THE EMISSION OF PHOTONS BY PLASMA FLUCTUATIONS

Miroslav Pardy
Department of Physical Electronics
and
Laboratory of Plasma physics
Masaryk University
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

August 15, 2020

Abstract

The totally ionized charged collisionless plasma at finite temperature is considered. Using the statistical and Schwinger field methods we derive the production of photons from the plasma by the Čerenkov mechanism. We derive the spectral formula of emitted photons by the plasma fluctuations. The calculation can be extended to the photon propagator involving radiative corrections.
PACS: 52.55, 52.20, 41.60B

1 Introduction

The production of photons by the plasma fluctuations is one of the problems which form the basic ingredients of the quantum field theory (QFT) at finite temperature. This theory has been formulated some years ago by Dolan and Jackiw (1974), Weinberg (1974) and Bernard (1974) and some of the first applications of this theory were the calculations of the temperature behaviour of the effective potential in the Higgs sector of the standard model.

Information on the systematic examination of the finite temperature effects in quantum electrodynamics (QED) at one loop order was given by Donoghue, Holstein and Robinett (1985). They treated about calculation of mass, charge, wave function renormalization and so on; they demonstrated the running of the coupling constant at finite temperature and discussed the normalized vertex function and the energy momentum tensor.

The similar discussion of QED was published by Johansson, Peressutti and Skagerstam (1986).

The finite-temperature QED, QFT and also quantum chromodynamics usually deal with the specific processses of these theories in the heat bath of photons or other particles.

On the other hand Ford (1982) and del Campo (1988), have considered production of gravitons by the fluctuations of the electromagnetic field of plasma.

Fluctuations are the integral part of the thermodynamical system. The physical quantities which characterize the macroscopic system in the equilibrium state are with great accuracy equal to their average values. Neverthless, they sometime deflect stochastically from their average values. In other words they fluctuate. The existence of fluctuations can be considered in certain sense as the fundamental feature of the thermal system. In such a way we can decide if the system is thermal or nonthermal. Such a criterium is important in particle physics where, for instance, the collisions of nucleons at low energies is nonthermal; however, in the high energy regime the plasma of particles is formed with the physical characteristics of the thermal system, where also thermodynamical fluctuations play important role.

In this article we slightly modify the approach of Ford and del Campo and compute the power spectral formula $P(\omega, t)$ of produced photons by the plasma fluctuations in the framework of the Schwinger field theory. We use here the statistical methods and Schwinger source theory for the determination of the plasma fluctuations. Because the thermal plasma has the electrodynamical index of refraction n, the resulting effect is the Čerenkov production of photons by plasma fluctuation. Such effect is considered in physics, to our knowledge, for the first time and in the source theory it was never solved. The relation to the experiments with the ionized plasma is evident and it is not excluded it will be sooner or later experimentally investigated.

The generalization of our mathematical procedure to the situation with the photon propagator with radiative corrections is possible (Schwinger, 1973; Pardy, 1994b; 1994c; 1994d). Then, we work with the high-temperature plasma, where the creation of pairs occurs. The relation of our theory to the experiments with the ionized plasma and with the plasma formed during the heavy-ion collisions is evident and can help to understand other processes inside the plasma.

2 Formulation of the problem in source theory

The basic formula of the Schwinger source theory is the vacuum-vacuum amplitude (Schwinger, 1970):

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W}, \tag{1}$$

where in case of the electromagnetic field the action W is defined as (Schwinger et al., 1976)

$$W = \frac{1}{2c^2}\int (dx)(dx')J_\mu(x)D_+^{\mu\nu}(x-x')J_\nu(x'), \tag{2}$$

where $J^\mu \equiv (c\varrho, \mathbf{J})$ being the conserved current and $D_+^{\mu\nu}(x-x')$ is the photon propagator in a medium with the index of refraction n, the magnetic permeability μ and the dielectric constant ε.

The explicit form of $D_+^{\mu\nu}(x-x')$ has been obtained as (Schwinger et al., 1976)

$$D_+^{\mu\nu} = \frac{\mu}{c}[g^{\mu\nu} + (1-n^{-2})\beta^\mu\beta^\nu]D_+(x-x'), \tag{3}$$

where $\beta^\mu = (1, \mathbf{0})$ and

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x-x')} \frac{1}{|\mathbf{k}^2| - n^2(k^0)^2 - i\epsilon} \tag{4}$$

with $k^\mu = (k^0, \mathbf{k})$. The Green function $D_+(x-x')$ can be further specified as

$$D_+(x-x') = \frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|} e^{-i\omega|t-t'|} \tag{5}$$

by the standard contour integral method (Schwinger et al., 1976).

The vacuum persistence probability follows from eq. (1) in the form (Schwinger et al., 1976):

$$| < 0_+|0_- > |^2 = e^{-\frac{2}{\hbar}\mathrm{Im}W}. \tag{6}$$

Using the definition of the spectral function $P(\omega, t)$

$$\frac{2}{\hbar}\mathrm{Im}W \stackrel{d}{=} \int \frac{d\omega dt}{\hbar\omega} P(\omega, t), \tag{7}$$

we get after some calculation (Schwinger et al., 1976):

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt' \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos\omega(t-t') \times$$

$$\times [\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t)\cdot\mathbf{J}(\mathbf{x}', t')], \tag{8}$$

where ϱ and $\mathbf{J}$ are connected with the four potential $A_\mu = (\varphi, \mathbf{A})$ according to the equation (Schwinger et al., 1976):

$$\Delta\varphi - \frac{\mu\varepsilon}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{1}{\varepsilon}\varrho \tag{9}$$

$$\Delta\mathbf{A} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{\mu}{c}\mathbf{J}. \tag{10}$$

As a consequence of the definition (7), $P(\omega, t)$ has the physical content of the power spectrum of emitted photons. Formula (8) was applied to many interseting cases, such as the linear motion of charge in a medium, the circular motion of a charge in this medium, and so on. Obviously it can be applied to other complicated motion of charges in the medium.

Formula (8) enables a determination of the spectrum of emitted photons in the case of the finite temperature regime. When the situation is such that a charge moves linearly in the finite temperature medium, we get so called the finite-temperature Čerenkov radiation which was discussed by Pardy (1989).

On the other hand it is possible to consider a case when the system is formed by charges with density ϱ and current density $\mathbf{J}$ and has the temperature T. Such system corresponds physically to the totally ionized plasma.

To our knowledge, such problem has been not so far solved in the framework of the source theory and it is our goal to determine the spectrum of photons emitted by the fluctuations of such system. Our article is based on the article by author (Pardy, 1991; 1994a; 1994b).

It is obvious that system in thermal equilibrium cannot produce photons by the average charge and current densities, however, it can do it by the thermal fluctuations which are involved in the correlation functions $< \varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') >$ and $< \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}', t') >$. A similar problem was solved by Ford (1982) and by del Campo (1988) who have been considered the emission of gravitons by the thermal fluctuations of the electromagnetic field of plasma where the electromagnetic fields fluctuations has been expressed by the correlation function of the tensor of energy and momentum $T_{\mu\nu}$ of the electromagnetic field.

Here we determine the correlation functions of the charge and current densities of totaly ionized hot plasma by the statistical methods.

3 The correlation functions

First, let us calculate the correlation function $< \varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') >$ at the temperature T. To achieve this goal let us consider the specific model of plasma, namely the plasma nonlimited in space, homogenous and totaly ionized in the state of the thermal equilibrium. Since the ions are assumed to have large masses, their motion can be neglected. The presence of ions forms only the necessary compensation of charge. A plasma, where the motion of ions is absolutely neglected is called the electron plasma. We suppose that such a plasma has an electromagnetic index of refraction and we will see later that it is the existence of this index of refraction, which is formed by the thermal motion of plasma particles that enables the production of Čerenkovian photons by plasma fluctuations.

The density of electrons in a plasma is given by the relation

$$N(\mathbf{x}, t) = \sum_{\alpha}^{\infty} \delta(\mathbf{x} - \mathbf{x}_\alpha(t)), \tag{11}$$

where $\mathbf{x}_\alpha(t)$ is the coordinate of the electron with number α. The summation concerns electrons within the unit volume. We will supoose there is no scattering between electrons.

Then we can write

$$\mathbf{x}_\alpha(t) = \mathbf{x}_\alpha + \mathbf{v}_\alpha t, \tag{12}$$

where $\mathbf{x}_\alpha$ and $\mathbf{v}_\alpha$ are the vector coordinate and velocity of the electron with number α at the initial time moment $t = 0$. The generalization to the case with the arbitrary initial conditions is obvious.

The statistical average value of the particle density can be introduced by the following definition:

$$N_0 =< N(\mathbf{x}, t) > . \tag{13}$$

Then, obviously:

$$N(\mathbf{x}, t) = N_0 + \delta N(\mathbf{x}, t), \tag{14}$$

or,

$$\delta N(\mathbf{x}, t) = N(\mathbf{x}, t) - N_0. \tag{15}$$

For the correlation function $< N(\mathbf{x}, t)N(\mathbf{x}', t') >$ we have:

$$< N(\mathbf{x}, t)N(\mathbf{x}', t') >= N_0 N_0 + < \delta N(\mathbf{x}, t)\delta N(\mathbf{x}', t') > . \tag{16}$$

As a consequence of the homogenity of the ion background, the correlation function of a charge density is related to the correlation function of particles in the following way:

$$< \varrho^2 >= e^2 < \delta N^2 >= e^2 < \delta N(\mathbf{x}, t)\delta N(\mathbf{x}', t' > \tag{17}$$

and the correlation function depends only on the space and time differences. Only correlation function of the charge density and the current density can contribute to the formula (8), determining the spectral density of photons emitted by the fluctuations of the charged plasma.

Obviously, for our plasma model we have (Sitenko, 1965)

$$< \delta N^2 >=< \sum_\alpha \delta(\mathbf{x} - \mathbf{x}_\alpha - \mathbf{v}_\alpha t)\delta(\mathbf{x}' - \mathbf{x}'_\alpha - \mathbf{v}_\alpha t') >, \tag{18}$$

which is a consequence of eq. (11) and eq. (15).

Now, let us introduce the one-particle distribution function $f(\mathbf{v})$ normalized by condition

$$\int d\mathbf{v} f(v) = 1, \tag{19}$$

where $v = |\mathbf{v}|$.

Then obviously (Sitenko, 1965)

$$< \delta N^2 > = N_0 \int d\mathbf{v} f(v)\delta(\mathbf{x} - \mathbf{x}' - \mathbf{v}(t - t'), \tag{20}$$

where N_0 is the density of thermal electrons.

The non-relativistic Maxwell distribution is of the form

$$f(v) = (m/2\pi kT)^{3/2} e^{-\frac{mv^2}{2kT}}, \tag{21}$$

where temparature T is measured in the Kelvin scale, and k is the Planck constant. The generalization of eq. (21) for the relativistic situation gives the Maxwell distribution in the following form (Sitenko, 1965):

$$f(v) = \frac{1}{\left(4\pi \frac{kT}{mc^2} K_2(\frac{mc^2}{kT})\right)} \times$$

$$\frac{c^2}{(c^2 - v^2)^{5/2}} \exp\{-\frac{mc^2}{kT} \frac{1}{\sqrt{1-(v/c)^2}}\}, \tag{22}$$

where $K_2(x)$ is so called MacDonald function with index 2 and it is defined by the equation (Tikhonov et al., 1977):

$$K_2(x) = \frac{1}{2}\left[\left(\frac{\partial I_{-\nu}}{\partial \nu}\right)_{\nu=2} - \left(\frac{\partial I_{\nu}}{\partial \nu}\right)_{\nu=2}\right] \tag{23}$$

with

$$I_\nu(x) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(k+\nu+1)} (\frac{x}{2})^{2k+\nu}. \tag{24}$$

Now, let us determine the current density correlation function if we know that the current density is given by the formula

$$\mathbf{J}(\mathbf{x}, t) = \sum_\alpha e\mathbf{v}_\alpha \delta(\mathbf{x} - \mathbf{x}_\alpha(t)). \tag{25}$$

It may be easy to show that using eq. (25) we get

$$< J_i J_j >= e^2 N_0 \int d\mathbf{v} v_i v_j f(v) \delta(\mathbf{x} - \mathbf{x}' - \mathbf{v}(t-t')). \tag{26}$$

4 The power spectral formula

At this moment we are prepared to write the power spectral formula $P(\omega, t)$ which expresses the distribution of emitted photons by the fluctuations of the totally ionized plasma. Using equations for density correlation function (20), and the current correlation function (26), and inserting them eq. (8), we obtain the general form for the spectrum of photons emitted by the plasma fluctuations:

$$P(\omega, t) = -e^2 N_0 \frac{\omega}{4\pi^2} \frac{\mu}{n^2} \int d\mathbf{x} d\mathbf{x}' d\mathbf{v} dt' \times$$

$$f(v)\delta(\mathbf{x} - \mathbf{x}' - \mathbf{v}(t-t')) \left(1 - \frac{n^2}{c^2} v^2\right) \frac{\sin \frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \cos\omega(t - t'), \tag{27}$$

where $f(v)$ is given by eq. (21) in the non-relativistic limit, or by eq. (22) in the relativistic case. It is obvious that the spectral formula does not depend on time as a consequence of combination $t - t'$ in this formula.

So, in the nonrelativistic situation we get after insertion of eq. (21) in the formula (27) and after the $\mathbf{x}'$-integration the power spectral formula of produced photons by the plasma fluctuations in the following form:

$$P(\omega, t) = \frac{-\omega}{4\pi^2}\frac{\mu}{n^2}e^2 N_o \times$$

$$\int d\mathbf{v}d\mathbf{x}d\tau f(v)\left(1 - \frac{n^2v^2}{c^2}\right)\frac{\sin\frac{n\omega v}{c}\tau}{v\tau}\cos\omega\tau, \tag{28}$$

where we have put $\tau = t - t'$.

As the quantity N_o is the number of particles in the unit volume, the $\mathbf{x}$-integration gives volume V of the plasma system; together with N_o, we get the total number N_V of particles in the volume V.

The second mathematical operation is the τ-integration. It is well known that

$$\int_{-\infty}^{\infty} d\tau \frac{\sin n\omega\beta\tau}{\tau}\cos\omega\tau = \pi;\ n\beta > 1;\ (\beta = v/c) \tag{29}$$

and the same integral is equal to zero for $n\beta < 1$.

In such a way it is necessary to evaluate integral

$$\int_{n\beta>1} dv_x dv_y dv_z\ \frac{1}{v}\left(1 - \frac{n^2v^2}{c^2}\right)e^{-\frac{mv^2}{2kT}}. \tag{30}$$

The last formula contains two integrals which can be evaluated in the spherical coordinates. For the first integral we have:

$$J_1 = \int_{c/n}^{\infty} dv v e^{-\frac{mv^2}{2kT}} = \frac{kT}{m}e^{-\frac{mc^2}{2kTn^2}}. \tag{31}$$

For the second integral we have:

$$J_2 = \int_{c/n}^{\infty} dv v^3 e^{-\frac{mv^2}{2kT}} = \frac{kTc^2}{mn^2}e^{-\frac{mc^2}{2kTn^2}} + \frac{2kT}{m}J_1, \tag{32}$$

as a result of the elementary integration.

Now, if we combine eqs.(28)–(32), we get the power spectral formula for the production of the photons by the plasma fluctuations in the following form:

$$P(\omega) = N_V\omega\mu\frac{e^2}{c^2}\left(\frac{kT}{2\pi^3 m}\right)^{1/2} e^{-\frac{mc^2}{2kTn^2}}. \tag{33}$$

In case of the relativistic Maxwell distribution involving the MacDonald functions, the evaluations of the integrals requires more complex technique of integration than in the nonrelativistic case. Nevertheless, the problem is solvable.

5 Discussion

The power spectral formula for the photons produced synergically by the thermal plasma fluctuations and by the Čerenkovian mechanism is here derived, to our knowledge, from the source theory and the plasma physics for the first time. It depends on the index of refraction of the plasma. The ω-dependence is the same as in case of the original Čerenkov radiation (Schwinger et al., 1976) and it proves that the photons are produced just by the Čerenkovian mechanism in plasma with the index of refraction $n > 1$.

Although the considered effect is only the ideal version of the physical reality, it can be related to the problems dealing with the heavy-ion colliosions in the high energy laboratories. During such processes the quark-gluon plasma is formed with the corresponding index of refraction. In such a way, during the thermal evolution of this plasma the Čerenkovian photons are produced by the plasma fluctuations during the late stage of the plasma evolution. While usually the proof of the plasma formation is the phase transformation of nuclear matter, we have determined the Čerenkovian photons as the possible signature of the late phase of the quark-gluon plasma formation.

The production of photons by the plasma fluctuations does not correspond to the production of photons from the energy loss dE/dx of a heavy leptons propagating through a high-temperature QED plasma (Braaten et al., 1991) because our process is soft process, while the high temperature plasma is formed during the first stages of the nucleus-nucleus collision as the very hard process.

If we relate the derived formula to the photon radiation in cosmology we deduce that probably during the late phase of big bang the situation occured that the photons were produced by the plasma fluctuations. On the other hand the measured dependence of the relic radiation on frequency is in a harmony with Planckian law and it means that the contribution by the plasma fluctuations is very small. Nevertheless, the contribution of plasma fluctuations cannot be a priori excluded. Thus, it gives us the serious problem on the formation of plasma at the beginning of the Universe.

At the same time the existence of the Čerenkovian spectrum during the explosion of the supernova can inform us on the existence of the plasma phase arising during the explosion.

References

Bernard, C. W. (1974). Feynman rules for gauge theories at finite temperature, Phys. Rev. D 9, 3312.

Braaten, E. and Thoma, M. H. (1991). Energy loss of a heavy fermion in a hot QED plasma, Phys. Rev. D 44, No. 4, 1298.

Campo, S. del and Ford, L. H. (1988). Gravitation emission by a Thermal Bath of Photons, Phys. Rev. D 38, 3657.

Dolan, L., Jackiw, R. (1974). Symmetry behavior at finite temperature, Phys. Rev. D 9, 3320.

Donoghue, J. F., Holstein, B. R. and Robinett, R. W. (1985). Quantum Electrodynamics at Finite Temperature, Ann. Phys. (NY) 164 No. 2, 233.

Ford, L. H. (1982). Gravitational radiation by quantum systems, Ann. Phys. (NY) 144, No. 2, 238.

Johansson, A. E., Peressutti, G. and Skagerstam, B. S. (1986). Quantum field theory at finite temperature, Renormalization and radiative corrections, Nucl. Phys. B 278, 324.

Pardy, M. (1989). Finite-temperature Čerenkov radiation, Phys. Lett. A 134, No. 6, 357.

Pardy, M. (1991). Production of photons by plasma fluctuations, Scripta Fac. Sci. Nat. Univ. Masaryk Brun. 21, Physics, 9.

Pardy, M. (1994a). The two-body potential at finite temperature, CERN-TH.7397/94.

Pardy, M. (1994b). Emssion of photons by plasma fluctuations, CERN-TH.7348/94.

Pardy M. (1994c). The Čerenkov effect with radiative corrections, Phys. Lett. B 325, 517.

Pardy, M. (1994d). The synchrotron production of photons with radiative corrections, Phys. Lett. A 189, 227.

Schwinger, J. *Particles, Sources and Fields*, Vol. I, (Addison-Wesley, Reading, Mass., 1970).

Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and Quantum Theory of Synergic Synchrotron-Čerenkov Radiation, Ann. Phys. (NY) 96, No. 2, 303.

Schwinger, J. *Particles, Sources and Fields* Vol. II, (Addison-Wesley, Reading, Mass., 1973).

Sitenko, A. G. *The electromagnetic Fluctuations in Plasma*, (Kharkov University Press, 1965). (in Russian).

Tichonov A. N. and Samarskii, A. A. *The equations of the Mathematical Physics*, (Nauka, Moscow, 1977). (in Russian).

Weinberg, S. (1974). Gauge and global symmetries at high temperature, Phys. Rev. D 9, 3357.