Einstein's theory resulting from a logical mistake by Lorentz

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Abstract

It is generally held that Lorentz transformation is superior to Galilean transformation. However, this paper reveals that the Lorentz's application of both transformations to derive the electromagnetic wave equation for two inertial reference frames is logically flawed. This logical mistake makes the superiority of Lorentz transformation questionable. The paper also demonstrates that both Lorentz and Galilean transformations can generate standard equations for electromagnetic waves, while both cannot keep the transformed Maxwell's equations consistent. Since the relativity theory is based on the belief that the Lorentz transformation can keep physics law in the same form for different reference frames, the findings of this paper have important implications: Either the relativity theory is not one hundred percent correct or our understanding of the theory needs updating in light of the new knowledge.

1 Introduction

Lorentz (1892)[1] found that Galilean transformation could not produce a proper wave equation for light and could not keep the Maxwell equations in the same form, so he proposed an additional transformation formula - the Lorentz transformation. Einstein regarded this transformation as a general one which keeps the general law of nature invariant with different reference frames. Based on the Lorentz transformation, Einstein developed the special theory of relativity. However, this paper shows that the Lorentz transformation is a by-product of a hidden logical mistake.

2 Seemingly correct wave equation obtained by Lorentz

The following is the procedure used by Lorentz (1892) to apply Galilean transformation to transform the standard electromagnetic wave equation. The general wave equation derived from Maxwell equations can be written as:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)K = P$$

Here only the x direction of the electromagnetic wave is displayed for simplicity. K is some wave functions, P is a function related to electromagnetic source. For electromagnetic waves, K represents magnetic field or electrical field.
Since Lorentz found that, only for charge free space, can his transformation formula lead to the same form of electromagnetic field equation for different reference frames (Miller, 1981, p35)[2], for simplicity, we consider only the standard source-free electromagnetic wave function, which does not affect the exposure of Lorentz's logical mistake.

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)K = 0 \tag{1}$$

Galilean transformation can be expressed as:

$$x_r = x - vt,$$

$$t_r = t$$

where x and t are distance and time, respectively, in the old reference frame. x_r and t_r are distance and time in the new reference frame.
The task is to find the following standard wave equation in the new reference frame:

$$\left(\frac{\partial^2}{\partial x_r^2} - \frac{1}{u^2}\frac{\partial^2}{\partial t_r^2}\right)K = 0 \tag{2}$$

Using Galilean transformation formula we can derive:

$$\frac{\partial t_r}{\partial x} = \frac{\partial t}{\partial x_r} = 0 \tag{3}$$

$$\frac{\partial x}{\partial t_r} = \frac{\partial x}{\partial t} = v \tag{4}$$

$$\frac{\partial x_r}{\partial t} = -v \tag{5}$$

$$\frac{\partial t}{\partial t_r} = \frac{\partial t_r}{\partial t} = \frac{\partial x}{\partial x_r} = \frac{\partial x_r}{\partial x} = 1 \tag{6}$$

Equation (3) indicates that time does not depend on the position of the object in any reference frame while equations (4) and (5) show that position depends on the speed of the frame as well as the time passed. This manifests the feature of Galilean transformation - the separation and independence of space and time. Here the reader should be reminded that, only for mutual dependence of x and t, we can have $\frac{\partial t_r}{\partial x} = /\frac{\partial x}{\partial t_r}$. As a result, equations (3)-(5) look contradictory but, in fact, are an alternative expression of Galileans transformation.

Based on equations (3)-(6), we can calculate the components in the wave equation (1):

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x_r}\frac{\partial x_r}{\partial x} + \frac{\partial}{\partial t_r}\frac{\partial t_r}{\partial x} = \frac{\partial}{\partial x_r} \tag{7}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x_r}\frac{\partial x_r}{\partial t} + \frac{\partial}{\partial t_r}\frac{\partial t_r}{\partial t} = -v\frac{\partial}{\partial x_r} + \frac{\partial}{\partial t_r} \tag{8}$$

Plugging equations (7) and (8) into the standard wave equation (1), we have:

$$\left(\frac{\partial^2}{\partial x_r^2} - \frac{1}{c^2}\big(\frac{\partial}{\partial t_r} - v\frac{\partial^2}{\partial x_r}\big)^2\right)K = 0 \tag{9}$$

This clearly is not a proper wave equation, so Lorentz thought the Galilean transformation is inappropriate for electromagnetic waves and proposed additional transform formulas to obtain a proper wave equation. However, there is a hidden mistake in the above attempt to transform the wave equation. Equation (2) requires that the function K is partial differentiated with respect to variable x_r and t_r. On the other hand, equation (9) results from differentiating K with respect to variable x and t, even though it seems that the equation is about variable x_r and t_r. As a result, these two equations mean different things and thus are not comparable. Consequently, the equation (9) derived using Lorentz's approach is valid, but it is not the required wave equation in the new reference frame.

To examine if the Galilean transformation can lead to a proper wave equation, we need to calculate the partial derivatives with respect to new variables x_r and t_r. Using equations (3) -(6), we have:

$$\frac{\partial}{\partial x_r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial x_r} + \frac{\partial}{\partial t}\frac{\partial t}{\partial x_r} = \frac{\partial}{\partial x} \tag{10}$$

$$\frac{\partial}{\partial t_r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial t_r} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_r} = v\frac{\partial}{\partial x} + \frac{\partial}{\partial t} \tag{11}$$

A consistency check shows that the above equations can be obtained by rearranging equations (7) and (8). Plugging equations (10) and (11) into equation (2), we obtain an improper wave equation similar to equation (9). It seems that we run into the same problem as before. However, with the initial standard wave equation (1), we can have:

$$\frac{\partial}{\partial x} = \pm\frac{1}{c}\frac{\partial}{\partial t} \tag{12}$$

The selection of the sign $\pm$ depends on the direction of v in the transformation formula. Plugging this into equation (11), we have:

$$\frac{\partial}{\partial t_r} = \pm\frac{v}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial t} = \left(1 \pm \frac{v}{c}\right)\frac{\partial}{\partial t} = \frac{c \pm v}{c}\frac{\partial}{\partial t} \tag{13}$$

letting $u = c \pm v$, from equations (10) and (13) we have:

$$\frac{\partial^2}{\partial x_r^2} - \frac{1}{u^2}\frac{\partial^2}{\partial t_r^2} = \frac{\partial^2}{\partial x^2} - \frac{1}{u^2}\frac{u^2}{c^2}\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = 0 \tag{14}$$

Here we see that the Galilean transformation can work perfectly to obtain the standard wave equation in the new reference frame. A simple example can drive this point home. Apply the Galilean transformation formulas to the following wave function:

$$K = Asin[\omega(t - x/c)]$$

We have:

$$K = Asin\{\omega[t_r - (x_r + vt_r)/c]\} = Asin\{\omega[(c-v)/c)][t_r - x_r/(c-v)]\}$$

Letting $u = c - v$, it is easy to verify that the transformed wave function satisfies the usual wave equation (2):

$$\left(\frac{\partial^2}{\partial x_r^2} - \frac{1}{(c-v)^2}\frac{\partial^2}{\partial t_r^2}\right)K = 0$$

Meanwhile, one can also verify that the transformed wave-function satisfies the equation (9). This proves that equation (9) derived by Lorentz holds, but it is a different one from what Lorentz was aiming for. As a result, the improper form of the equation derived by Lorentz does not mean the proper form of wave equation cannot be obtained from Galilean transformation.

One may wonder, if Lorentz approach is invalid, why could he derive a proper wave equation from the proposed Lorentz transformation? The answer is that, due to the same logical mistake, the proper form of wave equation derived from the Lorentz transformation formula is valid but is not the required wave equation for the transformed electromagnetic waves. An evidence comes from the inconsistency (or inaccuracy noticed by Lorentz himself) about the resultant speed of light in the new reference frame (Miller, 1981, p28). From the Lorentz transformation formula, one can easily verify that the speed of light c is unchanged after the transformation. However, Lorentz noticed that the speed of light from his proper wave equation is $\sqrt{c^2 - v^2}$, which is less than c.

Using the correct approach, we can find the required wave equation after Lorentz transformation.

The formulas for Lorentz transformation are:

$$x_r = \gamma(x - vt), y_r = y, z_r = z, t_r = \gamma(t - \frac{v}{c^2}x), \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Based on these equations we can derive:

$$\frac{\partial x_r}{\partial x} = \frac{\partial t_r}{\partial t} = \gamma$$

$$\frac{\partial x}{\partial x_r} = \frac{\partial t}{\partial t_r} = \frac{1}{\gamma}$$

$$\frac{\partial x_r}{\partial t} = -\gamma v$$

$$\frac{\partial t}{\partial x_r} = -\frac{1}{\gamma v}$$

$$\frac{\partial t_r}{\partial x} = -\frac{\gamma v}{c^2}$$

$$\frac{\partial x}{\partial t_r} = -\frac{c^2}{\gamma v}$$

It is of interest to note that Lorentz transformation satisfies $\frac{\partial a}{\partial b} = 1/\frac{\partial b}{\partial a}$. Using the above partial derivatives and equation (12), we can derive the following components of transformed wave equation:

$$\frac{\partial}{\partial x_r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial x_r} + \frac{\partial}{\partial y}\frac{\partial y}{\partial x_r} + \frac{\partial}{\partial z}\frac{\partial z}{\partial x_r} + \frac{\partial}{\partial t}\frac{\partial t}{\partial x_r} = \frac{\partial}{\partial x}\frac{1}{\gamma} + \frac{\partial}{\partial t}\frac{-1}{\gamma v} = \frac{\partial}{\partial x}\frac{1}{\gamma} + \frac{\partial}{\partial x}\frac{\pm c}{\gamma v} = \frac{\partial}{\partial x}\frac{1}{\gamma}(1 \pm \frac{c}{v})$$

$$\frac{\partial}{\partial t_r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial t_r} + \frac{\partial}{\partial y}\frac{\partial y}{\partial t_r} + \frac{\partial}{\partial z}\frac{\partial z}{\partial t_r} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_r} = \frac{\partial}{\partial x}\frac{-c^2}{\gamma v} + \frac{\partial}{\partial t}\frac{1}{\gamma} = \pm\frac{\partial}{\partial t}\frac{c}{\gamma v} + \frac{\partial}{\partial t}\frac{1}{\gamma} = \frac{\partial}{\partial t}\frac{1}{\gamma}(1 \pm \frac{c}{v})$$

It is then easy to verify that the wave equation after Lorentz transformation is a proper one:

$$\frac{\partial^2}{\partial x_r^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t_r^2} = \frac{1}{\gamma^2}(1 \pm \frac{c}{v})^2(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}) = 0$$

This wave equation shows a speed of light of c, which is consistent with the setting of Lorentz transformation.

3 Incompatibility of Lorentz transformation with Maxwell equations

A significant effort of physicists is paid to maintain the belief that different reference frames are equivalent—otherwise stated, that the natural laws are the same despite the reference frames chosen. For Lorentz, Einstein, and many others, the Lorentz transformation is an important relation to keep Maxwell equations invariant for all reference frames. However, due to the same logical mistake explained in the previous section, the invariant Maxwell equations are only an illusion of equivalence of different reference frames.
The usual way of using Lorentz's approach to derive invariant Maxwell equations can be demonstrated as follows. Based on Lorentz transformation relations, one can derive the following partial derivatives:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x_r}\frac{\partial x_r}{\partial x} + \frac{\partial}{\partial y_r}\frac{\partial y_r}{\partial x} + \frac{\partial}{\partial z_r}\frac{\partial z_r}{\partial x} + \frac{\partial}{\partial t_r}\frac{\partial t_r}{\partial x} = \gamma(\frac{\partial}{\partial x_r} - \frac{v}{c^2}\frac{\partial}{\partial t_r}) \qquad (15)$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y_r} \tag{16}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z_r} \tag{17}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x_r}\frac{\partial x_r}{\partial t} + \frac{\partial}{\partial y_r}\frac{\partial y_r}{\partial t} + \frac{\partial}{\partial z_r}\frac{\partial z_r}{\partial t} + \frac{\partial}{\partial t_r}\frac{\partial t_r}{\partial t} = \gamma(\frac{\partial}{\partial t_r} - v\frac{\partial}{\partial x_r}) \tag{18}$$

The task is then to apply these partial derivatives to standard Maxwell equations, for example,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

The expanded form of the above equation on the y axis is:

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \tag{19}$$

Plugging into equation (19) the partial derivatives in equations (15) -(18), we have:

$$\frac{\partial E_x}{\partial z_r} - \gamma(\frac{\partial E_z}{\partial x_r} - \frac{v}{c^2}\frac{\partial E_z}{\partial t}) = -\gamma(\frac{\partial B_y}{\partial t_r} - v\frac{\partial B_y}{\partial x_r})$$

Re-arranging the above equation, we can obtain:

$$\frac{\partial E_x}{\partial z_r} - \frac{\partial}{\partial x_r}\gamma(E_z + vB_y) = -\frac{\partial}{\partial t_r}\gamma(B_y + \frac{v}{c^2}E_z) \tag{20}$$

On the other hand, the required Maxwell equation after the Lorentz transformation should be written as:

$$\frac{\partial E'_x}{\partial z_r} - \frac{\partial E'_z}{\partial x_r} = -\frac{\partial B'_y}{\partial t_r} \tag{21}$$

Comparing equations (20) and (21), one makes a claim that Maxwell equation has the same form if:

$$E'_x = E_x$$

$$E'_z = \gamma(E_z + vB_y)$$

$$B'_y = \gamma(B_y + \frac{v}{c^2}E_z)$$

This approach is invalid for two reasons. First, the derived equation is based on the partial derivatives with respect to old coordinates (x,y,z,t), so it does not fit the definition of the transformed Maxwell equation. Putting it differently, although the equation uses the new coordinates (x_r, y_r, z_r, t_r), it is not the result of partial derivatives with respect to new coordinates, so it is incomparable to the required Maxwell equation after Lorentz transformation. Second, if the Maxwell equation works well after transformation, the functions for E_x, E_z and B_y should be transformed directly through the change of coordinates. As such, the functions should be essentially the same except that the coordinates have changed. Naming a set of new functions may generate equations that appear similar to the Maxwell equations in the old reference frame, but it does not make the transformed Maxwell equations consistent.

Resnick (1968)[3] compared the electrical and magnetic components of the transformed Maxwell equations with the results obtained by applying the Lorentz transformation to Lorentz force, finding that they agree with each other. However, this agreement is not surprising. Because Lorentz force is implicitly embodied in Maxwell equations, applying Lorentz transformation to Lorentz force is essentially the same as applying it to Maxwell equations. As a result, the consistency of two methods cannot varify the validity of the transformed Maxwell equations.

We can use the correct approach to verify if the Maxwell equations work well under Lorentz transformation. Using the Lorentz transformation formula, we can obtain:

$$\frac{\partial}{\partial x_r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial x_r} + \frac{\partial}{\partial y}\frac{\partial y}{\partial x_r} + \frac{\partial}{\partial z}\frac{\partial z}{\partial x_r} + \frac{\partial}{\partial t}\frac{\partial t}{\partial x_r} = \frac{\partial}{\partial x}\frac{1}{\gamma} - \frac{\partial}{\partial t}\frac{1}{\gamma v}$$

$$\frac{\partial}{\partial y_r} = \frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial z_r} = \frac{\partial}{\partial z}$$

$$\frac{\partial}{\partial t_r} = \frac{\partial}{\partial x}\frac{\partial x}{\partial t_r} + \frac{\partial}{\partial y}\frac{\partial y}{\partial t_r} + \frac{\partial}{\partial z}\frac{\partial z}{\partial t_r} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_r} = -\frac{\partial}{\partial x}\frac{c^2}{\gamma v} + \frac{\partial}{\partial t}\frac{1}{\gamma}$$

Plugging them into both sides of equation (21), the Maxwell equation after the Lorentz transformation, we have for the left hand side:

$$\frac{\partial E_x}{\partial z_r} - \frac{\partial E_z}{\partial x_r} = \frac{\partial E_x}{\partial z} - (\frac{\partial E_z}{\partial x}\frac{1}{\gamma} - \frac{\partial E_z}{\partial t}\frac{1}{\gamma v}) \tag{22}$$

and for the right hand side:

$$-\frac{\partial B_y}{\partial t_r} = \frac{\partial B_y}{\partial x}\frac{1}{\gamma}\frac{c^2}{v} - \frac{\partial B_y}{\partial t}\frac{1}{\gamma} \tag{23}$$

Since equation (21) must hold if Maxwell equation is consistent after transformation, we equalize the right-hand side of equations (22) and (23). This necessitates:

$$\gamma\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \frac{\partial E_z}{\partial t}\frac{1}{v} = \frac{\partial B_y}{\partial x}\frac{c^2}{v} - \frac{\partial B_y}{\partial t}$$

This equation is different from the original Maxwell equation. Deducting the common items in the original equation (19), we have:

$$(\gamma - 1)\frac{\partial E_x}{\partial z} + \frac{\partial E_z}{\partial t}\frac{1}{v} = \frac{\partial B_y}{\partial x}\frac{c^2}{v}$$

This is the extra condition required in order to maintain the Maxwell equation valid after the Lorentz transformation. This extra condition indicates that, just like the Galilean transformation, the Lorentz transformation is also not compatible with the Maxwell equations. In other words, the Maxwell equations will take on a different form after the Lorentz transformation.

4 Conclusion

Lorentz's logical mistake in transforming electromagnetic wave functions between different reference frames has not been detected for more than 100 years. This mistake has caused considerable misunderstanding and confusion in physics, especially in the area of electrodynamics. The positive side of the Lorentz mistake is that Einstein concluded that Lorentz transformation is superior to Galilean transformation. Based on this, Einstein derived relativity theory which had profound impact on the physics discipline.
Lorentz's mistake uncovered in this paper does not necessarily invalidate Einstein's theory because the Lorentz transformation is also valid and gives the constant speed of light. However, the identification of the mistake re-establishes the validity of Galilean transformation in the case of waves and light, and proves that even the Lorentz transformation cannot make Maxwell equations work consistently under different reference frames. Since Lorentz transformation is the foundation of the relativity theory, the limitation of the former necessitates the limitation of the latter. In light of this new knowledge, we should re-assess the relativity theory.

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