

Polynomials Generating Twin Prime Numbers

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Abstract

In the Ulam spiral, there are places where prime numbers appear continuously on line. Integers are arranged in a square spiral in the Ulam spiral. I thought that if integers are arranged differently, other continuous prime numbers would appear. Therefore, I arrange integers in the angles of 45, 90, 135, 180, 225, 270, 315, 153, 160 degrees, etc.. Then, prime numbers appeared continuously on line. And usually, integers are arranged, but I wonder what would happen if I arranged odd numbers. I arrange odd numbers in the angles of 45, 90, 135, 180, 225, 270, 315, 360, 153, 160 degrees, etc.. Then, twin prime numbers appeared continuously on line etc.. I found many polynomials generating 14 to 4 consecutive twin prime numbers.

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1 Introduction

I was interested in prime numbers looking at the Ulam spiral, I analyzed it myself. And I learned that Euler's polynomial generating prime numbers is simple and great. I thought that other polynomials generating prime numbers may be found in other arrangements, I investigate. In addition, I thought that polynomials generating twin prime numbers may be found by arranging odd numbers. I found many polynomials generating 14 to 4 consecutive twin prime numbers, and I collect the results.

These algebraic polynomials have the property that for $n = 0, 1, \dots, m-1$ value of the polynomial, eventually in module, are m primes.

2 Polynomials generating prime numbers

2.1 The Ulam spiral

In the Ulam spiral, there are places where prime numbers appear continuously on line. I noticed that there are places where prime numbers appear continuously in a certain pattern in the Ulam spiral, although they do not appear continuously on line. They are two polynomials, $P(n) = 4n^2 + 2n + 41$ and $P(n) = 4n^2 + 6n + 43$, generates 20 primes, see Figure 2.1. Each value is a value obtained by skipping one of Euler prime numbers. When the values of the two polynomials are inserted alternately, the values are the same as values of Euler prime numbers, see Figure 2.1.

2.2 Polynomial generating prime numbers 1

Integers are arranged in a square spiral in the Ulam spiral, but I thought that if integers are arranged differently, other continuous prime numbers would appear. Therefore, I arrange integers in the angles of 45, 90, 135, 180, 225, 270, 315, 153, 160 degrees, etc., using a computer. Then, prime numbers appeared continuously on line.

In 180 degrees arrangement, see Figure 2.2, 29 prime numbers appear continuously. It was prime numbers of Legendre polynomial [1798], $P(n) = 2n^2 + 29$, generates 29 primes: 29, 31, 37, 47, 61, 79, 101, 127, 157, 191, 229, 271, 317, 367, 421, 479, 541, 607, 677, 751, 829, 911, 997, 1087, 1181, 1279, 1381, 1487, 1597 .

2.3 Polynomial generating prime numbers 2

In 135 degrees arrangement, see Figure 2.3, 29 prime numbers appear continuously. It was prime numbers of Brox polynomial [2006], $P(n) = 6n^2 - 342n + 4903$ (or $6n^2 + 6n + 31$), generates 29 primes: 4903, 4567, 4243, 3931, 3631, 3343, 3067, 2803, 2551, 2311, 2083, 1867, 1663, 1471, 1291, 1123, 967, 823, 691, 571, 463, 367, 283, 211, 151, 103, 67, 43, 31 . Also, in Figure 2.3, prime numbers of polynomials, $P(n) = 6n^2 + 6n + p$, p are lucky numbers $p = 5, 7, 11, 17, 31$, are clearly appeared. In addition, prime numbers of polynomials, $P(n) = 6n^2 + p$, p are lucky numbers $p = 5, 7, 13, 17$, are clearly appeared.

2.4 Polynomial generating prime numbers 3

In 270 degrees arrangement, see Figure 2.4, 22 prime numbers appear continuously. It was prime numbers of Frame polynomial [2018], $P(n) = 3n^2 + 3n + 23$, generates 22 primes: 23, 29, 41, 59, 83, 113, 149, 191, 239, 293, 353, 419, 491, 569, 653, 743, 839, 941, 1049, 1163, 1283, 1409 .

2.5 Euler's polynomial generating prime numbers

In 90 degrees arrangement, see Figure 2.5 and the hexagonal 90 degrees arrangement, see Figure 2.6 (illustrated as a rectangle for simplification in Figure 2.6), 40 prime numbers appear continuously. It was prime numbers of Euler's polynomial, $P(n) = n^2 + n + 41$, generates 40 primes. Also, prime numbers of polynomial, $P(n) = n^2 + n + p$, p are Euler's lucky numbers $p = 3, 5, 11, 17, 41$, are clearly appeared.

2.6 Other polynomials generating prime numbers

I found polynomials generating prime numbers with small continuous numbers, and I will collect them in the future.

3 Polynomials generating twin prime numbers

Integers are arranged in the Ulam spiral, but I wonder what would happen if I arranged odd numbers. I arrange odd numbers in the angles of 45, 90, 135, 180, 225, 270, 315, 360, 153, 160 degrees, etc., using a computer. I mark the twin prime numbers. (In the figure of 360 degrees, I mark the prime numbers and twin prime numbers.) Then, twin prime numbers appeared continuously on line etc.. I found many polynomials generating twin prime numbers. The generating appearance of prime numbers are diagonal, vertical, and horizontal lines and evenly spaced, but the generating appearance of twin prime numbers are diagonal, vertical, horizontal, and curved lines and evenly spaced or not-evenly spaced.

3.1 Polynomial generating twin prime numbers 1

When odd numbers are arranged in 45 degrees arrangement, see Figure 3.1, continuous twin prime numbers appear.

The produce of polynomial is as follows. (Since the method of obtaining the polynomial in Section 3.1 is difficult to understand, so I recommend to refer to the method of obtaining the polynomial in Section 3.2.) The central values of the twin prime numbers are 12, 42, 102, 192, 312, 462, 642.

$\begin{array}{c} 12 \\ 30 \\ 42 \\ 60 \\ 102 \\ 30 \\ 90 \\ 192 \\ 30 \\ 120 \\ 312 \\ 30 \\ 150 \\ 462 \\ 30 \\ 180 \\ 642 \end{array}$	$\begin{array}{c} 12 \\ 42=(12+30) \\ =12+30\times 1 \\ 102=(12+30)+(30+30) \\ =12+30\times 2+30\times 1 \\ 192=(12+30)+(30+30)+(30+30+30) \\ =12+30\times 3+30\times 3 \\ 312=(12+30)+(30+30)+(30+30+30)+(30+30+30+30) \\ =12+30\times 4+30\times 6 \\ 462=(12+30)+(30+30)+(30+30+30)+(30+30+30+30) \\ +(30+30+30+30)=12+30\times 5+30\times 10 \\ 642=\dots \end{array}$	$\begin{array}{l} n=0 \\ \\ n=1 \\ \\ n=2 \\ \\ n=3 \\ \\ n=4 \\ \\ n=5 \end{array}$
	$f(n)=12+30n+30x(n-1)/2=12+30n+15n^2-15n=15n^2+15n+12$	

This polynomial is twin prime numbers even if $n = -1$ to -7 , so I insert $n=-7$,

$$\begin{aligned} f(n) &= 15(-7)^2+15(-7)+12=15n^2-15x2x7n+15x7x7+15n\cdot 15x7+12=15n^2-210n+735+15n-105+12 \\ &= 15n^2-195n+642 \end{aligned}$$

This is polynomial generating 14 twin prime numbers.

$P(n) = 15n^2 - 195n + 642 \pm 1$, generates 14 twin primes: 641/643, 461/463, 311/313, 191/193, 101/103, 41/43, 11/13, 11/13, 41/43, 101/103, 191/193, 311/313, 461/463, 641/643 .

But since the same twin prime numbers take twice each, so it is polynomial that 7 succession appear twice.

3.2 Polynomial generating twin prime numbers 2

When odd numbers are arranged in 180 degrees arrangement, see Figure 3.2, continuous twin prime numbers appear.

The produce of polynomial is as follows. The central values of the twin prime numbers are 60, 150, 270, 420, 600, 810, 1050, 1320, 1620, 1950, 2310.

$\begin{array}{c} 60 \\ 90 \\ 150 \\ 120 \\ 270 \\ 150 \\ 420 \\ 180 \\ 600 \\ 210 \\ 810 \\ 240 \\ 1050 \end{array}$	$\begin{array}{c} 60 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \\ 30 \end{array}$	$\begin{array}{l} 150=(60+90) \\ \quad =60+90\times 1 \\ 270=(60+90)+(90+30) \\ \quad =60+90\times 2+30\times 1 \\ 420=(60+90)+(90+30)+(90+30+30) \\ \quad =60+90\times 3+30\times 3 \\ 600=(60+90)+(90+30)+(90+30+30)+(90+30+30+30) \\ \quad =60+90\times 4+30\times 6 \\ 810=(60+90)+(90+30)+(90+30+30)+(90+30+30+30) \\ \quad +(90+30+30+30)=60+90\times 5+30\times 10 \\ 1050=\dots \end{array}$	$\begin{array}{c} n=0 \\ \\ n=1 \\ \\ n=2 \\ \\ n=3 \\ \\ n=4 \\ \\ n=5 \end{array}$
		$f(n)=60+90n+30xn(n-1)/2=60+90n+15n^2-15n=15n^2+75n+60$	

This is polynomial generating 11 twin prime numbers.

$P(n) = 15n^2 + 75n + 60 \pm 1$, generates 11 twin primes: 59/61, 149/151, 269/271, 419/421, 599/601, 809/811, 1049/1051, 1319/1321, 1619/1621, 1949/1951, 2309/2311 .

3.3 Polynomial generating twin prime numbers 3

When odd numbers are arranged in 270 degrees arrangement, see Figure 3.3,

continuous twin prime numbers appear.

The produce of polynomial is as follows. The central values of the twin prime numbers are 6, 12, 30, 60, 102.

$\begin{array}{c} 6 \\ \left[\begin{array}{c} 12 \\ 6 \\ 18 \end{array} \right] \\ 30 \\ \left[\begin{array}{c} 30 \\ 12 \\ 60 \end{array} \right] \\ 42 \\ 102 \end{array}$	$\begin{array}{c} 6 \\ 12 \\ 30 \\ 12 \\ 30 \\ 12 \\ 102 \end{array}$	$\begin{array}{lll} n=0 & & \\ n=1 & & \\ n=2 & & \\ n=3 & & \\ n=4 & & \end{array}$
	$\begin{aligned} 12 &= (6+6) \\ &= 6+6 \times 1 \\ 30 &= (6+6)+(6+12) \\ &= 6+6 \times 2+12 \times 1 \\ 60 &= (6+6)+(6+12)+(6+12+12) \\ &= 6+6 \times 3+12 \times 3 \\ 102 &= (6+6)+(6+12)+(6+12+12)+(6+12+12+12) \\ &= 6+6 \times 4+12 \times 6 \end{aligned}$	
	$f(n)=6+6n+12xn(n-1)/2=6+6n+6n^2 \cdot 6n=6n^2+6$	

This polynomial is twin prime numbers even if $n = -1$ to -4 , so I insert $n=n-4$

$$f(n)=6(n-4)^2+6=6n^2-6x2x4n+6x4x4+6=6n^2-48n+96+6=6n^2-48n+102$$

This is polynomial generating 9 twin prime numbers.

$P(n) = 6n^2 - 48n + 102 \pm 1$, generates 9 twin primes: 101/103, 59/61, 29/31, 11/13, 5/7, 11/13, 29/31, 59/61, 101/103 .

But since the same twin prime numbers take twice each, so it is polynomial that 5 succession appear twice.

3.4 Polynomial generating twin prime numbers 4

When odd numbers are arranged in 360 degrees arrangement, see Figure 3.4, continuous twin prime numbers appear.

The produce of polynomial is as follows. The central values of the twin prime numbers are 18, 12, 150, 432, 858, 1428, 2142, 3000, 4002.

$\begin{array}{c} 18 \\ \left[\begin{array}{c} -6 \\ 12 \end{array} \right] \\ 138 \\ 150 \end{array}$	$\begin{array}{c} 18 \\ 12 \\ 144 \\ 144 \\ 150 \end{array}$	$\begin{array}{lll} n=0 & & \\ n=1 & & \\ n=2 & & \end{array}$
	$\begin{aligned} 12 &= (18-6) \\ &= 18-6 \times 1 \\ 150 &= (18-6)+(-6+144) \end{aligned}$	

$$\begin{array}{lll}
 & 282 & =18 \cdot 6 \cdot 2 + 144 \cdot 1 \\
 & 432 & 432 = (18 \cdot 6) + (-6 + 144) + (-6 + 144 + 144) \quad n=3 \\
 & 426 & = 18 \cdot 6 \cdot 3 + 144 \cdot 3 \\
 & 858 & 858 = (18 \cdot 6) + (-6 + 144) + (-6 + 144 + 144) + (-6 + 144 + 144 + 144) \quad n=4 \\
 & 570 & = 18 \cdot 6 \cdot 4 + 144 \cdot 6 \\
 & 1428 & 1428 = (18 \cdot 6) + (-6 + 144) + (-6 + 144 + 144) + (-6 + 144 + 144 + 144) + (-6 + 144 + 144 + 144 + 144) \quad n=5 \\
 & 714 & + (-6 + 144 + 144 + 144 + 144) = 18 \cdot 6 \cdot 5 + 144 \cdot 10 \\
 & 2142 & 1050 = \dots \\
 & & f(n) = 18 \cdot 6n + 144n(n-1)/2 = 18 \cdot 6n + 72n^2 - 72n = 72n^2 - 78n + 18
 \end{array}$$

This is polynomial generating 9 twin prime numbers.

$P(n) = 72n^2 - 78n + 18 \pm 1$, generates 9 twin primes: 17/19, 11/13, 149/151, 431/433, 857/859, 1427/1429, 2141/2143, 2999/3001, 4001/4003 .

3.5 Polynomial generating twin prime numbers 5

When odd numbers are arranged in 60 degrees arrangement, see Figure 3.5, continuous twin prime numbers appear.

Since the method of obtaining the polynomial is the same as the method described above, so it will be omitted below.

This is polynomial generating 7 twin prime numbers.

$P(n) = 75n^2 - 345n + 420 \pm 1$, generates 7 twin primes: 419/421, 149/151, 29/31, 59/61, 239/241, 569/571, 1049/1051 .

3.6 Polynomial generating twin prime numbers 6

When odd numbers are arranged in 180 degrees arrangement, see Figure 3.6, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 3n^2 + 21n + 18 \pm 1$, generates 6 twin primes: 17/19, 41/43, 71/73, 107/109, 149/151, 197/199 .

3.7 Polynomial generating twin prime numbers 7

When odd numbers are arranged in 135 degrees arrangement, see Figure 3.7, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 3n^2 + 27n + 72 \pm 1$, generates 6 twin primes: 71/73, 101/103, 137/139, 179/181, 227/229, 281/283 .

3.8 Polynomial generating twin prime numbers 8

When odd numbers are arranged in 180 degrees arrangement, see Figure 3.8, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 3n^2 + 69n + 198 \pm 1$, generates 6 twin primes: 197/199, 269/271, 347/349, 431/433, 521/523, 617/619 .

3.9 Polynomial generating twin prime numbers 9

When odd numbers are arranged in 160 degrees arrangement, see Figure 3.9, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 6n^2 - 30n + 42 \pm 1$, generates 6 twin primes: 41/43, 17/19, 5/7, 5/7, 17/19, 41/43 .

But since the same twin prime numbers take twice each, so it is polynomial that 3 succession appear twice.

3.10 Polynomial generating twin prime numbers 10

When odd numbers are arranged in 60 degrees arrangement, see Figure 3.10, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 75n^2 - 165n + 102 \pm 1$, generates 6 twin primes: 101/103, 11/13, 71/73, 281/283, 641/643, 1151/1153 .

3.11 Polynomial generating twin prime numbers 11

When odd numbers are arranged in 225 degrees arrangement, see Figure 3.11, continuous twin prime numbers appear.

This is polynomial generating 6 twin prime numbers.

$P(n) = 153n^2 - 135n + 180 \pm 1$, generates 6 twin primes: 179/181, 197/199, 521/523, 1151/1153, 2087/2089, 3329/3331 .

3.12 Polynomial generating twin prime numbers 12

When odd numbers are arranged in 360 degrees arrangement, see Figure 3.12, continuous twin prime numbers appear.

This is polynomial generating 5 twin prime numbers.

$P(n) = 288n^2 - 180n + 30 \pm 1$, generates 5 twin primes: 29/31, 137/139, 821/823, 2081/2083, 3917/3919 .

3.13 Other polynomials generating twin prime numbers

I found many polynomials generating 4 twin prime numbers. The details of diagrams are omitted. The polynomials found in Figure 3.1 to Figure 3.12 are shown in the figures. The figures of polynomials found in the figures other than Figure 3.1 to Figure 3.12 are omitted.

3.13.1 $P(n) = 3n^2 + 69n + 1878 \pm 1$, generates 4 twin primes:
1877/1879, 1949/1951, 2027/2029, 2111/2113 .

3.13.2 $P(n) = 3n^2 + 141n + 1788 \pm 1$, generates 4 twin primes:
1787/1789, 1931/1933, 2081/2083, 2237/2239 .

3.13.3 $P(n) = 6n^2 + 222n + 2082 \pm 1$, generates 4 twin primes:
2081/2083, 2309/2311, 2549/2551, 2801/2803 .

3.13.4 $P(n) = 9n^2 + 3n + 18 \pm 1$, generates 4 twin primes:
17/19, 29/31, 59/61, 107/109 .

3.13.5 $P(n) = 12n^2 + 54n + 42 \pm 1$, generates 4 twin primes:
41/43, 107/109, 197/199, 311/313 .

3.13.6 $P(n) = 12n^2 + 174n + 1092 \pm 1$, generates 4 twin primes:
1091/1093, 1277/1279, 1487/1489, 1721/1723 .

3.13.7 $P(n) = 18n^2 + 240n + 600 \pm 1$, generates 4 twin primes:
599/601, 857/859, 1151/1153, 1481/1483 .

3.13.8 $P(n) = 18n^2 + 252n + 1032 \pm 1$, generates 4 twin primes:
 $1031/1033, 1301/1303, 1607/1609, 1949/1951$.

3.13.9 $P(n) = 27n^2 + 453n + 1788 \pm 1$, generates 4 twin primes:
 $1787/1789, 2267/2269, 2801/2803, 3389/3391$.

3.13.10 $P(n) = 33n^2 + 519n + 1998 \pm 1$, generates 4 twin primes:
 $1997/1999, 2549/2551, 3167/3169, 3851/3853$.

3.13.11 $P(n) = 48n^2 + 150n + 150 \pm 1$, generates 4 twin primes:
 $149/151, 347/349, 641/643, 1031/1033$.

3.13.12 $P(n) = 51n^2 + 657n + 570 \pm 1$, generates 4 twin primes:
 $569/571, 1277/1279, 2087/2089, 2999/3001$.

3.13.13 $P(n) = 78n^2 + 228n + 42 \pm 1$, generates 4 twin primes:
 $41/43, 347/349, 809/811, 1427/1429$.

3.13.14 $P(n) = 90n^2 + 150n + 822 \pm 1$, generates 4 twin primes:
 $821/823, 1061/1063, 1481/1483, 2081/2083$.

3.13.15 $P(n) = 99n^2 + 363n + 108 \pm 1$, generates 4 twin primes:
 $107/109, 569/571, 1229/1231, 2087/2089$.

3.13.16 $P(n) = 102n^2 + 72n + 18 \pm 1$, generates 4 twin primes:
 $17/19, 191/193, 569/571, 1151/1153$.

3.13.17 $P(n) = 150n^2 - 90n + 12 \pm 1$, generates 4 twin primes:
 $11/13, 71/73, 431/433, 1091/1093$.

3.13.18 $P(n) = 201n^2 + 57n + 570 \pm 1$, generates 4 twin primes:
 $569/571, 827/829, 1487/1489, 2549/2551$.

3.13.19 $P(n) = 255n^2 - 75n + 12 \pm 1$, generates 4 twin primes:
 $11/13, 191/193, 881/883, 2081/2083$.

3.13.20 $P(n) = 294n^2 - 462n + 348 \pm 1$, generates 4 twin primes:
347/349, 179/181, 599/601, 1607/1609 .

3.13.21 $P(n) = 375n^2 - 555n + 420 \pm 1$, generates 4 twin primes:
419/421, 239/241, 809/811, 2129/2131 .

3.13.22 $P(n) = 390n^2 + 90n + 138 \pm 1$, generates 4 twin primes:
137/139, 617/619, 1877/1879, 3917/3919 .

3.13.23 $P(n) = -12n^2 + 582n + 312 \pm 1$, generates 4 twin primes:
311/313, 881/883, 1427/1429, 1949/1951 .

3.13.24 $P(n) = -45n^2 + 555n + 348 \pm 1$, generates 4 twin primes:
347/349, 857/859, 1277/1279, 1607/1609 .

3.13.25 $P(n) = 90n + 1608 \pm 1$, generates 4 twin primes:
1607/1609, 1697/1699, 1787/1789, 1877/1879 .

Figure 2.1: The Ulam Spiral

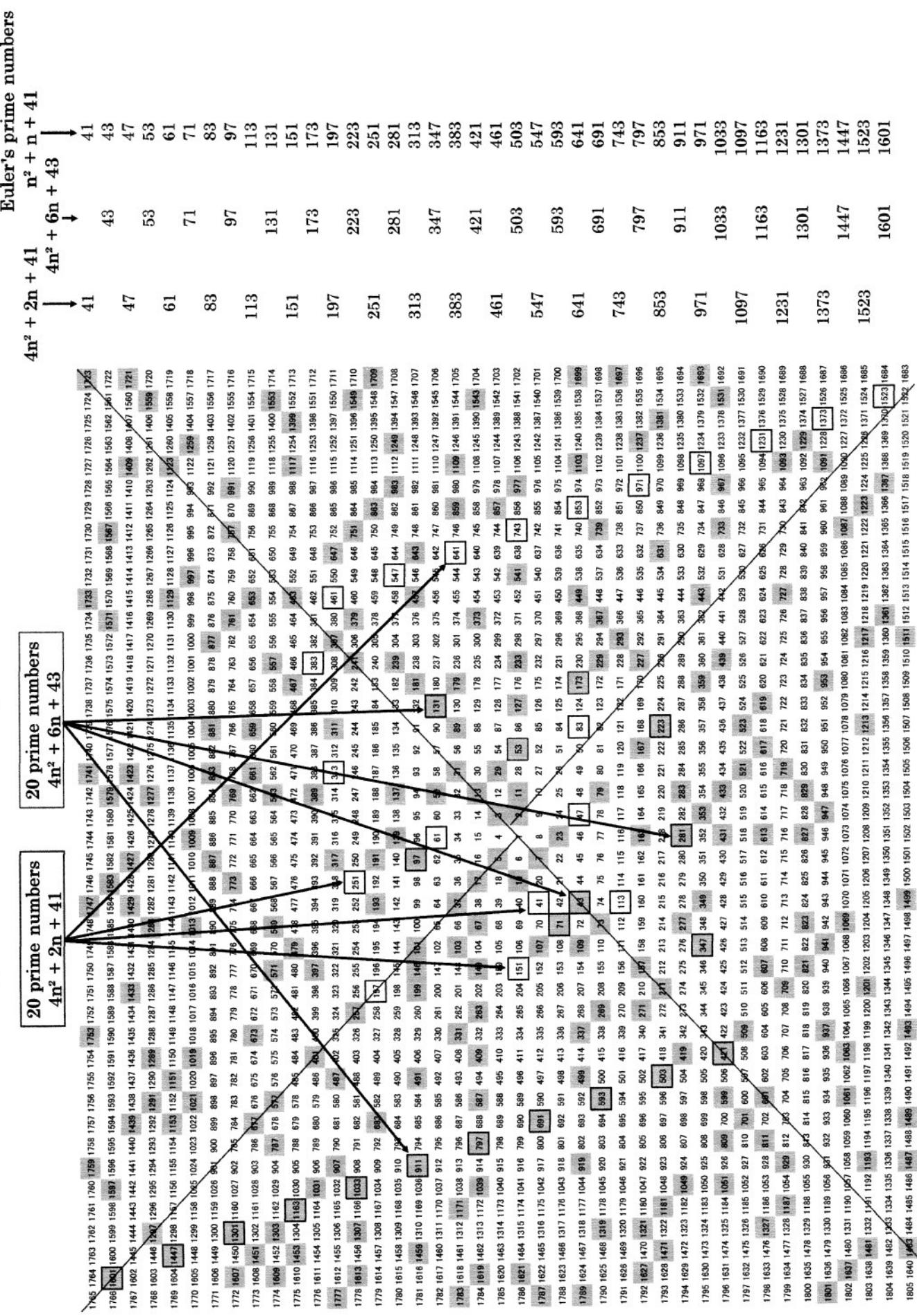
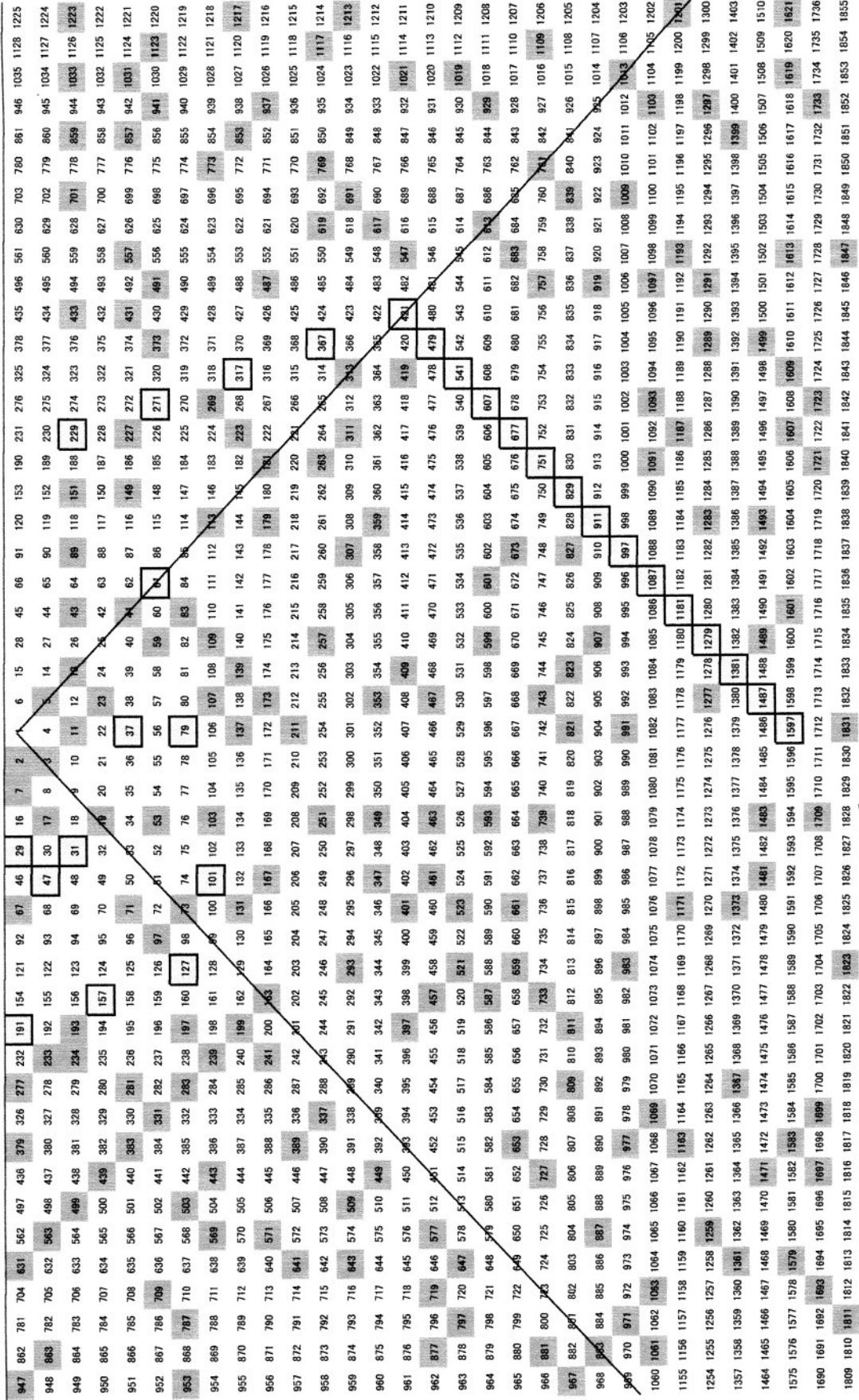


Figure 2.2: 180 degrees Arrangement Legendre Polynomial



Legendre polynomial $2n^2 + 29$

Figure 2.3: 135 degrees Arrangement Brox Polynomial



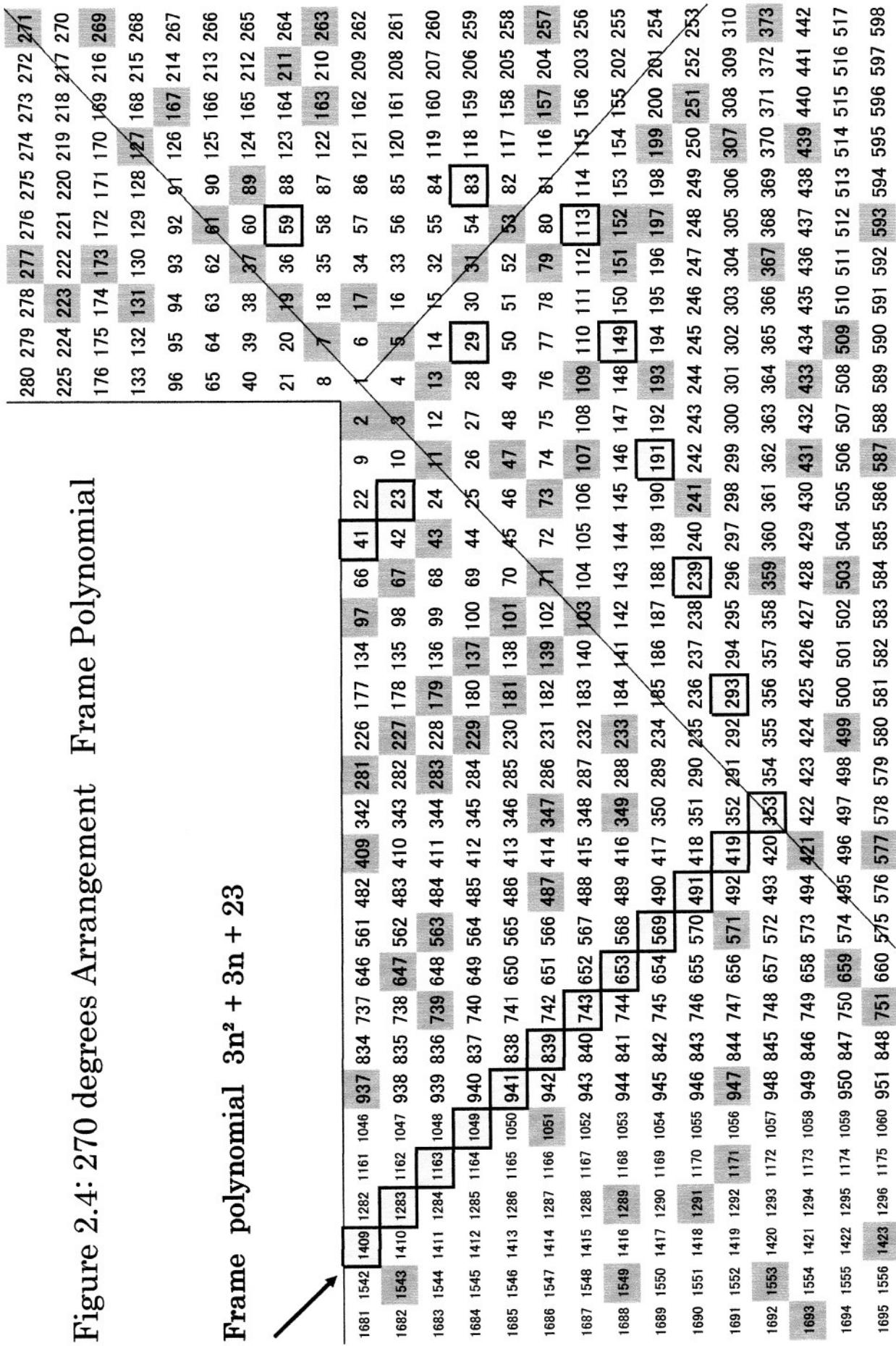
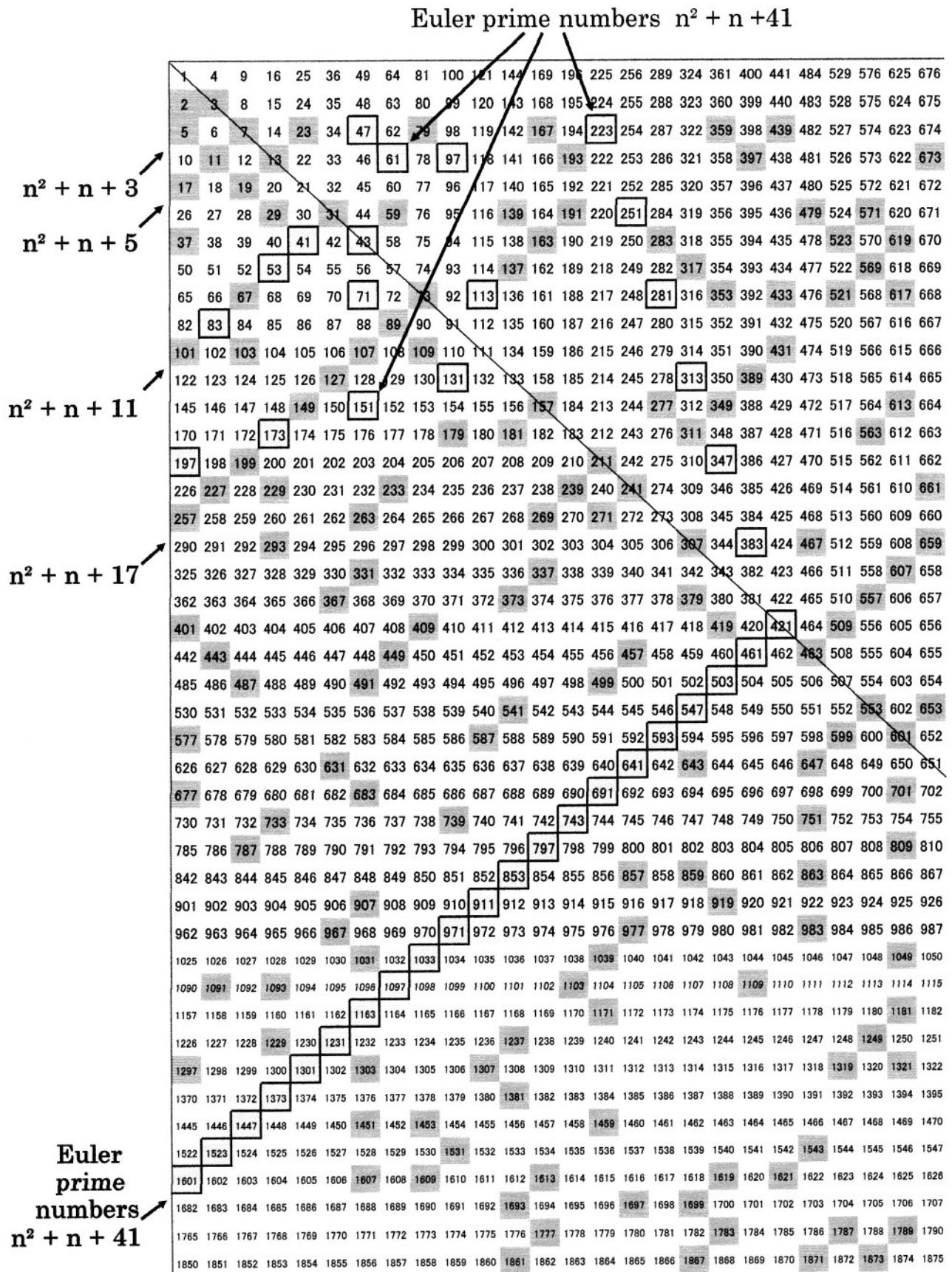


Figure 2.5: 90 degrees Arrangement Euler's Polynomial



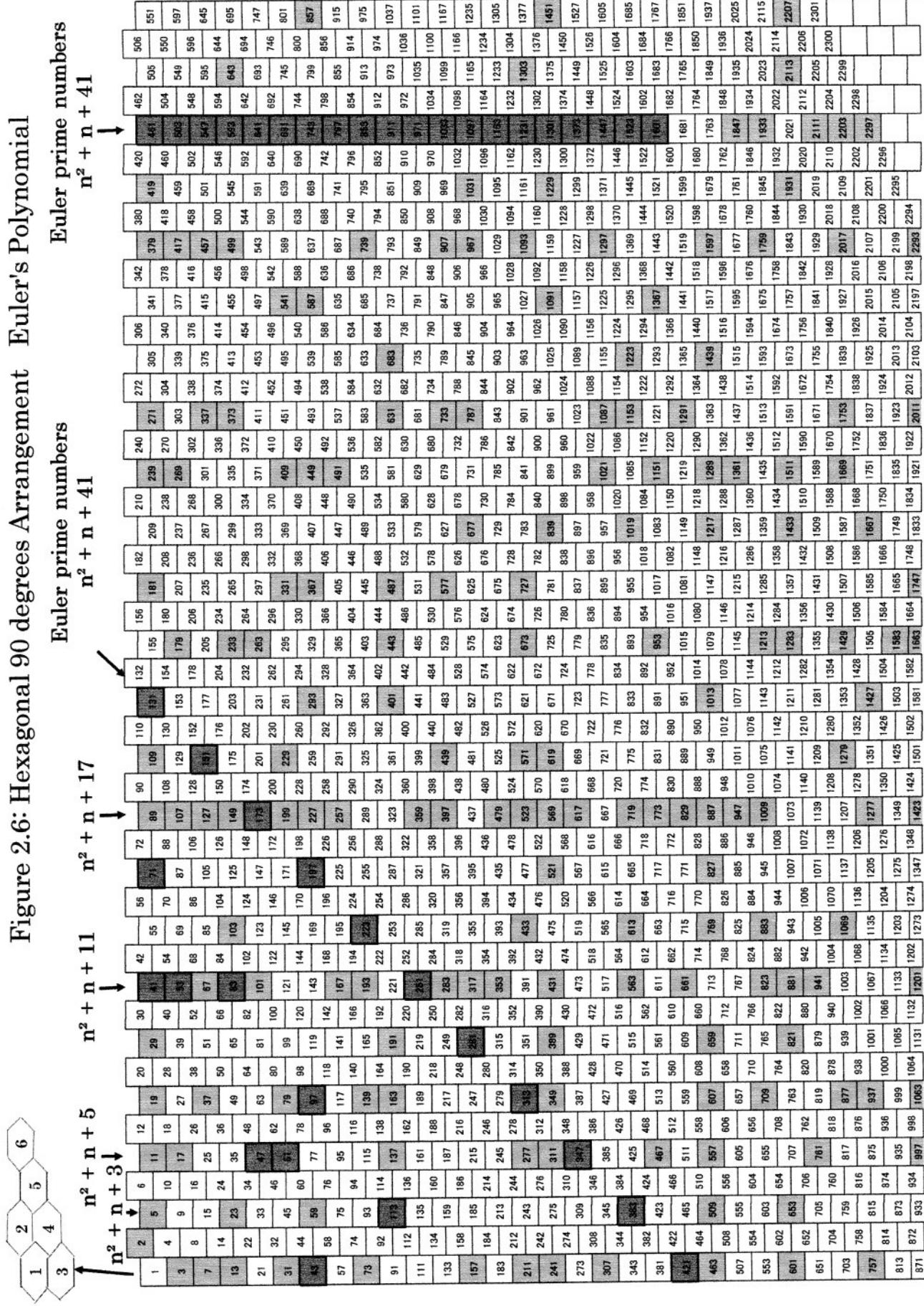


Figure 3.1: 45 degrees Arrangement

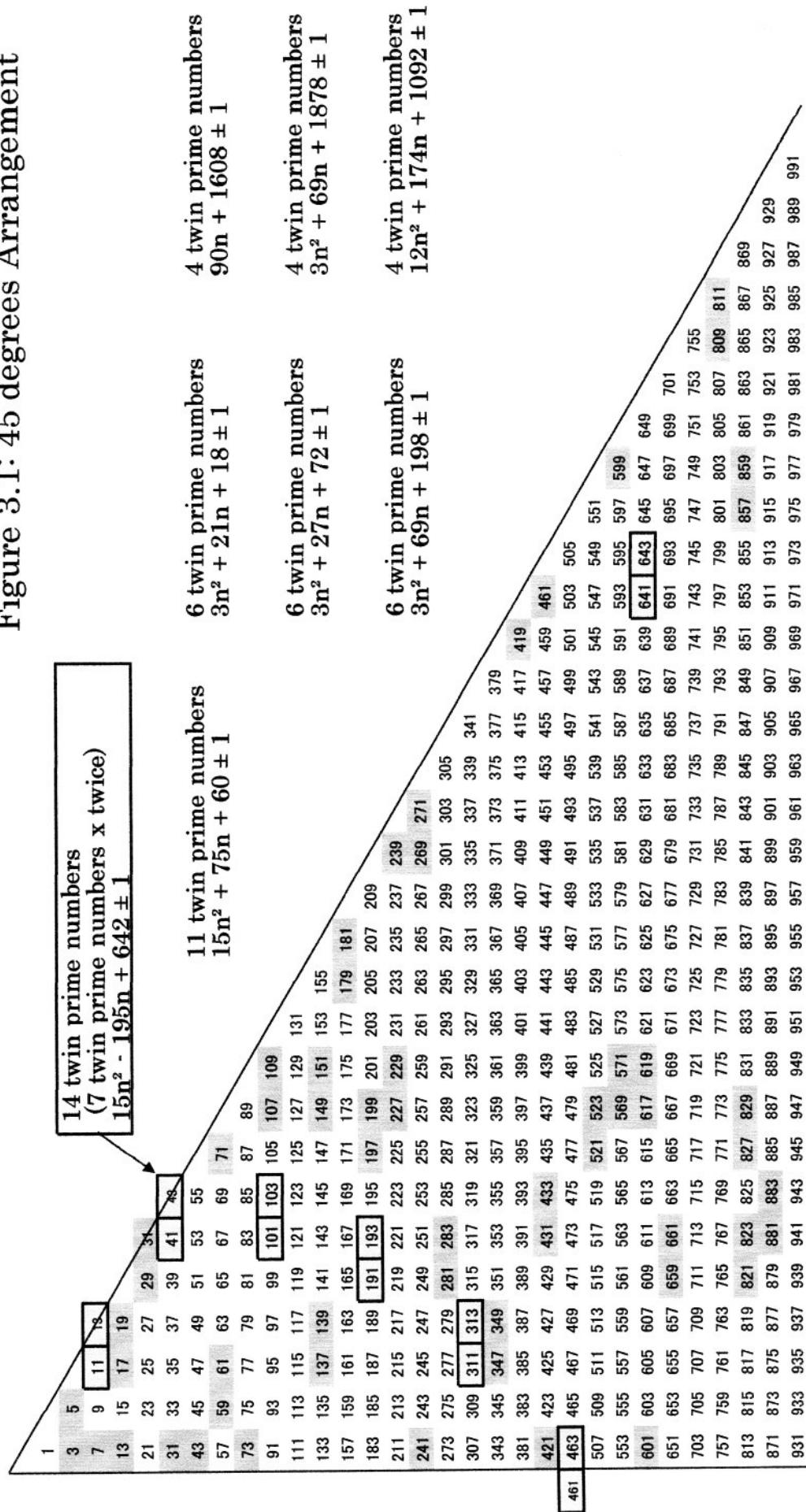
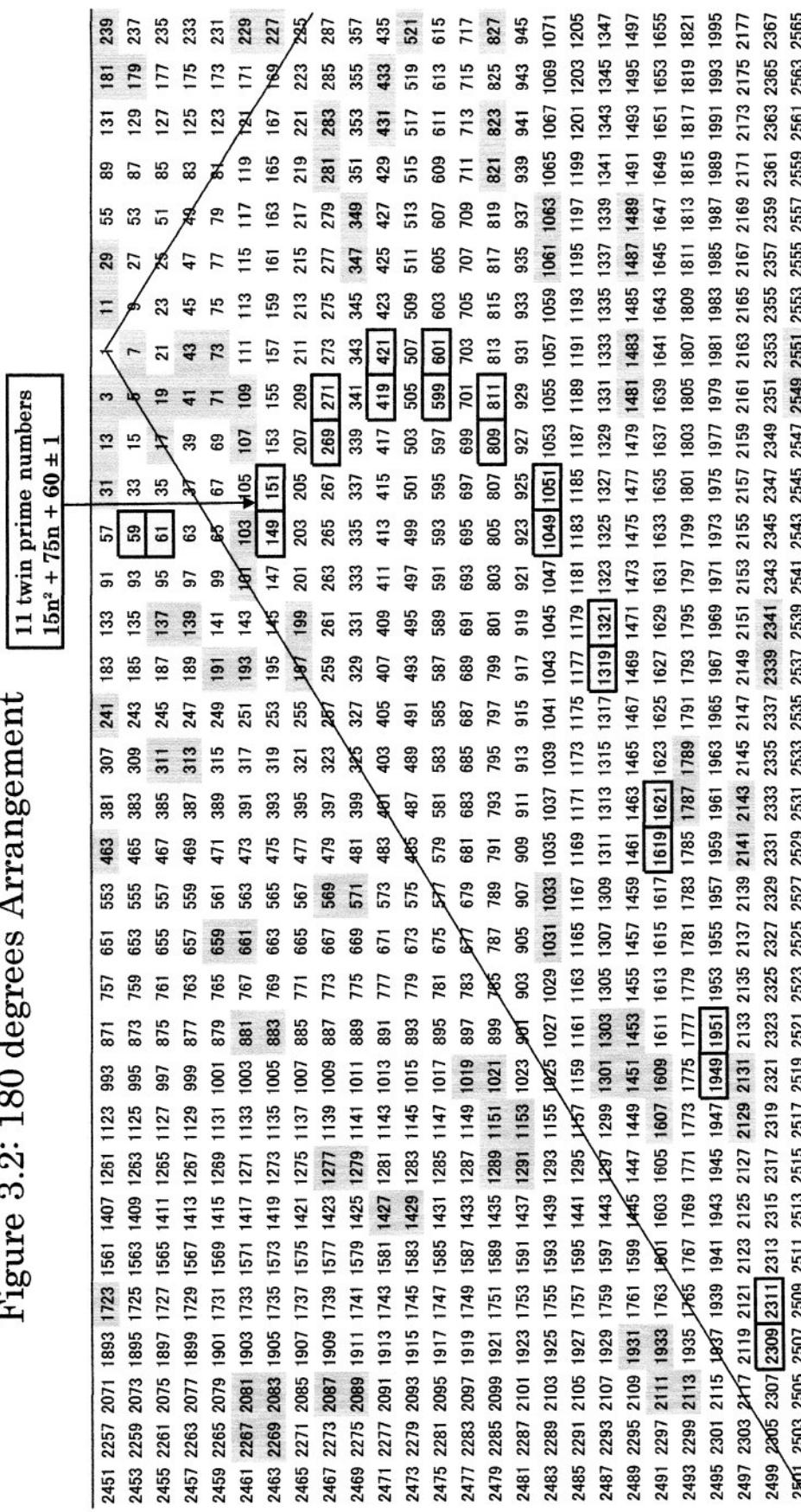


Figure 3.2: 180 degrees Arrangement

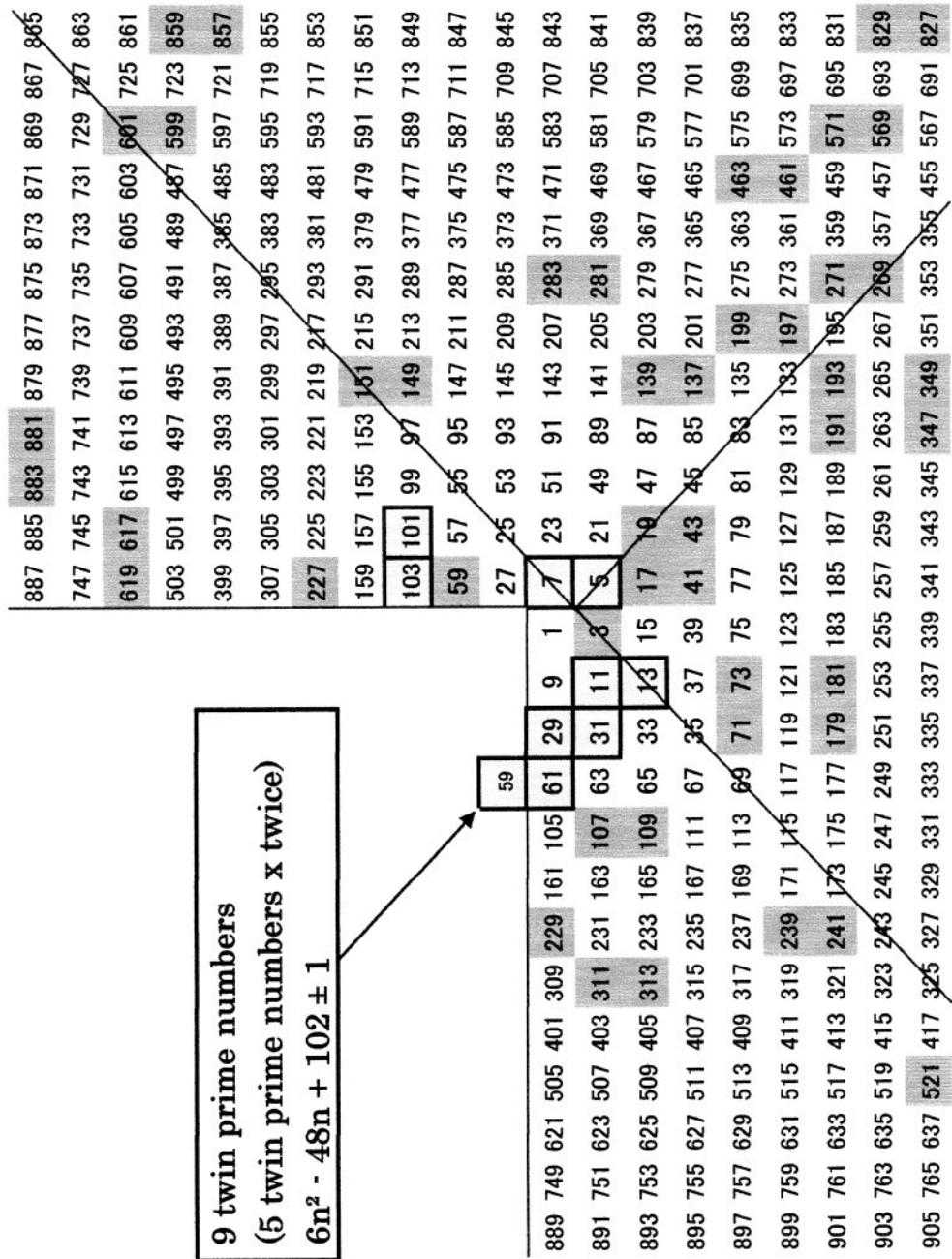


6 twin prime numbers
 $3n^2 + 21n + 18 \pm 1$

4 twin prime numbers
 $90n^2 + 150n + 822 \pm 1$

6 twin prime numbers
 $3n^2 + 69n + 198 \pm 1$

Figure 3.3: 270 degrees Arrangement



4 twin prime numbers
 $6n^2 + 222n + 2082 \pm 1$

Figure 3.4: 360 degrees Arrangement

4 twin prime numbers 5 twin prime numbers $288n^2 - 180n + 30 \pm 1$

9 twin prime numbers
 $72n^2 - 78n + 18 \pm 1$

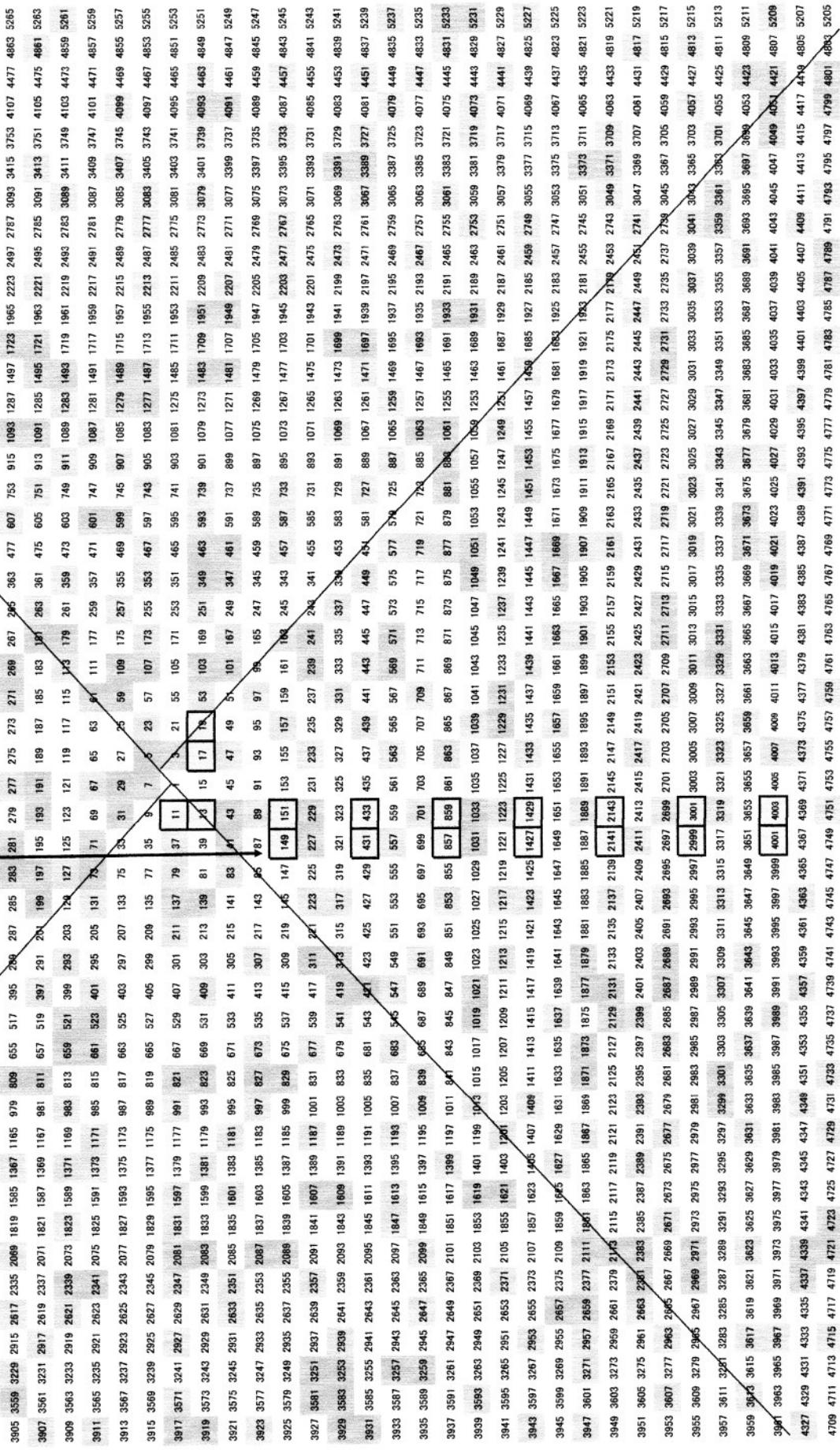


Figure 3.5: 60 degrees Arrangement

6 twin prime numbers $3n^2 + 27n + 72 \pm 1$		7 twin prime numbers $75n^2 - 345n + 420 \pm 1$		4 twin prime numbers $3n^2 + 141n + 1788 \pm 1$	
1					
3	5	7			
9	11	13	15	17	19
21	23	25	27	29	31
39	41	43	45	47	49
63	65	67	69	71	73
93	95	97	99	101	103
129	131	133	135	137	139
171	173	175	177	179	181
219	221	223	225	227	229
273	275	277	279	281	283
333	335	337	339	341	343
399	401	403	405	407	409
471	473	475	477	479	481
549	551	553	555	557	559
633	635	637	639	641	643
723	725	727	729	731	733
819	821	823	825	827	829
921	923	925	927	929	931
1029	1031	1033	1035	1037	1039
1143	1145	1147	1149	1151	1153
1263	1265	1267	1269	1271	1273
1389	1391	1393	1395	1397	1399
1521	1523	1525	1527	1529	1531
1659	1661	1663	1665	1667	1669
1803	1805	1807	1809	1811	1813
1953	1955	1957	1959	1961	1963
2109	2111	2113	2115	2117	2119
2271	2273	2275	2277	2279	2281
2283	2285	2287	2289	2291	2293
2303	2305	2307	2309	2311	2313
2315	2317	2319	2321	2323	2325
2327	2329	2331	2333	2335	2337
2339	2341	2343	2345	2347	

Figure 3.6: 180 degrees Arrangement

6 twin prime numbers $3n^2 + 21n + 18 \pm 1$																	
1261	1123	993	871	757	651	553	463	381	307	241	183	133	91	57	31	13	3
1263	1125	995	873	759	653	555	465	383	309	243	185	135	93	59	33	15	5
1265	1127	997	875	761	655	557	467	385	311	245	187	137	95	61	35	17	19
1267	1129	999	877	763	657	559	469	387	313	247	189	139	97	63	37	39	41
1269	1131	1001	879	765	659	561	471	389	315	249	191	141	99	65	67	69	71
1271	1133	1003	881	767	661	563	473	391	317	251	193	143	101	103	105	107	109
1273	1135	1005	883	769	663	565	475	393	319	253	195	145	147	149	151	153	155
1275	1137	1007	885	771	665	567	477	395	321	255	197	199	201	203	205	207	209
1277	1139	1009	887	773	667	569	479	397	323	257	259	261	263	265	267	269	271
1279	1141	1011	889	775	669	571	481	399	325	327	329	331	333	335	337	339	341
1281	1143	1013	891	777	671	573	483	401	403	405	407	409	411	413	415	417	419
1283	1145	1015	893	779	673	575	485	487	489	491	493	495	497	499	501	503	505
1285	1147	1017	895	781	675	577	579	581	583	585	587	589	591	593	595	597	599
1287	1149	1019	897	783	677	679	681	683	685	687	689	691	693	695	697	699	701
1289	1151	1021	899	785	678	687	789	791	793	795	797	799	801	803	805	807	809
1291	1153	1023	901	903	905	907	909	911	913	915	917	919	921	923	925	927	929
1293	1155	1025	1027	1029	1031	1033	1035	1037	1039	1041	1043	1045	1047	1049	1051	1053	1055
1295	1157	1161	1163	1165	1167	1169	1171	1173	1175	1177	1179	1181	1183	1185	1187	1189	1191
1297	1299	1301	1303	1305	1307	1309	1311	1313	1315	1317	1319	1321	1323	1325	1327	1329	1331

4 twin prime numbers
 $90n^2 + 150n + 1032 \pm 1$

6 twin prime numbers
 $3n^2 + 69n + 198 \pm 1$

4 twin prime numbers
 $90n^2 + 150n + 822 \pm 1$

6 twin prime numbers
 $3n^2 + 252n + 103 \pm 1$

Figure 3.7: 135 degrees Arrangement

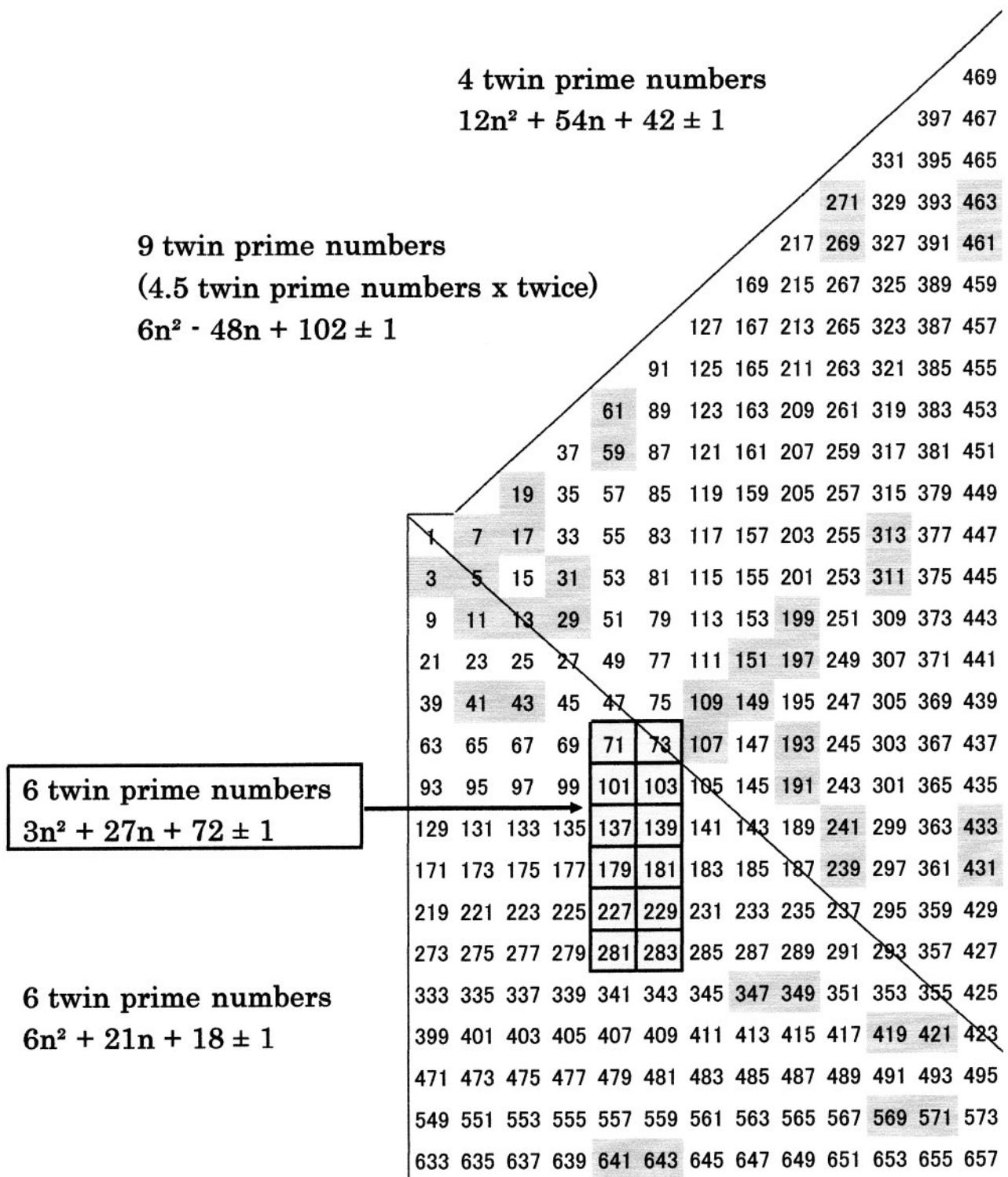


Figure 3.8: 180 degrees Arrangement

6 twin prime numbers
 $3n^2 + 69n + 198 \pm 1$

757	651	553	463	381	307	241	183	133	91	57	31	13	3	1	11	29	55	89	131	181	239	305	379	461	551	649
759	653	555	465	383	309	243	185	135	93	59	33	15	5	7	9	27	53	87	129	179	237	303	377	459	549	647
761	655	557	467	385	311	245	187	137	95	61	35	17	19	21	23	25	51	85	127	177	235	301	375	457	547	645
763	657	559	469	387	313	247	189	139	97	63	37	39	41	43	45	47	49	83	125	175	233	299	373	455	545	643
765	659	561	471	389	315	249	191	141	99	65	67	69	71	73	75	77	79	81	123	173	231	297	371	453	543	641
767	661	563	473	391	317	251	193	143	101	103	105	107	109	111	113	115	117	119	121	171	229	295	369	451	541	639
769	663	565	475	393	319	253	195	145	147	149	151	153	155	157	159	161	163	165	167	169	227	293	367	449	539	637
771	665	567	477	395	321	255	197	199	201	203	205	207	209	211	213	215	217	219	221	223	225	291	365	447	537	635
773	667	569	479	397	323	257	259	261	263	265	267	269	271	273	275	277	279	281	283	285	287	289	363	445	535	633
775	669	571	481	399	325	327	329	331	333	335	337	339	341	343	345	347	349	351	353	355	357	359	361	443	533	631
777	671	573	483	401	403	405	407	409	411	413	415	417	419	421	423	425	427	429	431	433	435	437	439	441	531	629
779	673	575	485	487	489	491	493	495	497	499	501	503	505	507	509	511	513	515	517	519	521	523	525	527	529	627
781	675	577	579	581	583	585	587	589	591	593	595	597	599	601	603	605	607	609	611	613	615	617	619	621	623	625
783	677	679	681	683	685	687	689	691	693	695	697	699	701	703	705	707	709	711	713	715	717	719	721	723	725	727
785	787	789	791	793	795	797	799	801	803	805	807	809	811	813	815	817	819	821	823	825	827	829	831	833	835	837
903	905	907	909	911	913	915	917	919	921	923	925	927	929	931	933	935	937	939	941	943	945	947	949	951	953	955
1029	1031	1033	1035	1037	1039	1041	1043	1045	1047	1049	1051	1053	1055	1057	1059	1061	1063	1065	1067	1069	1071	1073	1075	1077	1079	1081
1163	1165	1167	1169	1171	1173	1175	1177	1179	1181	1183	1185	1187	1189	1191	1193	1195	1197	1199	1201	1203	1205	1207	1209	1211	1213	1215
1305	1307	1309	1311	1313	1315	1317	1319	1321	1323	1325	1327	1329	1331	1333	1337	1339	1341	1343	1345	1347	1349	1351	1353	1355	1357	

4 twin prime numbers
 $18n^2 + 252n + 1032 \pm 1$
 $90n^2 + 150n + 822 \pm 1$

Figure 3.9: 160 degrees Arrangement

4 twin prime numbers
 $-45n^2 + 555n + 348 \pm 1$

4 twin prime numbers
 $375n^2 - 555n + 420 \pm 1$

6 twin prime numbers
 (3 twin prime numbers x twice)
 $6n^2 - 30n + 42 \pm 1$

6 twin prime numbers

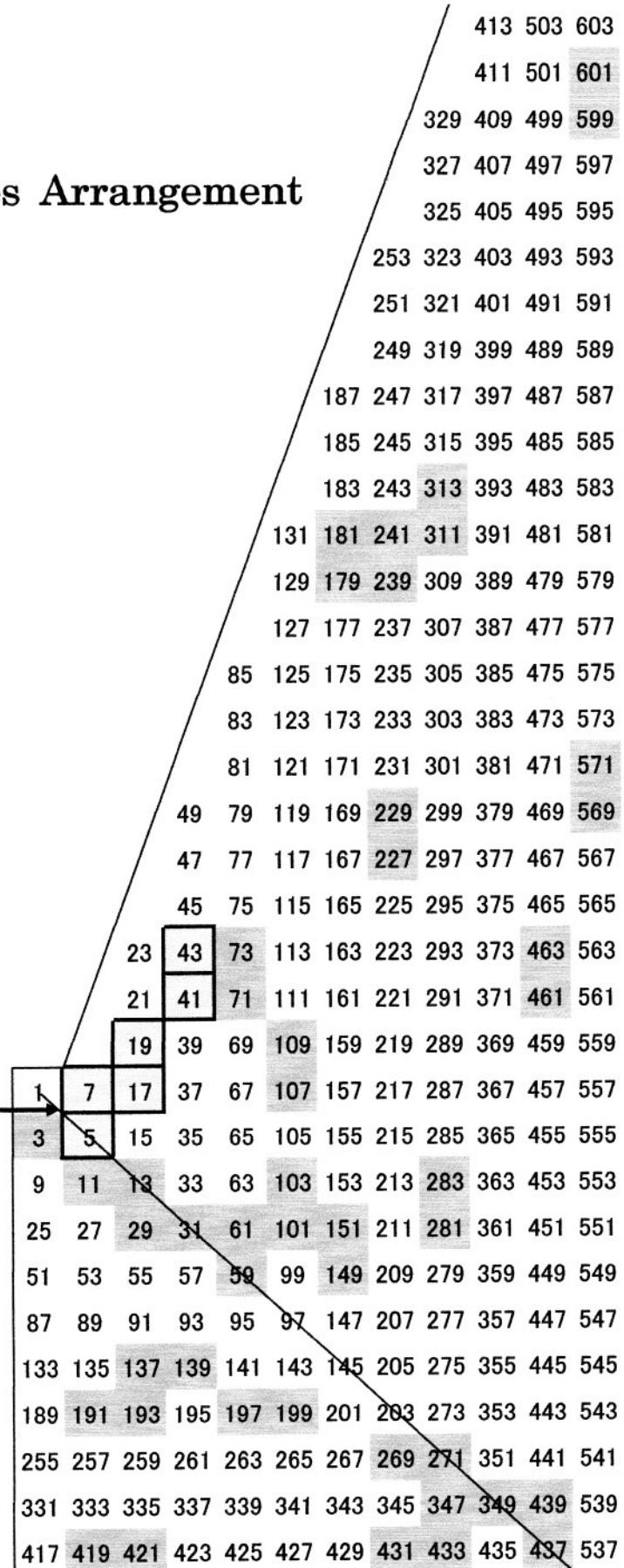
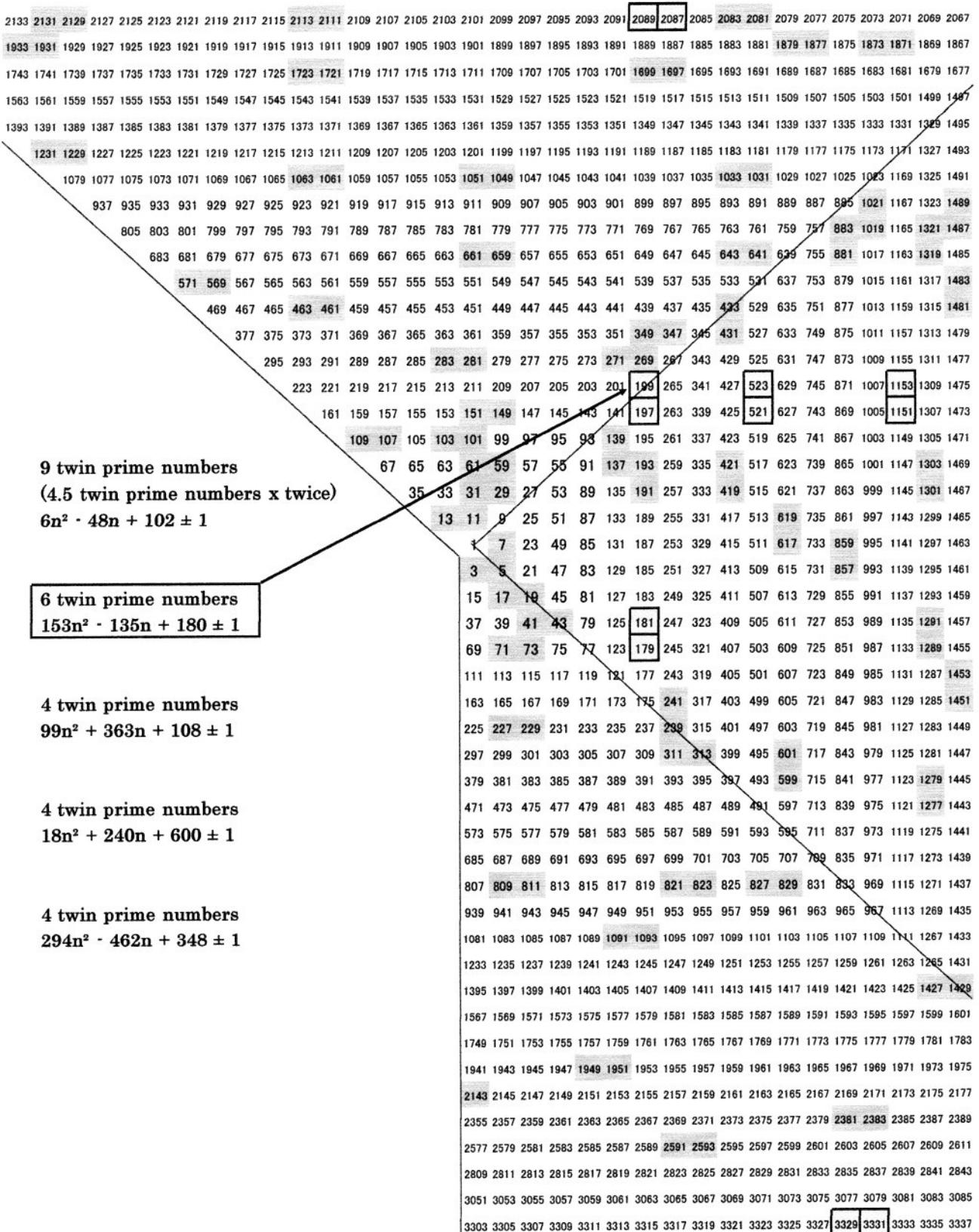


Figure 3.10: 60 degrees Arrangement

6 twin prime numbers $3n^2 + 27n + 72 \pm 1$			7 twin prime numbers $75n^2 - 345n + 420 \pm 1$			4 twin prime numbers $3n^2 + 141n + 1788 \pm 1$		
1								
3	5	7						
9	11	13	15	17	19			
21	23	25	27	29	31	33	35	37
39	41	43	45	47	49	51	53	55
63	65	67	69	71	73	75	77	79
93	95	97	99	101	103	105	107	109
129	131	133	135	137	139	141	143	145
171	173	175	177	179	181	183	185	187
219	221	223	225	227	229	231	233	235
273	275	277	279	281	283	285	287	289
333	335	337	339	341	343	345	347	349
399	401	403	405	407	409	411	413	415
471	473	475	477	479	481	483	485	487
549	551	553	555	557	559	561	563	565
633	635	637	639	641	643	645	647	649
723	725	727	729	731	733	735	737	739
819	821	823	825	827	829	831	833	835
921	923	925	927	929	931	933	935	937
1029	1031	1033	1035	1037	1039	1041	1043	1045
1143	1145	1147	1149	1151	1153	1155	1157	1159
1263	1265	1267	1269	1271	1273	1275	1277	1279
1389	1391	1393	1395	1397	1399	1401	1403	1405
1521	1523	1525	1527	1529	1531	1533	1535	1537
1659	1661	1663	1665	1667	1669	1671	1673	1675

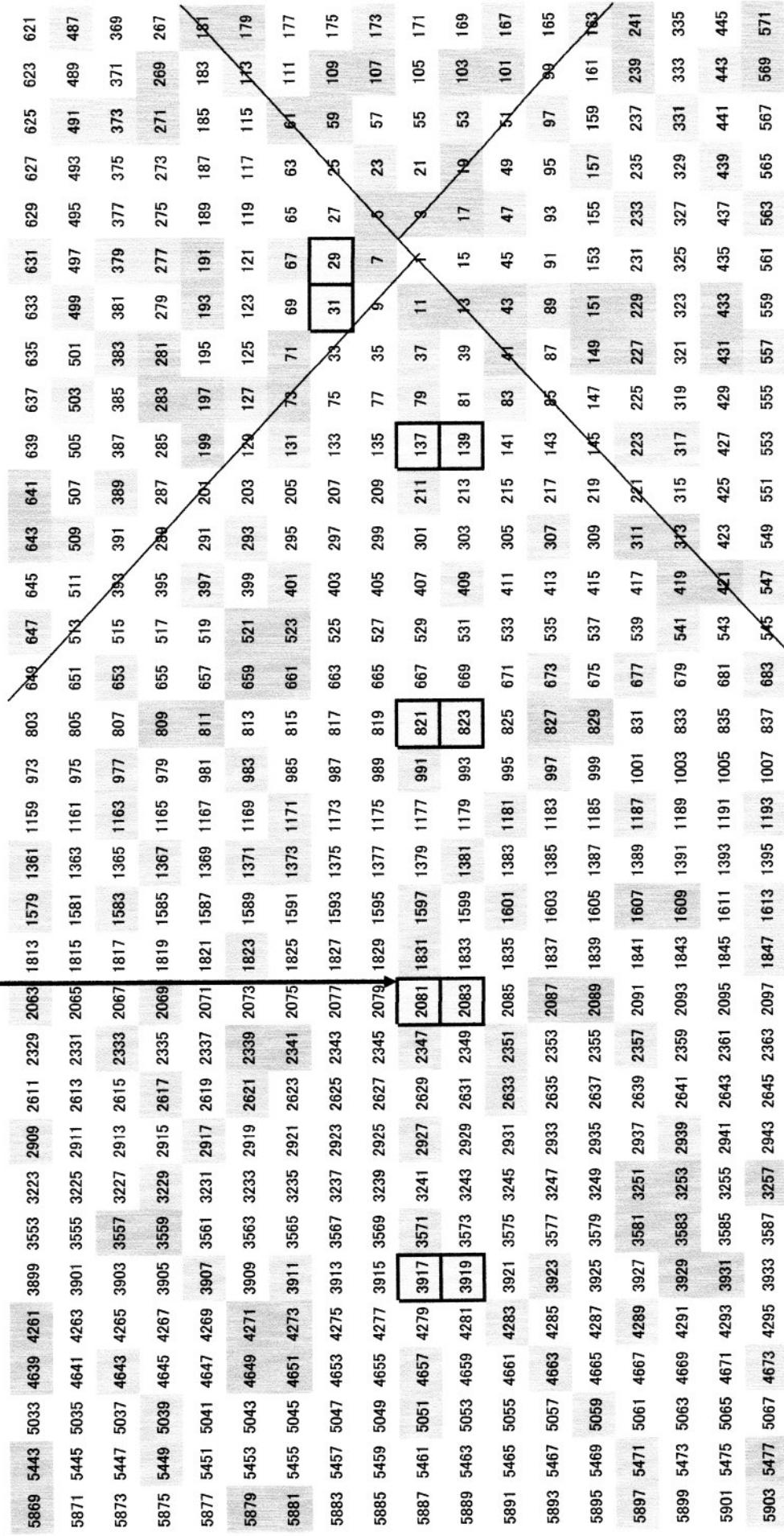
Figure 3.11: 225 degrees Arrangement



**4 twin prime numbers
 $33n^2 + 519n + 1998 \pm 1$**

5 twin prime numbers $288n^2 - 180n + 30 \pm 1$	
5869 5443 5033 4639 4261	3899 3553 3223 2909 2611

Figure 3.12: 360 degrees Arrangement



5 Consideration

It is expected that polynomials generating twin prime numbers may be found by devising other arrangements or by arranging up to large odd numbers by the method described in this research work etc..

6. Acknowledgment

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