Modified analog gravity from De Broglie matter wave

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Abstract

We have used the concept of De Broglie's matter wave associated with particles to derive a inverse square law of gravitation like the newton's law of gravitation at the plank's length with a slight modification. Obtaining the Newtonian form we have further extrapolated it to formulate Einstein's field equation, which generally is embedded with a slight modification.The Schwarzschild solution is calculated from the

modified field equation, which gives us the Schwarzschild radius of a miniature black hole. Using the solution, the entropy and the hawking temperature is also modeled theoretically for the miniature black hole at the plank's order.

1 Introduction

The unification of gravity and quantum mechanics has been a major challenge for scientists in search of the Unified Theory of all the four forces of nature for almost a century[1].The strong, weak and electromagnetic interaction have been unified using the quantum exchange of virtual bosons but gravity could not fit into this model. Although this has been shown to be theoretically possible considering the existence of gravitons(Quantum particle responsible for gravitational interactions), but no experimental research have yet confirmed the theory. A new quantum mechanical model for gravity was proposed by M.E Mc Calloch(2013), which assumed that the gravitational interactions only occurs between whole Plank masses and was showed that it gives Newton's gravitational law from Heisenberg's Uncertainity principle[2].

Louis De Broglie hypothesised the existence of matter waves in every quantum mechanical particle according to the wave-particle duality of the quantum objects formulated by Heisenberg [3, 4]. In this paper the basic concept is taken from, that the matter wave is in the order of plank's length. This order is very small relative to the macroscopic world that we observe and is the limiting case for the smallest length possible in the Universe. For this reason, the matter wave is localised and is considered behaving as a particle. Considering this case, we calculated an analogy of gravity from the basic quantum mechanics and have derived the newton's inverse square law of gravitation by a new approach.

In the year 1915, Albert Einstein gave his famous theory about the universe, where he proposed that the effect of gravity is due to curvature in space-time in his famous paper "General Theory of Relativity". Later on Carl Schwarzschild in his paper proposed a solution to the Einstein field equations(E.F.E). The existence of black holes in the macroscopic universe was theoretically found which was later confirmed in the year 2019 [5]. Stephen Hawking on the other hand showed that the black holes act like perfect black bodies and as all black-bodies emit thermal radiation[6], the black hole is also bound to emit thermal radiation due to quantum fluctuations, which was later termed as 'Hawking radiation'. In this paper also, we have followed the exact analytical methods to derive E.F.E for the obtained newton's gravitational law. Followed by the E.F.E, we can find out the Schwarzschild solution and the fundamental metric. The Schwarzschild solution is the pavestone to determine the Schwarzschild radius[7], which in this case is for the radius of a quantum particle or a miniature black hole which is in the order of planks dimensions, i.e the smallest possible black hole. Furthermore, we have calculated the entropy and hawking temperature of the miniature black hole[8].

2 Deriving Newton's Law of Gravitation

In 1924, De Broglie in his PhD thesis, proposed that electrons and other quantum mechanical particle is associated with a wave along with it, called the matter wave just like photons which posses both wave-like and particle-like nature[9]. The wavelength of the wave associated with this particles is given by the $\lambda = \frac{\hbar}{p}$, where λ is the de broglie wavelength, $\hbar$ is the reduced plank's constant and p is the momentum of the particle. Considering plank's length order wavelength of localised matter wave, which can be inferred as particle for being localised at an infinitesimally small space. This will be the smallest case possible in the universe due to considering of plank's length. Now, we have taken a particle of mass m in the vicinity of another mass M, where mass m is composed of n number of Plank masses and mass M composed of N number of Plank masses. These bodies are made up of locallised matter wave of wavelength of the order of Plank length. Sticking strictly to the quantum realm, by using Plank's units, we can apply De Broglie's equation for a single case,

$$\lambda \approx \frac{\hbar}{p} \tag{1}$$

Now, we consider summing over all possible interactions between the Planck masses in two bodies from equation(1)

$$\lambda \approx \frac{\sum_{i=1}^{n}\sum_{j=1}^{N}\hbar}{p} \tag{2}$$

where, $m = nm_p$ and $M = Nm_p$, So from this relation we can write

$$\begin{aligned} n &= \frac{m}{m_p}, \\ N &= \frac{N}{m_p} \end{aligned} \tag{3}$$

In the above equation, M_p =reduced Plank mass=$\sqrt{\frac{\hbar c}{8\pi G}} = 21.76\mu g$, [10]and some other constants are used like,
l_p= Plank length =$\sqrt{\frac{\hbar G}{c^3}} = 1.6{*}10^{-35}m$
t_p= Plank time=$\sqrt{\frac{\hbar G}{c^5}} = 10^{-43}s$

Since, we know Energy $E = pc$, where p is the momentum and c is the speed of light, we get by substituting the value of p in equation (2)

$$\lambda \approx \frac{\sum_{i=1}^{n}\sum_{j=1}^{N}\hbar c}{E} \tag{4}$$

As we have considered the matter wave of wavelength l_p, hence equation (3) can be rewritten as,

$$l_p \approx \frac{\sum_{i=1}^{n}\sum_{j=1}^{N}\hbar c}{E} \tag{5}$$

From equation (4), we get by alzebraic manipulation,

$$t_p \approx \frac{\sum_{i=1}^{n}\sum_{j=1}^{N}\hbar}{F.r} \tag{6}$$

Since $E = F.r$, where r is the distance of seperation between the two masses, and $t_p = l_p/c$.
Hence ,by substituting the value of n and N by removing the summation,

$$t_p \approx \frac{\hbar}{F.r}\frac{mM}{M_p^2} \tag{7}$$

Substituting the values of t_p and M_p,

$$\begin{aligned} F &= \frac{\hbar 8\pi mMGc}{r\hbar c}\frac{\sqrt{c^3}}{\sqrt{\hbar G}}, \\ &= \frac{8\pi GmM}{r.l_p} \end{aligned} \tag{8}$$

If we consider a limiting case for the spatial seperation between the two object is of the magnitude of Plank length then we can write $r = l_p$ therefore, equation (7) takes the form

$$F = \frac{8\pi GmM}{l_p^2} \tag{9}$$

This can be well inferred and understood as the newton's inverse square law of gravitation with a slight modification[11]. This formula draws an analogy to study the gravitational effects at the Plank's length which is more fundamental. This is a modified version of Newton's law for gravity using De Broglie equation.

3 Modified Einstein Field Equation

Einstein in his famous paper of relativity proposes that gravity is a consequence of the curvature of spacetime. He used the stress energy tensor and the Ricci tensor [12] to formulate his equation and the constant was derived from Newton's gravitational field. We will also use the modified newton's law to formulate E.F.E. We will start with the Christoffel symbol which is given by $\Gamma^a_{bc} = \frac{1}{2}g^{ad}[\partial_b\partial g_{dc} + \partial_c\partial g_{ab} - \partial_d\partial g_{bc}]$. This Christoffel symbol[13] in the newtonian limit represents force, i.e $\Gamma \equiv F$. Now, let us consider the gravitational effects between two objects in ordinary space time, $v << c$ and no presence of black holes. In this ordinary space time the derivatives of the metric tensor w.r.t the spatial coordinate is negligible except for the time component. Therefore, $\partial_b\partial g_{dc} + \partial_c\partial g_{ab} - \partial_d\partial g_{bc} \to 1$ and $g^{ad} = 1$ for $a, b, c, d = 1, 2, 3$. This gives us,

$$\Gamma = \frac{1}{2}\frac{\partial g_{00}}{\partial t} \equiv \bar{F} \tag{10}$$

We, know that $\bar{F} = -\nabla\phi$, and using the relation in equation 9, we get

$$\frac{1}{2}\partial_t g_{00} = -\nabla\phi \tag{11}$$

$$\implies g_{00} = -2\phi + constant$$

From the previous section, equation (7) we get ,

$$\int \bar{F}.dA = 32\pi^2 GM \tag{12}$$

Using divergence theorem $\int F.dA = \iiint \nabla\bar{F}dV$ to change from area integration to volume integration, $M = \iiint \rho(r)dV$ and other identities, equation 11 is written as

$$\nabla^2 g_{00} = 64\pi^2 G\rho(r) \tag{13}$$

Although, equation 12 is not yet a tensor equation since ρ is not a tensor. To change it to a tensor equation it can be written as,

$$G_{\mu\nu} = 64\pi^2 GT_{\mu\nu} \tag{14}$$

where, c=1 and $T_{\mu\nu}$ is the Stress-Energy tensor[14]. From the conservation of energy, we know that $\nabla T_{\mu\nu} = 0$, and to satisfy the R.H.S, the L.H.S must also be 0, i.e $G_{\mu\nu} = 0$. Therefore the E.F.E is derived to be,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 64\pi^2 GT_{\mu\nu} \tag{15}$$

$R_{\mu\nu}$ is the Ricci tensor which denotes the curvature of space time. R is the Ricci scalar. Equation (14) is the modified Einstein field Equation at the plank's length.

4 Schwarzschild Solution

The Schwarzschild metric is the simplest spherical symmetrical vaccum solution of E.F.E. This describes the gravitational field outside a spherical symmetrical body.[16] The metric is given by:

$$g_{\mu\nu} = \begin{pmatrix} B(r)c^2 & 0 & 0 & 0 \\ 0 & -A(r) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2Sin^2\theta \end{pmatrix} \tag{16}$$

where$g_{\mu\nu}$ is the fundamental metric,[15] B(r) and A(r) are constants. From the above metric, we get $g_{00} = B(r)c^2$, $g_{11} = -A(r)$,$g_{22} = -r^2$ and $g_{33} = -r^2Sin^2\theta$. Since all the cross terms are zero, therefore among the 64 Christoffel symbols, only 13 will be non-zero, which are given by,

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}, \tag{17}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} \tag{18}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = Cos\theta, \tag{19}$$

$$\Gamma^0_{10} = \Gamma^0_{01} = \frac{B'(r)}{2B(r)} \tag{20}$$

$$\Gamma^2_{33} = -Sin\theta Cos\theta \tag{21}$$

$$\Gamma^1_{11} = \frac{A'(r)}{2A(r)} \tag{22}$$

$$\Gamma^1_{22} = -\frac{r}{A(r)}, \tag{23}$$

$$\Gamma^1_{00} = \frac{c^2 B'(r)}{2A(r)}, \tag{24}$$

$$\Gamma^1_{33} = -\frac{Sin^2\theta}{A(r)} \tag{25}$$

Using the above Christoffel symbols the four Ricci tensors for the four dimensions are

$$R_{00} = -c^2 \frac{B'' r}{2A(r)} + c^2 \left[\frac{B'(r)}{4A(r)} \left(\frac{A'(r)}{A(r)} + \frac{B'(r)}{B(r)}\right)\right] - c^2 \frac{B'(r)}{rA(r)} \tag{26}$$

$$R_{11} = \frac{B''(r)}{2B} - \frac{B'(r)}{4B(r)} \left(\frac{B'(r)}{B(r)} + \frac{A'(r)}{A(r)}\right) - \frac{A'(r)}{rA(r)} \tag{27}$$

$$R_{22} = R_{33} = -1 - \frac{r}{2A(r)} \left(\frac{B'(r)}{B(r)} - \frac{A'(r)}{A(r)}\right) - \frac{1}{A(r)} \tag{28}$$

For the vacuum solutions of E.F.E, $G_{\mu\nu} = 0$, which implies that the Ricci tensors must be equal to 0. Therefore, Using the Schwarzschild trick,

$$\frac{R_{00}}{B(r)c^2} + \frac{R_{11}}{A(r)} = 0 \tag{29}$$

because the numerators are all individually zero. Upon Solving the equation and integrating it we get

$$A = \frac{1}{B} \tag{30}$$

Now, using this relation, in equation (28), we get,

$$B(r) = 1 - \frac{2S}{r} \tag{31}$$

where, S is a constant.

5 Formation of Black Hole

Black holes are regions in spacetime where the curvature of space tends to infinity. It exhibits strong gravitational force due to its infinite curvature, that even light cannot escape. The point from where, no particles can return is known as the event horizon[17].This was theoretically derived from the Schwarzschild solution for a non-rotating, uncharged body. In the previous section, the Schwarzschild solution was sought out for the modified field equation[18]. The solution can be written as

$$c^2(d\tau)^2 = \left(1 - \frac{2S}{r}\right) c^2 dt^2 - \left(1 - \frac{2S}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \quad (32)$$

In this equation when $r = 2S$, the equation blows up due to singularity. This singularity in the space time equation leads to the formation of black hole. For, cosmological black hole , $r = 2GM/c^2$ which is known as the Schwarzschild radius [19]. Although the Schwarzschild radius for a black hole at the plank's length[20] will be different due to the modified laws in the previous section. The geodesic equation is given by,

$$\frac{d^2x^\mu}{d\tau^2} = \Gamma^\mu_{\rho\nu} \frac{dx^\rho}{d\tau} \frac{dx^\nu}{d\tau} \quad (33)$$

From the equation, the acceleration can be calculated as

$$\begin{aligned} a_r &= \lim_{c \to +\infty} \frac{d^2x^1}{d\tau^2} \\ &= \lim_{c \to +\infty} \left(\Gamma^1_{\rho\nu} \frac{dx^\rho}{d\tau} \frac{dx^\nu}{d\tau} \right) \end{aligned} \quad (34)$$

From equation (32),we can write

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{2S}{r}\right) - \left(1 - \frac{2S}{r}\right)^{-1} c^2 \left(\frac{dr}{dt}\right)^2 - \frac{r^2}{c^2} \left(\frac{d\Omega}{dt}\right)^2 \quad (35)$$

Using the isotropic and homogenity of space [21] we can tell that all the coordinates are co-moving coordinates and are independent of each other. So, equation(35) can be written as

$$U_0 = \frac{d\tau}{dt} = \left(1 - \frac{2S}{r}\right)^{-1/2}$$

, where U_0 is the proper velocity along the time co-ordinate, and all the other proper velocities will be zero. Therefore,

$$U^\rho = \begin{pmatrix} ((1 - \frac{2S}{r})^{-1/2} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (36)$$

Now, using equation 34 and 36,

$$a^r = \lim_{c \to +\infty} \left(\Gamma^1_{00} U^0 U^0 \right)$$
$$= \frac{1}{2} \lim_{c \to +\infty} c^2 \frac{2S}{r^2} \quad (37)$$

where $r = 0, 1, 2, 3$ Using the modified newtons law we can write the following equation,

$$a^r = \frac{8\pi MG}{l_p^2} \quad (38)$$

Equating the equation 37 and 38, we can find out the value of S, which is the schwarzschild radius. So,

$$S = \frac{8\pi GMr^2}{l_p^2 c^2} \quad (39)$$

Substituting the value of S in the Schwarzschild solution, we get the full fledged space-time equation for a quantum particle, which is,

$$ds^2 = \left(1 - \frac{\frac{16\pi GM}{l_p^2 c^2}}{r^3} \right) - \left(\frac{1}{1 - \frac{\frac{16\pi GM}{l_p^2 c^2}}{r^3}} \right) dr^2 - r^2 d\Omega^2 \quad (40)$$

So, from the above equation of space time the singularity condition can easily be determined i.e the Schwarzschild radius of the body which is given by

$$r_s = \left(\frac{16\pi GM}{l_p^2 c^2} \right)^{\frac{1}{3}}$$

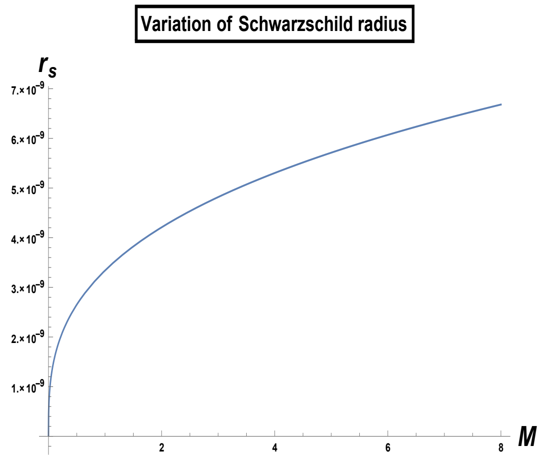

Fig 1.

Therefore, any miniature blackhole that can form will have the radius r_s and it is the limiting point for a blackhole formed in the quantum realm.

6 Entropy and Hawking Temperature

To find out the entropy of the miniature black hole, we need to start with two basic equations of the universe

$$E = h\nu \quad (41)$$

$$E = mc^2 \quad (42)$$

Using the broadness of equations(41) and (42), and considering the change in energy we can write simply, $\Delta m = \frac{h}{\lambda c}$, So the change in Schwarzschild radius can be written as

$$\Delta r = \left(\frac{16\pi Gh}{l_p^2 c^3 \lambda}\right)^{\frac{1}{3}} \tag{43}$$

Considering the microstates of the system the entropy of the system can be found out by the equation

$$dA = 4\pi r_s dr ds$$

Substituting the value of r_s and dr in the above relation and integrating both side we get,

$$S = \frac{l_p}{4\pi}\left(\frac{c}{16\pi MG}\right)^{\frac{2}{3}}\left(\frac{l_p \lambda}{h}\right)^{\frac{1}{3}} \tag{44}$$

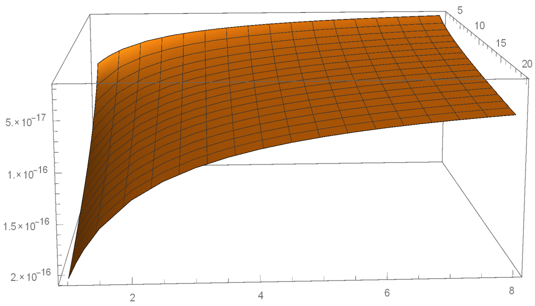

Fig 2. Variation of entropy with M and λ

The entropy (S) is plotted on the y-axis, mass on the x-axis and λ on the z-axis. From the above figure it is evident that the entropy will always keep on increasing, thus the existence of this kind of black hole does not violate the laws of thermodynamics and are stable, but they will evaporate much faster due to hawking radiation.[22, 23]

The temperature of this miniature black holes for hawking radiation is derived from the unruh temperature $E = kT = \frac{gh}{2\pi c}$, where k is the Boltzman constant[24]. The g, gravitational acceleration for the black hole is determined from our modified newton's law for the quantum particle, where the plank length is substituted for the Schwarzschild radius[25]. Thus,

$$g = \frac{8\pi GM}{r_s^2} \tag{45}$$

$$= 8\pi GM\left(\frac{l_p^2 c^2}{16\pi GM}\right)^{\frac{2}{3}}$$

From this , the hawking temperature of the miniature black hole is found to be,

$$T_b = \frac{(GMc)^{1/3} l_p^{3/4} \hbar}{(2\pi)^{2/3} k}$$

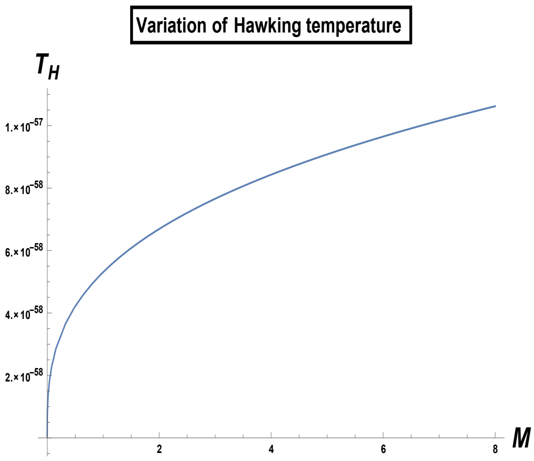

Fig 3.

The above figure shows the variation of the hawking temperature with the increase in mass of the particle. As the mass increases, there is abrupt increase first and then the increase in temperature decreases as the mass is increased further. In the figures we have used $G = 6.673 * 10^{-11} Nm^2 Kg^{-2}$, $c = 2.998 * 10^8 ms^{-1}$, $h = 6.626 * 10^{-34} Js$,
$k = 1.3806 * 10^{-23} m^2 Kg s^{-2} K^{-1}$.

7 Conclusion

In this paper we have found out a modified law of newton's law of gravity at the plank order. This gravity is assumed to act at the quantum level between particles. Further drawing out a modified E.F.E for the quantum order helps us to understand the nature of space time geometry in that vicinity[26]. We have also found out the radius of the black hole in that small order which is the smallest possible miniature black hole and is related to the plank length as obtained by the relation in this paper. From that we could also find out the entropy and the graph shows the possible existence of the blackhole at the quantum level. Furthermore, we have also calculated the hawking temperature for this types of black hole which is very small, near to absolute zero

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