# Neuro-Amorphic Function
# (NAF)

[P-S Standard]

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**Abstract.**
As a good try, we took liberty for formula derivation that would allow describing many physical phenomena. In Nature, we face many situations when a single magnitude is a reason for drastic changes in the whole physical process. A single flexible instrument for describing processes of any kind would help a lot.
We propose a function for generating mathematical models for a process behavior.
We introduce special parameters that will help researchers find an acceptable solution for their tasks. The Dynamics coefficient, as well as dynamic function, is crucial for graph change. It can be used for the dynamic corrections of the whole physical process.

## 1. Introduction

In practice, we see many tries to describe a complex process by usage of many formulas. But many of those formulas are good for one single process and not acceptable for another.

With high probability, there is no single solution for many complex processes and uniform formula as well. But it looks possible to formulate a mathematical transformer with the possibility to change its properties depending on changes in the process. Mathematically it can be realized as a transition from one formula to another. In other words, we use the core formula that changes their properties with changing dynamic function and relevant parameters. Unlike the spline function, the neuro-amorphic function (NAF) includes the unchangeable core part that is defined on the whole space of real numbers.

## 2. Problem

Describing the physical process, we need correct information about:

1. all factors that affect the process.
2. limits of factor space.
3. existing invariants.
4. space entropy
5. etc.

In the case when the process changes their properties drastically in time the task of mathematical description becomes difficult. **The problem is** to find a single relevant approach for many states of phenomena in question. *We will make a small try to do so below.*

## 3. Solution

$$\boldsymbol{m}(\boldsymbol{x}) = \boldsymbol{\alpha e}^{(\boldsymbol{\beta}\sqrt[3]{\boldsymbol{x}} - \boldsymbol{f_d})} \tag{1}$$

where

$\beta - form\ factor\ coefficient,$

$\gamma - dynamics\ coefficient,$

$\alpha - limit\ coefficient,$

$f_d - dynamic\ function\ in\ R^+$

The formula (1) was derived from a number of practical calculations for physical quantities that cannot be described by the normal distribution law.

In other words, in nature there are many cases when most of the values of the investigated quantity x do not lie in the vicinity of $M[x]$.

For example, the physical state of certain bodies changes dramatically when the limit values are reached. Accordingly, the distribution law of the studied values of these states changes. The formula (1) can describe a physical phenomenon that can have both one limit value and a number of limit values. Accordingly, the graph of a function $m(x)$ can be symmetric both with respect to one limit value (axis of symmetry) of a given function and with respect to the numerical series of such limit values.

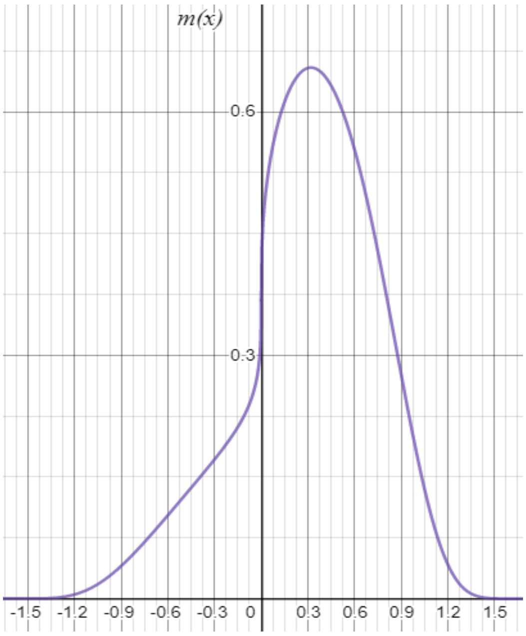
**Properties of $\boldsymbol{m(x)}$ ($\boldsymbol{f_d = e^{\frac{x^3}{\gamma x}}}$):**

1. The function $m(x)$ is defined and continuous on the entire number axis X.

$x = 0\ is\ a\ singularity\ or\ break\ point\ of\ the\ first\ kind.$

$$\lim_{x \to x_i} m(x) = m(x_i), \qquad (2)$$

$where\ x_i \in X^R$.

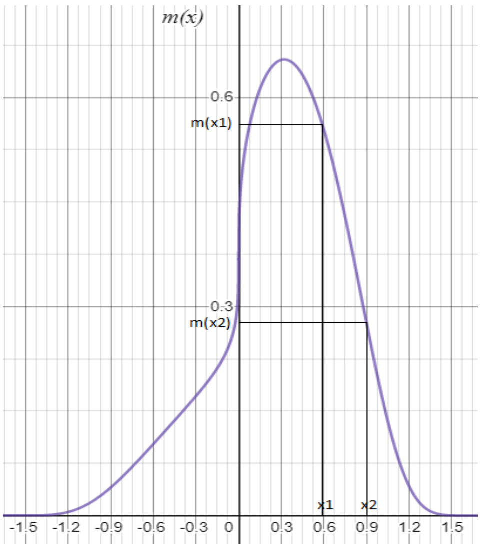



2. On the intervals $(-\infty; 0)$ $and$ $(0;\ x_{max})$, the function is strictly monotone (increasing) for $\alpha > 0, \beta > 0, \gamma > 0$.

$$\forall x_1, x_2 \in R, x_1 > x_2 \rightarrow m(x_1) > m(x_2) \qquad (3)$$

3. On the intervals $(0; +\infty)$ the function is strictly monotone (decreasing) for $\alpha > 0, \beta > 0, \gamma > 0$.

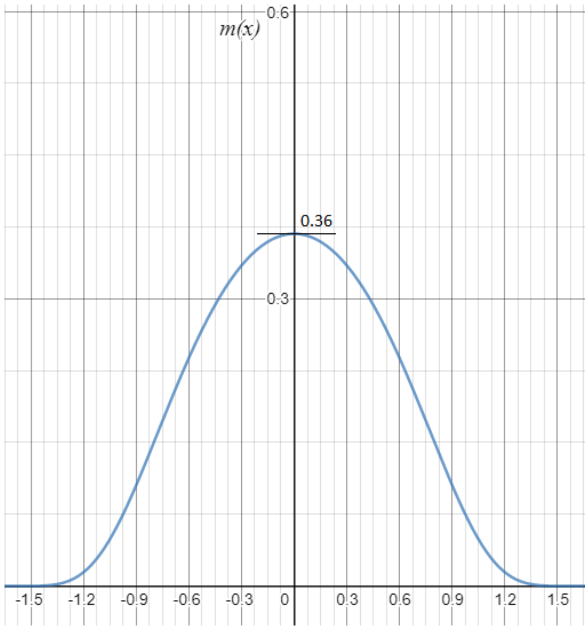
$$\forall x_1, x_2 \in R, x_1 < x_2 \rightarrow m(x_1) > m(x_2) \qquad (4)$$



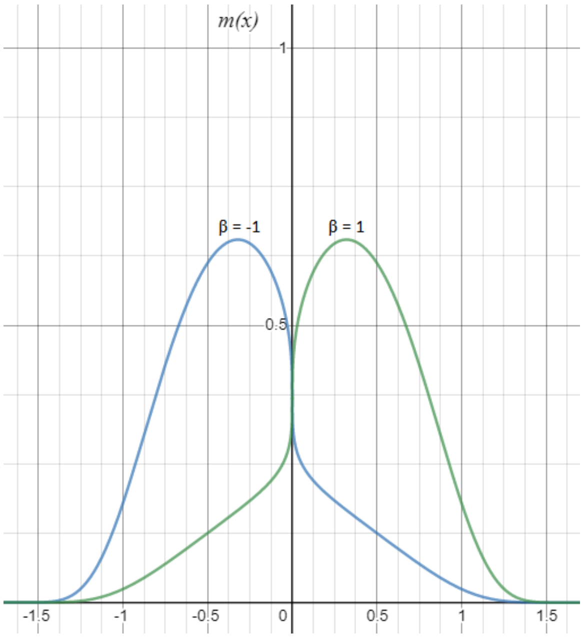
4. For two sets $(X, A)$ and $(Y, B)$, where $A, B - Borel\ \sigma - algebra\ of\ sets\ X\ and\ Y, accordinally, m(x): X \to Y\ is\ \frac{A}{B} measurable,$

$$\forall B' \in B, m^{-1}(B') \in A \quad (5)$$

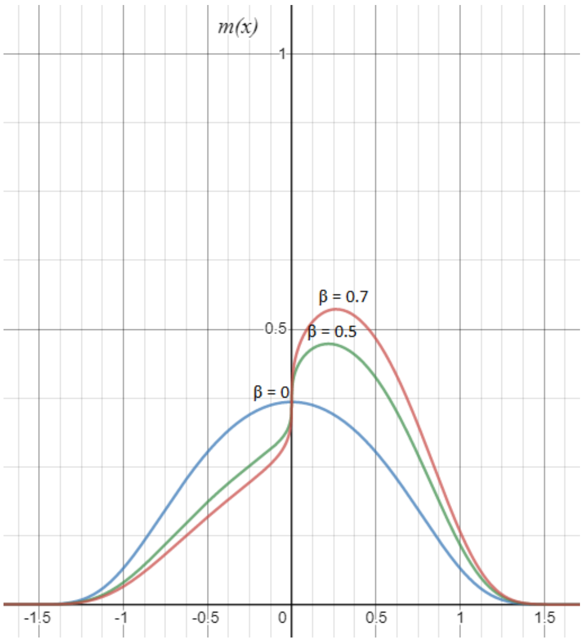
5. $Form\ factor\ coefficient$ is defined on the entire number axis X.




6. Provided that the form factor coefficient is zero $(\beta = 0)$, the graph of the function $m(x)$ on the interval $(-\infty; 0)$ is symmetric to the graph of the function $m(x)$ on the interval $(0; +\infty)$ about the y-axis. In other words, under these conditions, the function $m(x)$ is even.

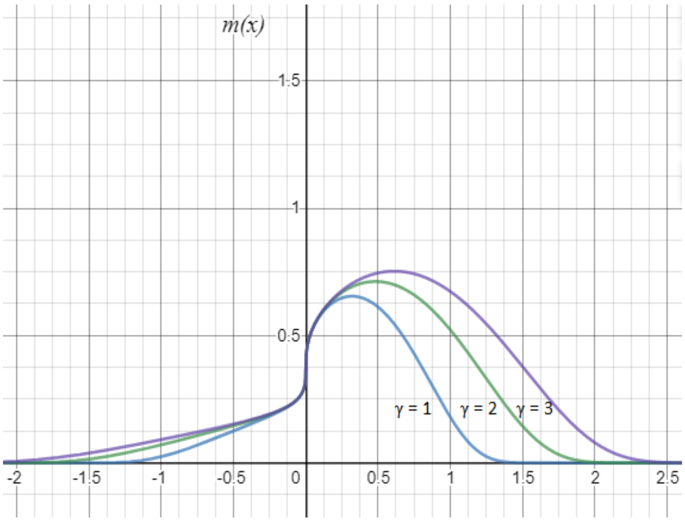


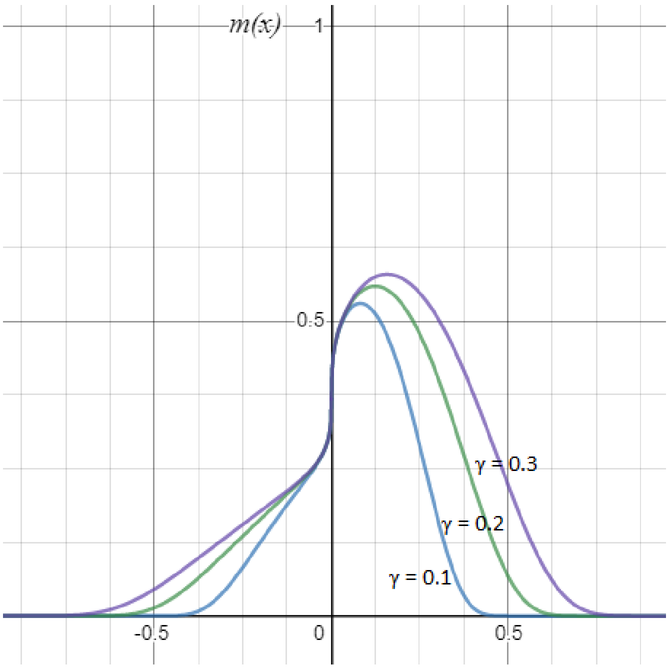

7. $For\ 0 < \beta < 1$, $m(x)$ is not an even or odd function.



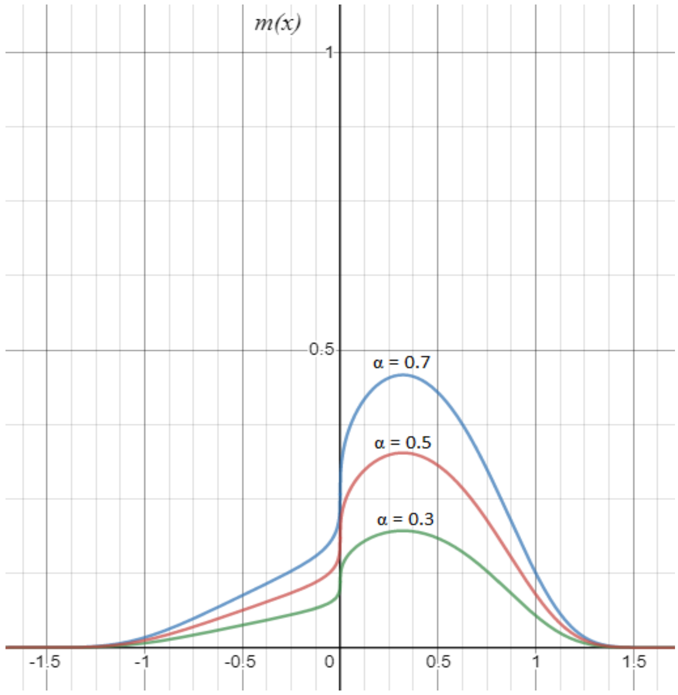

8. The dynamic function $f_d$ is defined and continuous on the entire positive x-axis which has a singular point or a break point of the first kind, $x = 0$.
9. $Dynamics\ coefficient\ \gamma$ of the dynamic function $f_d$ is defined on the interval $(0; +\infty)$. ***This coefficient plays an extremely important role in the dynamic response of***

***the function $m(x)$ to changes in the described phenomena.***



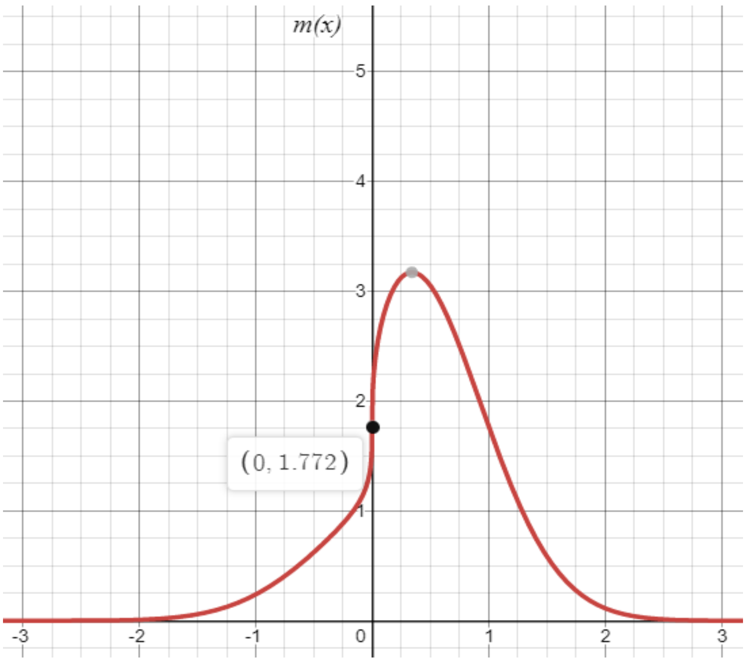

10. $Limit\ coefficient\ \alpha$ is defined on the interval $(0;+\infty)$ and defines the limiting maximum value of the function $m(x), m(x_{max})$.



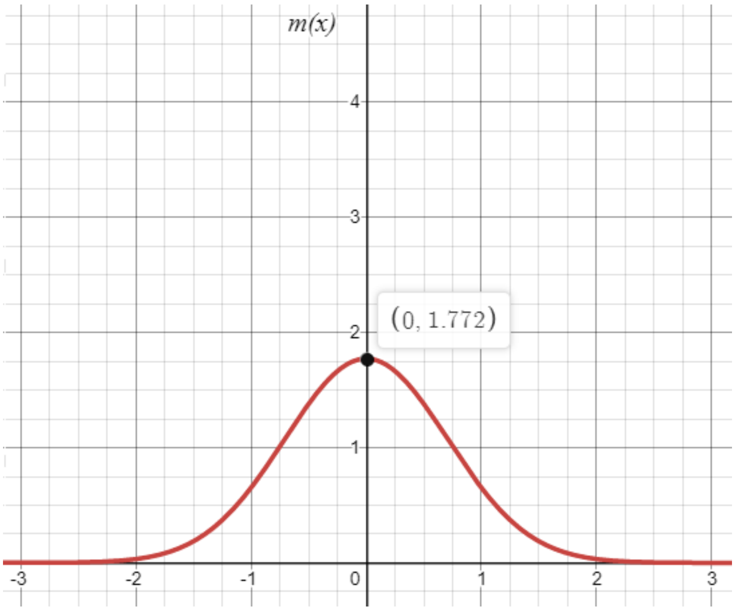

11. $\alpha \int_{-\infty}^{+\infty} e^{(\beta \sqrt[3]{x} - f_d)}\, dx \neq 1, \qquad for\ \alpha = 1, \beta = 1, \gamma = 1.$

**Properties of $m(x)$ ($f_d = \frac{x^2}{\gamma}, \alpha = \sqrt{\pi}$):**

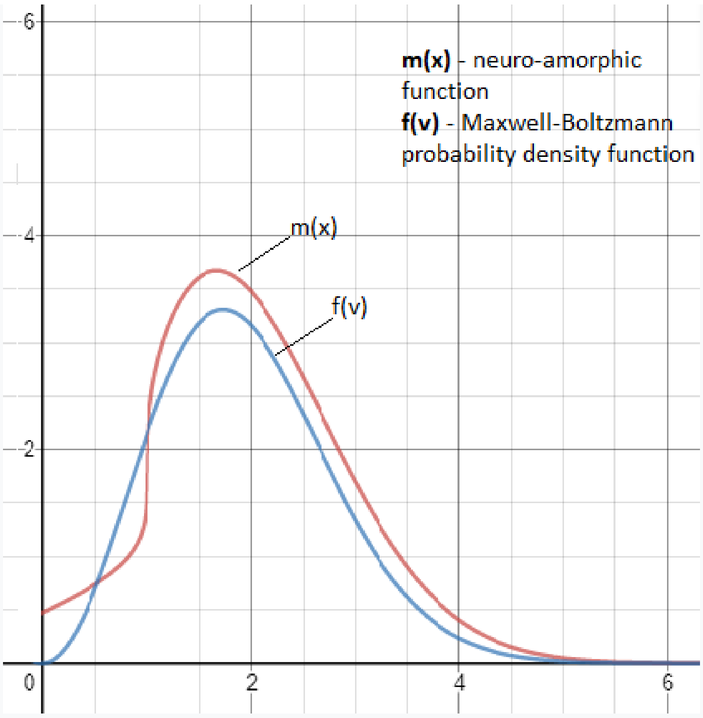
1. The function $m(x)$ is defined and continuous on the entire x-axis.



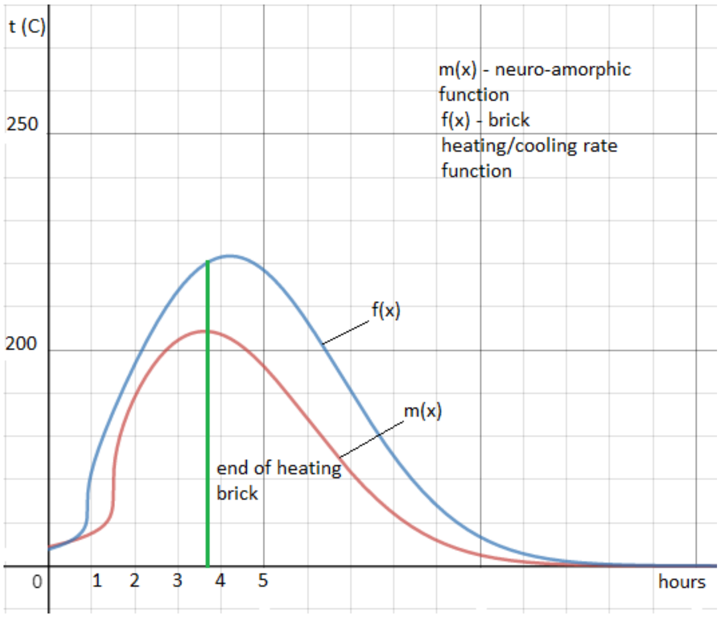
2. $\lim_{x\to 0} m(x) = m(x).$
3. The dynamic function $f_d$ is defined and continuous on the entire positive x-axis.
4. $\sqrt{\pi}\int_{-\infty}^{+\infty} e^{(\beta\sqrt[3]{x} - f_d)}\, dx \neq 1, \qquad for\ \alpha = \sqrt{\pi}, \beta = 1, \gamma = 1.$
5. $For\ \beta = 0, m(x)\ is\ even, m(0) = 1.7.$



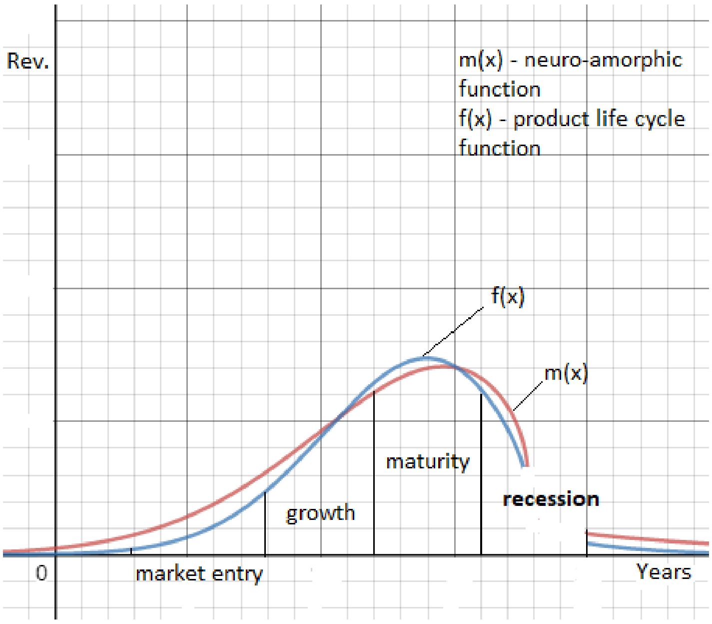

6. At a certain value of the dynamics coefficient $\gamma$ and a shift of the graph of the function $m(x)$ relative to the x-axis, the function $m(x)$ approximates with a certain accuracy the probability density function of the distribution of ideal gas molecules over velocities.




7. At a certain value of the dynamics coefficient $\gamma$ the function $m(x)$ approximates with a certain accuracy the brick heating speed function (refractory, ceramic brick).




8. At a certain value of the coefficients $\alpha,\ \beta\ and\ \gamma$ the function $m(x)$ approximates with a certain accuracy the product life cycle function in the market.




<u>Convolution function</u>

The convolution operation shows the degree of influence of two functions on each other, forming the third function.

It is important not to confuse the convolution of two different functions with an autocorrelation function. In the case of convolution, the range of one function may differ from the range of another one.

*The integrand convolution function* is a function defined on a fixed set M which is formed by multiplying two functions f and g defined on a set K and L, respectively.

Neuro-amorphic function $m(x)$ can be represented as two continuous functions:

$$f_1(x) = e^{\sqrt[3]{x}}, \qquad (6)$$

$$f_2(x) = e^{-e^{x^2}} \qquad (7)$$

However the convolution for the functions

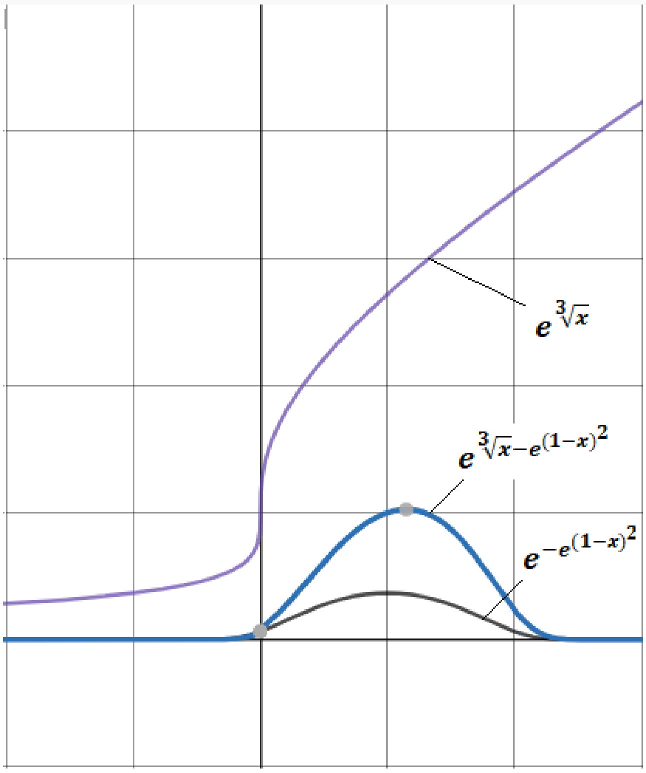
$$(f_1 * g)(x) = \int_{-\infty}^{+\infty} e^{\sqrt[3]{y}} e^{-e^{(x-y)^2}} dy = \int_{-\infty}^{+\infty} e^{\sqrt[3]{x-y}} e^{-e^{y^2}} dy \qquad (8)$$

will be a complex expression to calculate.

The convolution function

$$e^{\sqrt[3]{y}-e^{(x-y)^2}} \qquad (9)$$

will tend to the Gaussian function with shift x, provided $\sigma = 1$, $\mu = 0$.

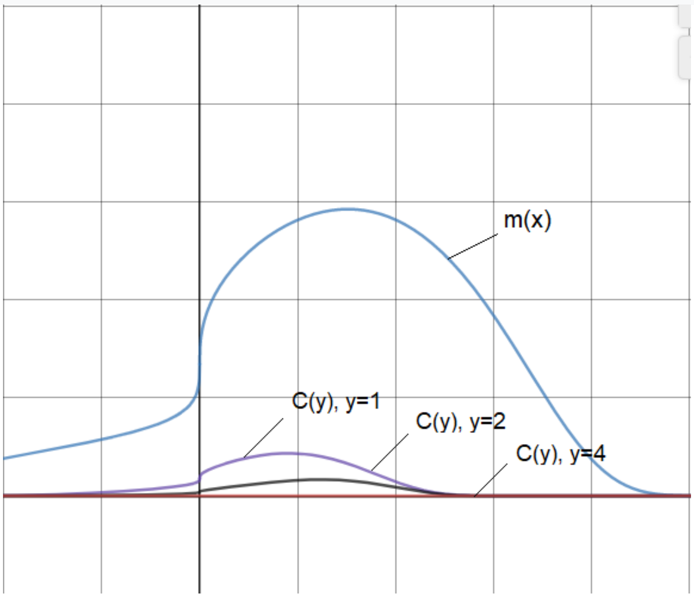



The Gaussian function $g(x)$ and $m(x)$ form an integrand convolution function

$$C(y) = g(y-x)m(x) = \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}\right)\left(e^{\sqrt[3]{x}-e^{x^2}}\right), \qquad (10)$$

where $\mu = 0, \sigma = 1, \alpha = 1, \beta = 1, \gamma = 1,$

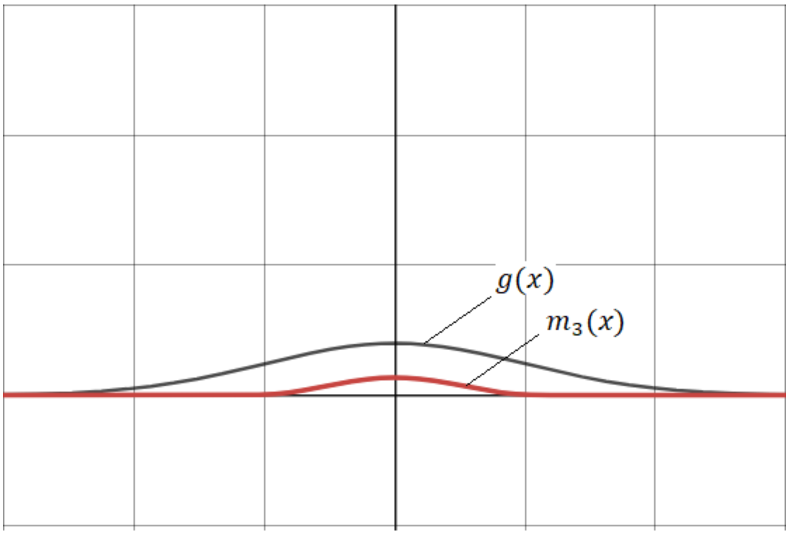
that degenerates into a straight line $y = 0$ at $y \to {}^{+\infty}_{-\infty}$.




The product of the two functions $m_1(x)\ for\ \alpha, \beta, \gamma = 1$ and $m_2(x)\ for\ \alpha = 1, \beta = -1, \gamma = 1$, where

$$m_2(x) = e^{-\sqrt[3]{x} - e^{x^2}}, \tag{11}$$

gives a function $m_3(x)$ tending to $g(x)\ for\ \mu = 0, \sigma = 1$.



Transformation formulas

$$\hat{f}(\gamma) = \int_{-\infty}^{+\infty} f(x)K(x,\gamma)dx\,, \quad K(x,\gamma) = e^{\sqrt[3]{\gamma x}} \qquad (12)$$

$$f(x) = \int_{-\infty}^{+\infty} K^{-1}(\gamma,x)\hat{f}(\gamma)d\gamma, \quad K^{-1}(\gamma,x) = \frac{1}{\gamma} ln^3 x \qquad (13)$$

$K(x,\gamma)$:

- $K(x,\gamma) \to f(x) = 1, for\ \gamma \to 0$
- $K(x,\gamma) \to \infty\ at\ x = 0, for\ \gamma \to \infty$
- $\int K(x,\gamma) = 0, for\ \gamma \to \infty$
- $K(x,\gamma) - generalized\ function, for\ \gamma \to \infty$
- $D(K)\ decreasing, for\ \gamma \to \infty$

## 4. Conclusion

The mathematical expectation cannot always qualitatively describe the behavior of the values of a quantity in a particular sample. As in many practical cases, the normal distribution can only be used to generalize the average results of the considered quantity. A good examples, the quantum physics or human neural net.

We offer a function that has a fairly flexible approximation mechanism as well as the ability to build dynamic systems of varying complexity.

We hope that our decent work will help other researchers in their life endeavors.

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