Cosmological Special Theory of Relativity

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ABSTRACT

In the Cosmological Special Relativity Theory, we study Maxwell equations, electromagnetic wave equation and function.

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1. Introduction

Our article's aim is that we make cosmological special theory of relativity.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time[6],

$$k = 0, t = t_0 >> \Delta t\ ,\ \ \Delta t \text{ is period of matter's motion} \tag{2}$$

Hence, the proper time is in cosmological time,

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dr^2 + r^2 d\Omega^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2] \\ &= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2), \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \tag{3}$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \tag{4}$$

Cosmological special theory of relativity's coordinate transformations are

$$\begin{aligned} c\bar{t} &= ct = \gamma(c\bar{t}' + \frac{v_0}{c}\Omega(t_0)\bar{x}') = \gamma(ct + \frac{v_0}{c}\Omega(t_0)x'\Omega(t_0)) \\ \bar{x} &= x\Omega(t_0) = \gamma(\bar{x}' + v_0\Omega(t_0)\bar{t}') = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t') \\ \bar{y} &= \Omega(t_0)y = \bar{y}' = \Omega(t_0)y', \\ \bar{z} &= \Omega(t_0)z = \bar{z}' = \Omega(t_0)z' \quad , \ \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \end{aligned} \tag{5}$$

Therefore, proper time is

$$\begin{aligned} d\tau^2 &= d\bar{t}^2 - \frac{1}{c^2}[d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2] \\ &= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2] \\ &= d\bar{t}'^2 - \frac{1}{c^2}[d\bar{x}'^2 + d\bar{y}'^2 + d\bar{z}'^2] \end{aligned} \tag{6}$$

Hence, velocities are

$$\frac{dx}{dt} = V_x = \frac{V_x' + v_0}{1 + \frac{\Omega^2(t_0)}{c^2} V_x' \cdot v_0}, V_x' = \frac{dx'}{dt'}$$

$$\frac{dy}{dt} = V_y = \frac{V_y'}{\gamma(1 + \frac{\Omega^2(t_0)}{c^2} V_x' \cdot v_0)}, V_y' = \frac{dy'}{dt'}$$

$$\frac{dz}{dt} = V_z = \frac{V_z'}{\gamma(1 + \frac{\Omega^2(t_0)}{c^2} V_x' \cdot v_0)}, V_z' = \frac{dz'}{dt'} \tag{7}$$

In cosmological special theory of relativity(CSTR)'s differential operators are

$$\frac{1}{c}\frac{\partial}{\partial \bar{t}} = \frac{1}{c}\frac{\partial}{\partial t} = \gamma(\frac{1}{c}\frac{\partial}{\partial \bar{t}'} - \frac{v_0}{c}\Omega(t_0)\frac{\partial}{\partial \bar{x}'})$$

$$= \gamma(\frac{1}{c}\frac{\partial}{\partial t'} - \frac{v_0}{c}\frac{\partial}{\partial x'})$$

$$\frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x}\frac{1}{\Omega(t_0)} = \gamma(\frac{\partial}{\partial \bar{x}'} - \frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial \bar{t}'})$$

$$= \gamma(\frac{\partial}{\partial x'}\frac{1}{\Omega(t_0)} - \frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t'})$$

$$\frac{\partial}{\partial \bar{y}} = \frac{\partial}{\partial y}\frac{1}{\Omega(t_0)} = \frac{\partial}{\partial \bar{y}'} = \frac{\partial}{\partial y'}\frac{1}{\Omega(t_0)}$$

$$\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial z}\frac{1}{\Omega(t_0)} = \frac{\partial}{\partial \bar{z}'} = \frac{\partial}{\partial z'}\frac{1}{\Omega(t_0)}, \gamma = 1/\sqrt{1 - \frac{{v_0}^2}{c^2}\Omega^2(t_0)} \tag{8}$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial \bar{t}^2} - \bar{\nabla}^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x})^2 + (\frac{\partial}{\partial y})^2 + (\frac{\partial}{\partial z})^2\}$$

$$= \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} - \frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x'})^2 + (\frac{\partial}{\partial y'})^2 + (\frac{\partial}{\partial z'})^2\} \tag{9}$$

The electric charge density ρ and the electric current density $\vec{j}$ are

$$j^\mu = \rho_0 \frac{dx^\mu}{d\tau}, j^0 = c\rho = c\gamma\rho_0, j^i = \vec{j} = \rho\vec{u}, \ i = 1,2,3 \quad (10)$$

In CSTR, transformations of the electric charge density and the electric current density are likely as coordinate transformations are

$$c\bar{\rho} = c\rho = \gamma(c\bar{\rho}' + \frac{v_0}{c}\Omega(t_0)\bar{j}_x') = \gamma(c\rho' + \frac{v_0}{c}\Omega(t_0)j_x'\Omega(t_0))$$

$$\bar{j}_x = j_x\Omega(t_0) = \gamma(\bar{j}_x' + v_0\Omega(t_0)\bar{\rho}') = \gamma(\Omega(t_0)j_x' + v_0\Omega(t_0)\rho')$$

$$\begin{aligned}\bar{j}_y &= \Omega(t_0)j_y = \bar{j}_y' = \Omega(t_0)j_y', \\ \bar{j}_z &= \Omega(t_0)j_z = \bar{j}_z' = \Omega(t_0)j_z'\end{aligned} \quad , \ \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (11)$$

2. Electrodynamics in CSTR

The electromagnetic potential A^μ is 4-vector potential. Hence, transformations of A^μ are

$$\bar{\phi} = \phi = \gamma(\bar{\phi}' + \frac{v_0}{c}\Omega(t_0)\bar{A}_x') = \gamma(\phi' + \frac{v_0}{c}\Omega(t_0)A_x'\Omega(t_0))$$

$$\bar{A}_x = A_x\Omega(t_0) = \gamma(\bar{A}_x' + \frac{v_0}{c}\Omega(t_0)\bar{\phi}') = \gamma(\Omega(t_0)A_x' + \frac{v_0}{c}\Omega(t_0)\phi')$$

$$\begin{aligned}\bar{A}_y &= \Omega(t_0)A_y = \bar{A}_y' = \Omega(t_0)A_y', \\ \bar{A}_z &= \Omega(t_0)A_z = \bar{A}_z' = \Omega(t_0)A_z'\end{aligned} \quad , \ \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (12)$$

In CSTR, electric field $\bar{\vec{E}}$ and magnetic field $\bar{\vec{B}}$ have to satisfy Maxwell equations of special relativity theory. Hence, in CSRT, Maxwell equations are likely as special theory of relativity,

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} \quad (13\text{-i})$$

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = 0 \quad (13\text{-ii})$$

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} \quad (13\text{-iii})$$

$$\bar{\vec{\nabla}} \times \bar{\vec{B}} = \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} \quad (13\text{-iv})$$

In this time, Eq(13-i) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} = 4\pi\rho \quad (14)$$

Hence, $\bar{\vec{E}} = \vec{E}\Omega(t_0)$ .According to special relativity, $\bar{\vec{B}} = \vec{B}\Omega(t_0)$

Eq(13-ii) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \cdot \vec{B} \Omega(t_0) = \vec{\nabla} \cdot \vec{B} = 0 \tag{15}$$

Eq(13-iii) is

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{E} \Omega(t_0) = \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \bar{\vec{B}}}{\partial \bar{t}} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \Omega(t_0) \tag{16}$$

Eq(13-iv) is

$$\begin{aligned} \bar{\vec{\nabla}} \times \bar{\vec{B}} &= \frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{B} \Omega(t_0) = \vec{\nabla} \times \vec{B} \\ &= \frac{1}{c} \frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c} \bar{\vec{j}} = \Omega(t_0)(\frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}) \end{aligned} \tag{17}$$

Hence, in CSTR, Maxwell equations are

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{18-i}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{18-ii}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \Omega(t_0) \tag{18-iii}$$

$$\vec{\nabla} \times \vec{B} = \Omega(t_0)(\frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}) \tag{18-iv}$$

Therefore, in CSTR, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are

$$\begin{aligned} \bar{\vec{E}} &= \vec{E}\Omega(t_0) = \Omega(t_0)(-\frac{1}{\Omega(t_0)} \vec{\nabla}\phi - \Omega(t_0) \frac{1}{c} \frac{\partial \vec{A}}{\partial t}) \\ &= -\vec{\nabla}\phi - \Omega^2(t_0) \frac{1}{c} \frac{\partial \vec{A}}{\partial t} = -\Omega(t_0) \bar{\vec{\nabla}} \bar{\phi} - \Omega(t_0) \frac{1}{c} \frac{\partial \bar{\vec{A}}}{\partial \bar{t}} \end{aligned} \tag{19}$$

$$\bar{\vec{B}} = \vec{B}\Omega(t_0) = \Omega(t_0) \vec{\nabla} \times \vec{A} = \Omega(t_0) \bar{\vec{\nabla}} \times \bar{\vec{A}} \tag{20}$$

3. Electromagnetic Wave in CSTR

Electromagnetic wave equation is in CSTR,

$$\begin{aligned} &\frac{1}{c} \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) = -\Omega(t_0) \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \\ &= \vec{\nabla} \times (\frac{1}{c} \frac{\partial \vec{E}}{\partial t}) = \vec{\nabla} \times (\frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{B}), \vec{\nabla} \times \vec{j} = \vec{0} \end{aligned}$$

$$= \frac{1}{\Omega(t_0)}\{-\nabla^2\vec{B} + \vec{\nabla}(\vec{\nabla}\cdot\vec{B})\} = -\frac{1}{\Omega(t_0)}\nabla^2\vec{B} \tag{21}$$

Hence, electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\vec{B} = \vec{0} \tag{22}$$

And,

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = \Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}, \frac{1}{c}\frac{\partial\vec{j}}{\partial t} = \vec{0}$$

$$= \vec{\nabla}\times(\frac{1}{c}\frac{\partial\vec{B}}{\partial t}) = \vec{\nabla}\times(-\frac{1}{\Omega(t_0)}\vec{\nabla}\times\vec{E})$$

$$= -\frac{1}{\Omega(t_0)}\{-\nabla^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E})\} = \frac{1}{\Omega(t_0)}\nabla^2\vec{E}, \vec{\nabla}(4\pi\rho) = \vec{0} \tag{23}$$

Hence, electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\vec{E} = \vec{0} \tag{24}$$

In CSTR, electromagnetic wave functions are

$$\vec{E} = \vec{E}_0\sin\Phi, \vec{B} = \vec{B}_0\sin\Phi$$

$$\Phi = \omega\{\frac{t}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c}(lx + my + nz)\} \tag{25}$$

Where,

$$l^2 + m^2 + n^2 = 1 \tag{26}$$

According to Maxwell equations are in CSTR,[1]

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{4\pi}{c}j_x\} = (\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_x}{\partial t} = (\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_y}{\partial t} + \frac{4\pi}{c}j_y\} = (\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_y}{\partial t} = (\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{4\pi}{c}j_z) = (\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_z}{\partial t} = (\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}) \tag{27}$$

Where,

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x'}{\partial t} + \frac{4\pi}{c}j_x\} = \{\frac{\partial}{\partial y}\gamma(B_z' + \frac{v_0}{c}\Omega(t_0)E_y') - \frac{\partial}{\partial z}\gamma(B_y' - \frac{v_0}{c}\Omega(t_0)E_z')\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t}\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z')+\frac{4\pi}{c}j_y\}=\{\frac{\partial B_x'}{\partial z}-\frac{\partial}{\partial x}\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y')\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t}\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y')+\frac{4\pi}{c}j_z\}=\{\frac{\partial}{\partial y}\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z')-\frac{\partial B_x'}{\partial z}\} \quad (28)$$

Where,[1]

$$\Omega(t_0)\frac{1}{c}\frac{\partial B_x'}{\partial t}=\{\frac{\partial}{\partial z}\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z')-\frac{\partial}{\partial y}\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y')\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t}\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z')=\{\frac{\partial}{\partial x}\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y')-\frac{\partial E_x'}{\partial z}\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t}\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y')=\{\frac{\partial E_x'}{\partial y}-\frac{\partial}{\partial x}\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z'\} \quad (29)$$

Hence, in CSTR, transformations of electromagnetic field are

$$E_x=E_x',\, E_y=\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z'),\, E_z=\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y') \quad (30)$$

$$B_x=B_x',\, B_y=\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z'),\, B_z=\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y') \quad (31)$$

In CSTR, electromagnetic wave functions are

$$E_x'=E_{x0}\sin\Phi',\, E_y'=\gamma(E_{y0}-\frac{v_0}{c}\Omega(t_0)B_{z0})\sin\Phi',\, E_z'=\gamma(E_{z0}+\frac{v_0}{c}\Omega(t_0)B_{y0})\sin\Phi' \quad (32)$$

$$B_x'=B_{x0}\sin\Phi',\, B_y'=\gamma(B_{y0}+\frac{v_0}{c}\Omega(t_0)E_{z0})\sin\Phi',\, B_z'=\gamma(B_{z0}-\frac{v_0}{c}\Omega(t_0)E_{y0})\sin\Phi' \quad (33)$$

In this time,

$$\Phi'=\omega'\{\frac{t'}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(l'x'+m'y'+n'z')\} \quad (34)$$

$$\Phi=\omega\{\frac{t}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(lx+my+nz)\} \quad (35)$$

If we compare Eq(34) and Eq(35),

$$\omega'=\omega\gamma(1-\Omega(t_0)l\frac{v_0}{c}),\, l'=\frac{l-\frac{v_0}{c}\Omega(t_0)}{1-\Omega(t_0)l\frac{v_0}{c}},\, m'=\frac{m}{\gamma(1-\Omega(t_0)l\frac{v_0}{c})},\, n'=\frac{n}{\gamma(1-\Omega(t_0)l\frac{v_0}{c})} \quad (36)$$

Where,

$$l'^2 + m'^2 + n'^2 = 1 \quad , \ \gamma = 1/\sqrt{1 - \frac{v_0{}^2}{c^2}\Omega^2(t_0)} \tag{37}$$

4. Conclusion

We know Maxwell equations, electromagnetic wave equations and functions in Cosmological Special Theory of Relativity.

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