A Set of General Principles Adhering to the Field of Complex Analysis

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Abstract;
The primary intentions of this paper revolve around an elementary introduction to a set of trivial, and nontrivial results in the field of complex analysis. It follows a previous abstract (a singular equality pertaining to Euler and De Moivre's complex formulations) I've written on the subject, and may be interpreted through the lens of conventional integral techniques, radicals, and the use of a natural logarithm.

Expression One:

The first, rather trivial conclusion adhering to this paper may be demonstrated through the mathematical sequence outlined below;

$$e^{i\theta} = cis\theta$$

$$i = i^5$$

$$e^{i^5\theta} = cis\theta$$

$$Since\ a^{b^c} may\ be\ redefined\ as\ being\ equivalent\ to\ a^{bc}$$

$$e^{i^5\theta} = e^{i5\theta} = e^{(i\theta)^5} = cis\theta$$

$$e^{(i\theta)^5} = cis\theta$$

$$e^{i\theta} = \sqrt[5]{cis\theta}$$

$$e^{i\theta} = \sqrt[5]{cis\theta} = \ cis\theta$$

$$\sqrt[5]{cis\theta} = \ cis\theta$$

Following this fact, exponentiating either term to the power 5 allows one to obtain;

$$cis\theta = (cis\theta)^5$$

It may be easily inferred that this iteration is applicable to equalities conjoining i and every one of its suitable exponents (such as $i^9 or\ i^{13}$), consequently allowing one to generalize the mathematical implication;

$$cis\theta = (cis\theta)^{1+4K}$$

wherein K constitutes an integer. [E1]

Expression Two:

Secondly, consider again the equality;

$$e^{i\theta} = cis\theta$$

$$e^{\vartheta^i} = cis\theta$$

$$e^{\theta} = \sqrt[i]{cis\theta}$$

Using [E1],

$$e^{\theta} = \sqrt[i]{(cis\theta)^{1+4K}}$$

[E2]

Expression Three:

Herein, we may demonstrate an integral consequence of two ubiquitous complex equalities.

To commence, we may first assume the equality confirmed in another, elementary paper - *'A Short Proof Pertaining to the Euler/De-Moivre Complex Identity';*
https://vixra.org/abs/2008.0200

$$e^{i\theta} = \frac{1}{2cos\theta - e^{i\theta}}$$

Since both functions are interchangeable, the same may be said with regards for their integrals.

$$\int e^{i\theta}\, d\theta = \int \frac{1}{2cos\theta - e^{i\theta}}\, d\theta$$

Since this integral is of a non-contour form, one may treat *i* as an analogous real constant;

$$\int e^{i\theta}\, d\theta = \frac{e^{i\theta}}{i} + C$$

$$\frac{e^{i\theta}}{i} + C = \int \frac{1}{2cos\theta - e^{i\theta}}\, d\theta$$

$$\frac{e^{i\theta}}{i} = \frac{1e^{i\theta}}{i} = \frac{i^4 e^{i\theta}}{i}$$

$\frac{i^4 e^{i\theta}}{i}$ may be simplified by virtue of an elimination of its multiples:

$$\frac{i^4 e^{i\theta}}{i} = \frac{i(i^3 e^{i\theta})}{i(1)} = \frac{i^3 e^{i\theta}}{1} = i^3 e^{i\theta} = -ie^{i\theta}$$

Thus, reverting back to our original expression yields:

$$\int \frac{1}{2cos\theta - e^{i\theta}}\, d\theta = -\, ie^{i\theta} + C$$

Since;

$$e^{i\theta} = \frac{1}{2cos\theta - e^{i\theta}} = cis\theta$$

$$\int e^{i\theta}\, d\theta = \int \frac{1}{2cos\theta - e^{i\theta}}\, d\theta = \int cis\theta\, d\theta = -\, ie^{i\theta} + C$$

[E3]

Expression Four:

The final constituent of this paper, is a simplistic inference made from the very same complex equality;

$$e^{i\theta} = cis\theta$$

If one were to impose an operation consisting of a natural logarithm on both sides of this expression, one might yield:

$$\ln e^{i\theta} = \ln cis\theta$$

$$\ln e^{i\theta} = i\theta \ln e = i\theta$$

$$i\theta = \ln cis\theta$$

$$i = \frac{\ln cis\theta}{\theta}$$

Since i^2 is, by definition, equal to -1;

$$i^2 = \left[\frac{\ln cis\theta}{\theta}\right]^2$$

$$\left[\frac{\ln cis\theta}{\theta}\right]^2 = -1$$

$$\frac{[\ln cis\theta]^2}{\theta^2} = -1$$

$$[\ln cis\theta]^2 = -\theta^2$$

Using [E1],

$$\left[\ln(cis\theta)^{1+4K}\right]^2 = -\theta^2$$

wherein K is an integer.

Additionally, repurposing the below expression may assist in delineating this logarithm more specifically.

$$e^{i\theta} = \frac{1}{2cos\theta - e^{i\theta}} = cis\theta$$

$$(cis\theta)^{1+4K} = (e^{i\theta})^{1+4K} = \left[\frac{1}{2cos\theta - e^{i\theta}}\right]^{1+4K}$$

$$\left[\ln(cis\theta)^{1+4K}\right]^2 = \left[\ln(e^{i\theta})^{1+4K}\right]^2 = \left[\ln\left[\frac{1}{2cos\theta - e^{i\theta}}\right]^{1+4K}\right]^2 = -\theta^2$$

wherein K is an integer.

[E4]

One may elect to reverse-engineer this result in order to generate its primary ancillary constituent:

$$i = \frac{\ln cis\theta}{\theta}$$

In order to accomplish this, one may initiate the endeavor by stating that -

$$\ln\frac{1}{2cos\theta - e^{i\theta}} = \ln 1 - \ln\left(2cos\theta - e^{i\theta}\right)$$

Neglecting K, and reverting back to the expression outlined in [E4],

$$\left[\ln\frac{1}{2cos\theta - e^{i\theta}}\right]^2 = -\theta^2$$

$$\left[\ln 1 - \ln\left(2cos\theta - e^{i\theta}\right)\right]^2 = -\theta^2$$

$$\ln 1 = 0$$

$$\left[-\ln\left(2cos\theta - e^{i\theta}\right)\right]^2 = -\theta^2$$

$$\left[-\ln\left(2cos\theta - e^{i\theta}\right)\right]^2 = \left[\ln\left(2cos\theta - e^{i\theta}\right)\right]^2$$

$$\left[\ln\left(2cos\theta - e^{i\theta}\right)\right]^2 = -\theta^2$$

$$\left[\ln\left(2cos\theta - e^{i\theta}\right)\right] = \sqrt{-\theta^2}$$

$$\sqrt{-\theta^2} = \sqrt{-1}\sqrt{\theta^2} = i\theta$$

$$\left[\ln\left(2cos\theta - e^{i\theta}\right)\right] = i\theta$$

Revisiting the familiar expression;

$$e^{i\theta} = \frac{1}{2cos\theta - e^{i\theta}} = cis\theta$$

$$\ln cis\theta = i\theta$$

$$i = \frac{\ln cis\theta}{\theta}$$

Naturally, the evaluation of these complex logarithms is not subject to the customary techniques executed whilst operating their real counterparts. In any event, it may be a plausible proposition that these conclusions be refocused onto a more substantive theme in the field of complex analysis.