A Conjecture On Some ds Periods On The Complex Plane

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Abstract: Here we will propose a simple and very difficult open question like the Fermat's problem on some ds periods on the complex pane. This very elementary problem will create a new field on the complex plane.

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1 Introduction

We would like to introduce a fairly simple open problem, however, its proof will be very difficult and so, we expect some new concept and fundamental development in complex analysis.

In order to state its essence, we will recall the elementary fact, first. Let D be an n ($n \geq 1$) ply connected bounded regular domain whose boundary components are $C_1, C_2, ..., C_n; C_n$ is the outer boundary component comprising of analytic Jordan curves. Then, for any given complex numbers $\alpha_1, \alpha_2, ..., \alpha_{n-1}$, there exist analytic functions $f(z)$ on $D \cup \partial D$ satisfying

$$\int_{C_j} f(z)dz = \alpha_j, \quad j = 1, 2, ..., n-1 \tag{1.1}$$

([1, 2]).

A new problem is to consider the above fundamental property for the case by changing dz by $ds = |dz|$. The result will be stated as follows:

ds **perods conjecture**: *For any given complex numbers* $\alpha_1, \alpha_2, ..., \alpha_n$, *there exist analytic functions* $f(z)$ *on* $D \cup \partial D$ *satisfying*

$$\int_{C_j} f(z)ds = \alpha_j, \quad j = 1, 2, ..., n, \tag{1.2}$$

except for the annulus case for the domain D.

For an annulus case for D, *for any complex number* α_1, *there exist analytic functions* $f(z)$ *on* $D \cup \partial D$ *satisfying*

$$\int_{C_1} f(z)ds = \alpha_1. \tag{1.3}$$

Of course, in the annulus case, the period

$$\int_{C_2} f(z)ds$$

is automatically determined by the period $\int_{C_1} f(z)ds$.

2 Comments

The conjecture and open problem may be contributed to the development of mathematics, as in many famous cases - for example, recall the Fermat's problem and ABC conjecture. In our case, we can expect some fundamental contributions, because its statement is very simple, however its solution may be very difficult as we see from the exceptional case of annulus.

dz normal periods vanishing cases may be considered as a typical subclass of analytic functions so-called exact differentials having single-valued integrals on the domain D. Indeed, for example, for the Bergman reproducing kernel, we consider the important exact Bergman kernel, see [2].

Therefore, similarly, we can consider the exact Szegö reproducing kernel for the usual Szegö kernel by the ds periods vanishing.

This open problem was discussed with Professor Nobuyuki Suita over 30 years ago, however we were not able to solve the problem. The author asked for his help for this problem.

References

[1] L. V. Ahlfors, Complex Analysis, McGraw-Hill Book Company (1966).

[2] Z. Nehari, Conformal Mapping, Graw-Hill Book Company, Inc. (1952).