CMB Anomalies from Self-Organized Criticality

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Abstract

The power spectrum of the cosmic microwave background (CMB) quantifies the distribution of relic radiation left over from the early Universe. As of today, CMB data acquired by Planck and WMAP satellites exhibit certain anomalies that challenge the standard model of cosmology (ΛCDM). The goal of this brief report is to sketch up an intriguing connection between CMB anomalies and self-organized criticality (SOC). Our proposal bypasses the interpretation of CMB anomalies based on Loop Quantum Cosmology (LQC).

Key words: CMB anomalies, power spectrum, ΛCDM, Self-organized criticality, Loop Quantum Cosmology.

1. Introduction

ΛCDM predicts that the primordial power spectrum is nearly scale-invariant and described by the power law [1]

$$P_R(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1} \qquad (1)$$

in which k is the wavenumber measured in Mpc^{-1}, A_s denotes the amplitude of the scalar mode of spectral index n_s and k_* represents the so-called pivot mode. According to the LQC model, (1) acquires a *suppression factor* $f(k)$ whose effect is negligible for large wavenumbers ($k >> k_0$), that is,

$$f(k) = O(1),\ k >> k_0 \tag{2}$$

with k_0 being a reference value. By contrast, suppression of the nearly scale-invariant spectrum (1) occurs if the wavenumber drops down near $k \leq 10k_0$. The modified primordial spectrum predicted by LQC is given by [1]

$$P_R(k) = f(k) A_s (\frac{k}{k_*})^{n_s - 1} \tag{3}$$

There is more than one way to display and simulate the power spectrum (1). For example, [2] brings up the following set of scaling relations inspired by LQC and inflation

$$P_R(k) = \begin{cases} A_s (k/k_*)^{n_s-1},\ k > k_* \\ A_s (k/k_*)^q,\ k_I < k \leq k_* \\ A_s (k_I/k_*)^q (k/k_I)^2,\ k \leq k_I \end{cases} \tag{3}$$

where k_* and k_I are the characteristic scales at primordial cosmological times and q is an exponent dissimilar in magnitude to the spectral index n_s.

The goal of this short report is to explore a scenario where (2) and (3) derive from a radically different approach to the CMB formation. Demanding *self-similarity* in the complex dynamics of large structures implies that CMB is the outcome of a *global SOC process*. Our preliminary analysis is consistent with earlier proposals where SOC is conjectured to assume a critical role in astrophysics and cosmology [3-7]. We caution the reader that this work is solely an early-stage effort, in need for further evaluation and development.

2. Mathematics of SOC: a short overview

Consider a large-scale ensemble of observables undergoing a second-order phase transition. The transition is driven by the control parameter λ as it approaches the critical value λ_c. Near the critical point and for systems of infinite extent ($L \to \infty$), the correlation length ξ diverges as

$$\xi \sim (\lambda - \lambda_c)^{-\nu} \ ; \ L \to \infty, \ \lambda \to \lambda_c \tag{4}$$

In the transition region, a relevant variable of the system is also a diverging quantity which scales as

$$A_\infty(\lambda) \sim |\lambda - \lambda_c|^{-\zeta} ; \ L \to \infty, \ \lambda \to \lambda_c \tag{5}$$

where ζ is a critical exponent. In what follows, we introduce the notation

$$\tau_s = -(\zeta / \nu) \tag{6}$$

There are two distinct cases associated with the power-law (4). If the size of the system greatly exceeds the correlation length, $L >> \xi$, by (4) and (5) we write

$$A_L(\lambda) \sim \xi^{-\tau_s} \ ; \ (L >> \xi, \ \lambda \to \lambda_c) \tag{6}$$

In the opposite case, $L << \xi$, the system size takes over the scaling behavior and (6) turns into

$$A_L(\lambda) \sim L^{-\tau_s} \ ; \ (L << \xi, \ \lambda \to \lambda_c) \tag{7}$$

Taken together, (6) and (7) define the *finite-size scaling* (FSS) ansatz

$$A_L(\lambda) = \xi^{-\tau_s}\Phi(L/\xi) \; ; \; (L \to \infty, \; \lambda \to \lambda_c) \tag{8}$$

where the scaling function controls the finite-size effects of critical behavior and is defined as

$$\Phi(x) = \begin{cases} const; & |x| >> 1 \\ x^{-\tau_s} \; ; & x \to 0 \end{cases} \tag{9}$$

To transition from the framework of critical phenomena to SOC, one simply identifies the correlation length with the concept of *avalanche-size*, i.e.,

$$s = \xi \; ; \quad s_c = L \tag{10}$$

The probability distribution defining the FSS ansatz in SOC is a natural extrapolation of (8) and takes the form of a probability distribution [8]

$$P(s,L) \sim s^{-\tau_s}\Phi(s/s_c) \text{ for } s >> 1, \; L >> 1 \tag{11a}$$

$$s_c(L) \sim L^{D_0} \text{ for } L >> 1 \tag{11b}$$

in which τ_s and D_0 are called the *avalanche-size exponent* and the *avalanche dimension*, respectively. Quite generally, (11) shows that, for a system of finite extent and large size avalanches, the avalanche-size probability behaves as a fractal function times a generic scaling function. To enable all moments of (11) to exist, the scaling function must decay sufficiently fast. One obtains the following representation of the scaling function upon power expanding it around zero,

$$\Phi(x) \sim \begin{cases} \Phi(0)+\Phi'(0)x+\frac{1}{2}\Phi''(0)x^2+\ldots, & x<<1 \\ \rightarrow 0, & x>>1 \end{cases} \tag{12}$$

The avalanche-size probability must be normalized to unity and its average be diverging along with $L\rightarrow\infty$, which leads to the following constraints

$$\sum_{s=1}^{\infty} P(s;L)=1 \qquad \text{for } L<\infty, \tag{13}$$

$$\langle s \rangle = \sum_{s=1}^{\infty} sP(s;L)\rightarrow\infty \ \text{ for } L\rightarrow\infty \tag{14}$$

Under the assumption that $\Phi(0)\neq 0$, the behavior of (11) for an infinite system size may be approximated as

$$\lim_{L\rightarrow\infty} P(s;L) \sim s^{-\tau_s}\Phi(0) \tag{15}$$

Furthermore, to comply with (13) and (14), the avalanche-size exponent must fall in the range

$$1<\tau_s\leq 2 \tag{16}$$

It is important to note that, while SOC has a clear *statistical* underpinning as described by (11), the power spectra (1)-(3) are based on *deterministic* measurements unrelated to probabilistic assumptions. A helpful analogy between (1)-(3) and (11) is nevertheless possible, with the caveat that (16) is not necessarily relevant insofar as the CMB spectrum is concerned.

With these considerations in mind, we set up next a parallel between (1)-(3) and the slowly driven evolution of SOC towards a *non-equilibrium steady state*.

3. CMB as non-equilibrium steady state

Since it is always convenient to work with dimensionless entities, we normalize the wavenumbers entering (1)-(3) according to

$$k^0 = \frac{k}{K}, \quad k_*^0 = \frac{k_*}{K} \tag{17}$$

in which K stands for a suitably chosen reference value. Comparative inspection of (1)-(3) and (11) suggests the term-by-term identification

$$s^0 = \frac{k^0}{k_*^0}, \quad k^0 = s^0 k_*^0 = \frac{s^0}{s_c^0} \tag{18}$$

$$s_c^0 = (k_*^0)^{-1} = \frac{K}{k_*} \tag{19}$$

Following [8], the critical avalanche size (19) scales with the maximal extension of the wavenumber space (or ultraviolet cutoff) Δ_{UV} as in

$$s_c^0 \sim (\Delta_{UV})^{D_0} \tag{20}$$

Labeling the scaling function and avalanche-size exponent by, respectively,

$$\Phi(\frac{s^0}{s_c^0}) = A_s\, f(k^0)\,,\;\; \tau_s = 1 - n_s \tag{21}$$

enables one to cast (2)-(3) in the same form as (11), namely,

$$\boxed{P_R(s^0) = s_0^{-\tau_s}\,\Phi(\frac{s^0}{s_c^0}),\;\text{for}\;\Delta_{UV} >> 1,\;\; s^0 >> 1} \tag{22a}$$

$$\boxed{s_c^0 \sim (\Delta_{UV})^{D_0}\,,\;\text{for}\;\Delta_{UV} >> 1} \tag{22b}$$

Note that the scaling function $\Phi(...)$ is nearly constant for $s^0 << s_c^0$ in the limit of unbounded wavenumbers $\Delta_{UV} \to \infty$ [8]. By default, this condition corresponds to the regime of infinitesimal spatial separations in the CMB map, where the spectra (1) and (2) are nearly scale-invariant.

We close with few remarks that (in our view) are important for future extensions of this work:

1) among the most straightforward scaling functions $\Phi(...)$ that may be considered in simulations are the Heaviside step-function and the exponential function, where the latter characterizes so-called "branching SOC processes" [8].

2) the CMB angular power spectrum displayed in [9] falls off at large multipole moments in a strikingly similar manner with the data collapse of [8, page 284]. Does this analogy further supports our approach or does it arise from a different rationale altogether?

3) To gain credibility, follow-up extensions of these ideas must successfully recover the large-scale power anomaly described by the parameter $S_{1/2}$, as well as the lensing amplitude A_L derived in [1].

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