Liquid surface tension.
Meniscus of liquid at a vessel wall

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Abstract

This article is a continuation of the previous study [1] devoted to the physical phenomena, connected with a surface tension of liquid. The meniscus of liquid at a vessel wall is one of the most known effects of such nature.

As well as the problem of the form of hanging and sitting drops of liquid, the analysis of meniscus form traditionally is based on the solution of Laplace equation [2].

In the given article the problem of meniscus is examined from the point of view of mechanical balance of a liquid under surface tension, gravity and wettability of a wall surface. The obtained solutions are convenient for computer calculations, and at the same time have necessary universality.

More and more exact and detailed knowledge about, apparently, well-known physical phenomena are indispensable for the development of modern technologies and technical equipment. In the full measure it concerns the study of liquid properties caused by surface tension [3].

1. Meniscus of liquid at an infinite straight wall

Perhaps it is the most simple problem to study meniscus under ideal circumstances when liquid has unlimited area of a surface and comes in contact with infinite straight vertical wall.

Depending on wettability of a wall surface the level of a liquid will be or above, or below the level of a liquid observed at a great distance from a wall.

On fig. 1 the variant with great wettability of wall surface is shown when liquid rises up the wall, and the surface tension draws upwards more remote areas of liquid.

It occurs due to negative curvature of liquid surface creating under the surface negative pressure *p* (in relation to surrounding atmospheric pressure).

Laplace equation in an examined variant has a most simple form:

$$p = \frac{\sigma}{R}. \qquad (1)$$

Here *R* - radius of curvature of a liquid surface,

σ - surface tension force.

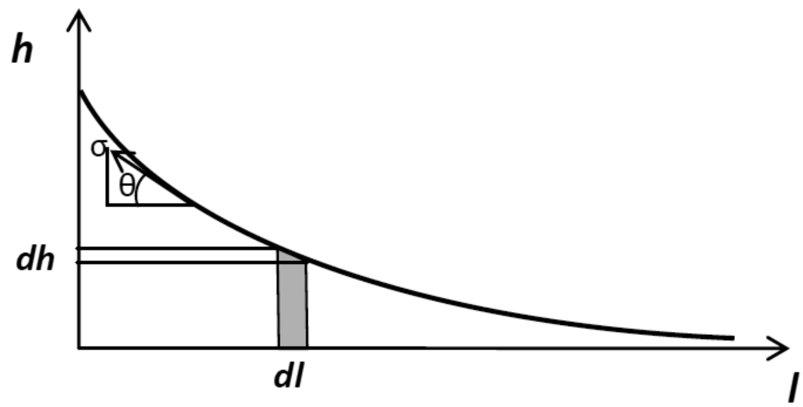

Fig. 1. Meniscus of liquid at a straight wall.

However the simplicity of equation notation does not always mean that it should have the simple analytical solution. It is quite true in respect of Laplace equation. Therefore we shall try to solve the formulated variant of meniscus form problem in a different way.

To set up an equation we shall use balance of forces which support mechanical equilibrium of a liquid (fig. 1):

$$\rho g h \frac{dl}{dh} dh = \frac{d}{dh}(\sigma \sin \Theta) dh. \qquad (2)$$

In the left part of the equation (2) is written down the weight of a layer of the liquid marked by dark color on fig. 1 which is above "zero" level (level at infinity). The height of this layer is equal *h*, the width is equal *dl*, and length we shall accept to be equal unit (in direction perpendicular to the plane of figure).

The derivative *dl/dh,* set in the left side of the equation, enables to regard *h* as an argument of the sought dependence that in this case is more convenient for the solution of a problem.

In the right side of (2) the force acting on the mentioned thin layer of a liquid (an increment of vertical component of surface tension force) is written down.

The solution of the equation (2) is convenient to stage.

In the first stage a solution at great distance from the wall is tried for.

In the second stage the solution in a near zone is searched in view of results, received at the first stage.

Far from a wall influence of a meniscus is small, hence the sine in the equation is so small that it can be replaced with tangent, that is, can be expressed through a derivative *dh/dl*.

So instead of the equation (2), it is possible to write down balance of forces and the meniscus equation will have a more simple form:

$$\rho g h dl = \frac{d}{dl}\left(\sigma \frac{dh}{dl}\right) dl. \qquad \frac{d^2 h}{dl^2} = \frac{\rho g}{\sigma} h. \tag{3}$$

The solution of this equation is the exponential function.

$$h = H \exp\left(-\sqrt{\frac{\rho g}{\sigma}} \cdot l\right) = H \exp\left(-\sqrt{\varsigma} \cdot l\right) = H \exp\left(-\frac{l}{l_0}\right). \tag{4}$$

Here *H* - some very small size chosen in view of the requirement of small *h* in the area distant from a wall (for this calculation it was taken *H* = 0,01 mm);

$\varsigma = \rho g / \sigma$ - so-called *capillary constant*;

$l_o = 1/\sqrt{\varsigma}$ - distance from a wall where the height of a meniscus decreases in *e* times (for water this distance is approximately *2,7 mm*).

The equation (3) can be set down, using dimensionless sizes

$$\frac{d^2 \alpha}{d\beta^2} = \Phi \alpha. \tag{5}$$

Here $\Phi = H^2 \rho g / \sigma$ - *Bond number*, $\alpha = h/H$, $\beta = l/H$ - relative coordinates.

The solution of the equation (5) accordingly looks like:

$$\alpha = \exp\left(-\sqrt{\Phi}\beta\right). \tag{6}$$

The first derivative of function (6) is

$$\frac{d\alpha}{d\beta} = -\sqrt{\Phi}\exp\left(-\sqrt{\Phi}\beta\right) = -\sqrt{\Phi}\cdot\alpha. \qquad (7)$$

The second derivative, obviously, coincides with (5).

Using the results of the first stage, we shall pass to the second stage of the solution of the equation (2), that is, in a near zone.

Let's set down the equation (2), using dimensionless quantities (similarly to the equation (5))

$$\frac{d}{d\alpha}(\sin\Theta) = \Phi\alpha\frac{d\beta}{d\alpha}. \qquad (8)$$

Let's transform the equation (8), using known trigonometric identity

$$\sin\Theta = \frac{1}{\sqrt{1+ctg^2\Theta}} = \frac{1}{\sqrt{1+\left(\frac{d\beta}{d\alpha}\right)^2}}. \qquad (9)$$

After differentiation of the left part the equation (8) gets a following form:

$$\frac{d^2\beta}{d\alpha^2} = -\Phi\alpha\left(1+\left(\frac{d\beta}{d\alpha}\right)^2\right)^{3/2}. \qquad (10)$$

The numerical solution of equation was carried out using program *Excel,* in view of results of calculation in the far zone (6), (7). Thus for a reference position we shall take exponential function (6), that is, the origin of coordinates proves to be far from a wall.

Hence, while filling the first table line of calculation in *Excel* are used following values (taken from the first stage of solution):

$\beta = 0$, $\alpha = 1$ ($H = 0{,}01mm = 10^{-5}m$), $d\beta/d\alpha = (d\alpha/d\beta)^{-1} = -(1/\Phi)\alpha$, $\Phi = 1{,}35\cdot 10^{-5}$.

Besides in the first line by means of formula (8) the second derivative is determined using values already set down in this line.

And, at last, in the same line multiplication by step value $\Delta\alpha = 0{,}65$ gives an increment of the first derivative (proceeding from the second derivative), and also an increment $\Delta\beta$, (proceeding from the first derivative $d\beta/d\alpha$). The step value $\Delta\alpha$ is selected to provide sufficient accuracy of calculation (it is discussed below).

The second line is to be filled similarly to the first line in view of the increments of values got in the first line.

Next lines are filled in by means of autofilling instrument.

Graphic representation of results of calculation is also carried out in computer program *Excel,* but the diagramme *β (α)* is more convenient to draw in natural view as it is shown on fig. 2 (*α* – standing axis, and *β* – horizontal axis).

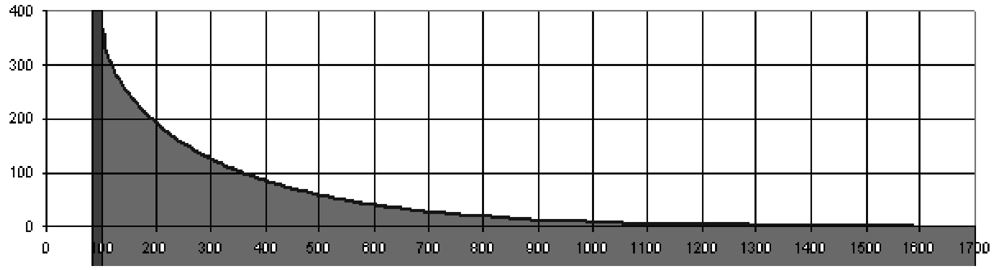

Fig. 2. Meniscus at an infinite straight wall
(In relative coordinates).

The universal curve of meniscus represented on fig. 2, can be used for any liquids, wetting angles and other conditions.

For example, the meniscus height of water at straight wall with best wettability does not exceed about *380 H = 380·0,01mm = 3,8mm.*

For other liquids or conditions it is necessary to select *H* so that the Bond number $H^2\rho g/\sigma$ stays constant.

In particular, in conditions of small free fall acceleration on the Moon ($g = 1{,}62\ m/s^2$) height of a meniscus can exceed *9 mm*, that is, approximately *2,4* times more (root of ratio *9,8/1,62*) and also in this proportion slower meniscus decreases when distance from a wall rises.

Universal curve (fig. 2) describes a case of meniscus at a wall with ideal wetting (becomes vertical at wall).

Thus, there was conditionally removed the restriction associated with wetting angle depending on a surface tension at interphase boundary (σ_{SG} - solid body - gas, σ_{SL} - solid body - liquid, σ_{LG} - liquid - gas):

$$\cos\Theta = \frac{\sigma_{SG} - \sigma_{SL}}{\sigma_{LG}}. \qquad (11)$$

Young equation (11) is a condition of mechanical equilibrium of a liquid and usually is a starting point for calculations.

However, in consideration of universality of the meniscus form, it is logical to impose boundary conditions (11) on the obtained full solution.

Really, putting a vertical plane (imitating wall) in places having various horizontal coordinates, it is possible to get different wetting angles at an imaginary wall (fig. 3).

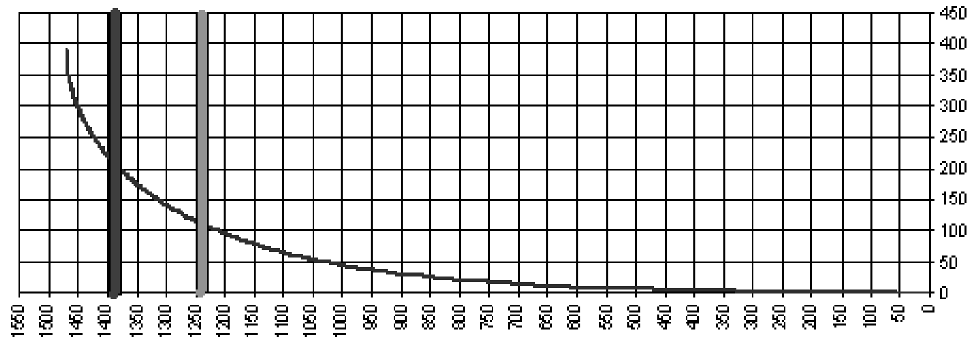

Fig. 3. Putting a vertical plane (imitating wall) in places with various horizontal coordinates, it is possible to get different wetting angles at an imaginary wall.

Let's also note that in case of wall surface poor wettability the universal curve of meniscus will be in negative region symmetric with respect to a horizontal axis of coordinates.

To simplify the notation in formulas and in the text is used the reduced writing of liquid surface tension σ though, strictly speaking, it is necessary to use more exact designation - σ_{LG} as in formula (11).

Turning back to the issue of accuracy of meniscus calculation, we shall mention simple but enough effective method of control.

Numerical integration of the area under a meniscus curve was carried out, then this value was multiplied by H^2, by water density and by free fall acceleration (the resulting amount is a specific per meter of wall length).

The received value is to be compared with the surface tension force of water. In our case the «weight of a meniscus» is about 0,072537 and turns out very close to known quantity of surface tension force 0,0728.

2. Meniscus of liquid at an external cylinder wall

Let's examine variant of meniscus at external wall of a cylinder facing infinite water surface surrounding it (fig. 4).

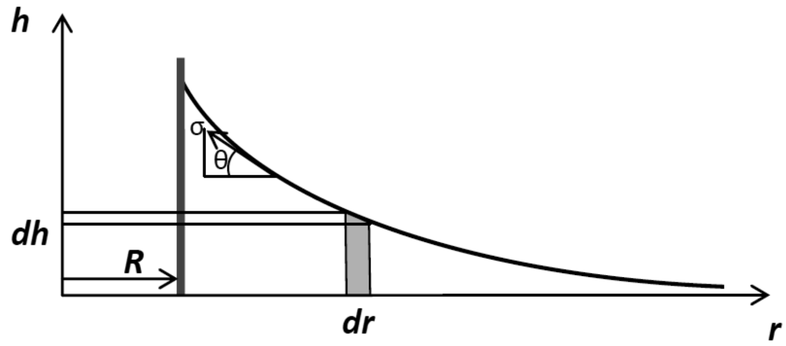

Fig. 4. Meniscus of liquid at external wall of a cylinder.

The equation of meniscus is set up similarly to the equation (2)

$$\rho gh \cdot 2\pi r \frac{dr}{dh} dh = \frac{d}{dh}(2\pi r \sigma \sin \Theta) dh. \tag{12}$$

Unlike the conditions of equation (2), here the equilibrium of a cylindrical thin liquid layer having height *h*, radius *r* and thickness *dr* is considered.

After differentiation of the right side of (12) in view of expression (9) the differential equation of meniscus takes on form:

$$\frac{d^2 r}{dh^2} = \left(1 - \frac{\rho g}{\sigma} hr\sqrt{1 + \left(\frac{dr}{dh}\right)^2}\right) \cdot \frac{1}{r}\left(1 + \left(\frac{dr}{dh}\right)^2\right). \tag{13}$$

Or in relative coordinates:

$$\frac{d^2 \beta}{d\alpha^2} = \left(1 - \Phi\alpha\beta\sqrt{1 + \left(\frac{d\beta}{d\alpha}\right)^2}\right) \cdot \frac{1}{\beta}\left(1 + \left(\frac{d\beta}{d\alpha}\right)^2\right). \tag{14}$$

Here $\Phi = H^2 \rho g/\sigma$ - *Bond number*, $\alpha = h/H$, $\beta = r/H$ - relative coordinates,

H - some very small size chosen in view of the condition that *h* is small in the area remote from a wall (here, as well as in case of the straight wall, examined in pt.1, was assumed *H = 0,01 mm*).

Unlike a meniscus at a straight wall in this case the solution of the equation depends on radius of the cylinder and consequently it is not unique. Thus, number of functions describing meniscus at an external surface of cylinder is unlimited.

Numerical solution of the problem of meniscus at external wall of a cylinder can be carried out, using the equation (14) similarly to pt. 1, but it is possible to do it in a different rather evident way.

Let's set down the equation (12) in other form:

$$\begin{aligned} &\rho gh \cdot 2\pi r dr = \frac{d}{dr}(2\pi r \sigma \sin\Theta)dr, \\ &\rho ghrdr = d(r\sigma \sin\Theta). \end{aligned} \qquad (15)$$

Or, using dimensionless quantities, we have:

$$\Phi\alpha\beta \cdot d\beta = d(\beta \sin\Theta). \qquad (16)$$

Integration of the equation (16) is carried out step by step from some remote point r_o, (or β_o) where very small value of height of meniscus *H* was set:

$$\int_{\beta_0}^{\beta} \Phi\alpha\beta \cdot d\beta = \beta \sin\Theta. \qquad (17)$$

For the reason that in this remote point the form of a meniscus approaches to exponent as at a straight wall (pt. 1), the initial value of sine in this conditions is set equal to the root of Bond number (7).

In process of numerical integration of the left and right sides of the equation the sine increases, as the general «weight of meniscus» grows proportionally to integral in the left side of the equation (17).

Accordingly the increment of height *dα* depends on *tg(θ)* or *dα/dβ,* that is, angle *θ*, which, in turn, is determined as *arcsin(θ)*.

In fig. 5 - 7 results of meniscus calculation for three different radiuses of cylinder are shown.

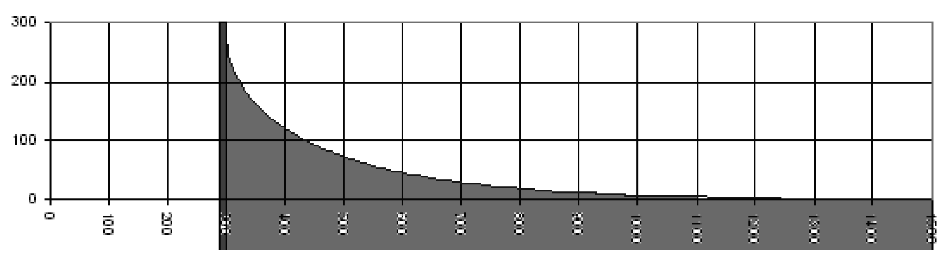

Fig. 5. Meniscus of water at an external wall of cylinder

(*R = 3mm*, height of meniscus is about *2,5mm*).

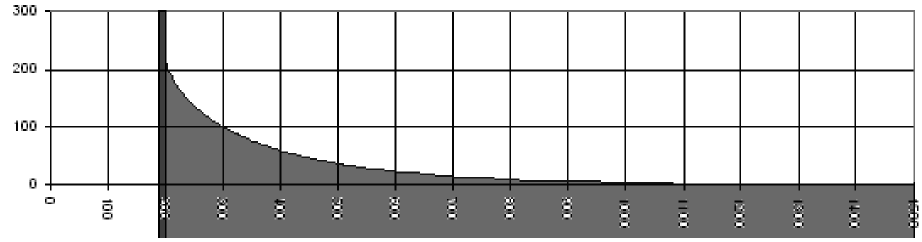

Fig. 6. Meniscus of water at an external wall of cylinder
(R = *2mm*, height of meniscus is about *2,2mm*).

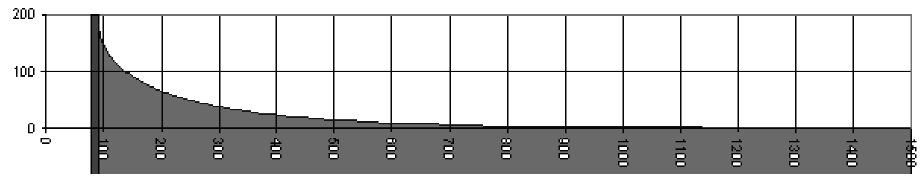

Fig. 7. Meniscus of water at an external wall of cylinder
(R = *0.9mm*, height of meniscus is about *1,7mm*).

Comparison of lines represented in these figures shows, that reduction of radius of the cylinder causes decrease of meniscus height.

It should be also noted that the height of meniscus at external wall of a cylinder in whole is less than at a straight wall (pt. 1) where it reaches in case of water about *3,8mm*.

3. Meniscus of liquid between two infinite straight walls

Setting up the meniscus equation of liquid between two infinite walls we shall reason by analogy with analyses of sitting and hanging drops [1].

In the lowermost area of meniscus we shall specify radius of curvature of liquid surface R. Counting of height h we shall do basing on a minimum level of a meniscus though in the sequel we shall also consider zero level determined by external pressure.

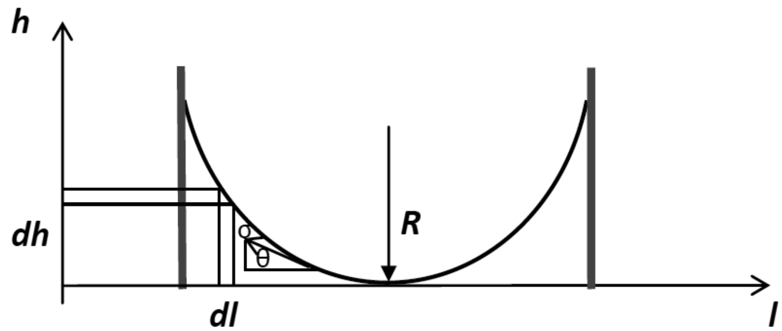

Fig. 8. Meniscus of liquid between two straight walls.

The equation describing equilibrium of a thin liquid layer (fig. 8) with height *h*, width *dl* and unit length (perpendicular to the plane of figure) is of the form:

$$\left(\frac{\sigma}{R}+\rho g h\right)\cdot\frac{dl}{dh}dh=\frac{d}{dh}(\sigma\sin\Theta)dh. \qquad (18)$$

Using expression (9) and differentiating the right side of equation, we get

$$\frac{d^2 l}{dh^2}=\left(\frac{1}{R}+\frac{\rho g}{\sigma}h\right)\cdot\left(1+\left(\frac{dl}{dh}\right)^2\right)^{3/2}. \qquad (19)$$

Or in relative quantities:

$$\frac{d^2\beta}{d\alpha^2}=(1+\Phi h)\cdot\left(1+\left(\frac{d\beta}{d\alpha}\right)^2\right)^{3/2}. \qquad (20)$$

The numerical solution of this equation is carried out in the same way as it was already done at the solution of the equation (10). But, in view of facilities of program *Excel*, it is possible to make it similarly to the solution of equation (15).

Then the equation (18) takes form:

$$(1+\Phi\alpha)\cdot d\beta=d(\sin\Theta). \qquad (21)$$

Here $\Phi = R^2\rho g/\sigma$ - *Bond number*, $\alpha = h/R$, $\beta = l/R$ - relative coordinates.

Integration of the equation (21) also is carried out step-by-step, starting from a minimum of a meniscus, where $h = 0$, (or $\alpha = 0$); $l = 0$ (or $\beta = 0$)

$$\int_0^\beta (1+\Phi\alpha)d\beta=\sin\Theta. \qquad (22)$$

Besides at filling the first line (first step) of calculation it is necessary to consider, that near the zero it is possible to write down:

$$h(2R-h)=l^2, \qquad h \approx \frac{l^2}{2R}, \qquad \alpha \approx \frac{1}{2}\beta^2. \qquad (23)$$

In process of numerical integration of the left and right sides of the equation the sine increases, as «the summed up part of meniscus weight» grows proportionally to integral in the left side of the equation (22).

As before, the increment of height *dα* is determined through *tg (θ)* (or *dα/dβ*), and a angle *θ*, in turn, is determined as *arcsin(θ)*.

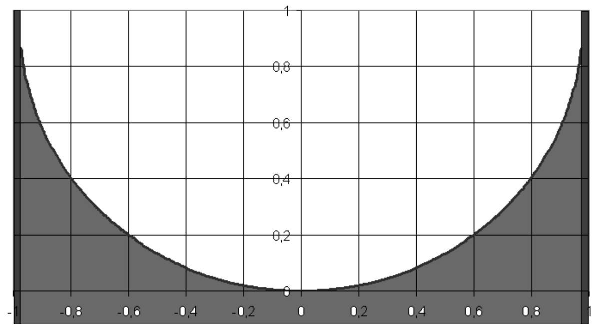

Fig. 9. Meniscus of liquid between two straight walls, $\Phi \approx 0,135$
(R = *1mm,* meniscus ≈*0,83 mm*, distance between walls ≈*2 mm*, $h_o \approx 7,4mm$).

In fig. 9 results of calculation of meniscus for R = *1mm* are shown.

It is clear, that at small distances between walls the meniscus section in form is close to semi circumference.

The lowermost area of meniscus proves to be above a zero-level (meeting external pressure). It means that curve of meniscus (fig. 9) as a whole is to be raised:

$$h_0 = \frac{\sigma}{R} \cdot \frac{1}{\rho g}, \qquad \alpha_0 = \frac{h_0}{R} = \frac{\sigma}{\rho g R^2} = \frac{1}{\Phi}. \qquad (24)$$

Thus, at distance between walls about 2 mm water level between them rises in addition to the meniscus represented in figure (≈0,83 mm) even more than by 7,4 mm ($\alpha_o = 1/\Phi \approx 7,43$).

At increase in distance between walls the form of meniscus more and more deviates from cylindrical (fig. 10 - 13), the radius R in meniscus minimum increases, and the zero-level of liquid decreases (24).

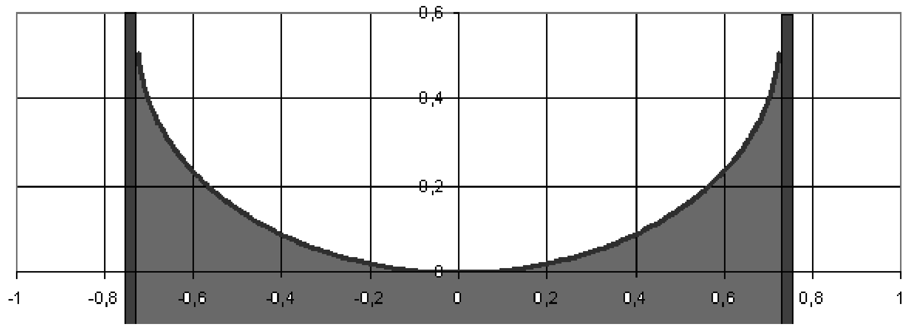

Fig. 10. Meniscus of liquid between two straight walls, Φ ≈ *3,36*

(*R = 5mm,* meniscus ≈*2,25 mm*, distance between walls ≈*7,25 mm*, $h_o \approx 1{,}49mm$).

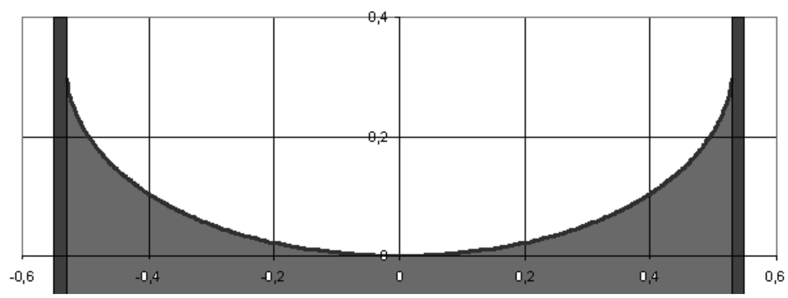

Fig. 11. Meniscus of liquid between two straight walls, Φ ≈ *13,46*

(*R = 10mm,* a meniscus ≈*3 mm*, distance between walls ≈*10,5 mm*, $h_o \approx 0{,}74mm$).

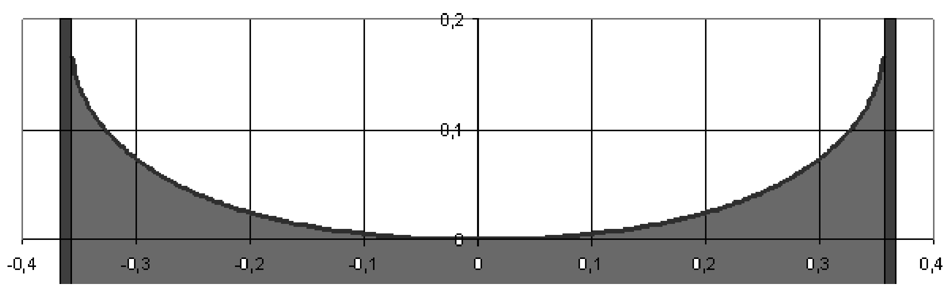

Fig. 12. Meniscus of liquid between two straight walls, Φ ≈ *53,85*

(*R = 20mm,* a meniscus ≈*3,2 mm*, distance between walls ≈*14,5 mm*, $h_o \approx 0{,}37mm$).

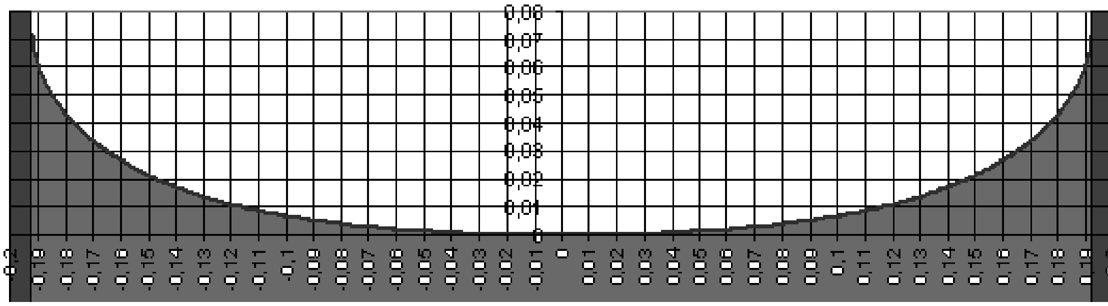

Fig. 13. Meniscus of liquid between two straight walls, Φ ≈ *336,5*
(*R = 50mm,* a meniscus ≈*3,5 mm*, distance between walls ≈*19,3 mm*, h_o ≈ *0,15mm*).

Simultaneously with it, the actual height of meniscus (the difference of liquid levels at a wall and in the middle between walls) increases and at the parameters shown in fig. 13, the meniscus nearly reaches limiting value at a single straight wall (pt. 1).

4. Meniscus of liquid at an inner surface of cylinder

The equation of meniscus of liquid inside of a cylinder (tube) is set up in similar terms of (12) and (18):

$$\left(\frac{2\sigma}{R}+\rho g h\right)\cdot r\frac{dr}{dh}dh=\frac{d}{dh}(r\sigma\sin\Theta)dh. \qquad (25)$$

Using expression (9) and differentiating the right side of the equation, we get

$$\frac{d^2r}{dh^2}=\left(\left(\frac{2}{R}+\frac{\rho g}{\sigma}h\right)+\sqrt{\frac{1}{1+\left(\frac{dr}{dh}\right)^2}}\cdot\frac{1}{r}\right)\cdot\left(1+\left(\frac{dr}{dh}\right)^2\right)^{3/2}. \qquad (26)$$

Or in relative quantities:

$$\frac{d^2\beta}{d\alpha^2} = \left((2+\Phi\alpha) + \sqrt{\frac{1}{1+\left(\frac{d\beta}{d\alpha}\right)^2}} \cdot \frac{1}{\beta} \right) \cdot \left(1+\left(\frac{d\beta}{d\alpha}\right)^2\right)^{3/2}. \qquad (27)$$

Interestingly enough that the equation of meniscus of liquid inside of a cylinder (27) actually agrees with the equation of a sitting drop form [1].

The equation (25) can be also set up in a following form:

$$(2+\Phi\alpha)\beta \cdot d\beta = d(\beta \sin\Theta). \qquad (28)$$

Here $\Phi = R^2\rho g/\sigma$ - *Bond number*, $\alpha = h/R$, $\beta = r/R$ - relative coordinates; R – radius of curvature of liquid surface in the center of a cylinder.

Integration of the equation (28), as before, is carried out step by step, starting from a minimum point of meniscus, where $h = 0$ $(\alpha = 0)$; $r = 0$ $(\beta = 0)$

$$\int_0^{\beta} (2+\Phi\alpha)\beta d\beta = \beta \sin\Theta. \qquad (29)$$

At filling in the first line (first step) of calculation table it is also necessary to keep in mind that near the zero it is trues the approximate expression:

$$\alpha \approx \frac{1}{2}\beta^2. \qquad (30)$$

In process of numerical integration of the left and right sides of the equation the sine increases, as «the summed up part of meniscus weight » grows proportionally to integral in the left side of the equation (29).

Thus the zero-level (h_o) proves to be twice more, than it was in case of two parallel straight walls (24) as curvature of liquid surface in this case exists in two mutually perpendicular directions:

$$h_0 = \frac{2\sigma}{R}\cdot\frac{1}{\rho g}, \qquad \alpha_0 = \frac{h_0}{R} = \frac{2\sigma}{\rho g R^2} = \frac{2}{\Phi}. \qquad (31)$$

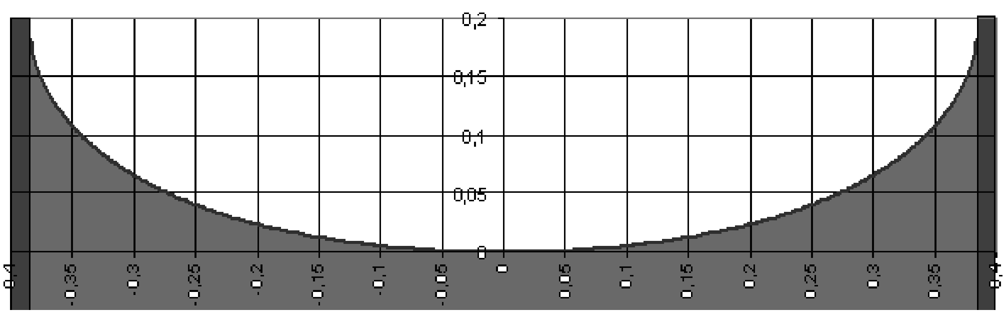

Fig. 14. Meniscus of liquid inside the cylinder, $\Phi \approx 53{,}84$ (*R = 20mm,* meniscus ≈*3,5 mm*, radius of the cylinder ≈*7,6 mm*, $h_o \approx 0{,}74mm$).

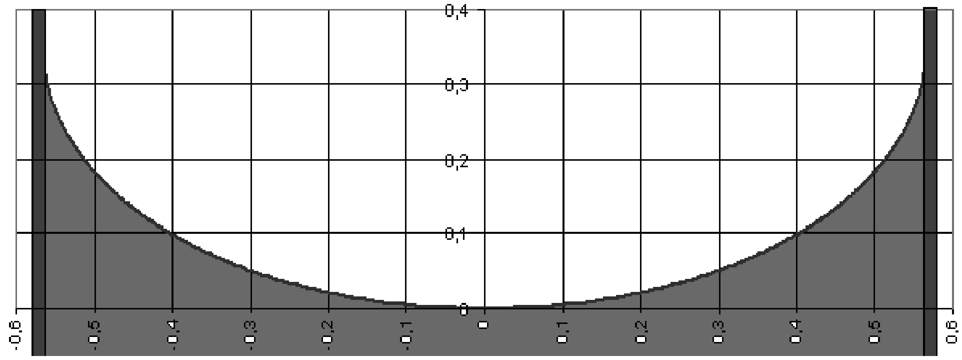

Fig. 15. Meniscus of liquid inside the cylinder, Φ ≈ *13,46*

(*R = 10mm,* meniscus ≈*3,1 mm*, radius of the cylinder ≈*5,7 mm*, h_o ≈ *1,48mm*).

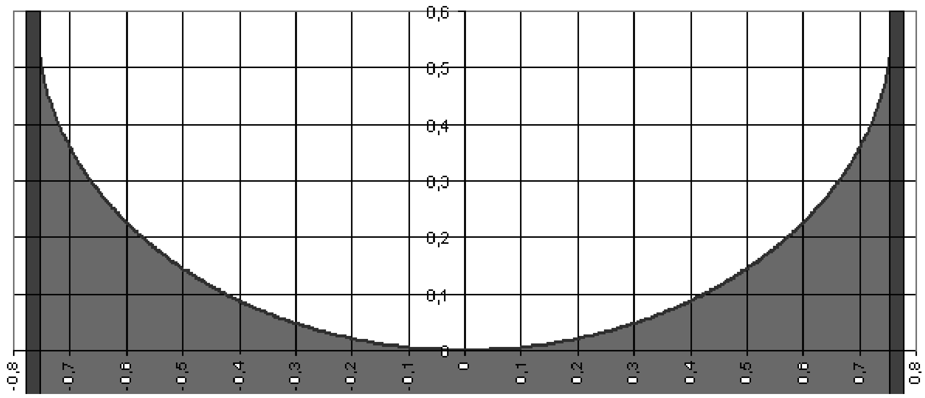

Fig. 16. Meniscus of liquid inside the cylinder, Φ ≈ *3,36*

(*R = 5mm,* meniscus ≈*2,5 mm*, radius of the cylinder ≈*3,8 mm*, h_o ≈ *2,97mm*).

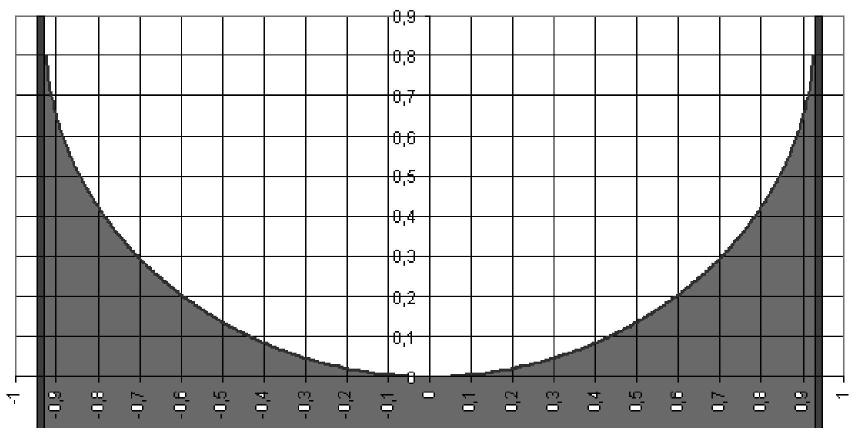

Fig. 17. Meniscus of liquid inside the cylinder, Φ ≈ *0,538*

(*R = 2mm,* a meniscus ≈*1,6 mm*, radius of the cylinder ≈*1,85 mm*, h_o ≈ *7,43mm*).

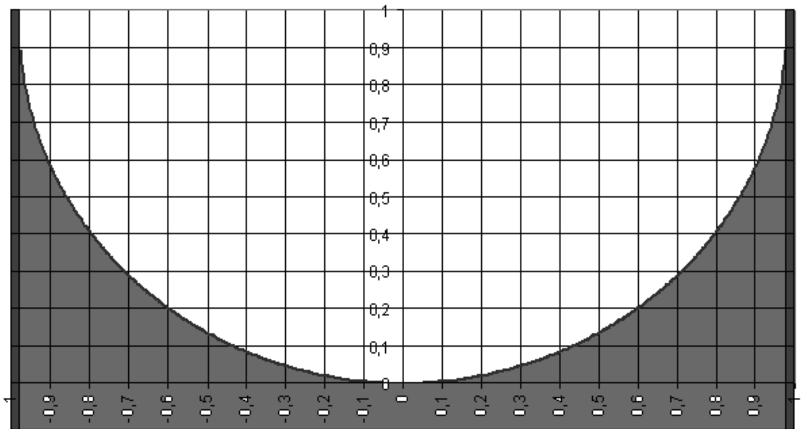

Fig. 18. Meniscus of liquid inside the cylinder, Φ ≈ *0,1346*
(*R = 1mm,* meniscus ≈*0,9 mm*, radius of the cylinder ≈*0,98 mm*, h_o ≈ *14,86mm*).

Just as it was in case of meniscus of liquid between two straight walls, the increment in curvature radius *R* (in meniscus minimum) more and more causes deviation of meniscus form the spherical from, and simultaneously the total level h_o decreases (fig. 14 - 18).

The above mentioned identity of equations of sitting drop and of liquid meniscus inside a cylinder appears, in particular, in the fact that the form of the top part of a drop coincides with the form of meniscus inside of the cylinder when there is poor wettability (fig. 19).

We have specially made calculation of meniscus with the same initial radius of surface curvature in the center *R = 8,8mm,* as well as of the sitting drop [1]. Results of calculations not only confirm the identity of these forms, but also indicate satisfactory calculation accuracy.

As before, in all the calculations as liquid was taken water that does not affect at all the generality of the received results due to the equations universality (via Bond number).

Let's also notice, that as well as in case of meniscus of liquid between two straight walls, the meniscus inside of the cylinder is described by different functions depending on concrete conditions, that is, these variants have unlimited number of solutions.

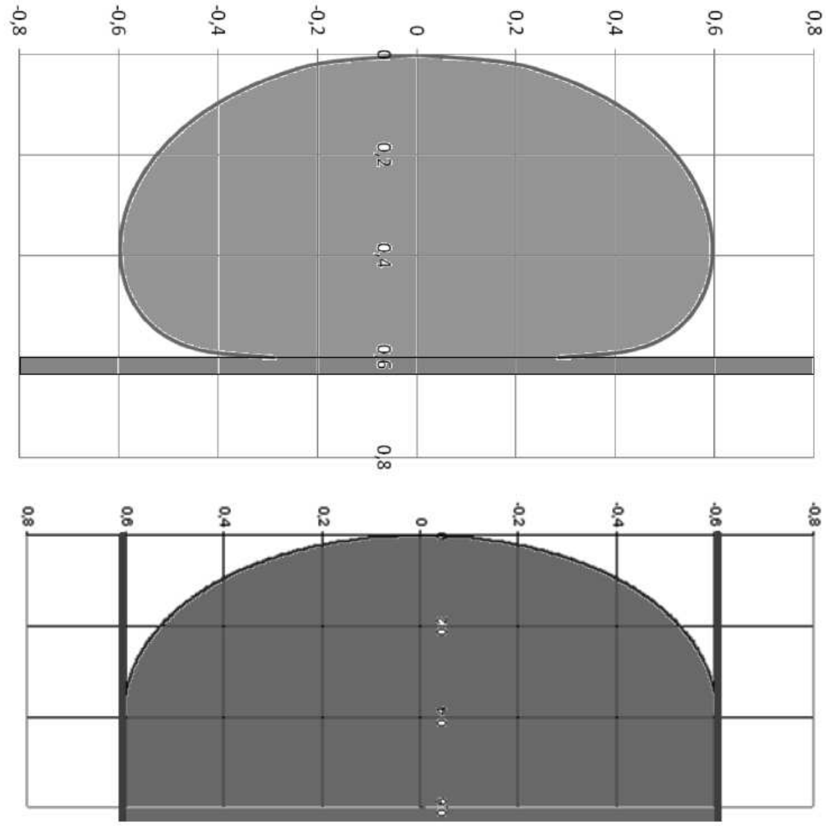

Fig. 19. At the bottom of the figure the meniscus inside of the cylinder with poor wetting is shown, $\Phi \approx$ *10,42* (*R = 8,8mm,* meniscus ≈*-3,5 mm*, radius of the cylinder ≈*5,28 mm*, h_o ≈*-1,69mm*). In the top of the figure as basis of comparison the sitting drop with similar parameters [1] is represented.

Depending on concrete conditions of contact of liquid with wall surface (wetting angle) on the basis of general solutions any specific solution providing conditions of liquid equilibrium (just as it is shown in fig. 3) can be got.

5. Conclusion

Opportunities of modern computer equipment make it possible to solve problems, which in rather recent past required a huge investment of time and laborious work.

Therefore in the last decades many problems not having the simple analytical solution have not been properly studied.

At the same time, in quite a number of cases detailed studying of physical phenomena about which there are approximate, insufficient knowledge, has not only educational, cognitive value. Development of modern technologies and techniques requires expanded, more detailed study of various physical phenomena and substance properties.

In the given paper the problem of meniscus of liquid is studied not by means of traditional solution of Laplace equation, but using analysis of mechanical equilibrium of a liquid under gravity, liquid surface tension and wettability at separation surface of solid and liquid. Such approach is not only obvious, but also is convenient for the numerical solution.

Analysis of liquid meniscus equations was carried out in various variants of contact (with a straight infinite wall, with the external side and with the internal side of cylinder, between two parallel straight walls). It was shown that meniscus equations in all cases can be expressed in dimensionless coordinates using Bond number (also dimensionless) and capillary constant.

Unlike the capillary constant, the Bond number depends on a choice of some characteristic parameter – linear segment or radius (in each case the choice is specific and almost arbitrary).

The opportunity to abstract from concrete boundary conditions (wetting angle) allows to derive universal solutions and to reveal on this basis general regularities.

Envelope lines of meniscus (solutions of equations) actually are *special mathematical functions* $\alpha_\Phi(\beta)$. In particular, in case of meniscus at a straight wall this special function at a great distance from a wall tends to exponential function.

It is necessary to emphasize especially, that the form of meniscus of liquid at a wall with poor wettability (other conditions being equal) is symmetric reflection of meniscus when there is ideal wettability (with respect to horizontal axis).

It is remarkable that form of meniscus of liquid inside the cylinder coincides with the form of top part of sitting drop (with the same Bond number).

Calculations and plotting of meniscus forms of liquid were realized on the basis of analysis of liquid equilibrium conditions with use of the most widespread and accessible computer program *Excel*.

Naturally, the proposed approach is not universal or replacing other methods of the analysis, but, undoubtedly, it can appear very convenient and useful addition to already known methods of studying of liquid properties.

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