A more elegant proof of Poincaré Conjecture

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Abstract

Besides the proof of the mathematical conjecture, a new form for the three-dimensional euclidean sphere is given. This sphere can be embedded into pseudo-euclidean metric, making the new description for the Universe.

Declarations of interest: none.

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I. AND BECAUSE MISTAKES AND FAKES SHALL ABOUND, THE WAY OF TRUTH WILL BE EVIL SPOKEN OF

This section can be removed from the paper on request of the referee. It is not meant as a proposal to modify the peer-review process, but as an argument for the referee to use goodwill.

The goal "to find mistakes" could be a bad attitude. The final goal should be to enjoy reading the publication. If flaws are seen, they must be reported. However, this report should be given without any laughs and sadistic enjoyment. Instead, the flaws should be reported with some sadness.

The psychologists have conducted a social experiment: they told the probants that the man on the photo is a serial killer. The probants testified that he is looking like one. The next day they told another group of probants that the man on the same photo is an American national hero; these probants have confirmed his heroic look.

In conclusion, having the "mistakes desire" as your default position while reading the manuscript of an unknown author increases the chances for the paper to be unjustly rejected. The scientific skepticism should be the readiness to deal with mistakes, but not the expectation – by desire – to find them.

Why do I ask as an author for detailed reports from the referee system? The referee must convince me that I have done mistakes. Otherwise, I would not accept them. Yes, it seems like living in an "utopian" perfect world. But I cannot repent a hypothetical mistake. I can only repent if the mistake is demonstrated to me and I am convinced that it is not the usual fake-news, trolling or bullying. This research principle is my personal "guiding star" during my quest for the objective truth. As an example, the absolute majority of scientists have accepted the proof for Goldbach's weak conjecture, but not all of the scientists have accepted it yet, mainly because it is not published in a journal. [1] Therefore, one needs to have personal convictions and opinions to move forward. [2]

To navigate in Science, you need to have a personal point of view and convictions you should not rush to abandon. Otherwise, you will soon be disoriented. Only then you will realize the objective truth. That is the subjective search for the objective truth because you are choosing what is right and what is not.

II. WHERE IS TREASURE OF PERELMAN?

The Poincaré Conjecture states:

Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.

Homeomorphisms are the isomorphisms in the category of topological spaces – that is, they are the mappings that preserve all the topological properties of a given space. Two spaces with a homeomorphism between them are called homeomorphic, and from a topological viewpoint they are the same.

On December 22, 2006, the journal Science honored Perelman's proof of the Poincaré conjecture as the scientific "Breakthrough of the Year", the first time this honor was bestowed in the area of mathematics. [3] As of 2020, the Poincaré conjecture is the only solved of the "Millennium problems", a set of seven problems in mathematics that were stated by the Clay Mathematics Institute on May 24, 2000.

Surely, there was great incompatibility between Dr. Perelman and the global Mathematical Society: the perfect genius has said some painful words against the decision-making of the Institutes, and has refused to collect the Prize money. [4] The Clay Institute subsequently used Perelman's prize money to fund the "Poincare Chair", a temporary position for young promising mathematicians at the Paris Institut Henri Poincare.

Indeed, all parties acted strictly in the framework of treaties and criminal laws. But within the law, there can be more or less worthy actions, more or less wise actions, and even legal unworthy actions and deeds. I can not sue the Clay Institute, because the laws and treaty were not broken: my remark is not the legal issue, but rather of a social one.

Referee: *"the actions and decisions of the Clay Institute and Dr. Perelman are not only fully legal, but worthy, noble, nice, and beautiful as well."*
My reply: *"I am not an all-knowing being, because I follow the "principle of sufficient reason" [5]. But you are exactly like one because the reader must believe you without any evidence. You have a fallacy of wishful thinking."*

Even if the Clay Institute would have given the Millennium Prize for solving the problem of the starving children in Africa, Perelman has not ordered such a noble charity, i.e. Perelman did not say: "distribute my Millennium Prize as you decide, e.g. give support to young talents.” For some skeptics then, the Clay Institute is indebted to the thinking community: the promised and deserved Prize was not awarded to the right person.

All this drives me to a natural conclusion that the deal with the Prize is unfinished; therefore, the Clay Institute can give the Prize to someone who is more grateful and less scandalous. The proof is not finished until the “champaign is opened”, i.e. the right social behavior is the expected part of the scientific process. Therefore, to still receive this Millennium Award one can give an alternative proof of the Poincaré Conjecture: “Do you not know that in a race all the runners run, but only one gets the prize? Run in such a way as to get the prize.” 1 Corinthians 9:24.

III. SOLUTION

Take the original manifold, which is simply connected and closed. Let us fill this manifold with points having a constant density, i.e. a homogeneous distribution. Every point in this manifold can be described in the spherical coordinates r, θ, ϕ. Therefore, the density of the corresponding points on the sphere (i.e. the image of the initial points) 1. does exist, and 2. is allowed to be non-homogeneous. Thus, there is a direct correspondence between the original manifold and the final sphere: any point on the original manifold corresponds to a point on the sphere.

Therefore, the Poincaré Conjecture is proven by now. [6]

A. Technical Part

Original manifold line element has general form

$$ds^2 = A(r,\theta,\phi)\,dr^2 + B(r,\theta,\phi)\sin^2\!r\,\sin^2\!\theta\,d\phi^2 + C(r,\theta,\phi)\sin^2\!r\,d\theta^2\,, \tag{1}$$

where in case of simply connected manifold for any triplet (r, θ, ϕ) the functions A, B, C have single value, i.e. they are “single-valued functions”.

The final 3-sphere would have

$$dS^2 = dr^2 + \sin^2\!r\,(\sin^2\!\theta\,d\phi^2 + d\theta^2)\,, \tag{2}$$

in which a cosmologist recognises the spatial part of the Friedmann-Lemaître-Robertson-Walker closed universe metric, latter is $dL^2 = -dt^2 + h^2\,dS^2$ with the scale factor h.

The constant density of the points on the original manifold is $\rho_0 = m/V_0$, where m is the total number of points inside the volume $V_0 = ds_r\, ds_\phi\, ds_\theta$, where $ds_r = dr\,\sqrt{A}$, $ds_\phi = \sqrt{B}\, d\phi \sin r\ \sin\theta$, $ds_\theta = \sqrt{C}\, d\theta \sin r$ are proper distances along the coordinate directions. The density of the corresponding points on the final sphere is $\rho = m/V$, where m has the same value as on the initial manifold, but the volume $V = dS_r\, dS_\phi\, dS_\theta$, where $dS_r = dr$, $dS_\phi = d\phi\ \sin r\ \sin\theta$, $dS_\theta = d\theta \sin r$ are proper distances along the coordinate directions on the sphere. On the final sphere and on the initial manifold we have the same values of $dr, d\theta, d\phi$, because the coordinates of points are the same. However, the distance (which is described by the metric tensor) between the points is different: $V \neq V_0$. In conclusion,

$$\rho = \rho_0\,\sqrt{ABC}\,. \tag{3}$$

As the initial manifold is regular, i.e. without singularities $0 < A, B, C < \infty$, one always has $\rho < \infty$. The latter means that such a correspondence between the initial manifold and the final sphere is allowed.

The possible infinities of B, C at $r = 0,\ \pi$ and at $\theta = 0,\ \pi$ are just the coordinate singularities (like one at Schwarzschild Black Hole event horizon), i.e. unphysical singularities because they would occur for various initial manifolds exactly at the same places.Therefore, such cases are absent, i.e. always holds $0 < A, B, C < \infty$.

Indeed, the final sphere can be expressed not in spherical, but in so-called "isotropic" coordinates:

$$dS^2 = f^2\,(dx^2 + dy^2 + dz^2)\,, \tag{4}$$

where $r^2 = x^2 + y^2 + z^2$, $f = f(r)$, $f(0) \neq 0$. Thus, the original manifold has regular metric

$$ds^2 = a(x,y,z)\, f^2\, dx^2 + b(x,y,z)\, f^2\, dy^2 + c(x,y,z)\, f^2\, dz^2\,, \tag{5}$$

with $a, b, c < \infty$.

I have presented the idea of a new metric in Eq. (4) for the closed universe. The function $f(r)$ is a solution of the equation

$$R = R_0 = \text{const}\,, \tag{6}$$

where R is the scalar curvature, or

$$2\,r\left(\frac{df}{dr}\right)^2 - 8\,f\,\frac{df}{dr} - 4\,f\,r\,\frac{d^2 f}{dr^2} = R_0\,f^4\,r\,. \tag{7}$$

This equation is exactly the same for the three dimensional metric (4) and for the four dimensional metric $dS^2 = -dt^2 + f^2\,(dx^2 + dy^2 + dz^2)$, where the time coordinate is included. This makes my theory esthetical and reliable.

The line element is local, i.e. infinitesimals $dx, dy, dz, dS \ll 1$. The solution can exist, according to the computational program Maple V; it is $f(r) = P(r)/r$, where

$$\ln(C_2\, r) = \int_{C_3}^{P(r)} \frac{\sqrt{6}}{\sqrt{6\, w^2 - R_0\, w^4 - w\, C_1}}\, dw\,, \tag{8}$$

where the constants of integration C_1, C_3 and $C_2 > 0$ are chosen so, that $P(0) = 0$ and $f(0) < \infty$. Obviously, $C_3 > 0$. The choice $C_1 = 0$ can be an obvious solution to the task: If $|\ln(0) + \int_{C_3}^{0} K(w)\, dw| < \infty$, the integral must be positive and infinite (i.e. diverging) to compensate the $\ln(0)$. But instead of the integral to be diverging, it is sufficient to have $K(w)\, w \neq 0$ at $w \to 0$. This trick is the "Limit Comparison test" from [7].

The $0 \leq r \leq r_m$, where r_m is the maximum coordinate distance from a given point. Even at $r = r_m$ one has $f \neq 0$; that follows from Eq. (8). Indeed, if $f(r_m) = 0$, then $P(r_m) = 0$, so the $\int_{C_3}^{0} K(w)\, dw = \infty$, then one has $\ln(r_m) = -\infty$, which implies that $r_m = 0$, which is unphysical (it points to an Universe with vanishing size).

Additionally, if $C_1 = 0$ then $6\, w^2 - R_0\, w^4 = w^2(6 - R_0\, w^2) \geq 0$. Therefore, $0 \leq w^2 \leq 6/R_0$. So, holds $P^2(r) \leq 6/R_0$ and $(C_3)^2 \leq 6/R_0$.

Let us control the validity of my theory. Take $R_0 = 0$, i.e. as a sphere of infinite radius (then the $r_m = \infty$). Therefore, the spacetime is locally flat. Indeed, from Eq. (8), where $C_1 = R_0 = 0$ is taken,

$$\ln(C_2\, r) = (\ln P - \ln C_3) = \ln(P/C_3)\,, \quad P = C_4\, r\,. \tag{9}$$

Thus, $f = (C_4\, r)/r = C_4$, and the metric in Eq. (4) is really a flat Universe metric and I am correct with my ideas. Because my theory is so well-working, one can conclude that if $R_0 < 0$, this metric would be the spatial part of an opened Universe metric (which Friedmann's constant is $k = -1$); and if $R_0 > 0$ it is a closed Universe metric ($k = 1$). Another conclusion is that we should always take $C_1 = 0$.

IV. SECOND PROOF

Let me study the line element

$$ds^2 = (a\,(1-\beta) + \beta\, f^2)dx^2 + (b\,(1-\beta) + \beta\, f^2)dy^2 + (c\,(1-\beta) + \beta\, f^2)dz^2\,, \tag{10}$$

where a, b, c are functions of x, y, z, and $f = f(r)$. For $\beta = 0$ this is the original manifold. For $\beta = 1$ it is the final sphere. Running from 0 to 1 is the continuous deformation of the manifold. Hereby the scalar curvature remains finite $|R| < \infty$, because $0 < a, b, c, f^2 < \infty$.

V. SUSPICIOUS POINT IN 2020-WIKIPEDIA

Quote from Wikipedia article "Poincare Conjecture":

"He wanted to cut the manifold at the singularities and paste in caps, and then run the Ricci flow again... In essence, Perelman showed that all the strands that form can be cut and capped..."

What are the caps and capping in the proof of Poincaré Conjecture and does the insertion of caps into initial manifold preserve homeomorphism? The caps do not belong to the original manifold; thus, there is no direct (one to one) correspondence between the original manifold and the final sphere.

Should somebody explain this point in Wikipedia? The public deserves to read a good article on Wikipedia.

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[1] Harald A. Helfgott, "The ternary Goldbach conjecture is true", arXiv:1312.7748 [math.NT].

[2] Massimiliano Proietti *et. al.*, Experimental test of local observer-independence, Science Advances 5(9), eaaw9832 (2019), arXiv:1902.05080 [quant-ph]; Ian T. Durham, Observer-independence in the presence of a horizon, arXiv:1902.09028 [quant-ph]

[3] Mackenzie D., "The Poincaré Conjecture–Proved", Science 314(5807), 1848–1849 (2006)

[4] Malcolm Ritter (July 1, 2010). "Russian mathematician rejects 1 million prize". AP on PhysOrg; "Maths genius declines top prize". BBC News. August 22, 2006

[5] Freeman C., The Greek Achievement: The Foundation of the Western World, Allen Lane, 1999, p. 152

[6] however, one can add to the proof the second step using Physics: to any point is assigned a dust particle. By thinking of the inner pressure or electric charge for the points image, the latter redistributes itself into points on the sphere of constant density, if

1. the points are allowed to be slipping-sliding on the perfect sphere,

2. any asymmetric distribution of charged points is unstable,

3. the physical laws drive the system to the stable, symmetric, state.

Hereby the original points on the initial manifold remain charge-less and pressure-less.

[7] G. B. Thomas, Thomas' Calculus, Addison-Wesley, New York 2001