Analysis on the non linearity of Time

Priyam Das

Amity University

M.Sc Physics final year

email id: das.priyam888@gmail.com

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Abstract

In this paper I have used the Schwarzschild solution for Einstein's field equation to find the change of proper time in a gravitational field with respect to time measured by gravitationally unaffected stationary clock. The equations tells us about the nature of temporal flow or temporal velocity for any body considering gravitational effect.I have further treated the solution to find a rate of change of the temporal velocity which signifies a temporal acceleration for any varying mass body. On the second phase I have used Einstein's time dilation equation of special relativity to find the temporal flow of a particle moving with certain speed, neglecting the gravitational effect of the particle. Further calculations on the nature of temporal velocity reveals us a temporal acceleration for a particle with different velocities at different instant of time. I have also found equations relating to the nature of time in extreme cases of the universe like black holes and particles like photons.

0.1 Acknowledgement

I would like to thank my parents for supporting me and encouraging me in writing this paper and Dr Swapan Ghosh as well as Mr Kripasindhu Kumar who always helped to dream bigger and extend my imagination into the next level. They always pushed me and encouraged me to do this kind of work. I would also like to thank my professor Dr. Goutam Chakraborty for helping me in editing this paper.I would also like to acknowledge my parents who have always supported me in my every step of this scientific journey

0.2 Introduction

Time is a very old concept introduced by early humans, we can trace its path to the era of Mayan civillization and other cultures. It is one of the fundamental quantities of physics and it is used to measure other quantities like velocity, acceleration, torque, etc. Although the origin of time is far greater than the age of Earth itself. It came into origin after Big Bang[1] and is said to flow[2] uninterruptedly in one direction. Early concept of time was that it is something constant, but Minkowski and Einstein mainly are the scientists that showed time and space are interconnected.The aim is to get a deeper insight into the true nature of the temporal dimension. The time as we know has an arrow according to many physicists and to some it is an illusion of our mind.[3] But in this paper, it is discussed about the non linear flow of time at certain gravity wells and relativistic velocities. I will prove theoretiically that what we comprehend about time and illusions and arrows is not entirely true. As we know illusions don't change or depend on particular spatial points, but the flow of time depends on curvature.[4].

According to Newton's first law-" An object will remain in its state of uniform motion along a straight line or in its state of rest until and unless an external force is applied on it". So let's think of us as objects that are moving in the four dimensional space-time. We can clearly see the effect of time on us(the aging process). So the concept of time force comes from here, that some force must act on us that pushes us in the temporal dimension. Moreover, people in different planets with different masses will age differently. This is a clear indication of non linear flow of time. First I will deduce a theoretical equation about the flow of time, taking in account how proper time will change at certain conditions of spac-time curvatures with respect to a stationary clock free from any gravitational effect. then we will consider bodies with variable mass. As mass will change, so the space-time curvature caused due to the body will change according to Einstein's general theory of relativity. This in turn shows us that the time dilation with respect to a stationary clock free from any gravitational effect will also varry for different instants. So as I have considered the change in proper time with respect to time measured by a gravitationally unafected clock as 'temporal velocity', therefore for the second case, the change in time dilation with respect to the time measured by the gravitationally unaffected stationary clock is termed as 'temporal acceleration' in analogy to spatial acceleration. As already discussed the concept of 'Time force', this is its origin. The equations derived by me tells us about the temporal velocity and acceleration for any body in the universe, and applying certain special conditions we can get an insight about the nature of time inside event horizons.

0.3 On the Schwarzschild time dilation

To find the rate of change of proper time with respect to time measured by a stationary clock unaffected from any gravitational time dilation, consider the Schwarzschild solution of Einstein field equation and differentiating with the time coordinate.

The Minkowski metric [5][6] is given by

$$[g_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The coordinates are

x =(x^0,x^1,x^2,x^3) =(ct , r)

Taking differential

dx =(cdt , r)

$$(1)$$

Therefore, the line element is

ds^2= (c^2dt^2 - dr.dr)

$$(2)$$

Using Schwarzschild solution of Einstein,s field equation

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{3}$$

where,

τ is the proper time

c is the speed of light

t is the time coordinate as measured by a stationary clock in space

r is the radial coordinate

θ is the co latitude

ϕ is the longitude[7]

Now dividing equation 3 by dt^2, which gives us a differential solution to find the rate of change of proper time with respect to time in an gravitationaly unaffected space, or simply the temporal flow or temporal velocity.

$$c^2\left(\frac{d\tau}{dt}\right)^2 = \left(1-\frac{r_s}{r}\right)c^2 - \left(1-\frac{r_s}{r}\right)^{-1}\left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\theta}{dt}\right)^2 - sin^2\theta\left(\frac{d\phi}{dt}\right)^2 \tag{4}$$

$$c^2\left(\frac{d\tau}{dt}\right)^2 = \left(1-\frac{r_s}{r}\right)c^2 - \left(1-\frac{r_s}{r}\right)^{-1}v^2 - r^2\omega^2 - sin^2\theta\left(\frac{d\phi}{dt}\right)^2 \tag{5}$$

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1-\frac{r_s}{r}\right) - \left(1-\frac{r_s}{r}\right)^{-1}\frac{v^2}{c^2} - \frac{r^2}{c^2}\omega^2 - \frac{sin^2\theta}{c^2}\left(\frac{d\phi}{dt}\right)^2 \tag{6}$$

$$\frac{d\tau}{dt} = \left[\left(1-\frac{r_s}{r}\right) - \left(1-\frac{r_s}{r}\right)^{-1}\frac{v^2}{c^2} - \frac{r^2}{c^2}\omega^2 - \frac{sin^2\theta}{c^2}\left(\frac{d\phi}{dt}\right)^2\right]^{-1/2} \tag{7}$$

This equation derived from the Schwarzschild solution of Einstein's field equations, gives us the velocity of time or tells us about the nature of flow of time for any object/body. This is a general equation derived to know about the nature or behaviour of time due to spac-time curvature for a body with any radius. For special cases like when the radius of a body(star) reaches its Schwarzschild radius [8], I have derived further and specialized this equation applying certain boundary conditions. Here the new term v is the instantaneous velocity of the body.

0.4 Considering specialized case

For any baryonic matter in the shape of sphere can either have one rotation along one of its axis θ or ϕ whereas other will be a cyclic coordinate.[9] But the significance of movement of both the axis with respect to t is of grave importance and brings down an important conclusion, which I will discuss later in the conclusion.

In our case we consider the body to be rotating along θ and let ϕ be the cyclic coordinate.

Therefore the equation 7 takes the form:

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{r_s}{r}\right) - \left(1 - \frac{r_s}{r}\right)^{-1} \frac{v^2}{c^2} - \frac{r^2}{c^2}\omega^2 \tag{8}$$

Since ϕ is a cyclic coordinate and does not depend on time.

therefore

$$\frac{d\phi}{dt} = 0$$

For any body the r_s << r, where r_s is the schwarzschild radius of any body given by $\frac{2Gm}{r}$

So using binomial expansion for the second term in R.H.S and neglacting higher degree terms, equation 8 becomes

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{r_s}{r}\right) - \left(1 + \frac{r_s}{r}\right)\frac{v^2}{c^2} - \frac{r^2}{c^2}\omega^2 \tag{9}$$

$$= -\frac{v^2}{c^2} - \frac{r_s}{r}\left(1 + \frac{v^2}{c^2}\right) - \frac{r^2}{c^2}\omega^2$$

$$= 1 - \frac{v^2}{c^2} - \frac{r_s}{r}\left(1 + \frac{v^2}{c^2}\right) - \frac{v^2}{c^2}$$

since v=rω

$$\frac{d\tau}{dt} = \left[\left(1 - \frac{2v^2}{c^2}\right) - \frac{r_s}{r}\left(1 + \frac{v^2}{c^2}\right)\right]^{1/2} \tag{10}$$

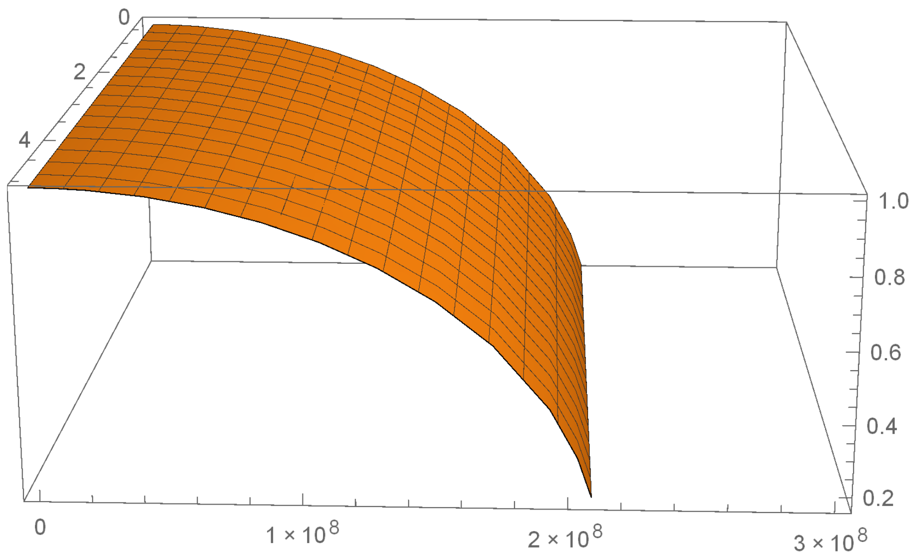

Fig 1.

The above graph shows the variation of the change in proper time w.r.t time coordinate considering the variation of velocity of the body and its radius. As the velocity and radius increases, the rate of time dilation decreases as it should be from experimental observations.

0.4.1 Case I

Now considering a speciasl case where the radius of a body is equal to its own schwarzschild radius i.e r_s = r.

equation 10 becomes

$$\frac{d\tau}{dt} = \left(1 - \frac{2v^2}{c^2} - 1 - \frac{v^2}{c^2}\right)^{1/2}$$

$$= \sqrt{3}\frac{v}{c}i$$

This is a very important conclusion for an object which is present on a body whose radius is equal to its own schwarzschild radius as this denotes how time will elapse for it. As seen in the result the term $\frac{v}{c}$ is very small except for photons or other particles having v nearly equal to c.This indicates that rate of flow of time for the object is very less. Moreover the appearence of i, i.e imaginary time coordinate it denotes that at the boundary of the singularity time must behave as minkowski time coordinate.[10]

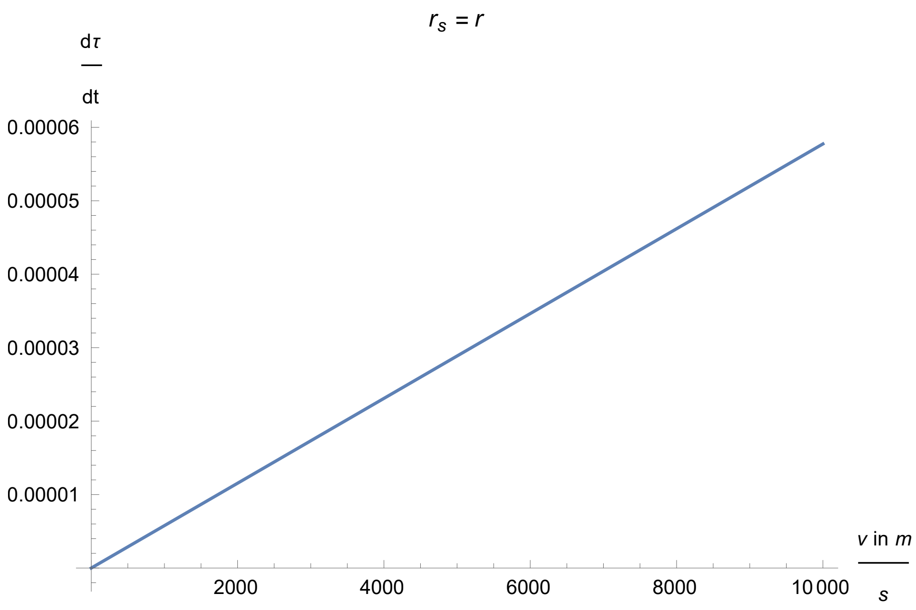

Fig 2.

0.4.2 Case II

For objects like Black Holes [11], the radius is much less than the schwarzschild radius r_s << r, the nature of time is unknown at this singular point. But applying the above condition to my equation, we get

$$\frac{d\tau}{dt} = \left(1 - \frac{r_s}{r}\frac{1+\frac{v^2}{c^2}}{1-\frac{2v^2}{c^2}}\right)^{1/2} \tag{11}$$

$$\frac{d\tau}{dt} = \left(1 - \frac{r_s}{2r}\frac{1+\frac{v^2}{c^2}}{1-\frac{2v^2}{c^2}}\right)$$

$$= 2 - \left[\frac{c^2+v^2}{c^2-2v^2}\right]$$

This is another important conclusion of this theory. It shows us how time behaves for a particle inside the event horizon[12] near the singularity. now for normal cases, v << c, so compared to c^2 it is very less and can be neglected. Therefore the result for

$$\frac{d\tau}{dt} = 1$$

which is a constant. this shows that the rate of time dilation for any particle inside a black holes will be constant and same.

Moreover the absence of any imaginary part shows that here time follows non - euclidian geometry For the final case, let us now concider photons inside event horizon moving with velocity c. therefore for v=c in eq 11 gives,

$$\frac{d\tau}{dt} = 4$$

this is same as that of the previous solution. Both are constans and this shows that as particles or photon approach singularity, the rate of change time dilation becomes constant, only due to the velocity of photons its a bit high, but it becomes same for all the photons that has, will or is approaching the space-time singularity.

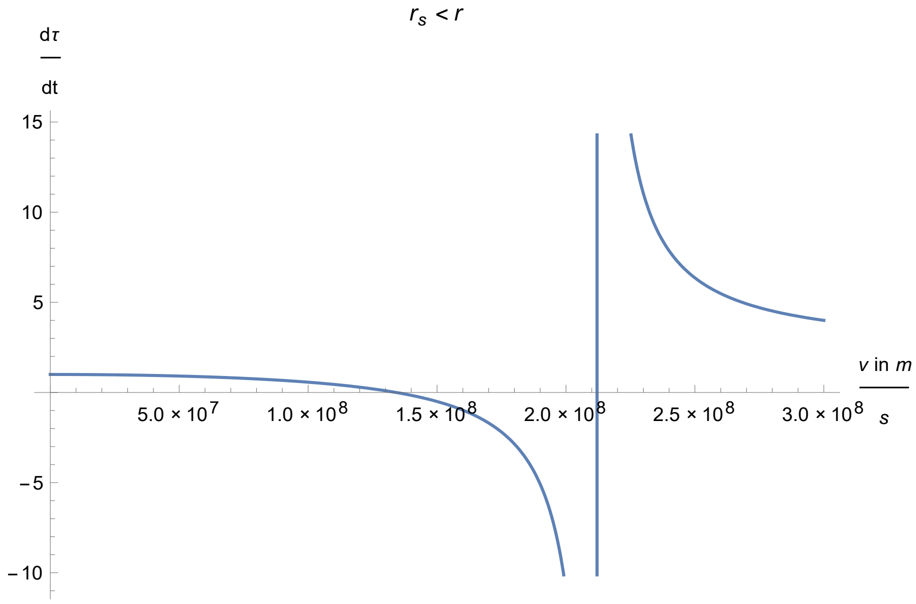

Fig 3.

0.5 Further analysis on time dilation

In the previous section, an equation is being developed showing how the flow of time depends on general for all heavy bodies and at certain extreme conditions of space, like black holes and event horizons. Now, since it is derived a difference in the flow of time. Therefore, let us consider a star or some other body with masses compareable to 1solar mass or far greater than that. now suppose the star has expended all its fuel and transitioning into the phase of red giant. So, slowly the mass of the star starts increasing.[13] According to general theory of relativity, the curvature increases and the time dialation due to it increases as well. Therefore the rate of change of time dialation with respect to the clock in a frame unaffected by curvature of space-time is givem by Equation (7) which denotes that the flow of time also starts varying. Suppose two binary stars[14] are in the verge of collapsing together then according to Einstein's general theory of relativity the value of dτ will change. It will attain different values before merging for the binary stars and different vallue after merging. This clearly indicates that the flow of time in the vicinity of the stars change with respect to a clock placed in a frame devoid of any gravitational effect. This can be compared analogous to the spatial acceleration and can be termed as temporal acceleration. This is done by taking in account my previous equation no 10 and furter differentiating it with the time coordinate t.

0.6 Theoritical proof

Squaring equation (10) and differentiating it with respect to time(t)

$$\frac{d}{dt}\left(\frac{d\tau}{dt}\right)^2 = \frac{d}{dt}\left(1-\frac{2v^2}{c^2}\right) - \frac{d}{dt}\left[\frac{r_s}{r}\left(1+\frac{v^2}{c^2}\right)\right] \tag{12}$$

$$2\frac{d\tau}{dt}\frac{d^2\tau}{dt^2} = -\frac{4va}{c^2} - \left[\frac{2r_s va}{rc^2} - \frac{r_s v}{r^2} - \frac{r_s v^3}{v^2c^2}\right] \tag{13}$$

substituting the value of $\frac{d\tau}{dt}$

$$\frac{d^2\tau}{dt^2} = \frac{-\frac{4va}{c^2} - \left[\frac{2r_s va}{rc^2} - \frac{r_s v}{r^2} - \frac{r_s v^3}{c^2r^2}\right]}{2\sqrt{\left(1-\frac{2v^2}{c^2}\right) - \frac{r_s}{r}\left(1+\frac{r^2}{c^2}\right)}} \tag{14}$$

equation 14 represents the double differentiation of the proper time τ with respect to t. we will further see what this equation reveals us about the nature of time in extreme cases of our universe

0.6.1 Case I

Let us consider the v=c. Since a particle moving with velocity c is moving with constant velocity, so the accelaration is 0 therefore equation 14 becomes

$$\frac{d^2\tau}{dt} = \frac{\frac{r_s c}{r^2} + \frac{r_s c^3}{c^2 r^2}}{2\sqrt{1 - 2 - \frac{r_s}{r}(1+1)}} \tag{15}$$

$$= \frac{\frac{2r_s c}{r}}{2\sqrt{-1 - \frac{2r_s}{r}}}$$

$$= \frac{\frac{2r_s c}{r}}{2i\sqrt{1 + \frac{2r_s}{r}}}$$

$$= -\frac{\frac{r_s c}{r}}{1 + \frac{r_s}{r}} i$$

For any body the schwarzschild radius r_s of a body is much less compared to the radius of the the body

therefore $\frac{r_s}{r} >>> 1, = -ci$ The -ve sign represents the acceleration in opposite direction, since the velocity of the particle is moving from v to c, the acceleration of the particle in temporal dimension is in opposite direction and the magnitude of it is equal to 3 * 10^8

0.6.2 Case IIa

Now we have to find the case of temporal acceleration for condition at r_s=r; It is important for a binary system of two bodies that have reached their Schwarzschild radius and is merging together, but the necessary condition is they increase their mass but still remains at their Schwarzschild radius. So the change in temporal velocity for the individual body before collapsing and after collapsing with respect to time flowing at a gravitationally unaffected space is given by:

$$\frac{d^2\tau}{dt^2} = \frac{-\frac{4va}{c^2} - \left[\frac{2va}{c^2} - \frac{v}{r_s} - \frac{v^3}{r_s c^2}\right]}{2\sqrt{\left(1-\frac{2v^2}{c^2}\right) - \left(1+\frac{v^2}{c^2}\right)}} \tag{16}$$

$$= -\frac{-\frac{6va}{c^2} - \left[-\frac{v}{r_s} - \frac{v^3}{r_s c^2}\right]}{2\sqrt{3}\frac{v}{c}} i$$

$$\frac{d^2\tau}{dt^2} = \frac{\frac{6va}{c^2} - \left[\frac{v}{r_s} + \frac{v^3}{r_s c^2}\right]}{2\sqrt{3}\frac{v}{c}} i \tag{17}$$

Equation 17 shows that for the above mentioned case the temporal acceleration will depend on the instantaneous velocity, acceleration and the Schwarzschild radius of the body. Moreover we can again see that the equation is generated in the imaginary part, which again tells us about eucladian geometry at the boundary of event horizon.

0.6.3 Case IIb

Let us consider a body whose r_s = r and is moving with an instantaneous velocity v = c. since c is constant, so the acceleration of the body will be 0 i.e a = 0. Now the acceleration of the body in time will be given by applying the above condition in equation 14.

$$\frac{d^2\tau}{dt^2} = \frac{\left[-\frac{c}{r_s} - \frac{c}{r_s}\right]}{2\sqrt{3}}i$$
$$= -\frac{2}{2\sqrt{3}}\frac{c}{r_s}i$$

Substituting the value of r_s

$$\frac{d^2\tau}{dt^2} = \frac{c^3}{2\sqrt{3}Gm}i \tag{18}$$

Equation 18 shows how temporal acceleration depends on a body for the case mentioned at case IIa. now we can see if there is no acceleration or the velocity is equal to c for the object, the temporal acceleration depends on r and inversely on mass. Now if $r << r_s$ then the equation 18, r_s is replaced by r. so in case of collapsing blackholes[15] the temporal acceleration varries inversely with the radius of the black hole.

0.7 On the special theory of relativity

I have derived the equation to find the temporal velocity and acceleration for any object by considering gravitational time dilation. Now for particles like photons, muons, electrons, etc we cannot apply general theory of relativity or the Schwarzschild solution of Einstein's field equations. For this almost massless particles possesing high velocity, I have to switch my theory on the time dilation caused due to moving particles. The theory remains same, that I have to find the flow of time, i.e the temporal velocity and then again the temporal acceleration for the object. Here there is no concept of gravity and let us assume for all cases that the experiments are taking space in a space free from any gravitational effect.

0.7.1 Treatment of time dilation

I have taken Einstein's equation for time dilation here which is given by,[16]

$$d\tau = dt\sqrt{1 - \frac{v^2}{c^2}} \tag{19}$$

where,

τ is the proper time

t is the time coordinate

v is the velocity of the particle with respect to some reference frame

c is the speed of light and dividing both the sides by the time measured in a gravitaionally effectless stationary space. For this we have to use an external reference frame A, which will be in a state of rest for the clock measuring t and will move with a velocity of -v w.r.t the particle.

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}} \tag{20}$$

This equation denotes how time will flow or rather the velocity of time for an object moving with a velocity of v w.r.t A. Now let us consider a particle like photon moving with a velocity c w.r.t A, therefor the equation changes to $\frac{d\tau}{dt} = 0$. This shows that for a particle like photon ,moving with a spatial velocity c have a temporal velocity 0. This indicates that time will not pass for any particle moving with velocity c.

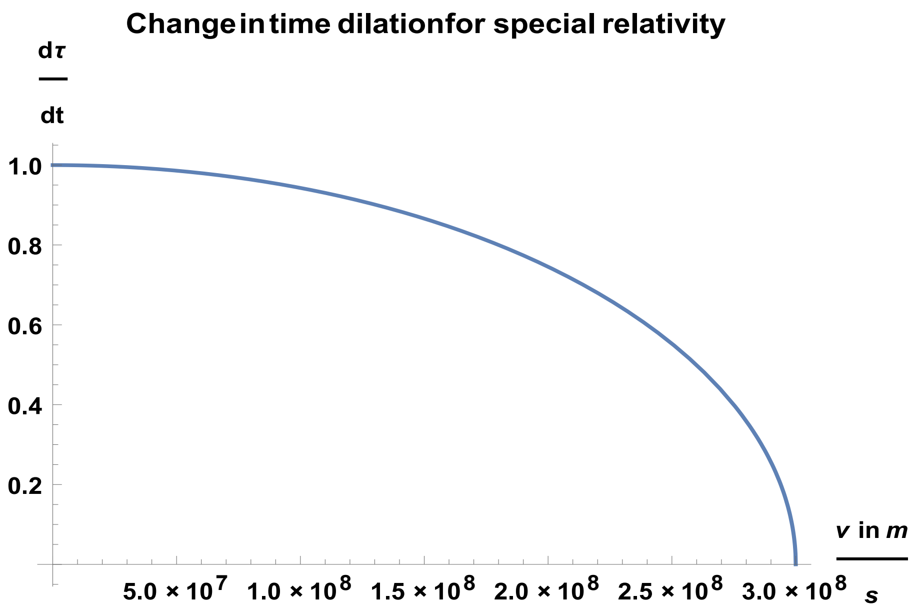

Fig 4.

0.7.2 Further treatment on time dilation

In the previous equation I have deduced an equation to describe the temporal velocity for a particle moving with a constant velocity. Now suppose a particle is moving with a variable velocity at different instants of time. Let a particle P moving with a velocity v_1 for a certain interval of time and then changes its velocity to v_2, $(v_1 < v_2)$ for another interval of time. Therefore the temporal velocity $\frac{d\tau}{dt}$ for v_1 will differ from the temporal velocity due to v_2 and this increase or decrease in temporal velocity, gives rise to the need of the concept of temporal acceleration. So differentiating equation 20 with respect to t we get,

$$\frac{d^2\tau}{dt^2} = \frac{d}{dt}\left(1 - \frac{v^2}{c^2}\right)^{1/2} \tag{21}$$

$$= \frac{1}{2c^2}\left(1 - \frac{v^2}{c^2}\right) 2v\frac{dv}{dt}$$

$$= \frac{2va}{2c^2\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

$$= \frac{va}{c^2\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

Since, $v << c$ using binomial expansion for the term in brackets and neglecting higher terms

$$= \frac{va}{c^2\left(1 - \frac{v^2}{2c^2}\right)}$$

$$\frac{d^2\tau}{dt^2} = \frac{2av}{2c^2 - v^2} \tag{22}$$

Equation 22 represents the general formula for a particle moving according to the case described above, where v is the mean velocity of v_1 and v_2 and a is the acceleration or deceleration of the particle depending on its velocities v_1 and v_2.

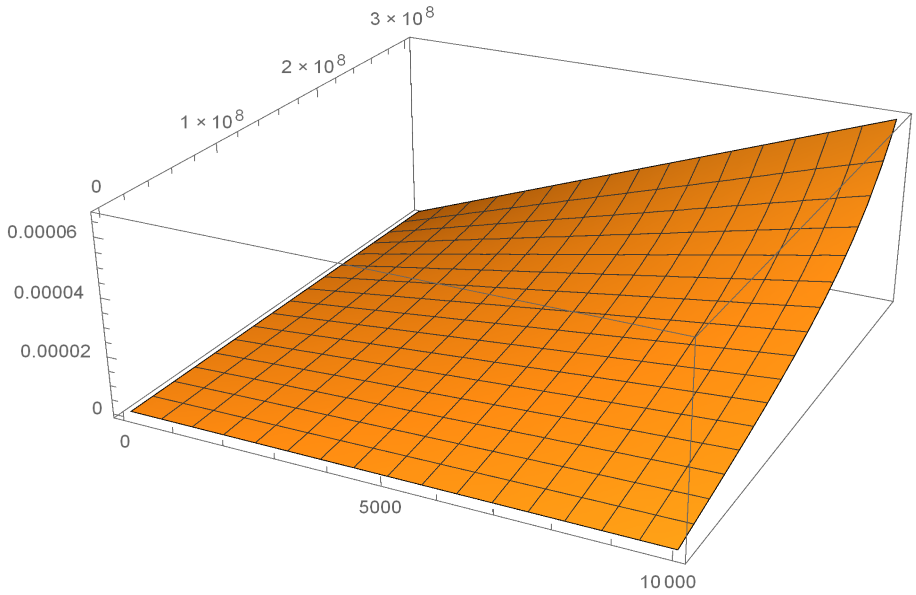

Fig 5.

For particles moving with velocity c from its birth to annihilation has mean velocity c and $a = 0$. therefore, this again shows $\frac{d^2\tau}{dt^2} = 0$. Which represents the temporal acceleration is 0 since there is no change in temporal velocity.Also shown in Fig 5. It also shows that the change in the expression happens due to both change in velocity and acceleration.

0.8 Conclusion

In this paper I have theoretically investigated on the nature of the temporal dimension, its flow and characters at different conditions. I have introduced new terms like temporal velocity and temporal acceleration and derived equations to represent them for general cases and special cases in the universe. The equations describing the temporal acceleration gives the idea of the change in temporal velocity for two different intervals of time or varying masses for a body. This change in temporal flow must be caused due to some force which can be considered as a temporal force. This temporal force must be extra dimensional and can be the cause of the formation of curvature in space-time due to the presence of mass and energy. In equation 7, I have considered ϕ as a cyclic coordinate independent of t. But if we consider ϕ and θ both changing in time, then the equation would indicate the temporal flow starting from a point and slowly increasing in magnitude. After attaining a maximum value the temporal flow starts collapsing again to a point as the similar case when it started. Considering the radius of our universe equal to the schwarzschild radius of the universe[17], the temporal flow for the universe can be represented by my equation showing the starting of temporal flow, attaining a maximum value and then collapsing to a single point. This can be refered to the Big Bang and Big Crunch for our observable universe. So, it can be concluded that time applies force on space and causes it to curve whereas following Newton,s 3rd law, space in turn applies an opposite force for time to slow down.

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