An array of mathematical results concerning polynomial, and inverse trigonometric expressions

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Abstract;
This paper is segregated into two components; one yielding a set of algebraically trivialized results regarding polynomial expressions, and the other delineating equalities between the derivatives of the three inverse trigonometric functions.

Part 1;

Firstly, consider the quartic polynomial;

$$ax^4 + bx^3 + cx^2 + dx + e$$

Note that integrating yields;

$$\int ax^4 + bx^3 + cx^2 + dx + e\,dx$$

$$= \frac{a}{5}x^5 + \frac{b}{4}x^4 + \frac{c}{3}x^3 + \frac{d}{2}x^2 + ex + f$$

i.e a polynomial of a degree 5.

Similarly, differentiating yields;

$$\frac{d}{dx}\,ax^4 + bx^3 + cx^2 + dx + e$$

$$4ax^3 + 3bx^2 + 2cx + d$$

i.e. a polynomial of degree 3.

Naturally, this phenomena can be generalized in the proposition that the first integral of a polynomial of a degree n, is another polynomial of a degree $[n + 1]$*, and that the first derivative of a polynomial of a degree n, is another polynomial of a degree* $[n - 1]$ *[with the latter being constrained to* $n \geq 3$*].*

Secondly, conceive of a classic quadratic expression;

$$ax^2 + bx + c$$

For any stationary points [its vertex];

$$\frac{d}{dx}ax^2 + bx + c = 0$$

$$2ax + b = 0$$

$$2ax = -b$$

$$x = \frac{-b}{2a}$$

In remapping this ubiquitous vertex formulation onto its domain;

$$ax^2 + bx + c$$

$$a\left[\frac{-b}{2a}\right]^2 + b\frac{-b}{2a} + c$$

$$\frac{b^2a}{4a^2} - \frac{b^2}{2a} + c$$

$$\frac{b^2}{4a} - \frac{b^2}{2a} + c$$

$$-\frac{b^2}{4a} + c$$

Any quadratic vertex of the form (h, k) subsequently equals;

$$\left[\frac{-b}{2a}, \frac{-b^2}{4a} + c\right]$$

Part 2;

Consider the inverse trigonometric function for sine values;

$$\frac{d}{dx}arcsinx = \frac{1}{\sqrt{1-x^2}}$$

Rearranging generates;

$$\sqrt{1-x^2} = \left[\frac{d}{dx}arcsinx\right]^{-1}$$

$$1 - x^2 = \left[\frac{d}{dx}arcsinx\right]^{-2}$$

$$-x^2 = \left[\frac{d}{dx} arcsinx\right]^{-2} - 1$$

$$x^2 = 1 - \left[\frac{d}{dx} arcsinx\right]^{-2}$$

[E1]

Consider the inverse trigonometric function for cosine values;

$$\frac{d}{dx} arccosx = \frac{-1}{\sqrt{1-x^2}}$$

Redefining facilitates;

$$\sqrt{1-x^2} = -\left[\frac{d}{dx} arcsinx\right]^{-1}$$

$$1 - x^2 = \left[\frac{d}{dx} arccosx\right]^{-2}$$

$$-x^2 = \left[\frac{d}{dx} arccosx\right]^{-2} - 1$$

$$x^2 = 1 - \left[\frac{d}{dx} arccosx\right]^{-2}$$

[E2]

Consider the inverse trigonometric function for tangential values;

$$\frac{d}{dx} arctanx = \frac{1}{1+x^2}$$

Reconstituting the above derivative in a manner commensurate with the formulations above, once again yields:

$$1 + x^2 = \left[\frac{d}{dx} arctanx\right]^{-1}$$

$$x^2 = \left[\frac{d}{dx} arctanx\right]^{-1} - 1$$

[E3]

Given that E1, E2 and E3 all represent derived equalities describing x^2, one may initiate a coalescence that relates them;

$$x^2 = \left[\frac{d}{dx} arctanx\right]^{-1} - 1 = 1 - \left[\frac{d}{dx} arccosx\right]^{-2} = 1 - \left[\frac{d}{dx} arcsinx\right]^{-2}$$

Reconstituting the above ultimately reveals, that for any combination of trigonometric correspondences;

$$\left[\frac{d}{dx} arctanx\right]^{-1} + \left[\frac{d}{dx} arccosx\right]^{-2} = 2$$

[E4]

$$\left[\frac{d}{dx} arctanx\right]^{-1} + \left[\frac{d}{dx} arcsinx\right]^{-2} = 2$$

[E5]

$$\left[\frac{d}{dx} arccosx\right]^{-2} = \left[\frac{d}{dx} arcsinx\right]^{-2}$$

[E6]

$$\left[\frac{d}{dx} arccosx\right]^{2} = \left[\frac{d}{dx} arcsinx\right]^{2}$$

[E7]