The quantum pythagorean fuzzy evidence theory based on negation in quantum of mass function

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Abstract

Dempster-Shafer (D-S) evidence theory is an effective methodology to handle unknown and imprecise information, due it can assign the probability into power of set. Quantum of mass function (QM) is the extension of D-S evidence theory, which can combine quantum theory and D-S evidence theory and also extended D-S evidence theory to the unit circle in complex plane. It can be seen that QM has the more bigger uncertainty in the framework of the complex plane. Recently, negation is getting more and more attention due it can analyze information from the another point. Hence, the paper firstly proposed negation of QM by using the subtraction of vectors in the unit circle, which can degenerate into negation proposed by Yager in startand probability theory and negation proposed by Yin. et al in D-S evidence theory. the paper proposed quantum pythagorean fuzzy evidence theory (QPFET), which is the first work to consider QPFET from the point of negation.

Keywords: Dempster-Shafer evidence theory, Quantum of mass function,

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1. Introduction

Uncertainty has been used in many fields, such as economics, social sciences, physical and artificial intelligence. However, how to express uncertainty is also an open and hot issue, that is to say, how to better express the information we get. To address this issue, there are many methodologies, such as probability theory [1], fuzzy sets [2], Dempster-Shafer (D-S) evidence theory [3, 4], quantum theory [5] and so on. D-S evidence theory can better handle unknown and imprecise information due it can assign probability into power set [3, 4, 6], which have been used extensively in many fields, such as decision-making [7, 8], information fusion [9], complex systems [10], evidential reasoning [11], classification [12, 13] and so on [8, 14, 15]. Although, D-S evidence theory satisfy non-additive and imprecise, there are some experiments have proved data can fluctuate at a given execution time, which is irreversible [16].

In another hand, quantum theory and D-S evidence theory also satisfy non-additive [17]. Similarly, QM also satisfy non-additive, hence,it has some connections with logic. In logic, negation plays an essential role [18]. Negation is an important mathematical tools [19], which can help us better understand the obtained information, such as, it very hard to prove $a + b > c$, however, it is easily to prove $a + b < c$. From this point, it can be be seen that negation is essential. Besides, there are many people make some work about the negation to better express the obtained information [20, 21, 22, 23]. Yager proposed the negation of probability theory, which can cause the increase of entropy [24]. Yin. et al proposed the negation of mass function, which can be used to consider conflict [25]. Torres-Blanc proposed the new negations and strong negations based on type-2 fuzzy sets [26]. Srivastava. et al discussed some properties of negation [27]. S-rivastava. et al used Shannon entropy function and the Kullback Leibler-s divergence to determine the uncertainty related with the negation [28]. Kang. et al proposed the negation of discrete Z-numbers [29]. Hence, s-tudying the negation of QM is a problem of value study, which is also an open issue.

2. Quantum Pythagorean Fuzzy Evidence Theory (QPFET)

In this section, the negation of QM can be introduced, which can be computed by using subtraction between vectors. Based on proposed negation, the QPFET can be presented, which consider amplitude and phase angle. Finally, there are some numerical examples can be used to further analyze the proposed method.

2.1. The Negation of Quantum Mass function

QM can expand the D-S evidence theory into two-dimensional space, exploring the negation of QM can provide us a new view to make decision by using the known information. In this section, the negation of QM can be firstly introduced. The proposed negation is computed from the view of vector, which made proposed method more reasonable.

Supposing there is QM $\mathbb{M}(|A_i>=\psi_i e^{j\theta_i}$, the negation of QM can be computed and explained as $Fig.$ 1. The specific steps of negation are introduced in details as follows.

$Step$ 1: Calculating the complementary of QM

Using $1 - \mathbb{M}(|A_i >$ to represent the complementary of $\mathbb{M}(|A_i >$, as follows.

$$\hat{\mathbb{M}}(|A_i >) = 1 - \mathbb{M}(|A_i >) = 1 - \psi_i e^{j\theta_i} \tag{1}$$

Step 2: Calculating amplitude and phase angle

From *Fig*. 1, it can be seen that QM, $Q\hat{M}$ and vector 1 can form a triangle, hence it is easy and reasonable by using trigonometric function to obtain amplitude and phase angle, which are shown as follows.

(i) Calculating amplitude

$$cos\theta = \frac{a^2 + b^2 - c^2}{2ab} \tag{2}$$

$$\Downarrow$$

$$cos\theta_i = \frac{1^2 + |\mathbb{M}(|A_i >)|^2 - |\hat{\mathbb{M}}(|A_i >)|^2}{2 * 1 * |\mathbb{M}(|A_i >)|} \tag{3}$$

$$\Downarrow$$

$$|\hat{\mathbb{M}}(|A_i >)| = \sqrt{1 + |\mathbb{M}(|A_i >)|^2 - 2 * 1 * |\mathbb{M}(|A_i >)|} \tag{4}$$

$$\Downarrow$$

$$\widehat{\psi_i} = \sqrt{|\hat{\mathbb{M}}(|A_i >)|} \tag{5}$$

(ii) Calculating phase angle

$$cos\widehat{\theta_i} = \frac{1 + |\hat{\mathbb{M}}(|A_i >)|^2 - |\mathbb{M}(|A_i >)|^2}{2 * 1 * |\hat{\mathbb{M}}(|A_i >)|^2} \tag{6}$$

According *Eq*. 14, 15 and 16, The negation of quantum mass function can be got as follows.

$$\hat{\mathbb{M}}(|A_i >) = \widehat{\psi_i} e^{j\widehat{\theta_i}} \tag{7}$$

Step 3 Normalization

Due the sum of amplitudes is not equal 1, hence it should normalize.

(i) Calculate the sum σ of negation of quantum mass function, namely.

$$\sigma = \sum |\widehat{\psi}_i|^2 \tag{8}$$

(ii) Noted the sum σ might not equal to 1, hence the negation results of normalization are as follows.

$$|\hat{\mathbb{M}}(|A_i>)| = \frac{|\widehat{\psi}_i|^2}{\sigma} \tag{9}$$

In fact, negation can be obtained by using the complement of each focal element. Besides, the negation in QM can be changed with the change of phase angle. Besides, when the QM degenerate the probability theory, the $Q\hat{M}$ is similar with the negation of Yager. When QM degenerate the mass function, the $Q\hat{M}$ and Yin.et al's negation are consistent.

2.2. Quantum Pythagorean Fuzzy Evidence Theory (QPFET)

Negation can be obtained by the above method, which can be understood as quantum probability which does not belong to A, namely, negation can be regarded the non-membership of A.

In this section, the quantum pythagorean fuzzy evidence theory (QPFET) can be introduced in detail.

Step 1 Calculating the negation of $\mathbb{QM}$

In this step, the negation can be get by using the $Eq.$ $11-19$.

Step 2 Constructing QPFET.

The paper considered membership as the probability that target is A, non membership as the probability that target is not A. In this step, $\mathbb{M}$

is considered the quantum membership degree, $\hat{\mathbb{M}}$ as the quantum non-membership degree. Hence, we can construct the QPFET is as follows.

Definition 2.1. *Supposing QFD is $|\Theta >= \{|A_1 >, |A_2 >, \cdots\}$, which is the non-empty set, the QPFET is defined as follows:*

$$|\Theta >= \{< (x, \mathbb{M}, \hat{\mathbb{M}}) > |x \in 2^{|\Theta>}\} \tag{10}$$

where $\mathbb{M}$ represents the quantum membership, $\hat{\mathbb{M}}$ represents the quantum non-membership, namely, the negation of $\mathbb{M}$.

Step 3 Obtaining quantum uncertainty or quantum hesitancy degree

(I) Calculate the sum of squares of $|\mathbb{M}|$ and $|\hat{\mathbb{M}}|$, namely, ρ.

$$\rho = |\mathbb{M}|^2 + |\hat{\mathbb{M}}|^2 \tag{11}$$

(II) Getting the amplitude of quantum uncertainty $|\iota|$.

In this step, if the ρ is more bigger than 1, the $|\iota|$ should be regarded as 0, otherwise $|\iota|$ should be computed by using the following equation.

$$|\iota| = (1 - \rho)^{0.5} \tag{12}$$

(III) Calculating the phase angle of quantum uncertainty ε_i

In this step, all phase angles should be the form $2\pi \cdot \theta_i$.

$$\zeta_i = \theta_i^2 + \hat{\theta}_i^2 \tag{13}$$

$$\varepsilon_i = (1 - \zeta_i)^{0.5} \tag{14}$$

This view is inspired by . Next, using numerical examples to explain the proposed method.

3. Conclusion

In this paper, the negation of QM can be discussed, which can be computed by using subtraction of vectors in the unit circle. The proposed negation is compatible with Yager's negation and Yin's negation when QM degenerate the probability and mass function, separately. In this paper, the probability of QM can be understood the membership of event A. Then, the probability after can be regarded as the non-membership of event A. From this point, the paper firstly proposed QPFET by using the negation to set the connection between fuzzy set and probability. In QPFET, the amplitude and phase angle can be considered. The amplitude in QPFET should satisfy that the sum of squares is 1, the phase angle also satisfy that the sum of squares is 1.

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