the Discussion

This section defines and prepares the scenario for the discussion that will be developed in the next section: the analysis of the speed of a photon through a standard fluid FL of refractive index $n_o > 1$ (for instance water at standard conditions) which is at rest in a container CO, in turn at rest in a frame RF_o that moves relative to another frame RF_v. As Figures 1 illustrates, the container CO of our discussion has a square section whose sides are placed parallel to the axis X_o and Y_o of its rest frame RF_o. CO is equipped with a laser source LS and a laser detector LD on its left side, where they can be adjusted. For the present

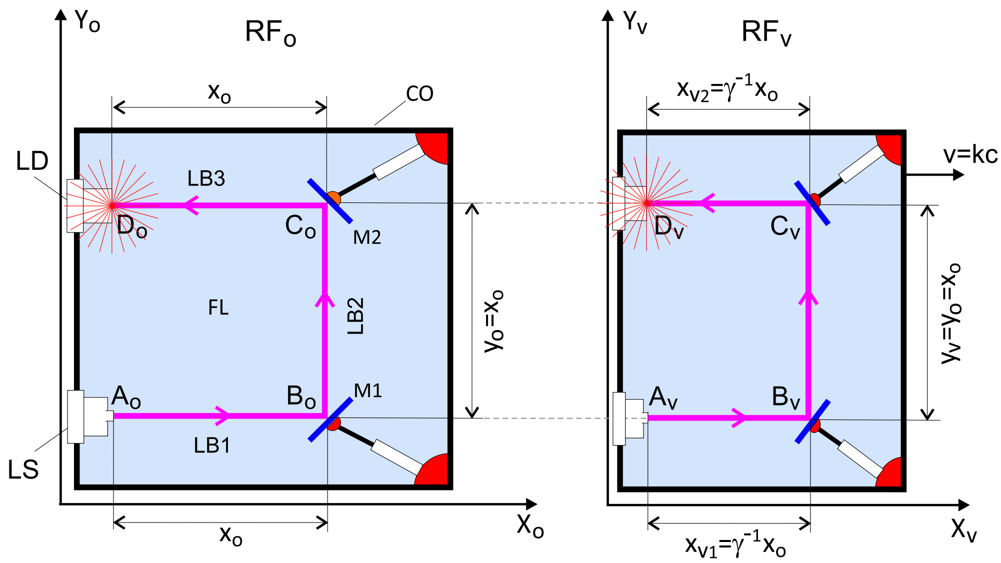

Fig. 1 – A section parallel to the plane X_oY_o of the standard fluid FL within its container CO in its rest frame RF_o (left) and in the frame RF_v (right), in which it its parallel to the plane X_vY_v. LS adjustable laser source; $LB1$, $LB2$ and $LB3$ mutually orthogonal parts of the visible laser beam trajectory; $M1$ and $M2$ adjustable mirrors; LD adjustable laser detector.

discussion, LS will be assumed to emit a visible laser beam $LB1$ parallel to X_o and so that it impacts on an adjustable mirror $M1$ that reflects it in a ray $LB2$ perpendicular to the incident $LB1$, i.e $LB2$ is parallel to Y_o. This ray $LB2$ reaches a second adjustable mirror $M2$ that reflects it in a third ray $LB3$ perpendicular to $LB2$, and then parallel to X_o. Finally, $LB3$ impacts on an the detector LD which emits an appropriate signal visible in all frames, whether at rest or in relative motion. The laser source LS, the mirrors $M1$ and $M2$ and the laser detector LD are adjusted in their rest frame RF_o in such a way that:

1. The trajectories A_oB_o and C_oD_o respectively of $LB1$ and $LB3$ are parallel to X_o and have the same length x_o.
2. The trajectory B_oC_o of $LB2$ is parallel to Y_o and has a length $y_o = x_o$.

According to the above established conventions, RF_v is a frame that coincides at a certain instant with RF_o, and from whose perspective RF_o moves parallel to X_v at a velocity $v = kc$, $0 < k < 1$. Therefore, and according to LT, from the perspective of RF_v:

1. The trajectories A_vB_v and C_vD_v respectively of $LB1$ and $LB3$ are parallel to X_v and have the same length $x_{v1} = x_{v2} = \gamma^{-1}x_o$.
2. The trajectory B_vC_v of $LB2$ is parallel to Y_v and has a length $y_v = y_o = x_o$.

The next section analyses the speed of a photon of the laser beam LB moving through the fluid FL, which, being a standard fluid, is an amorphous (non-crystalline) material and then isotropic with respect to the refractive index: the refractive index n_o of FL is the same in all directions through which light moves. Or in other words, light moves with the same velocity in all directions through FL, a conclusion of which we have the highest theoretical and empirical evidence. In particular, we will analyze the velocity of a photon ϕ of the laser beam LB through FL from the perspective of both RF_o and RF_v. But while the velocity c/n_o of ϕ through FL is the same as the velocity of ϕ with respect to RF_o, the velocity of ϕ through FL is different from the velocity of ϕ with respect to RF_v, which is the relativistic sum of the velocity c/n_o and the velocity kc of the container CO, and then of FL, with respect to RF_v.

According to the definition of velocity (scalar velocity, or module of the vector velocity), the velocity of a photon through FL is the ratio of the traversed distance through FL to the time the photon takes to traverse it. Both magnitudes, the distance and the time, can be measured in RF_o and in RF_v with their respective clocks and rulers. The distances measured in both frames can be directly transformed into each other by LT; and the times the photon travels through FL in RF_o and in RF_v will be proved to be the same as the respective times the photon travels with respect to RF_o and to RF_v (obvious as it may seem, it must be proved). In RF_o a photon of the laser beam LB always moves from A_o to B_o; then from B_o to C_o; and then from C_o to D_o. In RF_v the same photon always moves from A_v to B_v; then from B_v to C_v; and then from C_v to D_v. According to the adjustments in RF_o, it holds

$$A_oB_o = C_oD_o = x_o \tag{1}$$

$$B_oC_o = y_o \tag{2}$$

$$x_o = y_o \tag{3}$$

In RF_v, and according to LT, it holds:

$$A_vB_v = x_{v1} = \gamma^{-1}x_o \tag{4}$$
$$B_vC_v = y_v = y_o = x_o \tag{5}$$
$$C_vD_v = x_{v2} = \gamma^{-1}x_o \tag{6}$$

Since RF_o and RF_v are inertial reference frames, no force acts on them so that (1)-(3) and (4)-(6) hold wile performing all observations and measurements, and they are constants for each relative uniform velocity. They are, then, the distances a photon traverses *through* FL when going respectively from the source LS to the mirror $M1$, from the mirror $M1$ to the mirror $M2$, and from the mirror $M2$ to the detector LD. Obviously, these distances are different from the distances the photon traverses with respect to RF_v, as will be shown later.

With respect to time, and considering the isotropic nature of FL, light travels through FL at the same velocity in all directions. So, in RF_o a photon of the laser beam LB takes the same time t_o to go from A_o to B_o as to go from B_o to C_o as to go from C_o to D_o, i.e. it lasts a time $3t_o$ to go from A_o to D_o through a distance $3x_o$. In the case of RF_v, and denoting by t_{vab}, t_{vbc} and t_{vcd} the respective times a photon takes to go from A_v to B_v, from B_v to C_v and from C_v to D_v (times between the events start moving at A_v-end at B_v;start moving at B_v-end at C_v; start moving at C_v-end at D_v), all of them of the same duration t_o in RF_o, LT gives:

$$t_{vab} = \gamma t_o + \frac{\gamma x_o kc}{c^2} \tag{7}$$
$$= \gamma\left(t_o + \frac{x_o k}{c}\right) \tag{8}$$
$$t_{vbc} = \gamma t_o \tag{9}$$
$$t_{vcd} = \gamma t_o - \frac{\gamma x_o kc}{c^2} \tag{10}$$
$$= \gamma\left(t_o - \frac{x_o k}{c}\right) \tag{11}$$

Hence, in RF_v a photon of LB lasts a time $3\gamma t_o$ in going from A_v to D_v. The problem is that in RF_v the photon moves through FL a distance that is not $3\gamma^{-1}x_o$, but $(1 + 2\gamma^{-1})x_o$, which is related to the problem the next section examines.

Unnecessary as it may seem, it will be proved now the time t_{vab} a photon travels at a velocity c/n_v through FL when going from A_v to B_v, is the same as the time t_v it lasts in traversing the distance $A_vB_v + kct_v$ at the velocity c_v with respect to RF_v, which is the velocity resulting from the relativistic sum of the velocities c/n_o and kc, which is given by:

$$c_v = \frac{\dfrac{c}{n_o} + kc}{1 + \dfrac{kcc/n_o}{c^2}} = \frac{\dfrac{c + n_okc}{n_o}}{\dfrac{n_o + k}{n_o}} = \frac{c(1 + n_ok)}{n_o + k} \tag{12}$$

In consequence, it can be written:

$$t_{vab} = \frac{\gamma^{-1}x_o + kct_{vab}}{\dfrac{c(1 + n_ok)}{n_o + k}} = \frac{(\gamma^{-1}x_o + kct_{vab})(n_o + k)}{c(1 + n_ok)} \tag{13}$$
$$ct_{vab}(1 + n_ok) = (\gamma^{-1}x_o + kct_{vab})(n_o + k) \tag{14}$$
$$ct_{vab}(1 + n_ok) = \gamma^{-1}x_o(n_o + k) + kct_{vab}(n_o + k) \tag{15}$$
$$ct_{vab}(1 + n_ok - n_ok - k^2) = \gamma^{-1}x_o(n_o + k) \tag{16}$$
$$ct_{vab}(1 - k^2) = \gamma^{-1}x_o(n_o + k) \tag{17}$$
$$ct_{vab}\gamma^{-2} = \gamma^{-1}x_o(n_o + k) \tag{18}$$
$$t_{vab} = \gamma\left(\frac{n_ox_o}{c} + \frac{kx_o}{c}\right) \tag{19}$$

And being $c/n_o = x_o/t_o$:

$$t_{vab} = \gamma\left(t_o + \frac{x_ok}{c}\right) \tag{20}$$

that coincides with (8). For the case of t_{vbc} in which the photon moves at a velocity c/n_o parallel to Y_v, while the relative velocity kc is parallel to X_v, we will have a vector velocity $\vec{c_v}$ whose components result from the relativistic sum of the vectors $(kc, 0, 0)$ and $(0, c/n_o, 0)$:

$$\vec{c_v} = \left(\frac{0 + kc}{1 + \dfrac{kc \times 0}{c^2}}, \frac{\gamma^{-1}\dfrac{c}{n_o}}{1 + \dfrac{kc \times 0}{c^2}}, \frac{\gamma^{-1}0}{1 + \dfrac{kc \times 0}{c^2}} = 0 \right) \tag{21}$$

$$= (kc, \gamma^{-1}c/n_o, 0) \tag{22}$$

whose module c_v is

$$c_v = \sqrt{k^2c^2 + \gamma^{-2}\frac{c^2}{n_o^2}} = \sqrt{\frac{n_o^2c^2k^2 + \gamma^{-2}c^2}{n_o^2}} = \frac{c}{n_o}\sqrt{n_o^2k^2 + \gamma^{-2}} = \frac{c}{n_o}\sqrt{n_o^2k^2 + 1 - k^2} \tag{23}$$

$$= \frac{c}{n_o}\sqrt{1 + k^2(n_o^2 - 1)} \tag{24}$$

In this case, the photon moves with respect to RF_v a distance d_v:

$$d_v = \sqrt{k^2c^2t_{vbc}^2 + y_o^2} \tag{25}$$

at the velocity c_v given by (24). Hence, it holds:

$$t_{vbc} = \frac{\sqrt{k^2c^2t_{vbc}^2 + y_o^2}}{\dfrac{c}{n_o}\sqrt{1 + k^2(n_o^2 - 1)}} \tag{26}$$

$$t_{vbc}^2\frac{c^2}{n_o^2}(1 + k^2(n_o^2 - 1)) = k^2c^2t_{vbc}^2 + y_o^2 \tag{27}$$

$$t_{vbc}^2c^2(1 + k^2(n_o^2 - 1)) = n_o^2k^2c^2t_{vbc}^2 + n_o^2y_o^2 \tag{28}$$

$$t_{vbc}^2c^2(1 + k^2(n_o^2 - 1)) - n_o^2k^2) = n_o^2y_o^2 \tag{29}$$

$$t_{vbc}^2c^2(1 + n_o^2k^2 - k^2 - n_o^2k^2) = n_o^2y_o^2 \tag{30}$$

$$t_{vbc}^2c^2(1 - k^2) = n_o^2y_o^2 \tag{31}$$

$$t_{vbc}^2c^2\gamma^{-2} = n_o^2y_o^2 \tag{32}$$

$$t_{vbc}c\gamma^{-1} = n_oy_o \tag{33}$$

$$t_{vbc} = \gamma\frac{n_oy_o}{c} = \gamma\frac{y_o}{c/n_o} \tag{34}$$

$$t_{vbc} = \gamma t_o \tag{35}$$

that coincides with (9). Finally, in the case of the trajectory C_vD_v, the velocity c/n_o is parallel but in the opposite sense of kc, so that their relativistic sum is.

$$c_v = \frac{\dfrac{c}{n_o} - kc}{1 - \dfrac{kcc/n_o}{c^2}} = \frac{\dfrac{c - n_okc}{n_o}}{\dfrac{n_o - k}{n_o}} = \frac{c(1 - n_ok)}{n_o - k} \tag{36}$$

and the distance with respect to RF_v the photon traverses is $\gamma^{-1}x_o - kct_{vcd}$. So then, it can be written:

$$t_{vcd} = \frac{\gamma^{-1}x_o - kct_{vcd}}{\dfrac{c(1 - n_ok)}{n_o - k}} = \frac{(n_o - k)(\gamma^{-1}x_o - kct_{vcd})}{c(1 - n_ok)} \tag{37}$$

$$ct_{vcd}(1 - n_ok) = (n_o - k)(\gamma^{-1}x_o - kct_{vcd}) = (n_o - k)\gamma^{-1}x_o - (n_o - k)kct_{vcd} \tag{38}$$

$$ct_{vcd}(1 - n_ok + (n_o - k)k) = (n_o - k)\gamma^{-1}x_o \tag{39}$$

$$ct_{vcd}(1 - n_o k + n_o k - k^2) = (n_o - k)\gamma^{-1} x_o \tag{40}$$

$$ct_{vcd}(1 - k^2) = (n_o - k)\gamma^{-1} x_o \tag{41}$$

$$ct_{vcd}\gamma^{-2} = (n_o - k)\gamma^{-1} x_o \tag{42}$$

$$t_{vcd} = \gamma\left(\frac{n_o x_o}{c} - \frac{k x_o}{c}\right) \tag{43}$$

And being $c/n_o = x_o/t_o$:

$$t_{vab} = \gamma\left(t_o - \frac{x_o k}{c}\right) \tag{44}$$

that coincides with (11)

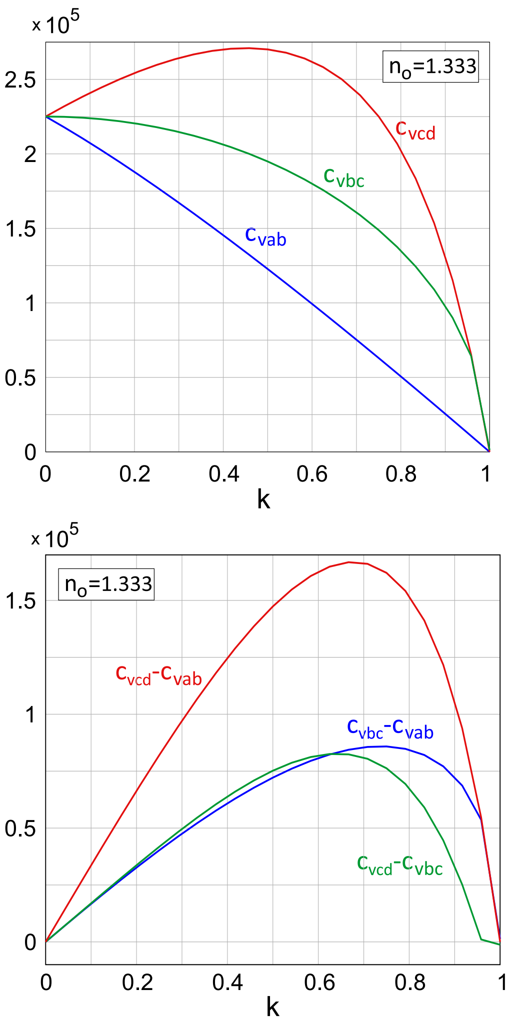

Fig. 2 – Top: the velocity of a photon through FL calculated from RF_v in each of its three mutually orthogonal trajectories. Bottom: Instantaneous changes of velocities of the photon ϕ after each reflection, as observed from RF_v.

4.-Inconsistent changes of velocity

This section examines the velocity of a photon ϕ of the laser beam LB from its emission by the source LS, which takes place at point A_o of RF_o (A_v in RF_v), to its detection by LD, which takes place at point D_o of RF_o (D_v in RF_v). To begin with, recall that what will be examined here is the velocity of a photon through a standard fluid FL with a refractive index $n_o > 1$, for instance water ($n_o = 1.333$) at standard conditions of pressure and temperature (obviously, FL could be any other fluid at many other thermodynamic conditions). As a standard fluid, FL is an amorphous material, i.e. a material without internal crystalline structure (without long-range order) and whose molecules move randomly. In consequence they are randomly distributed in its container CO, and the Law of Large Numbers ensures there is the same number of them in any direction (structural isotropy). Therefore, the number and types of the electromagnetic interactions between light and FL, responsible for the speed of the photon through FL, are the same in all directions. It is for this well known reason that fluids are isotropic with respect to the refractive index: the index of refraction is the same in all directions along which light propagates through them, obviously including the two senses of each direction (the refractive index is the same in both directions of any give direction even in anisotropic media). From the perspective of RF_o, the photon ϕ moves with the same velocity c/n_o along the three mutually orthogonal sections of its trajectory: A_oB_o, B_oC_o and C_oD_o. Things are quite different from the perspective of RF_v. Indeed, in the first part of its trajectory, from A_v to B_v, the photon travels with a velocity c_{vab} given by:

$$c_{vab} = \frac{\gamma^{-1} x_o}{\gamma\left(t_o + \dfrac{kcx_o}{c^2}\right)} = \frac{\gamma^{-1}}{\gamma\left(\dfrac{t_o}{x_o} + \dfrac{k}{c}\right)} \tag{45}$$

$$= \frac{\gamma^{-2}}{\dfrac{n_o}{c} + \dfrac{k}{c}} = \frac{\gamma^{-2} c}{n_o + k} \tag{46}$$

$$= c\,\frac{1 - k^2}{n_o + k} \tag{47}$$

In the second part of its trajectory, ϕ moves with a velocity c_{vbc} given by:

$$c_{vbc} = \frac{y_v}{\gamma t_o} = \frac{y_o}{\gamma t_o} = \frac{c}{\gamma n_o} = \frac{c\sqrt{1 - k^2}}{n_o} \tag{48}$$

And in the third part of its trajectory, from C_v to D_v, ϕ moves with a velocity c_{vcd} given by:

$$c_{vcd} = \frac{\gamma^{-1} x_o}{\gamma\left(t_o - \dfrac{kcx_o}{c^2}\right)} = \frac{\gamma^{-1}}{\gamma\left(\dfrac{t_o}{x_o} - \dfrac{k}{c}\right)} \tag{49}$$

$$= \frac{\gamma^{-2}}{\frac{n_o}{c} - \frac{k}{c}} = \frac{\gamma^{-2} c}{n_o - k} \tag{50}$$

$$= c\frac{1 - k^2}{n_o - k} \tag{51}$$

As Figure 2 (top) shows, the three velocities are different from one another (recall we are using the word ”velocity” for the module of the vector velocity). And the differences can be of several thousands of kilometers per second (even more than 150000 kilometers per second), as Figure 2 (bottom) shows. In consequence, from the perspective of the frame RF_v, the photon ϕ changes instantaneously its velocity after each reflection. But a simple reflection does not change the velocity of the reflected photon, only the direction of its trajectory is modified. And this is, in fact, what happens in the rest frame RF_o of the container CO. There are only two reason for which a photon freely moving through a standard fluid could change its velocity:

1. An appropriate force acts on the photon.
2. The photon begins to move in a new direction through the medium, in which it travels faster because of a decrement of the refractive index in that direction.

The problem is that none of them is the case. In fact, no force acted on ϕ in any point of its trajectory, nor there are special directions with less refractive indexes in the standard fluid FL through which ϕ moved. Notwithstanding, ϕ changed instantaneously its velocity after each reflection. And it was not an infinitesimal change, but one that could be of several thousand of kilometers per second, depending of the relative velocity kc. It is worth noting that these acausal changes are formal consequences of Fitzgerald-Lorentz contraction, time dilation and difference in phase synchronization (lack of simultaneity), i.e. consequences of the whole LT.

5.-Conclusions

The precedent section has proved the existence of unexplained changes in the velocity of a photon moving freely through a standard fluid when observed, via LT, in relative motion. Changes that are not random but regular: under the same conditions (the reflection of the photon by a mirror observed at the same relative velocity) they always happens the same way. But regular as they may be, they should not happen according to the known physical laws; they are incompatible with all of our knowledge on changes of velocity. In addition they do not happen in the rest frame of the mirrors that reflect the photons, which makes them special frames, and then frames that put to test the First Principle of relativity. It could be argued that the world resulting from applying LT to a rest frame is only apparent, unreal, as is unreal the bent of a rod partially and obliquely submerged in water. But even in such a case, the appearances are inconsistent with the known physical laws, at least for photons that are reflected by mirrors while moving freely through standard fluids. In consequence, LT should not be used to get physical conclusions on what happens in reference frames observed in relative uniform motion, in the same way that the observed bending of the rod partially submerged in water should not be used to draw conclusions on the internal structure of the rod. In short, LT gives an inconsistent description of the motion of a photon through a fluid at rest in its container when the container is observed in relative motion, which, at the very least, limits the set of consistent observations that can be transformed by LT.

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