Quantum annealing for solving a nurse-physician scheduling problem in Covid-19 clinics

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Abstract

Quantum annealing (QA) is a metaheuristic methodology that has recently been used to analyze the performance of first generation adiabatic quantum processors, like the one available from D-wave Systems, in solving combinatorial optimization problems. In this paper, we formulate QA representation of problems essential to mitigating the effects of the COVID-19 pandemic, namely, the Nurse Scheduling Problem (NSP), the Physician Scheduling Problem (PSP), and the Nurse-Physician Scheduling Problem (NPSP). The proposed objective functions for each problem are scripted to the Ising model and then transformed into the quadratic unconstrained binary optimization (QUBO) format. An optimal solution is sought for a set of constraints, including the number of nurses or physicians per shift and the maximum number of shifts for each nurse or physician. After reducing the proposed NSP, PSP, and NPSP QUBOs to a novel Ising type Hamiltonian, we obtain solutions from a classical (simulated) annealer, and then from the D-wave 2000Q forward and reverse annealer. Results from the reverse quantum annealer show a dramatic improvement in solution quality as compared to those from the classical annealer. We observe that the reverse annealing method gives us the most satisfactory output from among three heuristic processes used, for each scheduling problem.

Keywords: simulated annealing; forward annealing; reverse annealing; ground state Hamiltonian; QUBO; Ising model

1. Introduction:-

The origins of quantum computation (QC) lie in the ideas of Benioff relating to quantum mechanical versions of the Turing machine [1], and of Feynman pertaining the need of a simple quantum computational system that can efficiently simulate more complicated ones [2]. In the years that have ensued since these ideas were put forward, the notion of a quantum computer processing quantum information has been scientifically well established, with several equivalent models of quantum computing are available. Two most prominent ones are the quantum circuit model [3], which is popular due to its similarity to the electrical circuit model that prevails in engineering disciplines, and the adiabatic quantum computation (AQC) model [4], which is based on the adiabatic theorem of quantum mechanics in which a quantum system's lowest energy state stays invariant under a "slow" enough evolution of the system's Hamiltonian. Either model has advantages depending on the nature of the analysis one is interested in performing. The two models can be transformed into each other in polynomial time.

Simulated annealing was introduced by S. Kirkpatrick et. al. to solve real-world optimization problems [5].They used thermal fluctuation property to find a global minimum value by escaping local minimums that a system can get stuck in.Geman and Geman proved that if thermal fluctuation is decreased at the rate of $T=c/\ln t$ or slower, then it will reach the global minimum value (where the constant c depends on the system's size) [6]. Otherwise, it will stay stuck at some local minimum value. T sallis et. al. proposed another probabilistic method on the basis of conventional Boltzmann-type probability distributions to find out better convergence to solutions than the thermal fluctuation methodology [7].

In 1998, Kadowki et al. proposed quantum annealing [8] as a way to solve combinatorial optimization problems. Compared to the thermal fluctuation methodology, QA is a more efficient metaheuristic for detecting global minimum. However, there is no guarantee of getting global minimum from adiabatic process [3-4, 11-14].Many combinatorial optimization problems are reported as an application of QA [15-22], while similar efforts of our proposed work have been made regarding nursing scheduling problem [23].

D-wave Systems is the first commercial vendor of a quantum computer based on quantum annealing to solve combinatorial optimization problems. In this paper, we use this quantum computer to find an optimal solution of nurse and physician scheduling problem in the context of the COVID-19 pandemic. We first construct binary variable model or QUBO model and then convert it into the equivalent Ising or bipolar model. Basic QUBO formulation is expressed as

$$\text{QUBO: } Y = x^T Q x \quad \text{or} \quad f(x) = \sum_i Q_{i,i}\, x_i + \sum_{i<j} Q_{i,j}\, x_i\, x_j$$

where x is vector of binary variables and Q is N×N upper bound triangular matrix. The entries $Q_{i,i}$ of Q are the linear diagonal coefficient and off diagonal non zero coefficients are represented by $Q_{i,j}$. In this manuscript, upper bound triangular structure of Q matrix is used to define our constraints in QUBO model.

This paper is presented as follows. Section 2 consists of discussion regarding Hamiltonian function formation for nurse and physician. Hamiltonian function regarding nursing scheduling problem (NSP) is demonstrated in sub section 2.1. Physician scheduling problem (PSP) is illustrated in subsection 2.2. The subsection 2.3 is concentrated on Hamiltonian function of Nursing Physician Scheduling Problem (NPSP). The outcomes from D-Wave 2000Q processor for above mentioned scheduling problems are summarized in section 3. Again in this section, the importance of reverse annealing to find optimal results is also discussed and compared. Finally, we concluded in section 4.

2. **Scheduling Problem for Covid-19 Hospital**

The NSP, PSP, and NPSP deal with creating a rotating roster for both the nurse and the physician according to some given constraints for a clinic providing medical care to patients of COVID-19.

2.1 Nurse Scheduling Problem (NSP) for Covid-19 Hospital

We set a four-shift system per day: morning time (MT), day time (DT), early night (EN) and late night (LN). Each shift consists of 6 hours, for some specified N number of days. Due to overload of working shifts, we set as our first hard constraint the maximum number of shifts that each nurse can work per day, so as to ensure that the nurse can take sufficient amount of rest in-between shifts. Due to the high pressure working environment in the ward, we set the second constraint, a soft constraint, to be that each nurse/physician can do at most two consecutive shifts. According to WHO guideline, maintaining distance between individuals is extremely important to mitigate the spread of the virus that causes COVID-19, even for the front-line medical workers; hence, we set our third constraint, another hard constraint, to be the maximum number of nurses/physicians that can be present during each shift. Our constraints then are:

1. Hard Constraint 1: The maximum number of shifts, each nurse and physician can do per day/week.
2. Hard Constraint 2: The maximum number of nurse and physician, each shift can have.
3. Soft Constraint: The number of consecutive shifts each nurse and physician can do.

(2.1.a) Hamiltonian formulation of NSP:

For combinatorial optimization problem to be solved on a quantum annealer, it is a best practice to formulate it as a quadratic unconstrained binary optimization problem (QUBO). The QUBO can then be easily transformed into an Ising formulation which a quantum annealer like the one developed from D-Wave can accept as input.

In QUBO format, a number of nurses n ∈(1, 2, 3… N), are assigned to complete the set of working shift s, where s ∈ (1,2,3,……S). In our QUBO formulation, each day is divided into four shift. Hence duration of each shift is 6 hours. Now, binary vector variable of our proposed QUBO function $q_{t,s}$ is belongs to (0,1). If $q_{t,s}$ =1, then nurses is assigned for shift s. Otherwise scheduling process will not work. Now, QUBO formulation with hard constraint 1 (First constraint) is presented in equation 1. In this case, we assigned that each nurse can do maximum two shift in a day as a hard constraint 1. Means maximum duty hours of a nurse is 12 hours. The QUBO function with hard constraint 1 is depicted in equation 1.

$$\gamma\left[\sum_{n=1}^{N}\left(\sum_{s=1}^{S} hq_{(n,s)} - b\right)^2\right]$$

$= \gamma\left[\sum_{n=1}^{N}\left(h^2 \sum_{s=1}^{S} q_{(n,s)} + 2h^2 \sum_{s=1}^{S}\sum_{s1>s}^{S} q_{(n,s)}q_{(n,s1)} - 2hb\sum_{s=1}^{S} q_{(n,s)} + b^2\right)\right]$

$$= \gamma(\mathrm{h}^2 - 2hb)\sum_{n=1}^{N}\sum_{s=1}^{S} q_{(n,s)} + \gamma 2\mathrm{h}^2 \sum_{n=1}^{N}\sum_{s=1}^{S}\sum_{s1>s}^{S} q_{(n,s)}q_{(n,s1)} + \gamma \mathrm{h}^2. \quad (1)$$

The above constraints are satisfied when the cost function is minimum, which happens only when the left-hand side and the right-hand side terms of Eq. (1) are equal to zero for some unknown binary variables $q_{(n,s)}$. The variable γ is the positive Lagrange constant, set by the programmer, to tune the function. The expressions $\gamma(h^2 - 2hb)$and $\gamma 2\mathrm{h}^2$are the diagonal and off-diagonal terms of our Q matrix, respectively.

For our second hard constraint, we set a total number of hours that each shift can have, combining all the nurses' working hours that are placed in that shift to be a = h * m, where m is the maximum number of members that can be present in each shift. Hence the QUBO form of our constraint is:

$$\lambda\left[\sum_{s=1}^{S}\left(\sum_{n=1}^{N} hq_{(n,s)}\right) - a\right]^2$$

$$= \lambda\left[\sum_{s=1}^{S}\left(\sum_{n=1}^{N} hq_{(n,s)}\right)^2 - 2ah\sum_{n=1}^{N} q_{(n,s)} + 2h^2\sum_{n=1}^{N}\sum_{n1>t}^{N} q_{(n,s)}q_{(n1,s)} + a^2\right]$$

$$= \lambda h^2 \sum_{s=1}^{S}\sum_{n=1}^{N} q_{(n,s)} - \lambda 2ah\sum_{s=1}^{S}\sum_{n=1}^{N} q_{(n,s)} + \lambda 2h^2 \sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{n1>t}^{N} q_{(n,s)}q_{(n1,s)} + \lambda a^2$$

$$= \lambda(h^2 - 2ah)\sum_{s=1}^{S}\sum_{n=1}^{N} q_{(n,s)} - \lambda 2h^2 \sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{n1>t}^{N} q_{(n,s)}\, q_{(n1,s)} + \lambda a^2. \quad (2)$$

The above constraint is satisfied (i.e. the cost function is minimum) only when the left-hand side and the right-hand side terms of Eq. (2) equal zero, for some unknown binary variables $q_{(n,s)}$. The variable λ is the positive Lagrange constant, set by the programmer, to tune the function. Again, $\lambda(h^2 - 2ah)$and $\lambda 2h^2$are the diagonal and off-diagonal terms of our Q matrix, respectively.

For our third constraint (soft constraint), we take two composite indices, i(n,s) and j(n, s), as functions of the number of nurses and shifts. Hence the QUBO objective function's form for our constraint, that is, at most two consecutive shifts, is set for each team member and takes on the form

$\sum_{n=1}^{N}\sum_{s=1}^{S} Q_{i(n,s)j(n,s+1)} q_{(n,s)} q_{(n,s+1)} = c.$ (3)

To minimize the cost function, Equation (3) must be zero, otherwise the constant 'c' is set by the programmer to tune the objective functions for most optimal result.

The complete Hamiltonian of our problem is

$$H(q) = \gamma\left[\sum_{n=1}^{N}\left(\sum_{s=1}^{S} hq_{(n,s)} - b\right)^2\right] + \sum_{n=1}^{N}\sum_{s=1}^{S} Q_{(n,s)(n,s+1)}q_{(n,s)}q_{(n,s+1)} + \lambda\left[\sum_{s=1}^{S}\left(\sum_{n=1}^{N} hq_{(n,s)}\right) - a\right]^2. \quad (4)$$

We want to find the value of $q_{(n,s)}$ to minimize our cost function Eq. (4), therefore, we transform our QUBO form to the equivalent Ising model, where the binary variable $q_{(n,s)} \in \{0,1\}$areequivalent to the variables$q_{(n,s)} \epsilon$ {-1, +1}, representing bipolar spin variable $s_i = 2q_i - 1$. During the conversion of the QUBO form to Ising form for the D-Wave quantum annealer, the linear terms and quadratic terms are separated in the form

$$E_{ising}(s) = \sum_{i=1}^{N} h_i s_i + \sum_{i=1}^{N}\sum_{j=i+1}^{N} J_{i,j} s_i s_j$$

where, h_i are linear terms, $J_{i,j}$ are quadratic terms of our matrix, and $s_i$corresponds to a physical Ising spin from set {-1, +1}.

2.2 Hamiltonian formulation of PSP: For our physician scheduling problem, we use the same hard and soft constraint and the objective function that we formulated for our NSP. For the case of PSP, the upper limits and the number of shift per day may vary from the NSP. We take the number of physician to be p ∈ {1, 2, …, P} and working shift to be s ∈ {1,2,… , S}. From the Hamiltonian (Eq. 4) we follow our QUBO formalism for our PSP as expressed as equation 5.

$$H_1(q) = \gamma\left[\sum_{p=1}^{P}\left(\sum_{s=1}^{S} hq_{(p,s)} - b\right)^2\right] + \sum_{p=1}^{P}\sum_{s=1}^{S} Q_{(p,s)(p,s+1)}q_{(p,s)}q_{(p,s+1)} + \lambda\left[\sum_{s=1}^{S}\left(\sum_{p=1}^{P} hq_{(p,s)}\right) - a\right].^2 \quad (5)$$

From Eq. (5), we state that the constant *b* represents the total shift hours each physician can do per day, a = h * m, where h is each shift hour, m is the maximum number of physician allowed per shift. $q_{(p,s)}$ is binary variable from set {0,1} where $q_{(p,s)} = 1$ is physician 'p' is placed at shift 's'. The Lagrange constants γ, λ , are set by the programmer, to tune the Hamiltonian accordingly.

2.3 Hamiltonian formulation of NPSP: We assume a team of nurses and physicians, working for 'S' shift. The team consist of set of nurse and physician , t ∈ {1,2,3,... , T} , we assume the last two members of the set {1,2,3,…, T} ; T-1 and T be the physician and rest nurse. Let the matrix, for our QUBO form be Q. Let the number of working shift be s ∈ {1, 2,… ,S}, for our case we have four shift, each consist of h = 6 hour, to cover a day of 24 hour. Let $q_{t,s} \in \{0, 1\}$ be a binary variable, where if $q_{t,s} = 1$, then member 't' is placed on the shift s, otherwise zero. For our first constraint, we set the maximum number of shift, each nurse and physician can do per day, by total hour as b = 12; hence each nurse and physician can do at most 2 shift each day. Hence our QUBO form follows:

$$\gamma\left[\sum_{t=1}^{T}\left(\sum_{s=1}^{S} hq_{(t,s)} - b\right)^2\right]$$

$$= \gamma\left[\sum_{t=1}^{T}\left(h^2 \sum_{s=1}^{S} q_{(t,s)} + 2h^2 \sum_{s=1}^{S}\sum_{s1>s}^{S} q_{(t,s)}q_{(t,s1)} - 2hb\sum_{s=1}^{S} q_{(t,s)} + b^2\right)\right]$$

$$= \gamma(h^2 - 2hb)\sum_{t=1}^{T}\sum_{s=1}^{S} q_{(t,s)} + \gamma 2h^2 \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s1>s}^{S} q_{(t,s)}q_{(t,s1)} + \gamma b^2. \quad (6)$$

The above constraint is satisfied, i.e. the cost function is minimum, only when the left-hand side and the right-hand side terms of Eq. (6) become zero, for some unknown binary variables $q_{(t,s)}$. The variable γ is the positive Lagrange constant, set by the programmer, to tune the function. $\gamma(h^2 - 2hb)$ and $\gamma 2h^2$are the diagonal and off-diagonal terms of our Q matrix, respectively.

For our second constraint, we set a total duration of hour that each shift can have, combining all the team members' working hour, that are placed in that shift be a = h * m, where m, is the maximum number of member that can be present in each shift. Hence the QUBO form of our constraint is:

$$\lambda\left[\sum_{s=1}^{S}\left(\sum_{t=1}^{T} hq_{(t,s)}\right) - a\right]^2$$
$$= \lambda\left[\sum_{s=1}^{S}\left(\sum_{t=1}^{T} hq_{(t,s)}\right)^2 - 2ah\sum_{t=1}^{T} q_{(t,s)} + 2h^2\sum_{t=1}^{T}\sum_{t1>t}^{T} q_{(t,s)}q_{(t1,s)} + a^2\right]$$
$$= \lambda h^2\sum_{s=1}^{S}\sum_{t=1}^{T} q_{(t,s)} - \lambda 2ah\sum_{s=1}^{S}\sum_{t=1}^{T} q_{(t,s)} + \lambda 2h^2\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{t1>t}^{T} q_{(t,s)}q_{(t1,s)} + \lambda a^2$$

$$= \lambda(h^2 - 2ah)\sum_{s=1}^{S}\sum_{t=1}^{T} q_{(t,s)} - \lambda 2h^2\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{t1>t}^{T} q_{(t,s)}\, q_{(t1,s)} + \lambda a^2\,. \qquad (7)$$

The above constraint is satisfied, i.e. the cost function is minimum, only when the left-hand side and the right-hand side terms of Eq. (7), become zero, for some unknown $q_{(t,s)}$ binary variables. The variable λ is the positive Lagrange constant, set by the programmer, to tune the function. $\lambda(h^2 - 2ah)$and $\lambda 2h^2$are the diagonal and off-diagonal terms of our Q matrix, repsectively.

For our third constraint, we take two composite indices as,i(t,s) and j(t, s) as the function of team member and shift. Hence the QUBO Objective function form for our constraint, that is at most two consecutive shift is set for each team member is:

$$\sum_{t=1}^{T}\sum_{s=1}^{S} Q_{i(t,s)j(t,s+1)}q_{(t,s)}q_{(t,s+1)} = c \qquad (8)$$

To satisfy our objective function i.e. to minimize the cost function; Eq (7) must be zero otherwise 'c' which is a constant, set by the programmer, to tune the objective functions for most optimal result. Hence our complete Hamiltonian of our problem is:

$$H(q) = \gamma\left[\sum_{t=1}^{T}\left(\sum_{s=1}^{S} hq_{(t,s)} - b\right)^2\right] + \sum_{t=1}^{T}\sum_{s=1}^{S} Q_{(t,s)(t,s+1)}q_{(t,s)}q_{(t,s+1)} + \lambda\left[\sum_{s=1}^{S}\left(\sum_{t=1}^{T} hq_{(t,s)}\right) - a\right]^2 \;.(9)$$

We want to find the value of $q_{(t,s)}$ to minimize our cost function Eq. (9), hence we transform our QUBO form to equivalent Ising model, where the binary variable $q_{(t,s)} \in \{0,1\} are$ equivalent to $q_{(t,s)} \epsilon$ {-1, +1}, which are bipolar spin variable $s_i = 2q_i - 1$. During the conversion of QUBO form to Ising form for D-Wave Quantum Computer, the linear terms and quadratic terms are separated in the form:

$$E_{ising}(s) = \sum_{i=1}^{N} h_i s_i + \sum_{i=1}^{N}\sum_{j=i+1}^{N} J_{i,j}s_i s_j$$

where h_i are linear terms, $J_{i,j}$ are quadratic terms of our matrix, and $s_i$corresponds to a physical Ising spin from set {-1, +1}.

3. Result and Discussion

We use D-wave's quantum annealer to solve the NSP, PSP and NPSP. D-wave recently launched Leap 2, a classical-quantum hybrid sampler that supports up to 10,000 variables fully connected to the quantum processor. First we shall explore the result obtained from simulated annealing (SA), then we shall discuss the results from quantum forward annealing (FA) and quantum reverse annealing (RA) in the following subsections respectively. During our experiments, we have fixed the annealing time to be 200μs and total number of samples used as 1,000. Throughout our experiments we have fixed the positive Lagrange constants $\gamma = 5, \lambda = 1.2$.The standard schedule provided by D-wave is used for all forward annealing schedules. On the other hand, scheduled annealing process is illustrated for all the experiments as per Fig 1. The reverse scheduling process is started from time t= 0 μs with annealing parameter value 1. The process is continued till 2.75 μs and ended at the annealing parameter value of 0.45. Then annealing parameter value s is fixed at 0.45 till 83.3μs. Finally forward annealing process is started at time 83.3μs and completed the process at time 83.46μs with annealing parameter value s=1.

We present the results for NSP problem considering N=4, 5 and 6 nurses and number of shift per day being 4. The total number shifts for each case are S=4 to 20. The two hard constraints, namely the maximum number of nurses per shift are fixed at 2 and the maximum number of shifts for each nurse are fixed at S/2 for S=4 to 20.

The result of simulated annealing shows that the ground state Hamiltonian was found to be satisfying all constraints for the values of $\gamma = 5, \lambda = 1.2$, described in table 1 and fig 2. For demonstration, we consider a case of N=4 and S=12. The graphical representation of optimal ground state solution from simulated annealing for N=4, S=12 is shown in fig 3. It can be noted that the result of simulated annealing does not satisfy all the hard constraints, like in fig 3 (a) where the maximum number of shifts per nurse does not meet.

Now, let us discuss about FA and RA for the NPS with above mentioned values of N and S. The results of each type is shown in table 1 for the value of $\gamma = 5, \lambda = 1.2$. The graphical representation of optimal ground state solution from FA and RA, for N=4, S=12, is shown in fig 3(b) and (c).The schedules are satisfying all hard constraints.

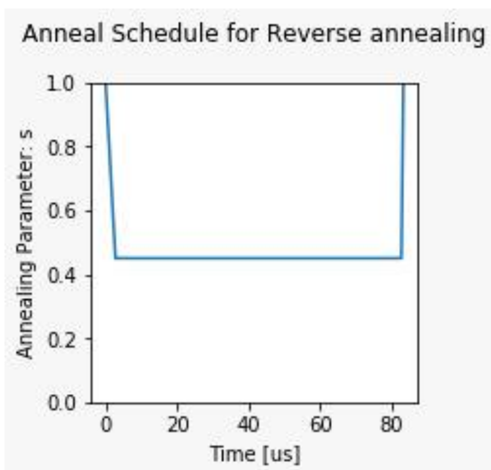
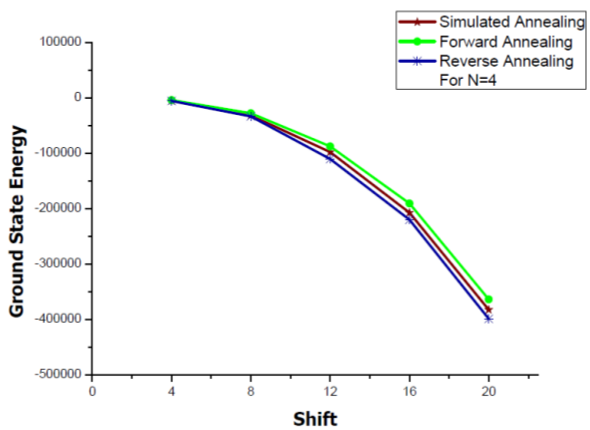


Fig 1: Annealing schedule. Reverse annealing schedule (t=0 to 2.75 μs and s= 1 to 0.45) , Pause state (t=2.75 to 83.3 μs and s= 0.45), Forward annealing process(t=83.3 to 83.46 μs and s= 0.45 to 1)

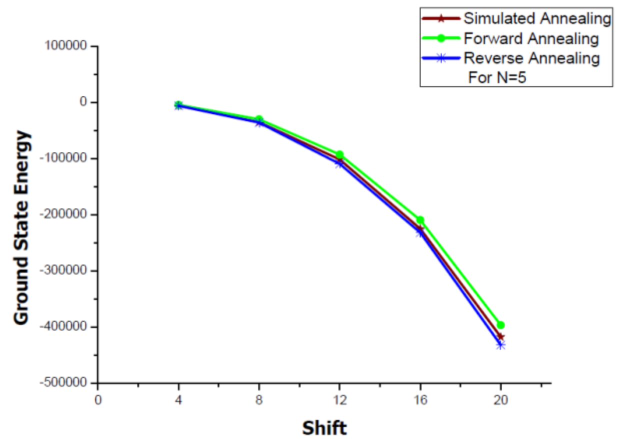
Table 1 : Generated ground states from different annealing process for different constraint.

Variables	Variables	Constraint 1	Constraint 2	Simulated Annealing	Forward Annealing	Reverse Annealing
Nurse	Shift	Max Nurse per shift	Max Shift for each Nurse	Ground State H	Ground State H	Ground State H
4	4	2	2	-5333.76	-4233.6	-5644.8
4	8	2	4	-32738.4	-27993.6	-33638.4
4	12	2	6	-97653.6	-87868.8	-100569.6

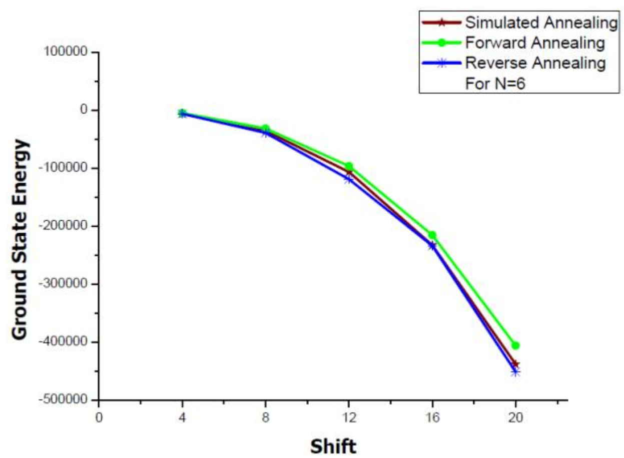
4	16	2	8	-219859.2	-200448	-223027.2
4	20	2	10	-414504	-382320	-417600
5	4	2	2	-5950.08	-4608	-6019.2
5	8	2	4	553.04	-30153.6	-35798.4
5	12	2	6	-103341.6	-92908.8	-105609.6
5	16	2	8	-228391.2	-209448	-232027.2
5	20	2	10	-427464	-396720	-432000
6	4	2	2	-6391.44	-4982.4	-6393.6
6	8	2	4	-36719.28	-31593.6	-37238.4
6	12	2	6	-106545.6	-96508.8	-109209.6
6	16	2	8	-36359.28	-31593.6	-37238.4
6	20	2	10	-435744	-406080	-441360



(a)



(b)



(c)

Fig 2: Ground state energy with respect to number of shift is achieved by simulated annealing, forward annealing and reverse annealing for (a) N=4, (b) N=5 and (c) N=6

NSP Schedule of 4 nurses for 12 shifts with SA when two hard constraints are applied

	1(MT)	2(DT)	3(EN)	4(LN)	5(MT)	6(DT)	7(EN)	8(LN)	9(MT)	10(DT)	11(EN)	12(LN)
Nurse 1	X	X		X					X	X		
Nurse 2				X	X	X	X		X		X	X
Nurse 3		X	X				X	X		X	X	X
Nurse 4	X		X		X	X		X				

(a)

NSP Schedule of 4 nurses for 12 shifts with FA when two hard constraints are applied

	1(MT)	2(DT)	3(EN)	4(LN)	5(MT)	6(DT)	7(EN)	8(LN)	9(MT)	10(DT)	11(EN)	12(LN)
Nurse 1		X		X		X	X			X		X
Nurse 2		X		X		X		X		X	X	
Nurse 3	X		X		X		X		X		X	
Nurse 4	X		X		X			X	X			X

(b)

NSP Schedule of 4 nurses for 12 shifts with RA when two hard constraints are applied

	1(MT)	2(DT)	3(EN)	4(LN)	5(MT)	6(DT)	7(EN)	8(LN)	9(MT)	10(DT)	11(EN)	12(LN)
Nurse 1	X		X		X		X		X		X	
Nurse 2	X		X	X		X			X			X
Nurse 3		X			X		X	X		X		X
Nurse 4		X		X		X		X		X	X	

(c)

Fig 3: NSP scheduling four nurses for 12 shifts (a) simulated annealing result (b) forward annealing result (c) reverse annealing result.

Next, we consider NPSP scheduling for COVID'19 hospitals. The NPSP problem considers two teams of Nurse-Physician working for 15 days each, after which time an entire team will be quarantined for the following 15 days. One Nurse-Physician team will remain as a buffer, in case any member of a given team tests positive for COVID'19. In such a situation, the entire effected team will be replaced by the buffer team. Let us next consider aNurse-Physician team consisting of 6 nurses and 2 physicians, on aschedule consisting of 5 days(or 20 shifts). The hard constraints for nurses are that two nurses must be present during a shift, and that each nurse can work for a maximum of 2shifts per day. On the other hand, a physician can work every alternative shift, and total two shifts per day. Since the team is working in a COVID'19 ward, team members are expected to maintain distancing measures, but not take a day off, as soft constraint, and every fifteen days the entire team will be quarantined or on leave.

We fix the positive Lagrange constants$\gamma = 5, \lambda = 1.2$ and the SA, FA, and RA scheduling setups remain the same as those described for NSP. The schedule for optimal ground state Hamiltonian from the three annealing regimens are described in fig 4 (a), (b) and (c). The graphical representation shows that in case of SA results, all the hard constraints are not satisfied. On the other hand, FA and RA results show that all of the hard constraints are satisfied. Fig 4(a),(b) and (c) demonstrate that the physician's duty is assigned every alternative shift, while a nurse can do two consecutive shifts. At the same time, for each shift, a maximum of two nurses and one physician are available.

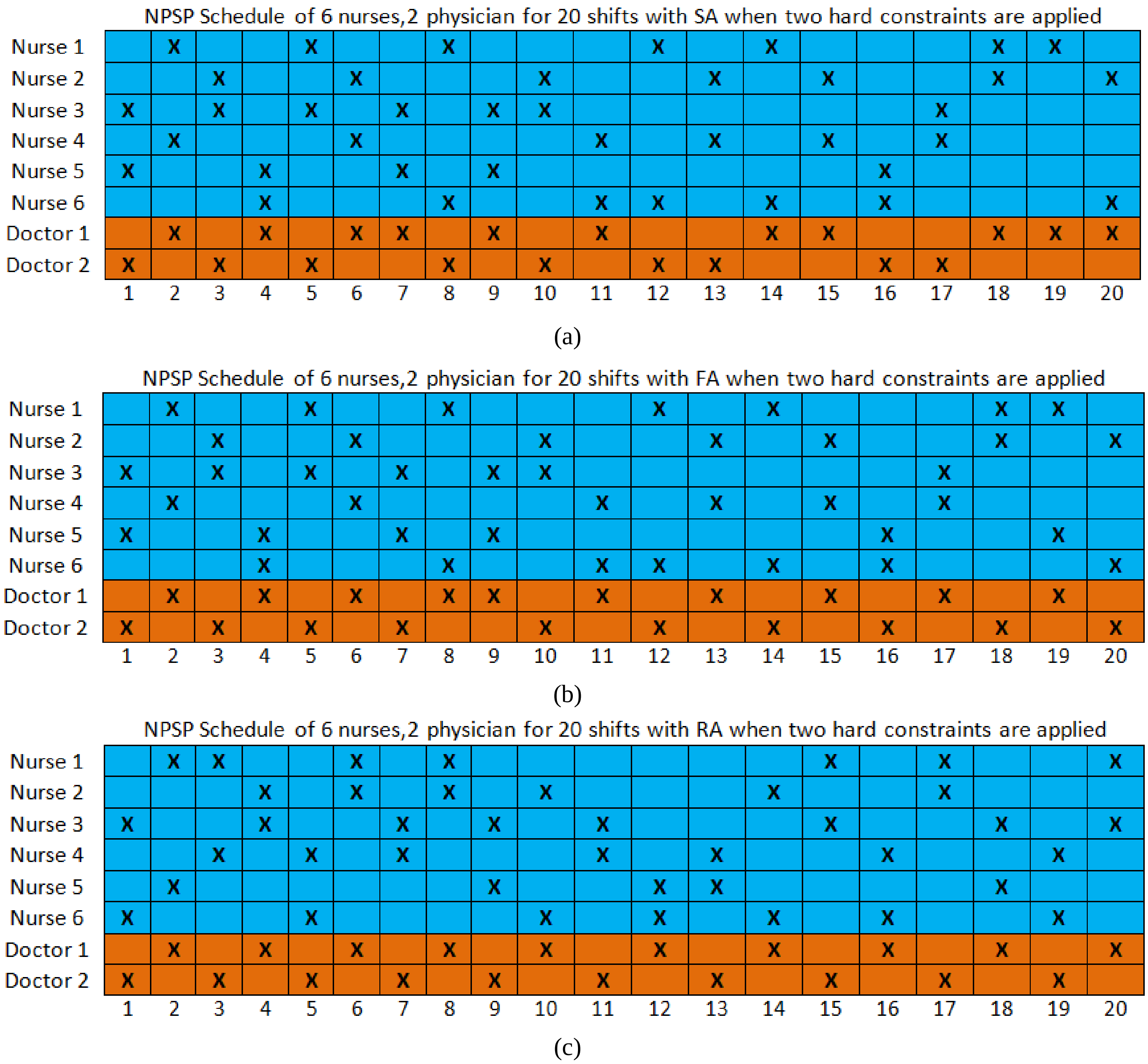


Fig 4: NPSP scheduling six nurses and two physician for 20 shifts (a) simulated annealing result (b) forward annealing result (c) reverse annealing result.

3.1 Comparative Study

Our examples of nurses and shifts per day comprised of 4 Nurse and 4 shifts per day for COVID'19 Hospitals with different hard and soft constraints as described in section 2.1. In this section, we compare our results with the previous report by Kazuki Ikeda et al in [23]. The comparison serves to demonstrate the advantage of using Leap2 D-wave hybrid solver with respect to just the D-wave solver. In this process we have considered the same example used in [23], that is, 3 nurses and 3 shifts per day, and a total of 6 shifts considered. The shift of a day labeled as day time (DT), early night (EN) and night time (NT), although both the three shift and four shift systems are commonly utilized in COVID'19 clinics. However, for the three shift system, same constraints are considered as stated earlier in section 2.1. Maximum number of nurses per shift is 2 and the maximum number of duty is 2 for given roster as a case study reported in [23], both are considered as hard constraints. The soft constraint is a day off i.e. three consecutive shifts off. A soft constraint is defined as follows.

Let $p(n, s)$ be the priority of day off i.e. three consecutive shifts off and defined as

$$p(n,s) = \begin{cases} 2 & high\ priority \\ 1 & middle\ priority \\ 0 & no\ priority \end{cases}$$

The soft constraint term $\eta \sum_{n,s} p(n,s) q_{(n,s)}$, the extra external Hamiltonian magnetic field is added with eq. 4 to allow the soft constraint as reported in [23]. The final Hamiltonian of the stated system becomes

$$H(q) = \gamma \left[\sum_{n=1}^{N} \left(\sum_{s=1}^{S} hq_{(n,s)} - b \right)^2 \right] + \sum_{n=1}^{N} \sum_{s=1}^{S} Q_{(n,s)(n,s+1)} q_{(n,s)} q_{(n,s+1)} + \lambda \left[\sum_{s=1}^{S} \left(\sum_{n=1}^{N} hq_{(n,s)} \right) - a \right]^2 + \eta \sum_{n,s} p(n,s) q_{(n,s)} \quad (10)$$

By heuristic tuning of positive Lagrange parameter η, the day off (or the three consecutive shifts off) requests are considered for the NSP. This constraint is treated as soft constraint, however, while higher priority is imposed, it is suppose to meet the day off criteria. On the other hand, hard constraints like the maximum number of nurses per shift, or the maximum number of shifts per nurse, are satisfied in the nurse-shift schedule. For the above experiment on D-wave Q2000TM, the hybrid sampler is used. As a result, only one best optimal lowest ground state energy sample with probability 1 is found. For this experiment, $p(n,s) = 1$, which means that the middle priority of day off is imposed. To demonstrate the higher priority day off for nurses, the experiment is also carried out for $p(n,s) = 2$ for specific nurse and from specific shift, the day off requested. In figure 5, we demonstrate the schedule of three nurses for six shifts using (a) SA simulator (b) QA and (c) RA with $p(1,4) = 1, p(2,5) = 1\ and p(3,2) = 1$, results shows that none of the schedule are satisfying 100% day off criteria as day off is soft constraints but all hard constraints are satisfied. On the other hand, when we impose $p(n,s) = 2$ for specific nurse and from specific shift for day off, the results shows in (d) SA simulator (e) FA and (f) RA and the entire schedule are stratifying day off criteria. Fig 6 is described the best optimal solution of above mentioned problem with minimum ground state energy -3194,-2304 and -3196 for SA, FA and RA, respectively.

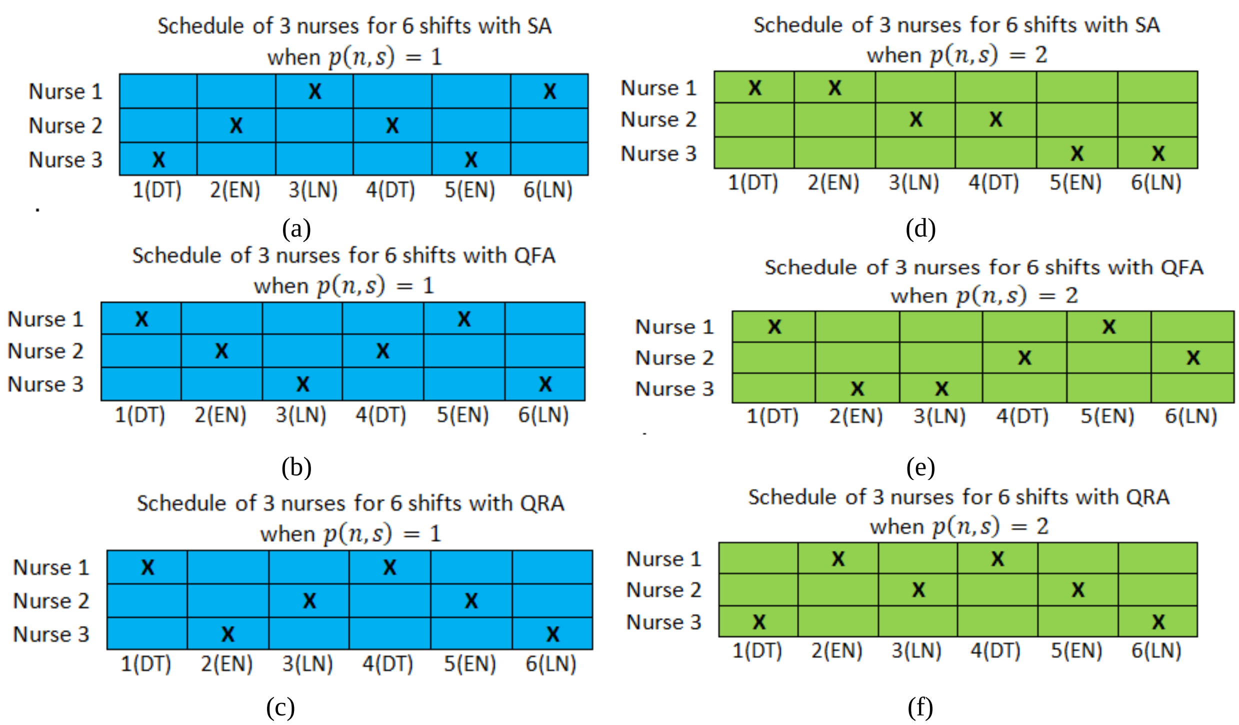


Fig. 5: Optimal scheduling solution for comparative study with kazukilkeda et al. [23], for three shift NSP with hard and soft constraints. For $p(n,s) = 1$, (a) SA result (b) FA result using D-wave hybrid solver (c) RA result

using D-wave hybrid solver. For $p(n, s) = 2$, (d) SA result (e) FA result using D-wave hybrid solver (f) RA result using D-wave hybrid solver.

Figure 5 shows the RA using D-wave hybrid solver to find lowest ground state energy for the optimal solution sample. The results show that ground state energy are G_{SA}= -3194, G_{FA}=-2304 and G_{RA}=-3196 respectively while the soft constraint is of middle priority as depicted in fig 5 (a), (b) and (c). Similarly, ground state energy of figures 5 (d)-(f) are G_{SA}= -4387, G_{FA}=-3600 and G_{RA}=-4752 respectively, while the soft constraint is of high priority.The ground state Hamiltonian for SA, FA and RA during middle priority and high priority soft constraint evaluation are shown in fig 6. During comparison with earlier reported NSP with D-wave [23], it can be observed that the day-off constraint is partially satisfied. It is also to be noted that the above mention report [23] used only the quantum sampler whereas we have used Leap2 hybrid sampler for these experiments.

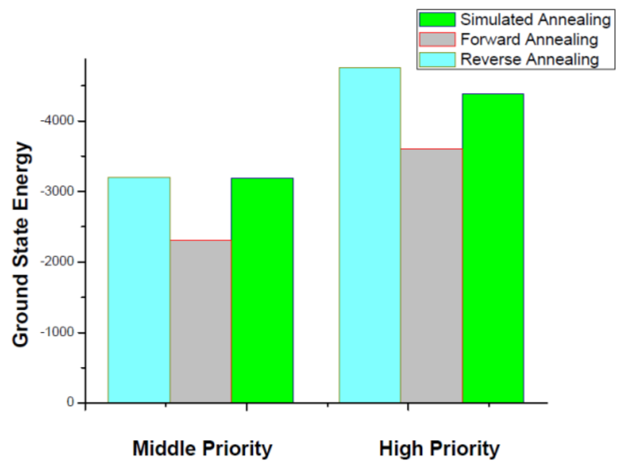


Fig. 6: Ground State energy for SA,FA and RA for the middle priority of day-off soft constraint and high priority day off soft constraint

4. Conclusion

We have presented nurse and physician scheduling problems specifically for the COVID-19 pandemic situation. The proposed Hamiltonian functions of NSP, PSP, and NPSP are reducing in the QUBO model first and then transform to the Ising Hamiltonian model. Hence Ising Hamiltonian functions easily solved by commercially available quantum annealer D-Wave 2000Q. In comparison with earlier reported NSP with D-wave 2000Q [23], it can be observed that the day off constraint is partially satisfied. It also to be noted that the above mentioned scientific report [23] used quantum sampler called D-wave sampler to get probabilistic optimized ground state. On the other hand, we have used Leap2 hybrid sampler for these experiments to get accurate and optimized global minima. In this paper, we fixed all constraints (Soft and hard both) following the WHO (World Health Organization) specified guidelines. The outcome of variety of quantum annealing processes proves that all assigned constraints satisfied successfully. Our result also demonstrates that the proposed Hamiltonian Ising function will provide an accurate, optimized, and uniform solution with increasing roaster and nurse or physician numbers in fixed annealing duration. It is also noted that among variety quantum annealing process, the reverse annealing process provides us the most optimized global minimum for all the above-mentioned scheduling problems. Adding extra soft or hard constraints, our proposed Hamiltonian Ising function can be tuned more as the quantum annealing process is in its early phase. Yet, our presented work will create a good impact on society in this pandemic situation.

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