Conjecture of the contribution of the Zero Point Energy to the Universe Expansion

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Abstract

The universe in expansion and with flat curvature requires to fits the cosmological parameters, like critical energy density. This is generated from the relation of kinetic versus gravity, and has been postulated within the great scale context of a homogenous distribution of matter. The dark energy at 70% contribution to density is one of the solutions proposed to compensate the observable density, and theoretical preserve a flat geometry for the universe. However, the universe geometry of the universe shows a heterogeneous distribution of matter-energy, which would turn the finding of iterative geometrical parameters a complex task to achieve. The present work studies the chronology lapse, from the Era of Last Dispersion to the present, and is centered in the evolution of voids as a function of temperature. The latter, parameter gives a homogenous character to the space-time. Thus, De Broglie's wavelength relationship to emission temperature, dimensions quantum properties like radiation pressure and zero point energy. Dark energy has not been described physically. Therefore, we treated the emergence of the Zero Point Energy, as the one which could be implicated in the re-acceleration of expansion. Using a superior limit, for the ZPE of 10% of the critic density and a universe density of 40%, the resulting calculation predicts re-acceleration at 4400 million years after the Big Bang. The accumulation of ZPE depends of the volume-voids evolution of the Cosmic Microwave Background (CMB) system. The chronology increase in volume of voids exerts a pressure that would reinforce the local gravity of filaments attraction of cold and hot matter. The formation of cumulus and super-cumulus, when draining voids of particles, favors and reinforces the joint action of ZPE-CMB distending the space. Thus, the adiabatic CMB-system allows formulating parameters of temperature-density and redshift, in relation to ZPE time-dependent accumulation.

Introduction

The observations, theory and computers simulations indicate that, the amount of matter in the universe is only 30 percent of the amount necessary to support a flat geometry of the universe, the latter, requires a critical density of $\rho_0 \approx 8.588\times10^{-10}\ \mathrm{J/m^3}$ [1]. However, the same combination of data indicates that the universe is flat. The observations and measurements of ordinary matter in the universe indicate a value of density of around 5% with respect to the critic. Also, a contribution of radiation of the 0,005% is considered, a 0.3% of hot dark matter and a 25% of cold dark matter [2, 3, 4].

In order to increase this value of density, the non-observed dark energy, was proposed to contribute to the Hamiltonian balancing kinetic versus gravity potential.

If the hypothesis of dark energy it's not accepted, the relation of kinetic versus gravity would not be in balance, and therefore the universe would show an open geometry. However, it could be postulated a system that would maintain such a balance at lower density than the one generally accepted [5, 6].

For such a purpose was evaluated the energy of emptiness or ZPE, predicted by quantum mechanics. The predicted lower density requirement could be fulfill by ZPE, because indirect measurements consider its superior limit value in the universe of about 10% of critical density [7, 8].

Results

Seeking a mechanism leading to flat geometry at 40% of critical density, 30% in the form of matter and 10% in the ZPE, it was consider a quantum expansion rather independent of gravitation. This system corresponds with the quantum effect photon multiplication, and elongation in the voids. The latter, are so large that we can assume that their interior is free of any relevant gravity, by surrounding galactic filaments.

Since the voids volume largely surpasses the volume of matter in the galactic filaments, CMB homogeneity is preserved when temperature evolution is evaluated in the sum of the voids volume [9, 10].

Hence, is possible to evaluate independently of gravitation and matter distribution, the photon elongation within the CMB system as the large scale thermodynamic parameter of the expansion [11, 12, 13].

CMB system and the parameter of the expansion

The volume of the galaxies has much lower magnitude that the one occupy by voids, accordingly the evolution of the universe expansion could be related to that of the voids, and because that one imply photon-elongation, the latter control space-expansion.

The totality of the universe could be idealized as galaxies immersed in a CMB-system, with a volume which could be individually open, but could be treated as close to evaluate cosmological events. The associated CMB spectrum, evolves in parallel to expansion parameter $a_{(t)}$, according to time t: $\frac{a_0}{a} = \frac{\lambda_0}{\lambda}$, where the sub-index "0" refers to the present. The λ-elongation of the CMB quantum system becomes a thermodynamic continuum.

The process of photon-multiplication with an energy drop to longer wavelengths, results of localization over the increasing space-time continuum domain. Hence, this one results from individual local events, resembling spontaneous parametric down-conversion (SPDC).

Accordingly, a photon γ become divided $\gamma \rightarrow \gamma_2 + \gamma_2$, conserving initial $E, E = \frac{1}{2}E + \frac{1}{2}E = E_2 + E_2$, duplicating De Broglie's wavelength $\lambda \rightarrow 2\lambda$ decreasing energy $\frac{1}{2}E = \frac{hc}{2\lambda}$. This treatment allows simulation to account for photon number, and the coupling of λ-elongation expansion parameter.

The increment $\Delta\lambda$ produces an increment in the confining volume $\Delta V = \frac{\pi}{6}\Delta\lambda^3$. This changes, increments ZPE within the voids $\Delta E_{ZPE} = \rho_v \, \Delta V$, where ρ_v is the vacuum density which remain constant along the chronology.

CMB-photon multiplication is equivalent to a SPDC quantum event, or the minimum action related to the process of space increment within void, and the events sum to conform universe expansion. The dimensioning over a quantum scale, allows a treatment for photon multiplication to become independent of gravitational effects.

The equations for the dynamics of universe expansion

The dynamic could be described by three independent equations with three parameters: energy density $\varepsilon_{(t)}$, pressure $P_{(t)}$ and the expansion parameter $a(t)$ [14].

$$\left(\frac{a'_{(t)}}{a_{(t)}}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon_{(t)} - \frac{kc^2}{R_0^2 a_{(t)}^2} \quad [1]$$

$$\varepsilon'_{(t)} + 3\frac{a'_{(t)}}{a_{(t)}}\varepsilon + 3\frac{a'_{(t)}}{a_{(t)}}P = 0 \quad [2]$$

$$P = \omega\,\varepsilon \quad [3]$$

Assuming flat universe, $k = 0$ simplify equation [1] $\left(\frac{a'_{(t)}}{a_{(t)}}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon_{(t)}$. Resolving this system to satisfies the three parameters. Substituting with [3] within [2], generates the differential equation $\varepsilon'_{(t)} + 3\frac{a'_{(t)}}{a_{(t)}}(1+\omega)\,\varepsilon_{(t)} = 0$, solving by the separate variable method

$\frac{d\varepsilon}{\varepsilon} = -3\,(1+\omega)\,\frac{da}{a}$ and $\varepsilon_{i(a)} = \varepsilon_{i,0} \times a^{-3(1+\omega)}$ [4]. Thus, replacing with [4] within [1] it is obtain $\left(\frac{a'_{(t)}}{a_{(t)}}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon_{i,0} \times a_{(t)}^{-3(1+\omega)}$, and $\frac{a'_{(t)}}{a_{(t)}} = \xi\, a^{-\frac{3}{2}(1+\omega)}$ [5], where $\xi = \left(\frac{8\pi G}{3c^2}\varepsilon_{i,0}\right)^{1/2}$ is a value.

Density evaluation

Radiation density, ε_r, within the voids, is regarded as only dependent of temperature $\varepsilon_r = \frac{8\pi^5 k^4 T^4}{15\, h^3 c^3}$, its uniform distribution; at the cosmological large scale allows treatment as an adiabatic system.

Matter density, ρ_m, chronologically becomes a function of the cosmological redshift z, $\frac{\rho_m}{\rho_{m_0}} = \left(\frac{a_{(t_0)}}{a_{(t)}}\right)^3 = (1+z)^3$, where the sub-index 0 indicates the present. Radiation density increases at a faster rate $\frac{\rho_r}{\rho_{r_0}} = \left(\frac{a_{(t_0)}}{a_{(t)}}\right)^4 = (1+z)^4$. The additional exponent reflexes that λ is proportional to the expansion parameter $a_{(t)}$ and this one to the redshift $\frac{a_{(t_0)}}{a_{(t)}} = 1+z$. The relative proportion between densities $\frac{\rho_m}{\rho_r} = \frac{\rho_{m_0}}{\rho_{r_0}}(1+z)^{-1}$, where in relationship to the present $\frac{\rho_{m_0}}{\rho_{r_0}} \approx 10^4$ and $1+z = T/T_0$, $\rho_m = 10^4 \frac{T_0}{T}\rho_r$. Density in energy terms is $\varepsilon = \rho\, c^2$, accordingly to $\varepsilon_m = 10^4 \frac{T_0}{T}\varepsilon_r$.

Since the value of vacuum density or ZPE changes at constant $\rho_V = \text{constant}$, in energy terms $\varepsilon_V = \text{constant}$, and the value of the vacuum energy E_V becomes only dependent of the space volume V, hence expansion increments the accumulation of ZPE.

Table 1: Composition of the universe. *Calculations for several ZPE levels were done the only included correspond to maximal contribution of 10% of critical density:* $\varepsilon_v = 0.1\,\varepsilon_0 \approx 8.588\times10^{-11}$ J/m^3.

Matter-energy	Particles	Mass o energy [eV]	Particle number	Total with regard to a flat universe
Ordinary Matter	Protons, electrons	10^9	1.85×10^{78}	5%
Radiation	Photons CMB	6.3×10^{-5}	3.78×10^{87}	0.005%
Hot dark matter	Neutrinos	≤1	3.175×10^{87}	0.3%
Cold dark matter	Particles super-symmetric?	10^{11}	10^{77}	25%
ZPE	¿?	¿?	¿ ?	10%

The pressure parameter in the resolution of the dynamics equations

The density $\varepsilon_{(t)}$ and pressure $P_{(t)}$ are additive quantities in equation [3] $P = \sum P_i = \sum \omega_i \varepsilon_i$, the total pressure becomes the sum of all partial pressures radiation, matter and vacuum $P_T = P_r + P_m + P_V$, where the general terms $P_{(t)} = P_T$.

Radiation pressure equal one third of radiation density $P_r = \frac{1}{3}\varepsilon_r$, the scalar factor omega $\omega = \frac{1}{3}$. The vacuum pressure $P_V = -\varepsilon_V$, when $\omega = -1$. The matter pressure $P_m = \frac{v^2}{3c^2}\varepsilon_m$, where v is the average speed of the particles with mass within the universe, with a velocity much lower than that of *c*, $v << c$, the factor $v^2/3c^2$ tends to cero $\frac{v^2}{3c^2} \to 0$, thus $\omega \to 0$.

However, if $\frac{v^2}{3c^2} \neq 0$, the expression of velocity $v^2 = \frac{k\,T}{\mu}$ allows $P_m = \rho\frac{k\,T}{\mu}$, where μ is the average mass of particles and T the average temperature. Numerically, the calculation shows that $\mu = 1\text{MeV}$ and $T = 10\,\text{K}$, thus $v^2 = \frac{1.38\times10^{-23}\,\text{J/K}\times10\text{K}}{1\text{MeV}\times1.783\text{Kg/Me V}}$ and $v \approx 8\times10^3$ m/s.

Accordingly, $P_T = \frac{1}{3}\varepsilon_r + \frac{v^2}{3c^2}\varepsilon_m - \varepsilon_v$, with $\varepsilon_m = 10^4\frac{T_0}{T}\varepsilon_r$, hence $P_T = \frac{1}{3}\varepsilon_r + \frac{v^2}{3c^2}10^4\frac{T_0}{T}\varepsilon_r - \varepsilon_v$. At 10% of critical density of $\varepsilon_v = 0.1\varepsilon_c$ equivalent to $\varepsilon_v = 8.58834\times10^{-11}\text{J/m}^3$; and radiation density $\varepsilon_r = \frac{8\pi^5k^4T^4}{15\,h^3c^3}$.

Table 2: the value of omega ω ***at the dominant density.*** *Reacceleration of the universe expansion at redshift of z=1.7 implicate a temperature of T=7.357 K, at present measure in CMB T=2.725.*

Dominant energy	ω	Temperature [K]
Radiation	1/3	10^{32}–3000
Matter	0	3000–7,357
CMB–Voids	–1	7,357–Present

The expansion parameter

The contribution of partial pressures to total $P_T = \sum P_i = \sum \omega_i\varepsilon_i$ could be replaced by a system of equivalent pressure with only one kind of particles $\sum \omega_i\varepsilon_i = \bar{\omega}\,\varepsilon_T$, where ε_T the total density is use for this work and $\bar{\omega}$ is the value that verifies equality, at 40% of critical $\varepsilon_T = 0.4\,\varepsilon_c$, with $\varepsilon_c = 8.40043\times10^{-10}\,\text{J/m}^3$.

Calculating $P_T = \frac{1}{3}\varepsilon_r + \frac{v^2}{c^2}\varepsilon_m + \varepsilon_v = 0.4\,\bar{\omega}\,\varepsilon_c$, where pressure of mass tends to zero $\frac{v^2}{c^2}\varepsilon_m \to 0$. Assuming that ZPE of 10%, $\varepsilon_v = 0.1\varepsilon_c$, hence $\frac{1}{3}\varepsilon_r - 0.1\,\varepsilon_c = 0.4\,\bar{\omega}\,\varepsilon_c$, which in temperature parameter becomes $\frac{1}{3}\frac{8\pi^5k^4T^4}{15h^3c^3} - 0.1\,\varepsilon_c = 0.4\,\bar{\omega}\,\varepsilon_c$, final solution is $\bar{\omega} = -0.24996 \approx -0.25$

Replacing in the equation $\frac{a'_{(t)}}{a_{(t)}} = \xi\,a^{-\frac{3}{2}(1+\omega)}$ [5], it is obtain $\frac{a'_{(t)}}{a_{(t)}} = \xi\,a^{-9/8}$, integrated $\int a^{1/8}da = \xi\int dt$, and with solution is $\frac{8}{9}a^{9/8} = \xi\,t$. Clearing the parameter $a = \frac{3}{4}\frac{3^{7/9}}{2^{2/3}}(\xi\,t)^{8/9}$ is obtain an expansion rate as $\frac{a_0}{a} = \left(\frac{t_0}{t}\right)^{8/9}$, that relate to redshift is $\frac{a_0}{a} = \left(\frac{t_0}{t}\right)^{8/9} = 1 + z$. Where present time t_0 estimated between 4.27921×10^{17} and 4.37388×10^{17} seconds.

Reported observations calculated reacceleration for a redshift $z \approx 1.7$. Introducing this value allows determining the time *t* of acceleration of expansion.

Thus for the lower value 4.27921×10^{17} s, becomes $\left(\frac{4.27921\times10^{17}}{t}\right)^{8/9} = 1 + 1.7$, and the time $t = 1.39984 \times 10^{17}$, or 4400 million light years after the Big Bang.

The higher value 4.37388×10^{17} s, $\left(\frac{4.37388 \times 10^{17}}{t}\right)^{8/9} = 1 + 1.7$, and the time $t = 1.43081 \times 10^{17}$, or 4500 million light years after the Big Bang.

The average value $t_0 = 4.32655 \times 10^{17}$ s, $\left(\frac{4.32655 \times 10^{17}}{t}\right)^{8/9} = 1 + 1.7$, and the time $t = 1.41533 \times 10^{17}$ s, or 4480 millions light years after the Big Bang.

The expansion for spontaneous parametric down conversion parameter

For this treatment the radiation pressure terms are replaced by SPDC-produce by the De Broglie density energy quantum or photon CMB as a function of either λ or T.

The photon energy $E_\lambda = \frac{hc}{\lambda}$, applying the derivate operator for λ becomes $\frac{dE_\lambda}{d\lambda} = -\frac{hc}{\lambda^2}$. The differential for ZPE is dependent of the volume differential $dE_\mathrm{v} = \varepsilon_\mathrm{v}\, d\mathrm{V}$. The differential for total energy within the voids and excluding baryons particles $dE_\mathrm{T} = dE_\lambda + dE_\mathrm{v}$.

The volume generate by the λ-De Broglie equals $\mathrm{V} = \frac{\pi}{6}\lambda^3$, and the differential $d\,\mathrm{V} = \frac{\pi}{2}\lambda^2\, d\,\lambda$, assuming that λ correspond to a diameter. Thus the differential for total energy $dE_\mathrm{T} = -\frac{hc}{\lambda^2} d\,\lambda + \varepsilon_\mathrm{v}\frac{\pi}{2}\lambda^2\, d\,\lambda$. Integrating $E_\mathrm{T} = \frac{hc}{\lambda} d\,\lambda + \varepsilon_\mathrm{v}\frac{\pi}{6}\lambda^3$ which is to be expressed as a function of temperature becomes $E_\mathrm{T} = k\,\mathrm{T} + \frac{\pi}{6}\frac{h^3c^3}{k^3\,\mathrm{T}^3}\varepsilon_\mathrm{v}$.

Voids total density $\varepsilon_T = \frac{6}{\pi}\frac{k^4\,\mathrm{T}^4}{h^3c^3} + \varepsilon_\mathrm{v}$, where $\varepsilon_\mathrm{dB} = \frac{6}{\pi}\frac{k^4\,\mathrm{T}^4}{h^3c^3}$ De Broglie density $\varepsilon_r = \frac{8\pi^5 k^4\mathrm{T}^4}{15\,h^3c^3}$ and the relation $\frac{\varepsilon_r}{\varepsilon_{dB}} = \frac{4\pi^6}{45}$, $\varepsilon_r = 85.5 \times\ \varepsilon_{dB}$.

Thus, total pressure $\mathrm{P}_T = \frac{1}{3}\varepsilon_\mathrm{dB} + \varepsilon_\mathrm{v} = 0.4\ \bar{\omega}\,\varepsilon_\mathrm{c}$, accordingly $\varepsilon_\mathrm{dB} = \frac{45}{4\pi^6}\varepsilon_r$ it is obtain $\frac{1}{3}\frac{45}{4\pi^6}\varepsilon_r - 0.1\varepsilon_\mathrm{c} = 0.4\ \bar{\omega}\,\varepsilon_\mathrm{c}$, or in temperature terms $\frac{2}{\pi}\frac{k^4\mathrm{T}^4}{h^3c^3} - 0.1\varepsilon_\mathrm{c} = 0.4\,\bar{\omega}\,\varepsilon_\mathrm{c}$, and solution $\bar{\omega} = -0.24999 \approx -0.25$. The prediction of expansion rate of the universe based in the λ-De Broglie reach similar conclusions to the one based in the previous treatment: $\frac{a_0}{a} = \left(\frac{t_0}{t}\right)^{8/9}$.

Therefore, either the quantum treatments by photon elongation or by SPDC could produce identical chronology for the evolution of voids than the one for assessing the evolution of the radiation spectrum treated as a continuum. Hence, both results allow similar rates when characterizing cosmic expansion either under the assumption of quantum or continuum properties.

Chronology of the expansion parameter

The results obtains under the treatments 5 and 6, allows to chronological asses the Eras according the predominance of the density parameter yielding similar dates for re-acceleration. Hence, also predicting that on a further away time the expansion parameter would maintain a constant tendency for the dissipative state of the density potential. The latter, may manifest the coupling between volume increase and ZPE accumulation at related constant rates.

Table 3: Chronology as a function of the dominant density. *Eras could be characterized as a function of density from the starting of re-acceleration 4400-4500 millions light years after the Big Bang.*

Dominant Periods		Radiation		Matter		CMB–Void		Future
Time	[s]	5.4×10^{-43}	1.2×10^{13}	1.2×10^{13}	1.415×10^{17}	1.415×10^{17}	4.33×10^{17}	$t\to\infty$
	[light-years]	×	$3,8\times10^{5}$	$3,8\times10^{5}$	4.5×10^{10}	4.5×10^{10}	1.372×10^{10}	
Temperature [K]		1.41×10^{32}	3000	3000	7.3575	7.3575	2.725	$T\to0$
RedShift: z		$z\to\infty$	1100	1100	1.7	1.7	×	×
Expansion Parameter		×	×	$\frac{a_0}{a}=\left(\frac{t_0}{t}\right)^{2/3}$		$\frac{a_0}{a}=\left(\frac{t_0}{t}\right)^{8/9}$		$\frac{a_0}{a}=k$
Ratios		×	×	$c\int_{1.2\times10^{13}}^{1.415\times10^{17}}\left(\frac{10^{17}}{t}\right)^{2/3}dt$		$c\int_{1.415\times10^{17}}^{4.326\times10^{17}}\frac{4.33\times10^{17}}{t}dt$		$c\int kdt$
Hubble		×	×	$H_0=2/3t_0$		$H_0=1/t_0$		×

Flatness of the universe

The photon multiplication in the voids allows inferring a thermal uniformity according to the evolution state of CMB spectrum along the universe chronology. CMB fluctuations may show disregard able values with the mean value of temperature smoothing by the photon multiplication which gives substance to evaluate a total density for the universe and to calculus finding it lower than critical.

Discussion

ZPE conforms a minimal potential unable to participate in thermal equilibrium an allowing a sustainable dissipation of the potential created by radiation density and allowing the theoretical formulation of a thermodynamic open system between inflation and ZPE.

The arrow of time allows for the dissipation of the potential from photons of higher to many more of lower energy by mechanisms like SPDC or other which allow decreasing potential but conservation of energy at lower levels. Accordingly, these processes become irreversibly integrated because dispended energy does not appear under the usual concept of entropy eliminating energy transfer by leveling potential. In the open system a potential is fully utilized because in the system not all products remain active to recreate reactant, in other words the system is maintained far away from equilibrium until escalated decreased potential becomes fully consumed.

This study centers in the chronology subsequent to the Last Dispersion Era, however becomes a part of a larger study based in the conjecture of a quantum Big Bang. In the latter, inflation is evaluated in term of the emergence of fundamental laws, allowing that energy space time could be confined within the Planck dimension. The sum of these events allows the accumulation of Planck particles to reach to the level of critical energy.

The latter process allows space to expand at a larger velocity than *c*, because each Planck particle could emerge within space within a time as short as that of the Planck stability 10^{-44} seconds. The uniformly distribution of accumulated particles, allows flatness and follows the exponential increase reached by the cooperative sum of all surging particles and their further multiplication through elongation.

The initial absence of gravity and a primordial implosion could be explained by analyzing interaction between two Planck particles separated by the Planck distance. The calculated gravitation wave function shows that the time it requires to act as a mediating force, exceeds by much the stability time of the Planck particles.

Conclusions

The initial cosmos could be described as a cooperative process between Planck particles accumulation and multiplication leading to particles of lower and lower energy. This allows a continuous increment in the energy quantum confinement space. The perspective of a continuum would allow the description of a thermodynamic integrated down sided escalation of potential levels, even when conserving total energy over a larger cosmos.

The chronology after the Last Dispersion Era allows to described expansion of the universe by the evolution of voids, integrating the quantum role with the dissipative thermodynamic of an open universe by ignoring the distribution of matter since filament occupy a much smaller volume than voids.

The λ-elongation was assayed by a simulation of the mechanisms of SPDC, this one shows that the evolution of radiation spectra and the hot Big Bang treatment could be homologated to characterize expansion in terms of the evolution of energy-space-time.

The results of the sections 5 and 6, allows to create a chronological order as a function density, which allows the finding of the reacceleration Era. Hence, it naturally predicts that the expansion parameter "*a*" defines a relationship $a_0 / a = k$ and appears link to the accumulation of ZPE.

The reacceleration at 4400 million light years after Big Bang within the contexts of voids evolution, allows to predicts that the effect of gravity becomes less and less significant because of the continuous increment of void volume, restricted how far could reach the influence of gravity.

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