The Mathematical Principle of the Influence of the Change of Space-time Structure on the Gravitational Constant

Zhi Cheng

gzchengzhi@hotmail.com

Abstract: This article attempts to use mathematical methods to solve the influence of the change of space-time structure on the gravitational constant. After adopting a purely mathematical method, it means that if there is a problem with the conclusion, you only need to find the reason in the axiom system proposed in this article. This can make the discussion of the problem more concise.

Keywords: space-time structure; gravitational constant; Schwarzschild radius

1 Introduction

In my last paper [1], I tried to analyze the experimental influence of Jupiter on the measurement of the gravitational constant on the earth's surface. Through specific calculations, it is found that theoretical estimates can significantly improve the results of many gravitational measurement experiments since 2000. However, after reviewing my papers [1~3], I feel that one of the issues is not clear enough. The question is how the gravitational constant will increase when the space is compressed. And why is the degree of space-time compression closely related to gravitational potential? This article attempts to use mathematical reasoning to discuss in depth the influence of the change of space-time structure on the gravitational constant.

Figure 1(a) shows a space-time coordinate, reflecting the situation where space-time is compressed from left to right. The scale on the axis indicates the length of the space unit.

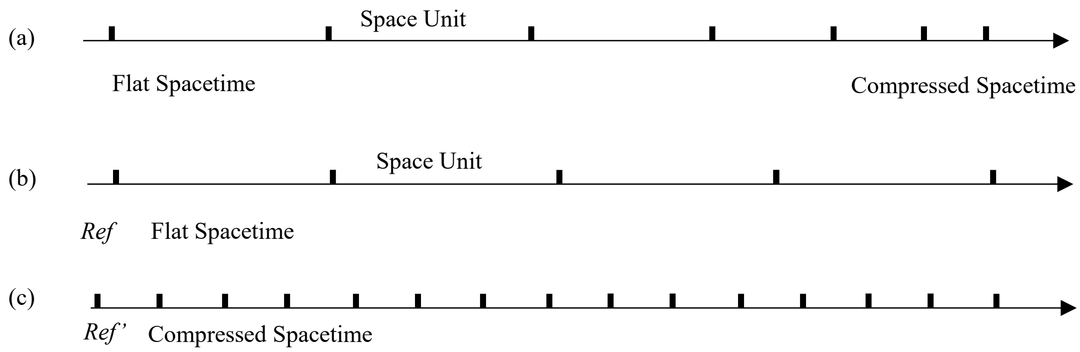

Figure 1. Flat and compressed spacetime

As can be seen from the top coordinate (a) in Figure 1, because space-time is compressed, it means that different spatial scales are displayed on the space-time coordinates. In the compressed space-time range, the space unit is relatively short, while in a relatively flat space-time, the space unit is relatively long. However, it is very inconvenient to deal with the problem of a coordinate system whose space unit length is changing. Therefore, (b) and (c) in Figure 1 introduce two coordinate systems with uniform scales, *Ref* and *Ref'*, respectively.

2 Changes in the length of different spacetime reference systems

Definition 1: Space-time reference system refers to a coordinate system with uniform scale with a specific space unit length as the unit of measurement, here referred to as the reference system.

In Figure 1(a), if we use the relatively flat space unit length on the left as a scale to mark the entire system, then this space-time can be called a flat space-time reference system, which is represented by *Ref* in Figure 1(b). It can be seen that the scale of the *Ref* reference system is uniform, but the scale interval is longer. And if the more severely compressed space unit length on the right is used as a scale to mark the entire system, then this space-time can be called a compressed space-time reference system. It is represented by *Ref'* in Figure 1(c). It can be seen that the scale of the *Ref'* spacetime reference system is relatively small.

Definition 2: The space unit length *I* represents the basic unit used to measure a length in a certain spacetime reference system.

Definition 3: Intrinsic length refers to the length measured in the space-time reference system whose space unit length is 1. The intrinsic space length can be expressed as a multiple of the

space unit length.

Definition 4: The intrinsic length is expressed in bold font. Letters in unbold font indicate the length measured in a certain spacetime reference system. The length measured in a spacetime reference system multiplied by the space unit length of the corresponding reference system is equal to the intrinsic length.

which is

$$\boldsymbol{a} = aI$$

Where $\boldsymbol{a}$ is the intrinsic length, I is the length of the space unit of the reference system, and a is the length measured in the reference system.

With the above definitions of some terms, let's determine the axiom system.

Axiom 1: Space-time is an elastic substance that can be squeezed by mass

This is basically consistent with the assumption of general relativity. In general relativity, the space-time around the mass will be curved due to the existence of mass. And this bending of time and space also means that time and space are compressed.

Axiom 2: The compressed space-time has a shorter space unit length. A completely flat space-time without any mass and energy has an infinite space unit length.

In Figure 1, for a relatively flat space-time, the space unit length is limited. But compared to those compressed space-time, the space unit length is much larger. But if there is a completely flat space-time at infinity, it means that there is no mass or energy in it, so this space-time has no physical meaning. For a space-time without any physical meaning, there is no spacetime unit length. Conversely, if a space-time has a finite spacetime unit length, it means that the space-time can be measured and has physical meaning.

Therefore, the term "flat space-time" is used in this article to mean space-time with a very small amount of mass and energy. And completely flat space-time means space-time without mass and energy at infinity.

According to the above axioms, we can further reason and obtain a series of meaningful theorems.

Theorem 1: If the same length result is measured in different spacetime reference systems, the intrinsic length corresponding to the measured length of the compressed spacetime reference system is shorter.

As shown in Figure 1, if the measured result in any reference frame is a, then the length is observed in the compressed space-time reference frame *Ref'*, and the length can be expressed as:

$$\boldsymbol{a}' = aI'$$

Now switch to the flat space-time reference system *Ref*. Since the unit length of the *Ref* reference system is I, therefore

$$\boldsymbol{a} = aI$$

Since

$$I > I'$$

Then

$$\boldsymbol{a} > \boldsymbol{a}'$$

It shows that although the measurement results of the two reference systems are the same, their intrinsic lengths are different. The intrinsic length of the flat space-time reference frame is longer.

Theorem 2: With the same intrinsic length, the result measured in the compressed space-time reference system is longer than that of the flat space-time reference system

Prove:

In *Ref*

$$\boldsymbol{a} = aI$$

In *Ref'*

$$\boldsymbol{a} = a'I'$$

Since

$$I > I'$$

Then

$$a' > a$$

Corollary 1: The Schwarzschild radius measured by the compressed space-time reference frame is longer

Consider the Schwarzschild radius solved by Einstein's field equation as an intrinsic length $\boldsymbol{r_s}$.

If we observe $\boldsymbol{r_s}$ in a relatively flat spacetime reference frame *Ref*

We can get

$$\boldsymbol{r_s} = r_s I$$

Now we switch to the compressed space-time reference system *Ref'* to observe $\boldsymbol{r_s}$.

We can get

$$\boldsymbol{r_s} = r_s' I'$$

Since the unit length of the *Ref* reference system is larger, it means

$$I > I'$$

Then

$$r_s' > r_s$$

3 Mass in different spacetime reference frames

Theorem 3: According to the requirements of conservation of energy, the rest masses observed in any space-time reference frame are equal.

Prove:

According to the relativistic mass-energy formula

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

Although the mass is located in a non-inertial frame of reference (such as a compressed space-time frame of reference), it is always possible to use an appropriate method, such as changing the inertial frame of reference so that p=0

therefore:

$$E = \sqrt{m_0^2 c^4} = m_0 c^2$$

It means that the static mass of the object will not change regardless of whether it is observed from the *Ref* or *Ref'* reference system.

4 The relationship between space unit length and gravitational potential

Conjecture: The length of space unit is inversely proportional to the gravitational potential

Regarding the relationship between space unit length and gravity, if space-time is regarded as an elastic substance, the squeezing of space-time by mass will compress the surrounding space-time. This space-time compression effect will cause a change in the spacetime unit length. According to the knowledge of elasticity, the displacement of space-time can be solved, and then the specific situation of space-time compression can be calculated, and then the relationship between space unit length and gravity can be calculated from this.

But such calculations will face some serious problems. For example, we don’t know if space-time is an elastic substance, what is its elastic modulus? In addition, space-time is four-dimensional, and solving with three-dimensional elasticity knowledge will also face the problem of incorrect results caused by deformation in the time dimension.

Here we understand the relationship between gravity and space unit length from Axiom 2 proposed in this article.

According to Axiom 2, the space unit length of a completely flat space-time at infinity is infinite. Therefore, the relationship between unit length and gravity can be expressed in many forms, including: reciprocal relationship, inverse square relationship, multiple inverse square relationship, exponential relationship, etc.

The simplest one is the reciprocal relationship, namely:

$$g = \frac{a}{r}$$

Here g is used to represent a physical quantity related to gravity. It can be force, field strength, gravitational potential, etc. In addition, considering that the magnitude of gravity is usually proportional to mass, the above formula can also be expressed as:

$$g = \frac{bM}{r}$$

If we let

$$b = -G$$

Then the above formula becomes

$$g = -\frac{GM}{r}$$

This is actually the formula for calculating gravitational potential.

If the above relationship adopts the reciprocal relationship of multiple powers, it can be expressed as:

$$g = -\frac{GM}{r^{\alpha}}$$

If expressed in exponential form, then

$$g = -GMe^{-r}$$

This article adopts the reciprocal relationship, that is, the gravitational potential is used to express the magnitude of gravity.

In this way, the relationship between space unit length and distance is proportional, and inversely proportional to gravitational potential. which is:

$$I = kr = -\frac{kGM}{V}$$

5 Gravitational constants in different spacetime reference frames

Theorem 4: The measured value of the gravitational constant of the compressed space-time reference frame is larger

Proof: For an object with a mass of *m*, the Schwarzschild solution can be obtained by using the Einstein field equation. This solution can be regarded as the intrinsic length of the Schwarzschild radius, namely

$$\boldsymbol{r_s} = \frac{\boldsymbol{Gm}}{c^2}$$

According to Theorem 2, the Schwarzschild radius measured in a flat space-time reference frame is expressed as

$$r_s = \frac{Gm}{c^2}$$

The Schwarzschild radius measured in the compressed space-time reference frame is expressed as

$$r_s{}' = \frac{G'm}{c^2}$$

According to Theorem 3, no matter which space-time reference frame is in, the static mass of the object will not change. And c is a constant, so the only thing that can be changed on the right side of the equation is the gravitational constant. Therefore, the gravitational constant of the compressed space-time reference frame is represented by G'.

considering

$$r_s' > r_s$$

Then we can get

$$G' > G$$

Theorem 5: The gravitational constant in different spacetime reference frames is proportional to the gravitational potential at that place

prove

$$\frac{G'}{G} = \frac{r_s'}{r_s} = \frac{\boldsymbol{r_s} I}{\boldsymbol{r_s} I'} = \frac{V'}{V}$$

6 Conclusion

This article attempts to prove mathematically the influence of space-time compression on the gravitational constant. This allows us to have a more concise understanding of the changing law of the gravitational constant. Of course, mathematics only provides us with a tool to make our analysis process more logical. Therefore, this analysis ultimately needs to rely on the verification of experimental data.

The conclusion of this paper relies on two axioms, namely, space-time can be compressed, and space-time compressed, the space unit length used to measure the length will be shorter, and the space unit length of completely flat space-time at infinity is infinite.

In order to analyze the influence of space-time compression on the gravitational constant without relying on the elastic modulus of the elastic material that constitutes space-time, this article has a conjecture that the space unit length is inversely proportional to the gravitational potential. Such a conjecture can make the calculation process very concise. From the specific analysis of some experimental data[1], this assumption is quite reasonable.

References

[1] Cheng, Z. (2020). A More Detailed Calculation of Jupiter's Influence on the Measurement of the Gravitational Constant. https://vixra.org/abs/2008.0035

[2] Cheng, Z. (2020). The Influence of the Spacetime Compression on the Gravitational Constant. https://vixra.org/abs/2008.0006

[3] Cheng, Z. (2020). The Influence of Jupiter's Orbit on the Measurement of Gravitational Constant. https://vixra.org/abs/2007.0142

时空结构变化对引力常数影响的数学原理

程智

gzchengzhi@hotmail.com

摘要：本文尝试用数学方法来解决时空结构的变化对引力常数的影响。采用了纯数学的方法之后，意味着如果结论存在问题，只需要在本文提出的公理系统寻找原因即可。这可以使得问题的讨论变得更加简洁。

关键词：时空结构；引力常数；史瓦西半径

1 引言

在我的上一篇论文中[1]，我尝试分析了木星对地球表面测量引力常数的实验影响。通过具体的计算发现理论的估算数值能够显著改善自 2000 年以来多次引力测量实验的结果。不过在回顾了我的这几篇论文之后[1~3]，我感到其中有一个问题阐述的不够清楚。这个问题就是为何时空被压缩之后，会提高引力常数的数值。而时空压缩的程度为何跟引力势能有密切的联系。本文尝试用数学推理的方法来深入探讨一下时空结构的变化对引力常数的影响。

图 1(a)显示了一个时空坐标，反映了从左到右时空被压缩的情况。坐标轴上面的刻度表示了单位空间长度。

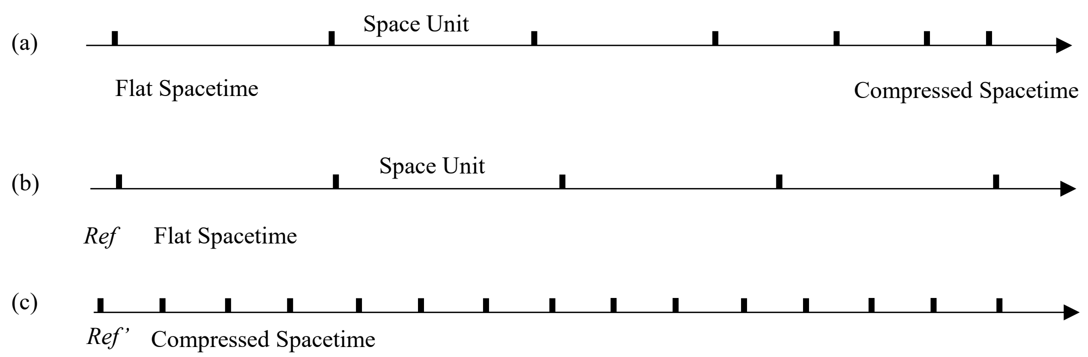

Figure 1. Flat and compressed spacetime

从图 1 最上面的一个坐标(a)可以看出，由于时空被压缩，意味着在时空坐标上面显示出了不同的空间刻度。被压缩的时空范围，空间单位比较短，而在较为平坦的时空中，空间单位比较长。但是单位空间长度是变化的坐标系处理问题非常不方便，因此在图 1 中的(b)和(c)分别引入了两个刻度均匀的坐标系 *Ref* 和 *Ref'*.

2 不同时空参照系长度的变化

定义 1: 时空参照系指的是以某个特定的单位空间长度为度量单位的一个刻度均匀的坐标系统，这里简称参照系。

在图 1(a)中，如果我们以左边比较平坦的单位空间长度作为刻度来标记整个系统，则可以将该时空称作平坦时空参照系，在图 1(b)中用 *Ref* 表示。可以看出，*Ref* 参照系的刻度均匀，但是刻度间隔较长。而如果以右边压缩的比较严重的空间单位长度作为刻度来标记整个系统，则可以将该时空称作压缩时空参照系。在图 1(c)中用 *Ref'* 表示。可以看出，Ref'时空参照系的刻度比较小。

定义 2：单位空间长度 *l* 表示在某个时空参照系中用来测量一个长度的基本单位。

定义 3: 固有长度指的单位空间长度为 1 的时空参照系中测量出来的长度。固有空间长度可以表示为单位空间长度的倍数。

定义 4: 固有长度用加粗字体表示。不加粗字体字母表示在某个时空参照系中测量到的长度，在一个时空参照系中测量到的长度乘以对应参照系单位空间长度等于固有长度。

即

$$\boldsymbol{a} = al$$

其中 $\boldsymbol{a}$ 为固有长度，*l* 为参照系单位空间长度，*a* 为该参照系中测量到的长度。

有了以上对一些术语的定义，下面我们再来确定其中的公理系统。

公理 1：时空是一种弹性物质，可以被质量挤压

这跟广义相对论的假设是基本一致的。在广义相对论中，由于质量的存在将导致质量周围的时空出现弯曲。而这种时空的弯曲也意味着时空被压缩了。

公理 2：被压缩的时空，其单位空间长度更短。没有任何质量和能量存在的完全平坦时空，其单位空间长度为无穷大。

在图 1 中，对于相对平坦的时空，其单位空间长度是有限的。但是相对于那些被压缩时空，其单位空间长度要大很多。但是如果对于无穷远处一个完全平坦的时空，意味着其中没有任何质量和能量的存在，因此这样的时空也就没有任何的物理意义。对于一个没有任何物理意义的时空，是不存在单位空间和时间长度的。反之，如果一个时空存在有限的单位空间和时间长度，则意味着这个时空是可以测量的，也就有了物理意义。

因此本文用"平坦时空"一词表示其中存在的质量和能量非常少量的时空。而完全平坦时空表示无穷远处完全没有质量和能量存在的时空。

根据上述公理，我们可以进一步推理，获得一系列有意义的定理。

定理 1：如果在不同时空参照系测量到相同的长度结果，则压缩时空参照系所测量长度对应的固有长度更短。

如图 1，如果在任意一个参照系中测量的结果是 *a*，则该长度在压缩时空参照系 *Ref'* 中观察，该长度可以表示为：

$$\boldsymbol{a}' = aI'$$

现在切换到平坦时空参照系 *Ref*，由于 *Ref* 参照系的单位长度为 *I*，因此

$$\boldsymbol{a} = aI$$

由于

$$I > I'$$

因此

$$\boldsymbol{a} > \boldsymbol{a}'$$

说明虽然两个参照系测量的结果相同，但是它们的固有长度不同。平坦时空参照系的固有长度更长。

定理 2：相同的固有长度，在压缩时空参照系中测量的结果要比平坦时空参照系更长

证明：

在 *Ref* 参照系有：

$$\boldsymbol{a} = aI$$

在 *Ref'* 参照系有

$$\boldsymbol{a} = a'I'$$

由于

$$I > I'$$

因此

$$a' > a$$

推论 1：压缩时空参照系测量出来的史瓦西半径更长

将爱因斯坦场方程求解出来的史瓦西半径看作是一个固有长度 $\boldsymbol{r_s}$.

如果在比较平坦的时空参照系 *Ref* 观察 $\boldsymbol{r_s}$

可以得到：

$$\boldsymbol{r_s} = r_s I$$

现在我们切换到压缩时空参照系 *Ref'* 来观察 $\boldsymbol{r_s}$.

可以得到：

$$\boldsymbol{r_s} = r_s' I'$$

由于 *Ref* 参照系空间单位长度更大，意味着：

$$I > I'$$

因此：

$$r_s' > r_s$$

3 不同时空参照系中的质量

定理 3：按照能量守恒的要求，在任何时空参照系观察到的静止质量都是相等的。

证明：

按照相对论质能公式

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

虽然该质量位于非惯性参照系（比如某个压缩时空参照系），但总是能够采用适当的方式，比如变换惯性参照系令 p=0

因此：

$$E = \sqrt{m_0^2 c^4} = m_0 c^2$$

说明无论从 *Ref* 还是 *Ref'* 参照系观察，物体的静止质量都是不会改变的。

4 单位空间长度与引力势能的关系

猜想：单位空间长度跟引力势成反比

对于单位空间长度与引力之间的关系，如果将时空看作是一种弹性物质，则质量对时空的挤压将压缩周围的时空。这种时空压缩效应将引起单位时空长度的变化。可以根据弹性力学的知识求解出时空的位移，进而计算出时空被压缩的具体情况，然后由此来推算单位空间长度与引力之间的关系。

但是这样的计算会面临一些比较严重的问题，比如我们不知道时空如果作为一种弹性物质，其弹性模量是多少？另外时空是四维的，用三维的弹性力学知识来进行求解，也会面临时间维度的变形而导致的结果不正确的问题。

这里从本文提出的**公理 2** 来理解引力和单位空间长度之间的关系。

按照公理 2，无穷远处的完全平坦时空其单位空间长度为无穷大。因此可以将单位长度与引力之间的关系表示为多种形式，包括：倒数关系、平方反比关系、多次方反比关系、指数关系等。

其中最简单的就是倒数关系，即：

$$g = \frac{a}{r}$$

这里用 g 来表示某个跟引力有关的物理量。可以是力，也可以是引力势等。另外考虑到引力的大小通常跟质量成正比，因此上述公式还可以表示为：

$$g = \frac{bM}{r}$$

如果让

$$b = -G$$

则上述公式变为：

$$g = -\frac{GM}{r}$$

这实际上就是引力势的计算公式。

如果上述关系采用多次方的倒数关系，则可以表示为：

$$g = -\frac{GM}{r^{\alpha}}$$

如果表示为指数形式，则

$$g = -GMe^{-r}$$

本文采用倒数关系，即用引力势来表示引力的大小。

这样单位空间长度跟距离之间的关系就是正比关系，跟引力势成反比。即：

$$I = kr = -\frac{kGM}{V}$$

5 不同时空参照系中的引力常数

定理 4：压缩时空参照系的引力常数测量值更大

证明：对于质量为 m 的物体，利用爱因斯坦场方程，可以获得史瓦西解。这个解可以被看作是史瓦西半径的固有长度，即：

$$\boldsymbol{r_s} = \frac{\boldsymbol{G}m}{c^2}$$

根据**定理 2**，在平坦时空参照系测量的史瓦西半径的表达式为

$$r_s = \frac{Gm}{c^2}$$

在压缩时空参照系中测量的史瓦西半径的表达式为

$$r_s{}' = \frac{G'm}{c^2}$$

再根据**定理 3**，无论在哪个时空参照系，物体的静止质量都是不会改变的。而 c 是常数，因此等式右边唯一能够改变的就是引力常数。故压缩时空参照系的引力常数用 G'表示。

考虑到：

$$r_s' > r_s$$

因此可以得到：

$$G' > G$$

定理 5：不同时空参照系中的引力常数跟该处引力势成正比

证明：

$$\frac{G'}{G}=\frac{r_s'}{r_s}=\frac{\boldsymbol{r_s}I}{\boldsymbol{r_s}I'}=\frac{V'}{V}$$

6 结论

本文尝试着从数学上来证明时空的压缩对于引力常数的影响。这样可以让我们对于引力常数的变化规律有一个比较简洁的理解。当然数学只是给我们提供了一种工具，使我们的分析过程更有逻辑性。因此这种分析最终还是需要依赖实验数据的验证。

本文的结论依赖两个公理，即分别是时空可以被压缩，以及被压缩的时空，其用来度量长度的单位空间长度将更短，且无穷远处完全平坦的时空的单位空间长度为无穷大。

为了能够在不需要依赖构成时空这种弹性物质的弹性模量等参数的情况下，分析时空压缩对引力常数的影响，本文有一个猜想，即单位空间长度跟引力势成反比。这样的猜想可以让计算过程变得非常简洁。从对一些实验数据的具体分析情况来看[1]，这种假设还是比较合理的。

参考文献

[1] Cheng, Z. (2020). A More Detailed Calculation of Jupiter's Influence on the Measurement of the Gravitational Constant. https://vixra.org/abs/2008.0035

[2] Cheng, Z. (2020). The Influence of the Spacetime Compression on the Gravitational Constant. https://vixra.org/abs/2008.0006

[3] Cheng, Z. (2020). The Influence of Jupiter's Orbit on the Measurement of Gravitational Constant. https://vixra.org/abs/2007.0142